Chapter 3: Using prices for coordination and motivation

*With specialization comes a keen need to plan and coordinate people’s activities.*

- Ex: to achieve efficiency, how much oil should be pumped from Saudi Arabian oil fields next month?
- The problems of (and possibilities for) organizing economic activity on a world scale are daunting, yet they are carried out, very smoothly much of the time.
The working of *unmanaged* markets

The *neoclassical* market *model*

The economy consists of consumers/resources suppliers, with needs and wants, and productive units (firms) that purchase resources (including labour services) from consumers, make the products consumers demand, and are owned by consumers (either directly or indirectly).
Market economies

• Markets perform a endless list of tasks (choosing e.g. production mix, levels of activities, savings, investments (also in human capital and new technologies)), and no single person (or computer) could determine an efficient allocation.

• How just to identify claims on resources, feasible plans, technological opportunities?
Market: an impossible task

• Collecting the necessary data, ensuring their accuracy, and keeping them continuously up to date would be impossible: it is clear that local decisions must rely to a large extent on knowledge of local circumstances.

• The organization problem is to provide people with the information they need to make decisions that are coherent, and to motivate them to carry out their parts of the overall efficient plan.
The market model at work

- The neoclassical model can be used to prove that a system of properly determined prices can solve the organization problem.
- In particular, under certain circumstances, prices provide people with all the additional information they need, and if individuals and firms take prices as given, and act out of pure self-interest, they will be motivated to undertake exactly those activities that lead to efficiency.
How far a price system can go in solving the coordination problem?

• A motivating example: The Department of Highway Safety …

• Suppose that your job is to save lives by directing the resources at your disposal to projects that reduce the number of fatal highway accidents..

• You are limited by the number of hours available from the work crew that carry out the projects (Table 3.1, p. 59).
Example: *continuation*

Life-Saving Projects (Estimates), last year:

<table>
<thead>
<tr>
<th>Project No.</th>
<th>Crew Hours</th>
<th>Lives Saved</th>
<th>Lives/100 0 Crew Hours</th>
<th>Project Accepted?</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>800</td>
<td>4</td>
<td>5.00</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>900</td>
<td>3</td>
<td>3.33</td>
<td>Yes</td>
</tr>
<tr>
<td>1</td>
<td>800</td>
<td>2</td>
<td>2.50</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>500</td>
<td>1</td>
<td>2.00</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>1300</td>
<td>2</td>
<td>1.54</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>700</td>
<td>1</td>
<td>1.43</td>
<td>No</td>
</tr>
</tbody>
</table>
Example: continuation

- It is clear from the table that last year project were not selected efficiently: project 2 should have been abandoned favouring projects 5 and 6.
- A selection problem arises because: 1) projects are not all available for review at once; 2) local staff members are involved in making the estimates and acceptance decisions.
- Suppose that you do have a good idea about which kind of projects will be available in advance.
Example: continuation

• How coordinate decisions made by different offices at different times?

• To start with, suppose first that you do know the projects, and can ranking them according to return-per-unit input (as in the previous Table).

• Then, it is very simple to provide information to local officers: the simple instruction to carry out projects with an index (number of lives saved per thousand crew hours) of at least two would do the job.
Example: *conclusion*

- The example illustrate a general lesson: no matter what the list of projects is, there is always a *number*, $P$, which would select optimally among them.
- This $P$ is a *price*, expressed in term of lives per 1000 hours.
- The unit of course does not matter: the only requirement is that projects can be ranked in term of a common unit which allows a cost/benefit analysis.
Example: conclusion

• Of course, in reality you do not know in advance what the value of $P$ should be, and you might end up either rejecting some projects that ought to have been accepted, or running out of resources for better projects later in the year.

• This is unavoidable, if decisions must be made before all the alternatives are known.

• But a great advantage of the “price system” is to eliminate the lack of consistency over time and among several decision makers.
A Market-Clearing Interpretation

- The determination of $P$ can be viewed as resulting from the working of a market (without a management defining “the price”):

  - 1) suppose that the project evaluators were able to bid for crew hours, with the object of maximizing *the number of the lives minus the cost of the crew hours*;

  - 2) plots the bids considering a fixed supply of 3(,000) hours.
Supply and Demand

Lives per Crew Hours

D

S

Crew Hours (Thousand)

0,8 1,7 2,5 3 4,3 5

Lives per Crew Hours

1,43 1,54 2 2,5 3,33 5

EOM: Chapter 3 (P. Bertoletti)
Equilibrium

- In the previous example, $P^* = 2$ is a market-clearing price at which each evaluator finds it most profitable to undertake precisely those projects that are part of the optimal plan.

- The example is special in that it considers a single resource, but it can be easily generalized with the use of several “prices” (in terms of lives saved), one for each resource.
Extensions (difficulties)

• In fact, each price can be expressed in money terms, once the benefits too are expressed in such a unit (“money value” of a life).

• A serious problem instead arises if there is no match between the size of the efficient plan and the available amount of hours (think of projects both large and indivisible).

• Ex: suppose that project 6 requires 1000 hours to save 4 lives. The optimal plan can still be easily identified if projects can be scaled down preserving the same “productivity” (e.g., perform project 6 at 50%). But its implementation would required additional communication.
The **Fundamental Theorem of Welfare Economics (FTWE)**. *If:*

- Each productive unit knows the prices and its technology, and maximizes its own profits for given prices;
- Each consumer knows the prices and her preferences, and maximizes her own utility for given prices and income;
- Prices are such that *supply equals demand* for each good;
- *Then the resulting allocation is Pareto efficient!* 

EOM: Chapter 3 (P. Bertoletti)
FTWE: implications

1. The only “global” information necessary to achieve coherence and efficiency is the system of prices: no need for central planning and extensive sharing of information.

2. Each agent is asked only to pursue her own interest, nobody needs to use information in ways contrary to her narrowly defined self-interest.
The Arrow-Debreu (AD) Model of a Private Ownership Economy

- Consumers

- Let an economy have a number $G$ of different goods and services (farm land, wheat, labour hours, shoes, …).

- Each consumers $n$ has a list (vector) $E^n$ of quantities of those goods which is called her endowment: $E^n = (E_1^n, E_2^n, \ldots, E_G^n)$, where $E_g^n$ is her endowment of good $g$.

- Endowment can be added together: $E = \Sigma_n E^n = (\Sigma_n E_1^n, \Sigma_n E_2^n, \ldots, \Sigma_n E_G^n)$ is the total economy endowment, where $E_g = \Sigma_n E_g^n$ is total endowment of commodity $g$. 
Consumers: continuation

- Consumers can sell their endowment, or consume it: let $S^n = (S_1^n, S_2^n, \ldots, S_G^n)$, where $S_g^n$ is the quantity of good $g$ that consumer $n$ is willing to sell (clearly, $S_g^n \leq E_g^n$).
- Consumer might also be willing to buy some commodity: $B^n = (B_1^n, B_2^n, \ldots, B_G^n)$, with obvious definition.
- Notice that many elements of the previous lists will of course be zero.
Consumers: continuation

- Let $P = (P_1, P_2, \ldots, P_G)$ the list of all prices.
- $PB^n = \sum_g P_g B_g^n$ is the total expenditure of consumer $n$.
- $PS^n = \sum_g P_g S_g^n$ is the income/revenue of consumer $n$.

- Let $F^n_j$ the fraction of the shares of firm $j$ that consumer $n$ owns (obviously, $\sum_n F^n_j = 1$).
- Accordingly, if firm $j$ pays a dividend/profit $D_j$, consumer $n$ receives $F^n_j D_j$, with total income from profits for her given by $FD^n = \sum_j F^n_j D_j$. 
Consumption Plans

• A consumption plan for consumer \( n \) is just a pair of lists \( (B^n, S^n) \).
• It is affordable if \( PB^n \leq PS^n + FD^n \).
• The resulting consumption level is:

\[
C^n = (C_1^n, C_2^n, \ldots, C_G^n) = E^n + B^n - S^n,
\]

where \( C_g^n = E_g^n + B_g^n - S_g^n \) is her consumption of good \( g \), which generates utility \( U^n(C^n) \).
• Consumers are assumed to care only about \( C^n \), choosing, at given prices, the affordable plan which maximizes \( U^n \).
Ex: Consumer Behaviour (per year, *extract*)

<table>
<thead>
<tr>
<th>Goods</th>
<th>Units</th>
<th>$P_g$</th>
<th>$E^*_g$</th>
<th>$S^*_g$</th>
<th>$B^*_g$</th>
<th>$C^*_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>Hours</td>
<td>15</td>
<td>2.600</td>
<td>2.000</td>
<td>0</td>
<td>600</td>
</tr>
<tr>
<td>Bread</td>
<td>Loaves</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Autos (new)</td>
<td>Number</td>
<td>8.000</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Autos (used)</td>
<td>Number</td>
<td>4.000</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Local non satiation

- It is also assumed that there is always some good or services of which the consumer would like to have (a little bit) more.
- This implies that actually:

\[ PB^n = PS^n + FD^n \]

(or, equivalently, \( PC^n = PE^n + FD^n \), where \( PC^n = \sum_g P_g C_g^n \) and \( PE^n = \sum_g P_g E_g^n \)) for all consumers (otherwise the consumers could improve their plans).
Firms

- A production plan for firm $j$ is just a pair of lists $(I^j, O^j)$, with $I^j = (I_{1j}^j, I_{2j}^j, \ldots, I_{Gj}^j)$ and $O^j = (O_{1j}^j, O_{2j}^j, \ldots, O_{Gj}^j)$, where $I_{gj}^j$ is the amount of good $g$ used as an input by firm $j$, while $O_{gj}^j$ is the amount of that commodity it produces as an output (again, many entries will be null).

- The set of technically feasible plan for firm $j$ is given by $T^j$, and feasibility is denoted by $(I^j, O^j) \in T^j$. 

EOM: Chapter 3 (P. Bertoletti)
Firms: continuation

• Firms and consumers are commonly on opposite sides of any market transaction (firms buy resources and sell products, and consumers sell resources and buy products).

• Firms are assumed to maximize profit:
  • \( D_j = PO^j - PI^j \),
  • where \( PO^j = \sum_g P_g O^j_g \) is total revenue of firm \( j \), and \( PI^j = \sum_g P_g I^j_g \) are its total costs.
Economies

- Formally, a private ownership economy is just:
  - A set of consumer \(N\), with preferences given by \(U^n\) and endowments \(E^n\);
  - A set of firms \(J\), with technologies \(T^j\) and ownership shares \(F_j = (F_j^1, F_j^2, \ldots, F_j^N)\).

- Notice that the model allows for a huge heterogeneity (in terms of preferences, endowments and technology) among consumers and among firms.
Allocations

- An **allocation** (for a given economy), is just a consumption plan for each consumer and a production plan for each firm that together are **feasible**: i.e.,

\[
\begin{align*}
S^n_g & \leq E^n_g, \\
(I^j, O^j) & \in T^j, \\
\sum_n B^n + \sum_j I^j & \leq \sum_n S^n + \sum_j O^j.
\end{align*}
\]
Price Formation

• The Model does not include, in its formal structure, any description of the mechanism by which prices are set and adjusted to changing conditions. It assumes that there are prices that are publicly known and at which everyone believe they can translate.

• In reality, of course many mechanisms are used (prices posted by stores, goods sold by auctions, wages set by negotiators, …), but excess demand should generally command a price rise, and excess supply a reduction of price, with a tendency to reduce the gap between demand and supply.
Competitive Equilibrium

• Suppose that the publicly known prices are set exactly as necessary to balance demand and supply: this point is called a competitive equilibrium.

• Formally, a competitive equilibrium is: 1) a price list, $P$; 2) a production plan for each firm, ($i^j$, $O^j$); and 3) a consumption plan for each consumer, ($B^n$, $S^n$), such that consumers maximize their utilities, firms maximize their profits, and for each commodity the quantity demanded equals the quantity offered for sale, i.e.,

\[ \sum_n B^n + \sum_j i^j = \sum_n S^n + \sum_j O^j. \]
FTWE:

• *The allocation produced by a competitive equilibrium is Pareto efficient.*

• Accordingly, any Pareto improvement must be *unfeasible.*

• Notice that just allocations are efficient (prices do not matter per se), and are judged in terms of consumers preferences (profits do not count).

• However, prices *serve to inform* the parties about what they should do, and *to motivate* them (of course, they also affect individual welfare).
Implications of the FTWE

• Notice that parties do not need to know why prices have changed to determine how to respond to changing circumstances, and there is no conflict among the owners of any firm about what it should do (all will agree with profit maximization).

• In spite of a tremendous variety among parties, behaviour is coherent enough so that no resources are wasted.
Scope of the Neoclassical Model

• The AD model is very general, and allows for a large number of interpretations.

• In particular, interpreting a commodity as located in time, address and contingencies (an umbrella, today, in Pavia, if it is sunny, is possibly a different commodity from an umbrella, today, in Pavia, if it is rainy), it can account for time (investment and saving), transport (location) and uncertainty (risky R&D).

• It also accounts for limited managerial information/ability, incorporated into $T^i$. 
Missing Markets

- However, the previous interpretations require that a (competitive) market (and a price) is available for all the relevant commodities.

- While in some case markets do exist also for “exotic” goods (as futures and options), it seems clear that there are many missing markets in reality (and that some existent markets are not competitive).

- Thus, the FTWE does not necessary apply.
The proof amounts to show that there cannot exist an *alternative, feasible* overall plan \{B^n', S^n', I^j', O^j'\} which *Pareto dominates* the competitive one \{B^n, S^n, I^j, O^j\}.

Suppose the contrary. Then there is (at least) a consumer \(i\) with \(U^i(C^i') > U^i(C^i)\), which implies that:

\[
PB^{i'} > PS^{i'} + FD^i,
\]

i.e., the alternative plan must have been *unaffordable* at *equilibrium* prices \(P\) (and previous income), otherwise the consumer would have chosen it.
FTWE: a proof (II)

• Similarly, since no consumer is worse off under the alternative plan, it must be the case that:

\[ PB^{n'} \geq PS^{n'} + FD^n, \]

otherwise non satiated consumers could have done better by exploiting the “saving” implied by \((B^{n'}, S^{n'})\).

• Adding up:

\[ \sum_n PB^{n'} > \sum_n PS^{n'} + \sum_n FD^n. \quad (i) \]

• Now consider firms: it must be the case that:

\[ PO^j - PI^j \geq PO^{j'} - PI^{j'}, \]

otherwise, to maximize its profit, firm \(j\) would have not chosen \((I^j, O^j)\) at prices \(P\).
FTWE: a proof (III)

• Adding up:
  \[ \sum_j (PO^j - PI^j) \geq \sum_j (PO^{j''} - PI^{j''}). \]  
  (ii)

• Also notice that:
  \[ \sum_n F D_n = \sum_n \sum_j F_j^n D_j = \sum_j D_j \sum_n F_j^n = \sum_j (PO^j - PI^j). \]  
  (iii)

• Finally, remember that feasibility also imply:
  \[ \sum_n B^n + \sum_j I^j \leq \sum_n S^n + \sum_j O^j. \]  
  (iv)
FTWE: a proof (IV)

- To complete the proof, note that:
  \[ \sum_j (PO^j - PI^j) \leq \sum_j (PO^j - PI^j) \quad \text{by (ii)} \]
  \[ = \sum_n FD^n \quad \text{by (iii)} \]
  \[ < \sum_n PB^{n'} - \sum_n PS^{n'} \quad \text{by (i)} \]
  \[ = \sum_n \sum_g P_g (B_g^{n'} - S_g^{n'}) \]
  \[ = \sum_g P_g \sum_n (B_g^{n'} - S_g^{n'}) \]
  \[ \leq \sum_g P_g \sum_j (O_g^{j'} - I_g^{j'}) \quad \text{by (iv) \& } P_g > 0 \]
  \[ = \sum_j \sum_g P_g (O_g^{j'} - I_g^{j'}) \]
  \[ = \sum_j (PO^{j'} - PI^{j'}) , \]

which is a contradiction.
Exercise no. 2, p. 87

- Consider an economy with two goods, $x$ ("money") and $y$ ("manna"), and two (types of) people (in equal numbers), 1 and 2, with $U_1 = x_1 + (3y_1 - y_1^2)$ and $U_2 = x_2 + (2y_2 - y_2^2)$.

- Each individual has an endowment given by 10 units of money and 1 unit of manna, i.e., $E^n = (10, 1)$. Notice that there are no wealth effects.

- Use the value-maximization principle to determine how manna must be allocated efficiently.

- Below, we compute the solution with reference to a pair of consumers of types 1 and 2.
Exercise 2: *continuation*

- Feasibility and efficiency requires that \((y_1 + y_2) = 2\), i.e., \(y_2 = 2 - y_1\).

- Accordingly, value maximization implies that \(y_1\) is chosen to maximize \(TV = 20 + [3y_1 - y_1^2] + [2(2 - y_1) - (2 - y_1)^2]\).

- The FOC and SOC are then \(TV'(y_1) = 5 - 4y_1\) and \(TV''(y_1) = -4 < 0\), which imply that the optimal level are \(y_1^* = 5/4\) and \(y_2^* = 3/4\).
Exercise 2: continuation

- What must the price of manna be (in money unit) in a competitive equilibrium?

- Let $P_x = 1$ be the “price” of money, and $P_y$ the price of manna (you can always normalize the price system like that, i.e., only relative prices matter).

- For each type of consumer, the optimal consumption bundle must satisfy the condition that the relevant indifference curve is tangent to the budget constraint.
Exercise 2: continuation

- Remember that, if you put the quantity of commodity \( y \) on the horizontal axis in a two-commodity space, the “slope” of the budget constraint is given by \( P_y/P_x \), and it measures how many units of good \( x \) you need to sell to buy an additional unit of \( y \).

- Also remember that the slope of the indifference curve at any point is called the *Marginal Rate of Substitution* (MRS).

- The MRS can be computed as the ratio of *marginal utilities*, i.e., \( MRS = (\partial U/\partial y)/(\partial U/\partial x) \), and it measures how many units of \( x \) are indifferent to an additional unit of \( y \) in terms of consumer preferences.
Optimal Consumption

\[ \tan \alpha = \frac{P_y}{P_x} \]

Budget line

Affordable consumption

EOM: Chapter 3 (P. Bertoletti)
Exercise 2: continuation

• The tangency condition for optimal consumption is then equivalent to the condition that MRS is equal to $P_y/P_x = P_y$.

• Since $\text{MRS}^1 = (\partial U^1/\partial y_1)/(\partial U^1/\partial x_1) = 3 - 2y_1$ and $\text{MRS}^2 = (\partial U^2/\partial y_2)/(\partial U^2/\partial x_2) = 2 - 2y_2$, it follows that the optimal consumption level of manna are given by $y_1 = (3 - P_y)/2$ and $y_2 = (2 - P_y)/2$.

• Notice that in a competitive equilibrium in which $\Sigma_n B^n = \Sigma_n S^n$ it must be the case that $\Sigma_n C^n = \Sigma_n E^n$: thus, $y_1 + y_2 = 2$. 
Exercise 2: conclusion

• Accordingly, in the competitive equilibrium $P_y = \frac{1}{2}$, $y_1 = y_1^*$ and $y_2 = y_2^*$.

• Notice that $B_y^1 = y_1^* - 1 = \frac{1}{4}$ and $S_y^2 = 1 - y_2^* = \frac{1}{4}$.

• Also notice that $S_x^1 = P_y B_y^1 = 1/8 = B_x^2 = P_y S_y^1$, and thus $x_1 = E_x^1 - S_x^1 = 10 - 1/8 = 79/8$ and $x_2 = E_x^2 + B_x^2 = 10 + 1/8 = 81/8$.

• The equilibrium can be illustrated in a so-called Edgeworth Box.
The Edgeworth Box I

• The **Edgeworth Box (EB)** is used to analyze the possible exchanges between **two** agents, 1 and 2, endowed with quantities of **two** commodities. Each agent is assigned either to the SW or to the NE corner of the box.

• The EB has sides whose dimensions are given by the total endowment of the two goods, and it is the set of all possible *distributions* (i.e., allocations without waste) of such an endowment.

• Once indicated the *initial endowment*, the indifference curves passing through it do identify the set of all the possible Pareto improvements.
The EB: *Pareto improvements*

EOM: Chapter 3 (P. Bertoletti) 46
The Edgeworth Box II

- Notice that (interior) Pareto efficient allocations in the EB do correspond to the set of tangency points among the families of indifference curves (so-called contract curve).

- The subset of points of the contract curve that are also Pareto improvements with respect to the initial endowment are called the CORE of the economy.

- The CORE is the set of all the allocations upon which two rational bargainers might agree upon. i.e., it identifies the set of all possible direct exchanges among rational consumers.
The EB: Contract Curve and the CORE

Contract curve

CORE
The Edgeworth Box III

• The EB can also be used to represent a competitive equilibrium in an economy of pure exchange with 2 (types of) consumers and 2 goods.

• Notice that the (common) budget constraint line must pass through the points which represent the consumer endowments and their optimal consumption plans (affordability can be written $PC^n \leq PE^n$).

• The equilibrium price system is nothing but the slope of the budget line.
The EB: Competitive equilibrium

\[ PC^2 \leq PE^2 \]

\[ PC^1 \leq PE^1 \]

\[ \tan \alpha = \frac{P_1}{P_2} \]
The Edgeworth Box IV

• Notice that the competitive equilibrium allocation (being Pareto efficient and resulting in a Pareto improvement with respect to the initial endowment) must belong to the CORE of the economy.

• This property further illustrates the connection between the result of explicit bargaining among agents and the allocation established by competitive prices.

• It can be proved that as the number of agents increases (for a given number of alternative types) the CORE shrinks, and that in the limit it contains only the competitive allocations.
The EB of Ex. no. 2: $tg \alpha = 2 = \frac{P_x}{P_y}$
Exercise no. 2, p. 87

• Notice that in this case the Contract curve is horizontal, reflecting the validity of the Value Maximization Principle: only the distribution of manna matters for efficiency.

• The reason of this is that the indifference curves have a slope (the MRS) which only depends on the amount of manna, due to the absence of wealth effects.
Exercise no. 3, p. 87

• Consider an economy with two goods, x and y, and two types of consumers (in equal numbers), “x-lovers”, with $U^1 = 2\ln x^1 + \ln y^1$, and “y-lovers”, with $U^2 = \ln x^2 + 2\ln y^2$.

• Each individual has an endowment given by 3 units of each good, i.e., $E^n = (3, 3)$.

• If $P_x = 1 = P_y$, how many units of each good will be supplied or demanded by each kind of person?

• Again, below we compute the solution with reference to a pair of consumers of types x and y.
Exercise 3: continuation

• Notice that $P_y/P_x = 1$, $MRS^1 = (\partial U^1/\partial y^1)/(\partial U^1/\partial x^1) = x^1/2y^1$ and $MRS^2 = (\partial U^2/\partial y^2)/(\partial U^2/\partial x^2) = 2x^2/y^2$.

• Thus, the “optimal” consumption plan for consumers of type $x$ (when prices are equal) will have a consumption ratio $x^1/y^1 = 2$, while the consumption ratio of type $y$ will be $x^2/y^2 = \frac{1}{2}$.

• This suggests that type $x$ will be willing to buy 1 unit of good $x$ and to sell 1 units of good $y$, and that type $y$ will do just the opposite.
Exercise 3: continuation

- With equal prices there exists actually a competitive equilibrium in which $B_x^1 = S_x^2 = 1 = S_y^1 = B_y^2$, $PB^n = PS^n$, $C^1 = (4, 2)$ and $C^2 = (2, 4)$.

- This allocation is efficient according to the FTWE.

- Notice that this allocation maximizes $W = \Sigma_n U^n = 2\ln x^1 + \ln y^1 + \ln x^2 + 2\ln y^2$ under the constraint that $x^1 + x^2 \leq 6$ and $y^1 + y^2 \leq 6$. 

EOM: Chapter 3 (P. Bertoletti)
Exercise 3: conclusion

• In fact, writing $W = 2\ln x^1 + \ln y^1 + \ln(6 - x^1) + 2\ln(6 - y^1)$, one gets (FOCs):

  • $\frac{\partial W}{\partial x^1} = \frac{2}{x^1} - \frac{1}{6 - x^1} = 0 \rightarrow x^1 = 4$
  • (i.e., $x^2 = 2$)
  • $\frac{\partial W}{\partial y^1} = \frac{1}{y^1} - \frac{2}{6 - y^1} = 0 \rightarrow y^1 = 2$
  • (i.e., $y^2 = 4$)

with (SOCs) $\frac{\partial^2 W}{(\partial x^1)^2}$, $\frac{\partial^2 W}{(\partial y^1)^2} < 0$ and $\frac{\partial^2 W}{(\partial x^1 \partial y^1)} = 0$. 

EOM: Chapter 3 (P. Bertoletti)
The EB of Ex no. 3: $\text{tg} \alpha = 1 = P_x/P_y$
Incentives in Markets

• The AD model *assumes* that producers and consumers take prices as given: if they do, the coordination problem is solved.

• But, will the agents find it individually optimal to take prices as given?

• In fact, if they have some “market power” (the ability to affect prices and/or manipulate information), they will generally have an incentive to do that, and the efficiency result may not hold.
Large Economies and Market Institutions

• A lot of research in economics has theoretically and empirically investigated agent behaviour under various “market institutions”.

• The main question is whether this behaviour will lead (approximately) to efficient outcomes.

• A common (but not uncontroversial) conclusion is that if the number of participants is sufficiently (but realistically) large competition will eliminate market power and produce efficient results.

• And that private property is a powerful engine for directing individual self-interest to produce welfare gains.
Markets’ Informational Efficiency

- In a market economy, only the relatively small amount of information represented by prices and offers (to buy and to sell) is transmitted.
- It can be proved (see Chapter 4) that this is the *minimal information transmission* consistent with efficiency.
- In actual market systems, more information is transmitted (think of marketing and regulatory activities), however, still prices provide much of it.
Prices and Socialism

• Abba Lerner once showed that a socialist system, with collective ownership of the means of production, could use prices to allocate resources efficiently.

• Accordingly, in principle, central planning could work “simply” by determining and announcing the “right” prices.

• However, their actual use have in practice found major difficulties (helping some groups and hurting others, the determination of prices by government is bound to become a political decision).
Market Results and Theories of Organization

• The AD model provides a benchmark under which “the invisible hand” result applies.
• However, if that result did always apply, there would be no need for other economic organizations (unless to pursue “political” goals).
• On the contrary, as observed by Alfred Chandler, historically new organizations arise when market outcomes were inefficient.
Market failures I

1. An obvious case is the exercise of *market power* by some participant.

2. *Increasing returns to scale*. For some market, a competitive equilibrium *cannot* exist. A case is the presence of significant *economies of scale* (which usually lead to *imperfectly* competitive markets).

Ex: suppose consumers’ willingness to pay is 16 up to 100 units, and zero onwards, while production require a fixed cost of 1,000, plus a variable unit cost of 5 up to capacity, which is equal to 200.
Economies of scale: *continuation*

- Notice that the *average cost* always decreases (which is how economies of scale are defined), down to 10 if 200 units are produced (it is equal to 1,005 for the first unit), while marginal cost is constant (equal to 5) up to capacity.

- As a result, the supply function is discontinuous, and no competitive equilibrium exists (see next Figure).
Economies of Scale: no Equilibrium
Economies of scale: continuation

• Notice that a “value-maximizing solution” does exist in the example: to produce 100 units and transfer them to the consumers generates a “total value” (i.e., social welfare) of $100 = 1600 - 1500$, where 1600 is gross consumer surplus and 1500 total cost.

• A “price” between 15 and 16 would then define the division of total value between net consumer surplus and producer surplus (profit).

• However, to implement such a solution it is necessary to know how many units the consumers are willing to buy (i.e., the demand function).
Economies of scale: conclusion

• Notice that the illustrated failure of the “price system” does not imply the failure of firms in the real world.

• For example, firms (think of producers with market power) try to keep in touch with their customers as much as possible, and base their plans on information/forecasts, provided by the marketing/sale offices, which concern the demand functions (i.e., quantities, qualities, product attributes, and much more).

• In fact, many organizational arrangements (sometimes internal to firms) are de facto used to replace the price system for purposes of coordination.
Market failures II

3. Externalities. Externalities are effects that the actions of one agent have on another’s welfare and that are “not regulated by the system of prices” (for which prices are not paid).

- Ex: smoke from a nearby factory, investments which create jobs and raise property values, inventions that provide foundation for further inventions by others.
- In all the previous case there are costs and benefits associated to agents other than those making decisions. Thus, the decision makers do not take full account of them, and inefficiency arises.
Missing Markets I

• The kind of market failure associated with externalities may be considered as a matter of missing markets.

• In fact, an externality can be considered as producing a good (or a “bad”) not traded into a market, which is then missing.

• Ex: you can think of pollution as the production of a bad for which the producer does not pay. Or of the new ideas provided by an inventor as commodities for which she receives no price.
Missing Markets II

- Notice that the patent system and the use of tradable “pollution permissions” go somewhat in the direction of creating similar missing markets (a general suggestion of the AD model).
- However, in many cases those markets cannot exist, or they would not be competitive (regarding too few traders).
- Moreover, markets may be missing for reasons other than externalities (think of future and contingent goods: see Chapter 5).
Search, Matching and Coordination

• The AD model assumes that all agents do know about all the markets and the prices (in fact, it assumes that well-organized markets are already at work), and that they will be able to buy and sell how much they like.

• In reality consumers search for products, jobs and prices, while firms search for employees and pay for advertising. And they are well aware that it might be difficult to buy and especially to sell (think of unemployment).

• This suggests that there may be self-fulfilling expectations, either optimistic or pessimistic.
Coordination failures

• The upshot is that, contrary to the conclusion of the neoclassical model, there could be multiple possible levels of economic activity which are internally consistent but inefficiently low.

• In similar situations, agents may have no incentive to alter their prices in the direction predicted by the usual mechanisms, and markets may turn out to be ineffective (the present economic crisis?).
Market Failures and Organization

• In presence of some market failure in achieving efficiency, the traditional view expects government to intervene with special policies.

• This might be necessary in some circumstances, as during an economic crisis.

• As an alternative, individual and firms themselves could take action to remedy, by creating arrangements to replace the price system, i.e., by creating some *nonmarket* organizational form.
The price system within organizations

• Senior managers of large organizations (in particular, *multidivisional* firms) in fact make extensive internal use of price systems, so decentralizing decisions.

• They use financial controls and performance measurements to introduce internal *transfer pricing* for transactions between units inside the organization.
Internal Organization in Firms I

1. The first large firms were organized *functionally* in a *centralized fashion* (ex: Ford in the 1920s), with an head office directing all activities and one department responsible for finance, one for production, and others for personnel, purchasing, logistics, sales, marketing, ....

• This form proved to be ill suited for coordinating activity in multiproduct firms operating over broad geographical areas: too much time and information lost in communication (as in the HBC story).
Internal Organization in Firms II

2. At the opposite extreme were the *holding companies* that emerged in the 19th century, decentralized collections of separate firms under a common ownership.

- The head office uses to play no managerial role, simply collecting the profit, with no gain from coordinated decisions across units.

3. After the end of the First World War many firms independently introduced a *multidivisional organization*. 
Multidivisional Organization

- The idea essentially is to create mini-companies – divisions within the firm, each responsible for a particular product, market, region, or technology, under a single division manager, but with relatively strong central offices to coordinate (and to raise outside capital, allocating resources, appointing and monitoring divisional managers).

- Pioneers included Du Pont (a chemical firm which separated its explosive business from fertilizer business) and General Motors, which was created as a combination of independent car makers.

- Today also non-manufacturing firms or non-profit organizations (like Universities) have similar forms.
Transfer Pricing I

• The performance of divisions (or of smaller responsibility centres) is always measured at least partly in financial terms.

• Moreover, products and services are frequently supplied by one division to another: this crucially requires transfer prices (TP), which can be the most important determinant of the measured performance.

• Ex: think of an integrated petroleum company with divisions which extract, transport and refine oil.
Transfer Pricing II

- Notice that, given the volumes actually transferred, TP do not affect overall corporate profit.

- However, they might if managers have autonomy over the determination of the quantities, either internally or in external dealings.

- Badly chosen TP can also misdirect corporate decisions, misleading central executives about relative profitability.
Transfer versus Market Prices I

• In general, firms have to rely on internal standard cost estimates to set TP properly.

• However, TP are easily determined if there is an outside *perfectly competitive* market, with no additional corporate costs or benefits from dealing with it.

• In such a case adopting market prices as TP directs divisional decisions to maximize corporate profits and provides the right signal to assess profitability (*it actually makes no difference whether the goods are bought and sold internally or externally*).
Transfer versus Market Prices II

• Of course, perfect substitutes for the firm’s products will be rarely available on a competitive market (unless in the case of standardized commodities).

• Still, a competitive market for similar goods can be a useful reference: see the example of Bellcore (p. 81), where a mistakenly TP of 50$ per page (!) for internal typing service was discovered once compared with the much lower external price.
Transfer versus Market Prices III

• Indeed, when nearly perfectly competitive markets are available, there is little point in vertical integration (see Chapter 16).

• In practice, there might be severe transaction costs to use outside markets, because of informational differences (see Chapter 5).

• However, without an external alternative source, to improve profitability of their own division, managers have an incentive to manipulate TP, perhaps by assigning large overhead cost to specific products (very much as monopolists do).
Transfer versus Market Prices IV

• We now prove the previous claim about the use of external prices as TP.

• Consider the following Figure, depicting the marginal cost (supply function) of the selling division, and the marginal revenue (demand function) of the buying division.

• Just by chance it can happen that the external market price (either \( P_1 \) or \( P_2 \)) is equal to the one that would clear the internal market (\( P_3 \)).
TP with an external market

![Graph showing the relationship between price, quantity, marginal revenue (MR), and marginal cost (MC). Points A, B, C, D, E, F, G, P₁, P₂, P₃, Q₃, x₁, y₁, y₂, x₂ represent various price and quantity levels.]

EOM: Chapter 3 (P. Bertoletti)
Transfer versus Market Prices V

• It is easy to prove that the largest profit is obtained by the corporation if TP are set at the level of the external market price.

• For example, suppose that the market price is $P_1$.

• If the TP is set equal to $P_3$ (the best alternative), and external dealings are forbidden, then quantity $Q_3$ is internally exchanged and overall profit are given by the area $ADE (= ADP_3 + P_3DE)$. 
Transfer versus Market Prices VI

• If, on the contrary, *external dealings are admitted*, then the buying division will buy quantity $y_1$ externally by paying $P_1$, making a profit given by the area $P_1CE$, while the selling division will sell externally $x_1$, making a profit given by $ABP_1$ (the overall corporate profit is $ABCE$).

• If, finally, $P_1$ is set as a TP, then the buying division will buy externally only $(y_1 - x_1)$, while the selling division sells $x_1$ internally, with the same profits.
Transfer versus Market Prices VII

• Exercise no. 1, p. 87: notice that, with respect to the “internal solution”, the profit of the selling division decreases of $P_1BDP_3$, while the profit of the buying division increases of $P_1CDP_3$ (corporate profit overall increases of $BCD$).

• Similarly, if the market price is $P_2$, by setting an identical TP the selling division will sell $y_2$ internally and $(x_2 - y_2)$ externally, and the overall profit amounts to $AGFE (= AGP_2 + P_2FE)$.

• In the latter case, the profit of the selling division increases of $P_3DGP_2$, while the profit of the buying division decreases of $P_3DFP_2$ (corporate profit overall increases of $DGF$).
Transfer versus Market Prices VIII

• There is an alternative (and somehow deeper) proof, which uses the Value-Maximization Principle and the FTWE.

• Consider the following “artificial” economy that can be associated to the situation we are examining:

a) two goods: money \((x)\), with a price normalized to 1, and the product of the selling division \((y)\), whose price is \(P\);

b) three agents/consumers, with no wealth effects, and utilities given by profit: the selling and buying divisions, and “the market”.

EOM: Chapter 3 (P. Bertoletti) 89
Transfer versus Market Prices IX

- The selling division has utility given by $U^S = x^S - v^S(y^S)$, where $v^S(y^S)$ is the cost of selling $y^S$.
- The buying division has utility given by $U^B = x^B + v^B(y^B)$, where $v^B(y^B)$ is the profit from buying $y^B$.
- The market (i.e., all the other consumers) has utility given by $U^M = x^M + py^M$, where $p$ is the outside market price and $y^M$ the amount transacted with the corporation. The linearity of $U^M$ implies that the market will be willing either to buy or to sell an unbounded amount of $y^M$, unless it costs exactly $py^M$ (in such a case the market will be indifferent among any value of $y^M$).
Transfer versus Market Prices X

• Last assumption is equivalent to assume that the market is very large with respect to the corporation we are considering (an hypothesis similar to the small-country assumption in international trade, by which the international market (price) equilibrium is unaffected by local behaviour).

• It immediately implies that $P = p$ in a competitive equilibrium in which it must be the case that:

  • $y^S - y^B = y^M$. 

EOM: Chapter 3 (P. Bertoletti)
Transfer versus Market Prices XI

- It follows that the selling division will choose \( y^S \) to maximize \( p y^S - v^S(y^S) \), the buying division will choose \( y^B \) to maximize \( v^B(y^B) - p y^B \) and the resulting allocation will be Pareto efficient.

- This also implies that total value will be maximized, which is given by the total profit for the corporation: \( p(y^S - y^B) + v^B(y^B) - v^S(y^S) \).

- Notice that this apply, by construction, even if \( p \) is not (part of) the “real” competitive equilibrium of the outside economy, which thus need not to be Pareto efficient.
Transfer versus Market Prices XII

- Also notice in our example that:
  1. no TP different from $p$ can achieve a larger corporate profit (efficiency);
  2. by regarding division managers as consumers, we proved that their efficient behaviour can be guided by prices;
  3. the previous efficiency (maximum-value) result actually applies only to the divisions rather than to the overall economy (by construction, the welfare of the market does not depend on the allocation).