Advanced Macroeconomics II
Business Cycle and Labor Market

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These slides are heavily based on Jordi Galì slides for Macroeconomia Avanzada II
Walrasian Equilibrium

- Perfect competition implies the following Labor Demand (LD):
  \[ LD : \omega_t = a_t - \alpha n_t = mpn_t \]

- where \( \omega_t \equiv w_t - p_t \) is the real wage.
- The Labor Supply (LD) is
  \[ LS : \omega_t = \sigma c_t + \varphi l_t = mrs_t \]

- where \( l_t \) is the labor force: all the individuals willing to work at the current labor market conditions

Walrasian Equilibrium:

\[ mrs_t = mpn_t \]

\( \iff \) No involuntary unemployment

- The labor force \( l_t = n_t \)
Walrasian Equilibrium

Gali (2013 JEEA)
Unemployment and Labor Market Distortions

- Imperfect Competition or other distortions. The wage schedule

\[ WS : \omega_t = \mu_t^w + \sigma c_t + \varphi n_t \]

where \( \mu_t^w > 0 \) is the wage mark up.

- The labor demand under imperfect competition in the good market

\[ LD : \omega_t = a_t - \alpha n_t - \mu_t^P = mpn_t - \mu_t^P \]

- Competitive Labor supply

\[ \omega_t = \sigma c_t + \varphi l_t \]

- Unemployment rate

\[ u_t \equiv l_t - n_t \]

- The implied relationship between wage markup and unemployment is:

\[ u_t = \frac{1}{\varphi} \mu_t^w \]
Figure 1. The Wage Markup and the Unemployment Rate

\[ w_t - p_t \]

Labor supply 
\( mrs_t \)

\[ \mu_t^w \]

Employment 
Labor force

\[ n_t \]

\[ l_t \]
Example (I): Efficiency wages

- There are a large number of identical competitive firms, $L$. The representative firm production function is:

$$Y_t = F(E_t N_t),$$

- with $F'(E_t N_t) > 0$ and $F''(E_t N_t) < 0$.
- $N_t$ is the number of workers hired by the representative firm;
- $E_t$ is workers effort
- $E_t N_t$ is the Effective Labor
Example (I): Efficiency wages

- Workers effort is a positive function of the real wage, that is.

\[ E_t = E(W_t), \]

and \( E'(W_t) > 0 \). Thus,

\[ Y_t = F(E(W_t)N_t) \]

- There are \( \bar{N} \) identical workers, each one supplies one unit of labor inelastically.
- Then, \( \bar{N} - LN_t = U_t \) are **unemployed workers**
Example (I): Efficiency wages

- The representative firm seeks to maximize its real profit, given by:

\[ \Pi = Y_t - W_t N_t \]

- where \( W_t \) is the real wage paid and \( N_t \) is the amount of labor hired.

- Thus, firm’s maximization problem is

\[ \max_{\{W_t, N_t\}} F\left( E\left( W_t \right) N_t \right) - W_t N_t \]

- Firms first order conditions (FOCs):

\[
\begin{align*}
N & : \quad F'\left( E\left( W_t \right) N_t \right) E\left( W_t \right) - W_t = 0 \\
W_t & : \quad F'\left( E\left( W_t \right) N_t \right) E'\left( W_t \right) N_t - N_t = 0
\end{align*}
\]

- combining the FOCs

\[
\frac{W}{E\left( W \right)} E'\left( W \right) N - N = 0
\]
Example (I): Efficiency wages

- Rearranging, the optimality conditions imply:

\[ E'(W_t) \frac{W_t}{E(W_t)} = 1 \quad \text{(Solow Condition)} \]

- **Solow Condition**: the elasticity of effort with respect to the real wage is 1.

\[ F'(E(W_t)N_t) = \frac{W_t}{E(W_t)} \quad \text{(MPN=MC)} \]

- The marginal productivity of effective labor, \( F'(E(W_t)N_t) \), is equal to its marginal cost \( \frac{W_t}{E(W_t)} \) (cost per unit of effective labor).

\[ \implies \text{Firms hire workers up to the point where MPN=MC} \]
Efficiency wages: interpretation of the Solow Condition

- Consider a firm that wants to minimize the cost per unit of effective labor

\[
\min_{\{W_t\}} \frac{W_t}{E(W_t)}
\]

- Foc with respect to \( W_t \) is

\[
W_t : - \frac{W_t}{(E(W_t))^2} E'(W_t) + \frac{1}{E(W_t)} = 0
\]

- Rearranging, we find the Solow Condition

\[
E'(W_t) \frac{W_t}{E(W_t)} = 1
\]

- Notice: the Solow Condition implies that \( \frac{W_t}{E(W_t)} \) is constant.
Efficiency wages: the determination of the real wage:

Figure: Romer (2006): Advanced Macroeconomics, III Edition, chp. 9.2, Figure 9.1(a)
Example (II): Monopolistic Unions

\[
\max_{W_t} U(C_t, N_t)
\]

on \( U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \), subject to:

\[
C_t = W_t N_t + \Pi_t
\]

\[
N_t = W_t^{-\epsilon_w} Q_t
\]
Example (II): Monopolistic Unions

\[
\max_{W_t} U\left(W_t^{1-\epsilon_w} Q_t + \Pi_t, W_t^{-\epsilon_w} Q_t\right)
\]

\[
U_c(C_t, N_t) (1 - \epsilon_w) W_t^{-\epsilon_w} Q_t - \epsilon_w U_n(C_t, N_t) W_t^{-\epsilon_w - 1} Q_t = 0
\]

\[
W_t = \frac{\epsilon_w}{(1 - \epsilon_w)} \frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = 0 = \frac{\epsilon_w}{(\epsilon_w - 1)} \left[ - \frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} \right]
\]

Optimality condition:

\[
W_t = \frac{\epsilon_w}{\epsilon_w - 1} MRS_t
\]

where we have defined \( MRS_t \equiv -\frac{U_{n,t}}{U_{c,t}} = C_t^\sigma N_t^\varphi \)

Define \( \mu^w \equiv \log \frac{\epsilon_e}{\epsilon_w - 1} \) ("optimal wage markup"). In logs, with \( \omega_t = w_t - p_t \):

\[
\omega_t = \mu^w + \sigma c_t + \varphi n_t
\]

\[
u_t = u = \frac{\mu^w}{\varphi}\]
Example (III): Wage Rigidities

- Desired wage
  \[ \omega_t^* = \mu^w_t + \sigma c_t + \varphi n_t \]

- Effective wage
  \[ \omega_t = \mu^w_t + \sigma c_t + \varphi n_t \]

- Implies that:
  \[ \omega_t - \omega_t^* = \varphi (u_t - u^*) \]

with \( u^* = \frac{\mu^w}{\varphi} \) ("natural rate of unemployment").

- With a wage-norm
  \[ \omega_t = \gamma w_{t-1} + (1 - \gamma) w_t^* \]

Thus,
\[ u_t - u^* = \gamma (u_{t-1} - u^*) - (\gamma / \varphi) \Delta w_t^* \]

\[ \Rightarrow \text{Slow adjustment of the unemployment rate} \]
Example (IV): Hysteresis (Blanchard-Summers)

Labor Demand

\[ \omega_t = a_t - \alpha n_t \]

In this model wages are predetermined, remain fixed for one period and are function of the employment target \( n^*_t \):

\[ \omega_t = E_{t-1}\{a_t\} - \alpha n^*_t \]

Model "insiders/outsiders": \( n^*_t = n_{t-1} \)

\[ \omega_t = E_{t-1}\{a_t\} - \alpha n_{t-1} \]
Example (IV): Hysteresis (Blanchard-Summers)

Thus:

\[ a_t - \alpha n_t = E_{t-1}\{a_t\} - \alpha n_{t-1} \]

or

\[ n_t = n_{t-1} + \frac{1}{\alpha} (a_t - E_{t-1}\{a_t\}) \]

\[ = n_{t-1} + \frac{1}{\alpha} \epsilon^a_t \]

HP: the labor supply is inelastic and equal to \( l \), then the unemployment rate is:

\[ u_t = u_{t-1} - \frac{1}{\alpha} \epsilon^a_t \]

\[ \implies \text{Even temporary shock have permanent effects! A model for Europe?} \]
Search and Matching Model

- Mortensen-Pissarides (RES 94), Pissarides (2000)
- Law of motion of the employment rate

\[ N_{t+1} = (1 - \delta) N_t + H_t \]

where \( \delta \) is the workers separation rate and \( H_t \) are new matches.

- Unemployment rate is

\[ U_t = 1 - N_t \]

- Matching function (usually a Cobb-Douglas)

\[ H_t = M(V_t, U_t) = mV^\alpha U^\beta \]

- Fixed vacancy cost \( k \)
Search and Matching Model

- The probability at which a worker finds a job in period $t$ is (job finding rate):
  \[ \frac{H_t}{U_t} = M \left( \frac{V_t}{U_t}, 1 \right) \equiv \phi(x_t) \]
  where $x_t \equiv V_t / U_t$ is an index of labor market tightness, and $\phi'(x_t) > 0$

- The probability at which a firm fills a vacancy in period $t$ is:
  \[ \frac{H_t}{V_t} = M \left( 1, \frac{U_t}{V_t} \right) \equiv \rho(x_t) \]
  with $\rho'(x_t) < 0$.

- **Beveridge Curve**
  \[
  M(V_t, U_t) = \delta N_t + \Delta N_{t+1} = \delta(1 - U_t) - \Delta U_{t+1}
  \]
  Steady state
  \[ M(V, U) = \delta(1 - U) \]
Search and Matching Model

The Beveridge Curve

United States, December 2000–September 2011

2008-2011 data in red

vacancy rate (percent)

unemployment rate (percent)

Source: Shimer
Search and Matching Model

- The law of motion of employment is:

\[ N_{t+1} = (1 - \delta) N_t + \phi(x_t) U_t \]

\[ U_{t+1} = [1 - \delta - \phi(x_t)] U_t + \delta \]

- Firm’s net surplus of existing worker

\[ S_t^F = A_t - W_t + \beta(1 - \delta) E_t \{ S_{t+1}^F \} \]

\[ = E_t \left\{ \sum_{k=0}^{\infty} (\beta(1 - \delta))^k (A_{t+k} - W_{t+k}) \right\} \]

- **Optimal vacancy posting policy:** cost of posting a vacancy equals the benefits derived from finding a new worker

\[ k = \rho(x_t) \beta E_t \{ S_{t+1}^F \} \]
Search and Matching Frictions

- Steady state
  
  \[ U = \frac{\delta}{\delta + \phi(x)} \]

  \[ S^F = \frac{A - W}{1 - \beta(1 - \delta)} \]

  \[ \frac{k}{\rho(x)} = \frac{\beta(A - W)}{1 - \beta(1 - \delta)} \]

- Implications
  
  \[ \uparrow A \implies \uparrow x \implies \downarrow U \]

  \[ \uparrow W \implies \downarrow x \implies \uparrow U \]

  \[ \uparrow k \implies \downarrow x \implies \uparrow U \]

  \[ \uparrow \delta \implies \downarrow x \implies \uparrow U \]