The BMW Model: A New Framework for Teaching Monetary Economics

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Abstract: Although the IS/LM-AS/AD model is still the central tool of macroeconomic teaching in most macroeconomic textbooks, it has been criticized by several economists. Colander (1995) demonstrated that the framework is logically inconsistent; Romer (2000) showed that it is unable to deal with a monetary policy that uses the interest rate as its operating target, and Walsh criticized that it is not well suited for an analysis of inflation targeting. The authors present a framework that develops the Romer approach into a very simple but, at the same time, comprehensive macroeconomic model. In spite of its simplicity, it can carry the main insights of the New Keynesian macroeconomics to an intermediate level and deal with issues like inflation targeting, monetary policy rules, and central bank credibility.

Key words: inflation targeting, monetary policy, New Keynesian macroeconomics, optimal interest rate rules, simple rules
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Although the IS/LM-AS/AD model is still the central tool of macroeconomic teaching in most macroeconomic textbooks, it has been criticized by several economists. Colander (1995) demonstrated that the framework is logically inconsistent; Romer (2000) showed that it is unable to deal with a monetary policy that uses the interest rate as its operating target, and Walsh (2002) argued that it is not well suited for an analysis of inflation targeting. In this article, we present the so-called BMW (Bofinger, Mayer, and Wollmershäuser 2002) model that develops the Romer (2000) approach into a very simple but, at the same time, comprehensive macroeconomic model. In spite of its simplicity, it can carry the main insights of the New Keynesian macroeconomics (Clarida, Gali, and Gertler 1999) to an intermediate level and deal with issues such as inflation targeting, monetary policy rules, and central bank credibility. An important contribution of our model is the explicit comparison of Taylor-style reaction functions with optimal monetary policy. Moreover, our approach has the advantage of modeling expectations in a
more general way so that issues of credibility that play a pivotal role in inflation targeting can be addressed. In this respect, a main innovation of the model is that we integrate the Barro-Gordon (1983) time-inconsistency problem into our analysis. Finally, our model explicitly treats the reaction of monetary policy to demand shocks. This issue is crucial to an understanding of central banking but has been neglected in the graphical analysis by Walsh (2002).

THE BMW MODEL

Main Building Blocks

The model consists of three building blocks: an IS curve, a Phillips curve, and a monetary policy rule. The IS curve makes the output gap y depend on autonomous demand components a, the real interest rate r, and a demand shock \( \varepsilon_1 \).

\[
y = a - br + \varepsilon_1.
\]  

(1)

The output gap \( y \) is defined as the deviation of (the logarithm) aggregate output from its potential, or full capacity, level. This approach is very much in line with Romer (2000). It is clear from this equation that, in the absence of shocks, the output gap (which is zero then) depends on the real interest rate, which is given by \( a / b \). In accordance with Blinder (1998, 31), this rate is called the neutral, real short-term, interest rate \( r_0 \). From a New Keynesian perspective, the IS curve depicts the optimal allocation of consumption of households in equilibrium over time.

The second building block is the expectations-augmented Phillips curve.

\[
\pi = \pi^e + dy + \varepsilon_2.
\]  

(2)

The inflation rate is determined by expectations about inflation \( \pi^e \), the output gap \( y \), and a supply shock \( \varepsilon_2 \). The parameter \( d \) is nonzero and positive. For reasons of simplicity, we assumed in the most basic model that the central bank is credible, that is, that private sector inflation expectations are identical with the central bank's inflation target \( \pi_0 \). This automatically translates into \( \pi^e = \pi_0 \) so that the Phillips curve can be rewritten as

\[
\pi = \pi_0 + dy - \varepsilon_2.
\]  

(3)

In a later section, we discuss inflation expectations in a more general way. In particular, we show that our approach of modeling the Phillips curve in equation (3) can be regarded as a special case of the New Keynesian perspective, in which expectations are formed rationally, and the current inflation rate is related to the expected future inflation rate. Walsh (2002), by contrast, assumed that the private sector has adaptive expectations, and he did not show under which condition inflation fluctuates on average around the central bank's inflation target.1

As a third building block, we specify the way in which monetary policy is conducted. We assume that the instrument of monetary policy is the real interest rate. Even though the central bank directly controls the nominal interest rate on the money market, it indirectly controls the real interest rate as long as the central
bank is able to predict the impact of its monetary policy action on the inflation rate. This assumption, however, is standard in rational expectations models (see Appendix A for details). In the most basic version, we assume that the central bank decides on its interest rates without any specific monetary policy strategy. This assumption enables us to describe the fundamental principles of monetary policy in a simple and, at the same time, comprehensive graphical treatment. In practice, however, central banks usually follow a monetary-policy strategy that provides the guideline of how to set the interest rate instrument. Over the past decade, the strategy of inflation targeting has become very popular. The central bank uses all available information, and it determines the optimal interest rate on the basis of a loss function that includes its preferences toward its target variables, inflation and output (Svensson 1999). Alternatively, the central bank may conduct monetary policy on the basis of a simple policy rule. In contrast to an inflation targeting strategy, a central bank that follows a simple policy rule changes its instrument only in response to a small subset of the available information. The most prominent simple rule is a Taylor rule (Taylor 1993).

A Simple Approach to an Analysis of Supply and Demand Shocks

We start with the basic version, where the central bank sets the real interest rate in a discretionary way. In the spirit of the IS/LM-AS/AD approach, the graphical treatment requires two diagrams (Figure 1). The IS curve and the representation of monetary policy are depicted in the $y - r$ space. The IS curve is downward sloping, and the interest rate policy is depicted by a horizontal monetary policy line. In equilibrium, at the intersection of $r_0$ and IS, the output gap $y$ is closed. The Phillips curve is depicted as an upward sloping curve in the $y - \pi$ space. If $y = 0$, inflation is at its target level. If the economy is hit by a negative demand shock $(\varepsilon_2 < 0)$, the IS curve is shifted to the left (Figure 1) so that at a constant real interest rate, $r_0$, the output gap becomes negative ($y_1 < 0$). In the lower panel, this is translated into an inflation rate, $\pi_1$, that is below the central bank’s target rate, $\pi_0$. If the central bank lowers the real interest rate from $r_0$ to $r_1$, the output gap is closed, and inflation returns to its target level. Thus, the model shows that there is no tradeoff between output and inflation in the situation of a demand shock. In addition to the IS curve, we could also depict an aggregate demand (AD) curve in the $y - \pi$ space. Without any specified monetary policy rule, however, this curve would be analytically identical to the IS curve. Graphically, this would imply a vertical line in the lower panel, located at the output level determined by the IS curve and the real interest rate policy in the upper panel.

If the economy is confronted with a positive supply shock $(\varepsilon_2 > 0)$, the lower panel of Figure 2 shows that the Phillips curve is shifted upward. If the central bank decides to remain passive, one can see from the upper panel that at a constant real interest rate, the output gap remains zero. The inflation rate increases from $\pi_0$ to $\pi_B$ (point B). It is important to note that this requires an equivalent increase in the nominal rate because inflation has gone up. Alternatively, the central bank can increase the real interest rate to keep inflation at its target level.
this case, it has to accept a negative output gap $y_A$ (point A). Of course, the central bank can also decide to target intermediate combinations of $y$ and $\pi$ that lie on the Phillips curve between points A and B.

Thus, the basic version of the model helps us to understand the underlying principles of monetary policy in a very simple way. Monetary policy that responds to demand and supply shocks directly affects output (shown in the $y - r$ space in Figure 2) and indirectly affects inflation (shown in the $y - \pi$ space). Compared with the IS/LM-AS/AD model, this basic version of the model is at the same time (a) simpler because it does not require readjustments of the LM curve, which take place if the price level changes the real money supply; (b) more powerful because it allows explanation of changes in the inflation rate; and (c) more comprehensive because it includes the expectations-augmented Phillips curve.

Compared with the model by Walsh (2002), our approach has the advantage of explicitly dealing with demand shocks, thereby showing that monetary policy is not always confronted with a tradeoff between output and inflation. In addition,
it demonstrates graphically the interest rate response of the central bank in the $y - r$ space. Both issues are crucial for an understanding of monetary policy.

In our view, for an introductory textbook, this version of the model would be sufficient. It could even be simplified by omitting the shock terms $\varepsilon_1$ and $\varepsilon_2$. A comparable approach has been presented by Taylor (2001, ch. 24). In contrast to Taylor, however, we present a closed-form graphical analysis that endogenously determines all variables of interest simultaneously. The standard presentation of fiscal policy also could be included easily in the model.\(^2\)

**Inflation Targeting**

A monetary policy strategy facilitates the internal decisionmaking process of a central bank as well as its transparency and accountability in relation to the public. The most prominent strategy is inflation targeting. It can be described by introducing a loss function of the central bank, an element to which Walsh (2002) only implicitly refers.

\[
L = (\pi - \pi_0)^2 + \lambda y^2, \quad \text{with } \lambda \geq 0.
\]
Accordingly, the central bank aims at stabilizing squared deviations of the inflation rate from the inflation target, while also being concerned with economic activity. If \( \lambda > 0 \), such preferences are defined as a policy of flexible inflation targeting; if \( \lambda = 0 \), they are defined as strict inflation targeting or an "inflation nutter" (Svensson 1999).

Given the transmission structure of a change in the monetary policy stance, which runs from the real interest rate over economic activity to the inflation rate, optimal monetary policy can be derived by applying the following two-step procedure. First, we insert the Phillips curve in equation (3) into the loss function in equation (4), and second, we minimize the modified loss function with respect to \( y \). The solution gives an optimal value of the output gap.

\[
y = -\frac{d}{d^2 + \lambda} \varepsilon_2.
\]  

(5)

This solution method is in line with the standard New Keynesian approach, where monetary policy is conducted via an optimal control of the output gap. If we insert equation (5) into the Phillips curve in equation (3), we can derive the following reduced form expression for the deviation of the inflation rate from its target:

\[
\pi - \pi_0 = \frac{\lambda}{d^2 + \lambda} \varepsilon_2.
\]  

(6)

Because the demand shock \( \varepsilon_1 \) does not appear in equations (5) and (6), demand shocks can be perfectly compensated by the central bank.

Under a strategy of inflation targeting, one way to conduct monetary policy is to follow an instrument rule (Svensson and Woodford 2003). Such a rule makes the reaction of the instrument of monetary policy depend on all the information available at the time the instrument is set (i.e., the exogenous variables \( \varepsilon_1 \) and \( \varepsilon_2 \) and the structure of the economy). In our framework, the instrument rule can be derived by inserting equation (5) into equation (1) and by solving the resulting expression for \( r \).

\[
r^{opt} = \frac{a}{b} + \frac{1}{b} \varepsilon_1 + \frac{d}{b(d^2 + \lambda)} \varepsilon_2.
\]  

(7)

The rule shows the following characteristics: the optimal response to demand shocks \( \varepsilon_1 \) does not depend on the central bank's preferences \( \lambda \). As the interest rate changes according to \( (1/b) \varepsilon_1 \), the output gap remains zero, irrespective of the preference type (see also equation [5]). Thus, as long as demand shocks are part of the information set of the central bank, they do not inflict any costs on society. The reaction of the central bank to supply shocks depends on preferences \( \lambda \). A central bank that only cares about inflation (\( \lambda = 0 \)) requires a strong real rate response and, accordingly, a large output gap (point A, Figure 2). With an increasing \( \lambda \), the real interest rate response declines (point B, Figure 2). In equilibrium (\( \varepsilon_1 = \varepsilon_2 = 0 \)), the real interest rate will be given by the neutral, real short-term interest rate, \( r_0 = a/b \).
The strategy of inflation targeting can also be presented with our graphical analysis (Figure 3). The instrument rule enters as a horizontal line in the $y - r$ space, marked by $r(\varepsilon_1, \varepsilon_2)$, to highlight the shift parameters of the monetary policy line. As before, the AD curve that could be derived from inserting the policy rule equation (7) into the IS curve in equation (1) would be a vertical line. The loss function of the central bank can be illustrated by circles around a bliss point in the $y - \pi$ space. The bliss point that represents the first best outcome with a loss of zero is defined by an inflation rate $\pi$ equal to the inflation target $\pi_0$ and an output gap of zero. We can derive the geometric form of the circle by transforming the loss function in equation (4) into

$$1 = \frac{(\pi - \pi_0)^2}{(\sqrt{L})^2} + \frac{(y - 0)^2}{(\sqrt{L})^2}$$

where $(0; \pi_0)$ is the center of the circle, and the radius is given by $\sqrt{L}$.

In the case of a demand shock, monetary policy is always able to maintain the bliss combination (Figure 3). In the case of a supply shock, the loss function helps
to identify the optimum combination of $\pi$ and $y$. Here, the shifted Phillips curve serves as a constraint under which the radius of the circle has to be minimized. The optimum combination is graphically given by the locus on the Phillips curve $(y_1; \pi_1)$ that is tangent to an isoquant of the loss function (Figure 4). To attain this point, the central bank will adjust its instrument to realize the optimum output gap, $y_1$.

An alternative view of inflation targeting is given by the so-called “targeting rule” of the central bank (Svensson and Woodford 2003), which gives a high-level specification of monetary policy that can be directly derived from the central bank’s strategy. Targeting rules are an important device to describe actual central banks as the institutional changes that took place during the last two decades aimed at committing central banks at the target level (i.e., specifying a concrete inflation target). By eliminating the supply shock $\varepsilon_2$ from equations (5) and (6), we arrive at the following consolidated first-order condition:

$$\pi = \pi_0 - \frac{\lambda}{d} y.$$  \hspace{1cm} (9)

![Figure 4. Supply shock under inflation targeting.](image-url)
In contrast to instrument rules, targeting rules are a linear relationship between endogenous target variables that will have to hold with equality if monetary policy is conducted optimally. By the very definition of a first-order condition, this ensures that, for a given value of private sector expectations and thus for any given location of the Phillips curve, the loss function in equation (4) is minimized.

Graphically, the optimal outcome is thus described by the intersection of the Phillips curve \( PC_1 \) with the targeting rule of the central bank (Figure 5).\(^5\) In equation (9), an increasing \( \lambda \) (i.e., an increasing weight on output stabilization) leads to a steepening of the reaction function \( RF(\lambda) \).

**The Taylor Rule: Monetary Policy Guided by a Simple Instrument Rule**

Instead of relying on all available information, a central bank can also restrict its information to a small subset of directly observable variables. At the very heart of simple instrument rules lies the notion that they are not derived from an optimization problem. Instead, the coefficients are chosen ad hoc, based on the experiences and skills of the monetary policymakers.\(^6\)

The most prominent version of a simple instrument rule is the Taylor (1993) rule. According to this rule, the actual real interest rate is defined as the sum of the equilibrium real interest rate \( r_0 \) and two additional factors accounting for the actual economic situation that is assumed to be observable by movements in the inflation rate \( e \) and in the output gap \( f \):

\[
r = r_0 + e(\pi - \pi_0) + fy, \quad \text{with } e, f > 0.
\]  

(10)

**FIGURE 5. Supply shock and the targeting rule.**
In our model, the Taylor rule can be represented by an upward-sloping monetary policy (MP) line in the \( y - r \) space (Figure 6). Whereas movements in the output gap lead to changes in the real interest rate, which constitute movements along the MP line, the inflation rate represents a shift parameter.

In contrast to previous sections, an explicit construction of an AD curve is required. Algebraically, the AD curve can be easily derived by inserting the Taylor rule in equation (10) into the IS curve in equation (1), by replacing \( r_0 \) with \( al/b \), and by solving the resulting equation for \( \pi \).

\[
\pi = \pi_0 + \frac{\varepsilon_1}{be} \times \frac{1 + bf}{be} y. \tag{11}
\]

Graphically, it can be constructed in the same spirit as the aggregate demand curve in the AS/AD model. We start with an MP line for an inflation rate equal to \( \pi_0 \) and an output gap of zero (Figure 6). This combination of output and inflation gives point A in the lower panel. Then we derive an MP line for an inflation rate

**FIGURE 6.** Simple instrument rules and the aggregate demand curve.
\( \pi_1 > \pi_0 \). According to the Taylor principle (Taylor 1999), which states that real interest rates should be raised in response to an increase in the inflation rate, this line is associated with higher real interest rates than \( MP(\pi_0) \). Hence, the new equilibrium is characterized by a negative output gap \( y_1 \). The combination of \( y_1 \) and \( \pi_1 \) gives the point B in the lower panel. Connecting point A with point B results in a downward-sloping AD curve, \( AD_0 \).

Because of the downward-sloping AD curve, the graphical analysis of shocks under a Taylor rule is more complex than under inflation targeting. If the economy is hit by a negative demand shock, the IS curve in the upper panel of Figure 7 shifts leftward. In response to the decrease of the output gap from 0 to \( y' \), the central bank lowers real interest rates by moving along the \( MP(\pi_0) \) line from \( r_0 \) to \( r' \), which leads to the output gap \( y' \). In the lower panel, the AD curve has to shift. Its

![Figure 7. Demand shock and a Taylor rule.](image-url)
new locus is obtained by its having to go through point A, which is defined by the new output gap \( y' \) and the so far unchanged inflation rate \( \pi_0 \). The new equilibrium is reached by the intersection of the shifted aggregate demand curve with the unchanged Phillips curve in point B. It is characterized by an output decline to \( y_1 \) (which is less than \( y' \)) and an inflation rate \( \pi_1 \). The decline of the output gap from \( y' \) to \( y_1 \) and the inflation rate to \( \pi_1 \) (instead of \( \pi' \)) is a result of the central bank further reducing the real interest rate, because the Taylor rule requires a lower real rate as a result of the decline in the inflation rate. In the upper panel, this is reflected by a downward shift of the MP line, which intersects with the IS\(_1\) line at the same output level, the result of the intersection of the AD\(_1\) line with the Phillips curve in the lower panel. This may sound somewhat difficult, but the mechanics of the shifts are completely identical with the shifts in the IS/LM-AS/AD model in the case of the same shock. Although in our model the decline in inflation implies an expansionary monetary impulse because it lowers the real interest rate, in the IS/LM-AS/AD model, the decrease in the price level increases the real money stock, which also has an expansionary effect because it lowers the nominal interest rate.

For a graphical discussion of a supply shock, one only need consider the \( y - \pi \) space (Figure 8). The Phillips curve is shifted upward, which increases the inflation rate to \( \pi' \). In this case, the Taylor rule requires a higher real interest rate that leads to a negative output gap \( y_1 \). The reduced economic activity finally dampens the increase of inflation rate to \( \pi_1 \).

A final remark should be made on the choice of the response coefficients in the Taylor rule. The intensity with which the central bank reacts to movements in \( \pi \) and \( y \) provides the central bank with a certain degree of flexibility for which the
following important relationships can be established: An increasing weight on the output gap $f$ leads to a steepening of the AD curve and, accordingly, to smaller output gaps. An increasing weight on the inflation rate $e$ leads to a flattening of the AD curve and, hence, to smaller inflation gaps $(\pi - \pi_0)$. The Taylor principle holds as long as $e > 0$. If we are dealing with a “passive Taylor rule” ($e < 0$), the AD curve becomes upward sloping, and monetary policy itself is the source of destabilizing effects on economic activity (Figure 9).

**Inflation Targeting versus Taylor Rules**

We have shown that demand shocks $\varepsilon_1$ have an impact on the AD curve and the output gap if a central bank follows a Taylor rule. This result is in contrast to the

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**FIGURE 9.** Destabilizing effects of a passive Taylor rule.
results that we obtained for an optimal monetary policy rule under inflation targeting. With our simple model, we are able to identify the mechanisms that prevent simple rules from being as efficient as optimal rules. This insight is of crucial importance for an understanding and evaluation of the differences between simple rules and optimal monetary policy.

Although the Taylor rule is a linear relationship between two endogenous variables, it can be transformed in a way that shows the implicit reaction of the central bank to exogenous demand and supply shocks. By inserting the AD curve in equation (11) into the Phillips curve in equation (3) and by solving the resulting equation for $y$, we get the reduced form of the output gap, which only depends on the two exogenous shock terms. Inserting this expression back into the Phillips curve in equation (3) yields the equivalent for the inflation rate. Substituting these two reduced-form equations into the Taylor rule in equation (10) then gives the following interest rate rule in terms of $\varepsilon_1$ and $\varepsilon_2$:

$$r_{\text{Taylor}} = r_0 + \frac{ed + f}{1 + bf + bed} \varepsilon_1 - \frac{e}{1 + bf + db e} \varepsilon_2. \quad (12)$$

In the case of demand shocks, optimal and simple monetary policy rules can only be identical if the reaction coefficients of $a_i$ in equations (7) and (12) were identical. Thus, the following equation has to hold:

$$\frac{ed + f}{1 + bf + bed} = \frac{1}{b}, \quad \text{or, equivalently,} \quad \frac{1}{b + 1/(ed + f)} = \frac{1}{b}. \quad (13)$$

Equation (14), however, is only true for values of $e$ or $f$ approaching infinity. With realistic reaction coefficients, for example, $e = f = 0.5$ as has been originally assumed by Taylor (1993), this equality is violated, and the interest rate adjustment will go in the right direction, but it will be too weak to restore the equilibrium marked by the bliss point. The intuition behind this result is that simple rules, which use only a subset of information, cannot be identical with a policy that has perfect information about the shocks $\varepsilon_1$ and $\varepsilon_2$.

In the case of a supply shock, it is possible that both types of rules yield identical results. This can be shown by equating the reaction coefficients of the supply shock $\varepsilon_2$ in equations (7) and (12).

$$\frac{d}{b (d^2 + \lambda)} = \frac{e}{1 + bf + db e}, \quad \text{or,} \quad \lambda = \frac{d(1 + bf)}{eb}. \quad (15)$$

Thus, under certain conditions, a Taylor rule can lead to an optimum response to supply shocks. According to equation (16), a high weight on output stabilization in the loss function $\lambda$ requires a Taylor rule that attaches a high value $f$ to the
output gap and a low value \( e \) to the inflation gap. In addition, as shown in equation (16), for reasonable specifications of preferences \( \lambda > 0 \), the Taylor principle \( e > 0 \) has to hold.

Although it is conventional wisdom that simple monetary policy rules perform worse than optimal rules in terms of losses, our analysis showed that this applies only to the case of demand shocks, whereas in principle, an optimum response to supply shocks is possible.

**Three Approaches for the Specification of Inflation Expectations**

So far, we have modeled inflation expectations \( \pi^e \) in a very simple way. For a more general analysis, we had to specify in detail how expectations may be formed. Thereby, we distinguished three cases: adaptive expectations, \( \pi^e = \pi^e(\pi_{t-1}) \); rational expectations, \( \pi^e = E(\pi_{t+I}) \), where \( I \) is defined as the private sector's information set; and rational expectations and a credible central bank, \( \pi^e = \pi_0 \).

Adaptive expectations were at the core of the model developed by Walsh (2002). He implicitly assumed that expectations are initially exogenously given. This can be seen in his graphical analysis (Walsh 2002, Fig. 1, 335) where he started with inflation expectations that were higher than the inflation target of the central bank. Although Walsh did not explain how these initial expectations were formed, his case can be translated into our framework as an unexpected shift of the Phillips curve to the left. He then assumed a sluggish adjustment in wages and prices so that the economy eventually moved back to its long-run equilibrium \( \pi_0 \); \( \pi_0 \). The lack of a clear specification of expectations also affected his analytical exposition, as he did not endogenize expectations but carried \( \pi^e \) through his equations.

In our view, it seems useful to endogenize expectations. To map the standard New Keynesian Phillips curve in which current inflation is determined by rational expectations about future economic conditions (Calvo 1983) into a static framework, we had to impose that the disturbance term \( \varepsilon_2 \) was purely white noise. Under this assumption, the private sector expects inflation to return immediately to equilibrium in the period following a shock. This equilibrium is exactly defined by the central bank's inflation target, provided that the inflation target is credible (see Appendix B for an analytical treatment). Thus, assuming that the private sector's inflation expectations are identical to the central bank's medium-term inflation target, it is simply a special case of rational expectations.

**A Central Bank with an Inflation Bias**

The rational expectations solution also allows a discussion of a central bank with an inflation bias, which correspondingly suffers from low credibility (Barro and Gordon 1983). For this purpose, we had to modify the central bank's loss function as follows:

\[
L = (\pi - \pi_0)^2 + \lambda (y - k)^2, \quad \text{with} \quad k > 0.
\]
By introducing the parameter $k$, the central bank targets an output gap that is above zero. This could be rationalized by monopolistic distortions in goods and labor markets that keep potential output below an efficient level. Compared with the loss function that we have used so far, the bliss point $k; \pi_0$ has moved to the right (Figure 10).

In line with Barro and Gordon (1983), the game between the private sector and the central bank can be modeled as follows. The private sector forms its inflation expectations, which enter the contracts settled on the goods and labor market. Using these private sector expectations, the central bank chooses an inflation rate that minimizes its loss function so that one arrives at the following targeting rule:

$$\pi = \pi^e + \frac{\lambda}{d} k - \frac{\lambda}{d} y. \quad (18)$$

In comparison to equation (9), the new reaction function of the central bank has shifted to the right because of the inflationary bias.

If the private sector forms expectations rationally, it minimizes the following loss function:

$$L = [\pi(\pi^r) - \pi^e]^2. \quad (19)$$

Accordingly, it will take the first order condition of the central bank in equation (18) into account while building its expectations. Thus, our framework indicates that monetary cheating does not pay off as the economy will end up with no gains in output $y^{nat} = 0$ but with higher rates of inflation $\pi^{nat} > \pi_0$ (point B, Fig. 10). Compared with surprise inflation (point A with $\pi = \pi^e$ and $y = y^e$, which is based on a Phillips curve with $\pi^e = \pi_0$), this solution is clearly inferior because it leads
to a higher inflation rate without a positive gain in output. The loss circle lies outside the circle attached to the solution with surprise inflation. Even if the central bank announces an inflation target, rational agents will realize that it has a strong incentive to renege on its announcement. To avoid the high social loss under discretion, a mechanism is required that credibly commits the central bank to the inflation target $\pi_T$.

Thus, our framework can be easily extended for an analysis of the issues that are related to the Barro and Gordon (1983) model. Although these results are well known in literature, the model has the advantage that it provides a coherent framework for a discussion of monetary policy, which includes both traditional stabilization issues and the topics related to time inconsistency.

**SUMMARY**

The BMW model offers important advantages over the IS/LM-AS/AD model. In its most basic version, it is at the same time simpler and more powerful than the IS/LM-AS/AD model. In its more complex versions, it can analyze important concepts, such as inflation targeting and monetary policy rules, that have become standard tools in New Keynesian macroeconomics. Compared with other recent approaches (Romer 2000; Taylor 2001; Walsh 2002), it deals explicitly with the central bank's reaction to demand shocks and focuses on the concept of central bank credibility, which plays a pivotal role in inflation targeting. In addition, it allows an easy comparison of optimal and simple monetary policy rules, which is a novelty in the framework of a static model.

**NOTES**

1. Walsh is vague about this central question: "The economy eventually reaches equilibrium with an output gap and inflation equal to $\pi_T$" (Walsh 2002, 339; emphasis added).
2. It would also be possible to present the IS curve in terms of output levels $Y$ instead of output gaps $y$. In this case, equation (1) can be stated as
   \[ Y = \tilde{a} - \tilde{b}r + \tilde{\varepsilon}_1. \] (1a)
   As a benchmark, we would then have to define potential output $Y_p$, which can be derived from a neoclassical labor market. The Phillips curve would be formulated as
   \[ \pi = \pi' + d \left( \frac{Y - Y_p}{Y_p} \right) + \varepsilon_2. \] (6a)
3. In Appendix B, we solved the optimization problem of monetary policy in terms of the nominal interest rate.
4. For the more general case of an ellipse ($\lambda \neq 1$), see Bofinger, Mayer, and Wollmershäuser (2002).
5. Figure 5 corresponds to the way Walsh (2002) set up his graphical analysis. Instead of focusing our analysis on the $y - \pi$ space, however, we preferred a two-panel graph (one in $y - r$ space, the other $y - \pi$ space) for two reasons. First, we can explicitly depict the reaction of the central bank's instrument $r$ to demand and supply shocks. Second, the two-panel approach makes clear that inflation can only be indirectly controlled by the central bank via aggregate demand.
6. There is an enormous literature on optimal simple rules that derives the coefficients of the simple rule by minimizing the central bank's loss function (Rudebusch and Svensson 1999). Although such an approach would also be feasible within our model, we think that optimal simple rules are too complicated to be taught in intermediate macro courses.
7. At the end of this section, we provide a rationale for the Taylor principle.
REFERENCES


APPENDIX A

ON THE EQUIVALENCE OF EXPRESSING MONETARY POLICY ACTIONS IN TERMS OF REAL AND NOMINAL INTEREST RATES: USING ACTUAL REAL RATES

So far, we have assumed that the real interest rate is the operating target of the central bank. In this appendix, we show that our model can also be applied if a central bank uses the nominal interest rate. For this analysis, we have to distinguish two different cases.

(1) The aggregate demand depends on the actual real interest rate \( r \), that is, nominal interest rate \( i \) minus actual inflation \( \pi \),

\[
r = i - \pi .
\]

(20)

In this case, the central bank has to calculate the optimal real interest rate first. It then knows the optimum inflation rate that it adds to the optimal real interest rate to calculate the required nominal interest rate.

(2) The aggregate demand depends on the expected real interest rate,

\[
r = i - \pi^e .
\]

(21)

In this case, the private sector forms its inflation expectations \( \pi^e \) on the basis of the loss function of the central bank and the structure of the economy. The central bank then determines the optimal real interest rate. The nominal interest rate is calculated as the sum of the expected inflation rate and the optimum real interest rate.

Starting with the first case, the IS equation can equally be written as

\[
y = a - b(i - \pi) + \varepsilon_1 .
\]

(22)

We again assume that monetary policy is guided by the loss function in equation (4) and that the structure of the economy is given by equations (22) and (3). Inserting the Phillips curve into the loss function and solving the optimization problem of the central bank yields
again the output gap equation (5). Inserting equation (5) back into the Phillips curve results in the reduced form equation (6). With these equations at hand, we can easily compute the corresponding nominal interest by inserting equations (5) and (6) into equation (22).

\[ i = \frac{a}{b} + \pi_0 + \frac{1}{b}e_1 + \frac{d + b\lambda}{b(d^2 + \lambda)}e_2. \]  

(23)

This equation can be rewritten as

\[ i = r^{opt} + \pi_0 + \frac{\lambda}{(d^2 + \lambda)}e_2 = r^{opt} + \pi. \]  

(24)

Hence, one can always switch from nominal to real interest rates by adding or subtracting the endogenously determined inflation rate, which the central bank can predict, as the supply shock \( e_2 \) is an element of the central bank’s information set. In addition, one can easily show by subtracting equation (6) from equation (24) that the Taylor principle holds.

\[ r = \frac{a}{b} + \frac{1}{b}e_1 + \frac{d^2 + \lambda}{bd\lambda}(\pi + \pi_0), \]  

as

\[ (d^2 + \lambda/bd\lambda) \geq 0. \]  

(25)

For the second case, we inserted equation (21) into the IS curve.

\[ y = a - b(i - \pi^e) + e_1. \]  

(26)

Let us assume the following sequence of events:

1. Given that the structure of the economy (Phillips curve, IS curve) and the loss function of the central bank are common knowledge, the private sector builds its expectations.
2. The central bank observes supply and demand shocks and sets the optimal real interest rate to minimize its loss function.
3. Given its knowledge on the optimal real interest rate and on private inflation expectations, the central bank will choose its preferred nominal interest rate accordingly.

As we show in Appendix B, it will have to hold that under rational expectations, the expected inflation rate will be equal to the inflation target of the central bank (see equation [35]). Therefore, we can rewrite the IS equation as follows:

\[ y = a - b(i - \pi_0) + e_1. \]  

(27)

Making use of this result, we can calculate the following expression for the optimal nominal interest rate:

\[ i = \frac{a}{b} + \pi_0 + \frac{1}{b}e_1 + \frac{d}{d(d^2 + \lambda)}e_2, \]  

which can equally be written as

\[ i = r^{opt} + \pi_0 = r^{opt} + \pi^e. \]  

(29)

Thus, under scenario (2), one can always switch from real to nominal interest rates by adding inflation expectations \( \pi^e \).

**APPENDIX B: RATIONAL EXPECTATIONS IN THE MODEL**

We now show how to endogenize expectations in our model. We assume that the central bank is guided by the loss function in equation (4), and the structure of the economy
is given by equations (1) and (2). Substituting the Phillips curve into the loss function and deriving the optimal output gap yields

\[ y = - \frac{d}{d^2 + \lambda}(\pi^\tau - \pi_0) - \frac{d}{d^2 + \lambda} \varepsilon_2. \]  

(30)

Inserting equation (30) into the Phillips curve in equation (2), we get the following optimal inflation rate for the central bank as a function of private sector expectations:

\[ \pi(\pi^\tau) = \frac{\lambda}{d^2 + \lambda} \pi^\tau + \frac{d^2}{d^2 + \lambda} \pi_0 + \frac{\lambda}{d^2 + \lambda} \varepsilon_2. \]  

(31)

At the beginning of a period, private agents settle goods and labor market contracts. Therefore they have to build expectations on the inflation rate. We assumed that the private sector was guided by the following loss function:

\[ L = [\pi(\pi^2) - \pi^\tau]^2. \]  

(32)

The first order condition of the private sector is given by

\[ \pi^{\text{opt}}(\pi^\tau) = \pi^\tau. \]  

(33)

When forming its expectations, the private sector takes the optimal inflation rate of the central bank, equation (31), into account. Accordingly, it has to hold that

\[ \pi^\tau = \frac{\lambda}{d^2 + \lambda} \pi^\tau + \frac{d^2}{d^2 + \lambda} \pi_0 + \frac{\lambda}{d^2 + \lambda} \varepsilon_2. \]  

(34)

Solving for the private sector expectations yields

\[ \pi^\tau = \pi_0, \]  

(35)

because the current supply shock is not an element of the information set of the private sector. Equation (35) equally holds true for a Taylor rule if the private sector forms its expectations according to the loss function in equation (32).