Advanced Macroeconomics II
Monetary Models (II): Models with Nominal Rigidities

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Part of these slides are based on Jordi Gali slides for Macroeconomia Avanzada II.
BASIC NEW KEYNESIAN MODEL

NK model has become the workhorse for:

- the analysis of monetary policy;
- the study of business cycle dynamics;
- welfare evaluation.
- interaction between monetary and fiscal policy
EMPIRICAL EVIDENCE: CEE (1999)

IRFS to an EXOGENOUS TIGHTENING of MONETARY POLICY
MAIN FEATURES OF NEW KEYNESIAN MODELS

RBC FRAMEWORK with:

- **Imperfect competition in goods market**: each firm produces a differentiated good for which it sets the price. Firms are not \textit{price-taker}.
- **Sticky prices**
- **Walrasian Labor Market**
- **Closed Economy**
- **No capital accumulation**
Despite the fact that there are rational expectations (RE), money is not neutral in the short run.

**WHY?**

- Goods markets are not Walrasian $\Rightarrow$ Monopolistic Competition: is the necessary condition to introduce price-stickyness and demand driven fluctuations!!
- Price-stickyness $\Rightarrow$ money has real effects in the short run. $\Rightarrow$ The classical dichotomy does not hold.
- Since the basic NK model does not consider any form of capital accumulation $\Rightarrow$ the money transmission channel is via consumption smoothing.
- The NK model is Keynesian in the short-run and classical in the long-run!
FIRST INTUITION

- Suppose that the economy is hit by a positive monetary policy shock \( \Rightarrow \) the nominal interest rate decreases.
- If prices are sticky the real interest rate decreases
- Consumers decide to consume more today than tomorrow
- The aggregate demand increases \( \Rightarrow \) since firms that cannot revise their price will increase their supply of goods \( \Rightarrow Y \uparrow \), and money has real effects in the short run
- In the long run price are flexible and money is neutral!
LABOR MARKET

In the basic New Keynesian model

- LABOR MARKET is still WALRASIAN: As in RBC model there is no involuntary unemployment!
- Differently from Keynesian models, real wages are not countercyclical but procyclical (as in RBC models) \( \implies \) EVIDENCE: real wages seems to be acyclical
THE MODEL

HOUSEHOLDS I

The lifetime utility:

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \text{ with } 0 < \beta < 1 \]  

(1)

where

\[ C_t \equiv \left( \int_0^1 C_t(i) \frac{\varepsilon - 1}{\varepsilon} \, di \right)^{\frac{\varepsilon}{\varepsilon - 1}} \]

is a Dixit-Stiglitz consumption basket. \( C_t(i) \): there exists a continuum of goods represented by an interval \([0, 1]\). The quantity of good \(i\) consumed by the household in period \(t\).
HOUSEHOLDS II

- Households’ period budget constraint is:

\[ P_t C_t + Q_t B_t = B_{t-1} + W_t N_t + T_t + D_t \]

\( t = 0, 1, 2... \) plus solvency constraint.

- \( P_t C_t = \int_0^1 P_t \,(i) \, C_t \,(i) \) and \( P_t \,(i) \) is the price of good \( i \).
- \( W_t \): nominal wage;
- \( B_t \): quantity of one-period, nominally riskless discount bonds purchased in period \( t \) and maturing in \( t + 1 \).
- Each bond pays one unit of money at maturity and its price is \( Q_t = \frac{1}{R_t} \).
- \( T_t \): lump-sum transfer
- \( D_t \): dividends from ownership of firms (Households are the owner of firms)
SOLVING THE HOUSEHOLDS PROBLEM

Optimal intertemporal problem

\[
\max_{\{C_t, N_t, B_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t),
\]

s.t. \( P_t C_t + (1 + i_t)^{-1} B_t = B_{t-1} + W_t N_t + T_t + D_t \)  \hspace{1cm} (2)

and the solvency constraint.
The period utility is: \( U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \).

The Langrangean:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t, N_t) - \lambda_t \left[ P_tC_t + \frac{B_t}{1+i_t} - B_{t-1} - W_tN_t - T_t - D_t \right] \right\}
\]

the first order conditions are:

\[
\begin{align*}
\frac{\partial L}{\partial C_t} &= 0 : \quad C_t^{-\sigma} = \lambda_t P_t \implies C_{t+1}^{-\sigma} \lambda_{t+1} P_{t+1} \\
\frac{\partial L}{\partial B_t} &= 0 : \quad -\lambda_t (1+i_t)^{-1} + \beta E_t \lambda_{t+1} = 0 \\
\frac{\partial L}{\partial N_t} &= 0 : \quad N_t^{\varphi} = W_t \lambda_t
\end{align*}
\]
Combining the first order conditions, then as usual we get:

**Labor supply**: \[ \frac{W_t}{P_t} = N_t^\varphi C_t^\sigma \]  \hspace{1cm} (3)

**Euler equation**: \[ R_t^{-1} = \beta E_t \left[ \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \right) \right] \]  \hspace{1cm} (4)
FIRMS
Monopolistic Competition

- The basic NK model abstracts from capital accumulation and assume that there is a continuum of intermediate good-producing firms $i \in (0, 1)$ which hire $N_t(i)$ units of labor from the representative household and produce $Y_t(i)$ units of the intermediate good using the following technology:

$$Y_t(i) = A_t N_t^{1-\alpha}(i). \quad (5)$$

where $\ln(A_t/A) = a_t$ is an exogenous process of technological progress (or total factor productivity TFP), which evolves according to:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim N(0, \sigma_a^2) \quad i.i.d.$$ 

Since $a_t$ is unknown agents must form expectations about it. They adopt the RE to do so.
Costs minimization

Before choosing the price of its goods, a firm chooses the level of \( N_t (j) \) which minimizes its costs, solving the following costs minimization problem:

\[
\min_{\{N_t(i)\}} TC_t (i) = W_t N_t (i)
\]

subject to (5)

The first order condition with respect to \( N_t (i) \) is given by:

\[
\Psi_t (i) = \frac{W_t}{(1 - \alpha) A_t (N_t (i))^{-\alpha}},
\]

where \( \Psi_t (i) \) represents firm’s \( i \) marginal costs and the Lagrangian multiplier of the costs minimization problem.
Firms maximize

$$\max_{\{P_t(i)\}} P_t(i) Y_t(i) - \Psi_t(i) Y_t(i)$$

s.t. 

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t$$

where $\Psi_t(i)$ are nominal marginal costs faced by firm $i$. F.o.c.

$$(1 - \epsilon) \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t + \epsilon \Psi_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} \frac{1}{P_t(i)} Y_t$$

then the optimal price is:

$$P_t(i) = \frac{\epsilon}{\epsilon - 1} \Psi_t(i)$$

(7)

or

$$P_t(i) = \mathcal{M} \Psi_t(i)$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1} = 1 + \frac{1}{\epsilon - 1} > 1$ is the gross markup. The optimal price is a markup $\frac{\epsilon}{\epsilon - 1}$ over nominal marginal costs $\Psi_t(i)$. 
Combining the equation of marginal costs, i.e:

$$\Psi_t(i) = \frac{W_t}{(1 - \alpha) A_t N_t(i)^{-\alpha}}$$

with that of the optimal price, in logs

$$p_t(i) = \mu + \psi_t(i)$$

$$= \mu + w_t - (a_t - \alpha n_t(i) + \log(1 - \alpha))$$

where $\mu \equiv \log M$ i $\psi_t(i) \equiv \log \Psi_t(i)$. Notice that in log-deviation would be:

$$\hat{p}_t(i) = \hat{\psi}_t(i)$$

$$= \hat{w}_t - (a_t - \alpha \hat{n}_t(i))$$
Equilibrium (in logs)

- **Aggregate Demand**

  \[ y_t(i) = c_t(i), \quad i \in [0, 1] \quad \implies \quad y_t = c_t \]

  \[ y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \]

- **Labor demand (prod. function solved for n):**

  \[ n_t = \frac{1}{1 - \alpha}(y_t - a_t) \]

- **Aggregate supply:**

  \[ p_t = \mu + w_t - (a_t - \alpha n_t + \log(1 - \alpha)) \]

  \[ w_t = p_t + \sigma c_t + \varphi n_t \]

- **Bond Market:**

  \[ b_t = 0 \]
Equilibrium of real variables:

\[ n_t = \frac{(1 - \sigma)a_t + \log(1 - \alpha) - \mu}{\sigma(1 - \alpha) + \varphi + \alpha} \]

\[ y_t = c_t = \frac{(1 + \varphi)a_t + (1 - \alpha)(\log(1 - \alpha) - \mu)}{\sigma(1 - \alpha) + \varphi + \alpha} \]

\[ w_t - p_t = \frac{(\sigma + \varphi)a_t + (\sigma(1 - \alpha) + \varphi)(\log(1 - \alpha) - \mu)}{\sigma(1 - \alpha) + \varphi + \alpha} \]

\[ r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho - \frac{\sigma(1 + \varphi)(1 - \rho_a)}{\sigma(1 - \alpha) + \varphi + \alpha} a_t \]

\[ \implies \text{effect of an increase in } (\mu). \text{ Notice that since the markup is constant it affects logs and long-run but not the log-deviations, it means that it does not affect the short run.} \]

\[ \implies \text{Money Neutrality long-run but also short-run.} \]

\[ \implies \text{The Optimal monetary policy is indeterminate} \]

\[ \implies \text{The equilibrium is inefficient} \]

\[ \implies \text{Monetary policy: affects nominal variables} \]
Monopolistic Competition and Constant Prices

Assumptions:

- Prices are constant: \( p_t = p = 0 \) (normalization), \( t = 0, 1, 2, \ldots \)
- Then the markup is: \( \mu_t = -\psi_t \geq 0 \), \( t = 0, 1, 2, \ldots \)

Equilibrium (omitting constants)

\[
y_t = c_t
\]

\[
y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - \rho)
\]

\[
n_t = \frac{1}{1 - \alpha}(y_t - a_t)
\]

\[
w_t = \sigma y_t + \varphi n_t
\]

\[
\mu_t = w_t - (a_t - \alpha n_t)
\]
Monopolistic Competition and Constant Prices

- Interest rate rule:

\[ i_t = \rho + \phi_\pi \pi_t + \nu_t = \rho + \nu_t \]

with

\[ \nu_t = \rho_v \nu_{t-1} + \varepsilon_t^\nu \]

Equilibrium:

Iterating forward the IS and excluding bubbles:

\[ y_t = \frac{1}{\sigma(1 - \rho_v)} \nu_t \]

From the production function:

\[ n_t = \frac{1}{\sigma(1 - \rho_v)(1 - \alpha)} \nu_t - \frac{1}{1 - \alpha} a_t \]

\( \Rightarrow \) Monetary policy is not neutral

\( \Rightarrow \) A technology shock reduced labor hours if it is not accompanied by an expansionary monetary policy.
Money Supply
\[ m_t = \rho_m m_{t-1} + \varepsilon_t^m \]

Money Demand
\[ m_t = p_t + y_t - \eta i_t = y_t - \eta i_t \]

Equilibrium:
\[
y_t = \frac{1}{1 + \sigma \eta} \sum_{k=0}^{\infty} \left( \frac{\sigma \eta}{1 + \sigma \eta} \right)^k E_t\{m_{t+k}\}
\]
\[
= \frac{1}{1 + \sigma \eta (1 - \rho_m)} m_t
\]

\[
n_t = \frac{1}{(1 - \alpha)(1 + \sigma \eta (1 - \rho_m))} m_t - \frac{1}{1 - \alpha} a_t
\]

\[\implies\] Monetary Policy non-neutrality
\[\implies\] A technology shock reduced labor hours if it is not accompanied by an expansionary monetary policy.
The Basic New Keynesian Model

- **NK Phillips Curve (NKPC)**

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t \]

with \( \tilde{y}_t \equiv y_t - y_t^n \) ("output gap").

- **Dynamic IS Curve**

\[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - r^n_t) + E_t \{ \tilde{y}_{t+1} \} \]

- **Interest rate rule**

Example:

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \]
The Basic New Keynesian Model

- **Assumption:** probability that a firm can revise its price is \(1 - \theta\) (independent from time and from what done by other firms).

\[ \Rightarrow \text{average price duration } \frac{1}{1-\theta} \]

\[ \Rightarrow \text{share of firms that keep price constant: } \theta \]

\[ \Rightarrow \theta \in [0, 1] : \text{index of price rigidity} \]

- **Price level evolution**

\[ p_t = \theta p_{t-1} + (1 - \theta) p_t^* \]

- **Optimal price**

\[ p_t^* = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \{ \psi_{t+k} \} \]

- **Inflation: New Keynesian Phillips Curve**

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} - \lambda (\mu_t - \mu) \]

with \( \mu_t \equiv p_t - \psi_t \) (markup) and \( \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \)
- **Price Markup** (assuming: $\alpha = 0$)

\[ \mu_t = p_t - (w_t - a_t) \]

- **Labor market equilibrium**

\[ w_t - p_t = \sigma c_t + \varphi n_t \]

\[ n_t = y_t - a_t \]

- **Good Market equilibrium**

\[ y_t = c_t \]
Combining together the previous equations

\[ \mu_t = (1 + \varphi) a_t - (\sigma + \varphi) y_t \]

Under Flexible price equilibrium \( \mu = \frac{\varepsilon}{\varepsilon - 1} \), and then

\[ \mu = (1 + \varphi) a_t - (\sigma + \varphi) y_t^n \]

\[ \Rightarrow y_t^n = -\frac{\mu}{\sigma + \varphi} + \frac{1 + \varphi}{\sigma + \varphi} a_t \]

Combining the two:

\[ \mu_t - \mu = -(\sigma + \varphi) \tilde{y}_t \]

where \( \tilde{y}_t = y_t - y_t^n \)

NKPC becomes

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t \]

with \( \kappa \equiv \lambda (\sigma + \varphi) \)
Properties of the NKPC

(i) "Forward-looking"

\[ \pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t \{ \tilde{y}_{t+k} \} \]

\[ \Rightarrow \text{past inflation do not affect inflation today} \]

(ii) No trade-off between stabilizing inflation and output gap..

\[ \Rightarrow \text{"divine coincidence" (Blanchard-Galí)} \]

\[ \Rightarrow \text{No cost of disinflation.} \]

(iii) Not easy to measure the output gap empirically (Galí-Gertler 1998).
Properties of the IS Curve

- Consumption Euler equation + good market equilibrium

\[ y_t = E_t \{ y_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho) \]

Subtracting at both sides \( y^n_t \) and \( y^n_{t+1} \), with \( \tilde{y}_t \equiv y_t - y^n_t \)

\[ \tilde{y}_t = -\frac{1}{\sigma} \left( i_t - E_t \{ \pi_{t+1} \} - r^n_t \right) + E_t \{ \tilde{y}_{t+1} \} \]

with

\[ r^n_t \equiv \rho + \sigma E_t \{ \Delta y^n_{t+1} \} \]

\[ = \rho + \frac{\sigma(1 + \varphi)}{\sigma + \varphi} E_t \{ \Delta a_{t+1} \} \]
Monetary Policy

- **Taylor Rule**

\[ i_t = \rho + \phi_{\pi_t} \pi_t + \phi_{y_t} \hat{y}_t + \nu_t \]

- Role of monetary aggregates
  - Money Demand (ad hoc):

\[ m_t - p_t = y_t - \eta i_t \]

  - Implications for the growth rate of the money supply:

\[ \Delta m_t = \pi_t + \Delta y_t - \eta \Delta i_t \]
Basic NK model

- **NKPC**

\[ \pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \]

with \( \tilde{y}_t \equiv y_t - y^n_t \) ("output gap").

- **IS Curve**

\[ \tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r^n_t) + E_t\{\tilde{y}_{t+1}\} \]

- **Interest rate rule**

Example:

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \]

- **Exogenous variables**

\[ \nu_t = \rho_\nu \nu_{t-1} + \varepsilon^\nu_t \]

\[ a_t = \rho_a a_{t-1} + \varepsilon^a_t \]
Example with exact solution

- Assume that:
  
  (i) \( \{v_t\} \) and \( \{a_t\} \) are white noise (\( \rho_a = \rho_v = 0 \))
  
  (ii) \( i_t = \rho + \phi_\pi \pi_t + v_t \)
  
  (iii) log. utility in consumption (\( \sigma = 1 \))

\[ \Rightarrow \hat{r}_t^n = -a_t \]

\[ \Rightarrow \hat{y}_t^n = a_t \]

Conjecture ("method of undetermined coefficient"). The solution must be of the form:

\[ \tilde{y}_t = \psi_{ya} a_t + \psi_{yv} v_t \]

\[ \pi_t = \psi_{\pi a} a_t + \psi_{\pi v} v_t \]
Solution:

\[
\tilde{y}_t = \frac{1}{1 + \kappa \phi_{\pi}} a_t - \frac{1}{1 + \kappa \phi_{\pi}} \nu_t
\]

\[
\pi_t = -\frac{\kappa}{1 + \kappa \phi_{\pi}} a_t - \frac{\kappa}{1 + \kappa \phi_{\pi}} \nu_t
\]

\[
\hat{y}_t = \tilde{y}_t + \tilde{y}_t^n = \frac{\kappa \phi_{\pi}}{1 + \kappa \phi_{\pi}} a_t - \frac{1}{1 + \kappa \phi_{\pi}} \nu_t
\]

\[
\hat{n}_t = \hat{y}_t - a_t = -\frac{1}{1 + \kappa \phi_{\pi}} a_t - \frac{1}{1 + \kappa \phi_{\pi}} \nu_t
\]

\[
i_t = \rho - \frac{\kappa \phi_{\pi}}{1 + \kappa \phi_{\pi}} a_t + \frac{1}{1 + \kappa \phi_{\pi}} \nu_t
\]

\[
m_t = \frac{\kappa (\phi_{\pi} (1 + \eta) - 1)}{1 + \kappa \phi_{\pi}} a_t - \frac{1 + \kappa + \eta}{1 + \kappa \phi_{\pi}} \nu_t + p_{t-1}
\]

- Discussion
Model Calibration and Simulation (Galí 2008/ rev2015)

- Calibration:
  \[ \beta = 0.99, \sigma = \phi = 1 \]
  \[ \alpha = 1/3 \]
  \[ \phi_{\pi} = 1.5, \phi_y = 0.5/4 \]
  \[ \theta = 3/4 \]
  \[ \eta = 4 \]
  \[ \rho_v = 0.5, \rho_a = 0.9 \]

- Effects of Monetary Shocks
- Effects of Technology Shocks
Responses to a Monterey Policy Shock: Interest rate rule
Responses to a Technology Shock: Interest rate rule