Exercise 1

*The consumer problem with an alternative utility function.* The representative consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to the following budget constraint

$$C_t + Q_t B_t = W_t N_t + B_{t-1} + D_t \quad \forall t$$

where the utility function is given by

$$U(C_t, N_t) = \left[ \frac{C_t (1 - N_t)^\gamma}{1 - \sigma} \right]^{1-\sigma}$$

1. Determine the optimality conditions of the consumer’s problem.
2. Derive a log-linear approximation of the optimality conditions around a steady state with consumption $C$ and employment $N$.

Exercise 2

*Technology shocks in a RBC model.* Consider a real economy where the consumer’s utility is given by

$$U(C, N) = \ln \left( C - \frac{1}{1 + \varphi} N^{1+\varphi} \right)$$

where $C$ represents consumption and $N$ are the hours of work. *Note:* the utility function is different from the one we normally use in class.

The budget constraint is

$$C = W N + D$$

where $W$ represents the wage and $D$ are corporate profits (which are distributed as dividends). The firms operate under perfect competition with the production function $Y = AN^{1-\alpha}$, where $Y$ is production and $A$ represents the level of technology. All of the production is consumed.

1. Derive the labor supply and demand functions.
2. Determine the equilibrium level of employment $N$ and production $Y$ as a function of the technology parameter $A$ and represent in a graph the impact of a positive technology shock on the labor market.
3. Describe the empirical evidence on the effects of technology shocks on employment and compare it with the predictions of the model. What is the importance of the wealth effect in the model and how does it explain the difference between the theoretical predictions and empirical evidence?
Exercise 3

Labor market and technology shocks. Consider a RBC model without capital or government. The utility function of the representative consumer is

$$U(C_t, N_t) = \ln C_t - \frac{1}{1 + \phi} N_t^{1+\phi}$$

where $C_t$ is consumption and $N_t$ employment. The production function for the representative firm is $Y = AN^{1-\alpha}$, where $Y$ is production and $A$ the technology level. All of the production is consumed ($Y_t = C_t$). The optimality conditions of the consumer’s and firm’s problem are given respectively by (in logarithmic terms and ignoring constants):

$$w_t^s = c_t + \phi n_t^s$$
$$w_t^d = a_t - \alpha n_t^d$$

where $w_t$ represents the logarithm of wages.

1. What is the economic interpretation of (1) and (2)?
2. Determine the logarithmic equilibrium values of employment $N$ and production $Y$.
3. Describe how the economy responds to a positive technology shock ($\uparrow a_t$) using a graph representing the labor market.
4. Compare the predictions of the model with the empirical evidence on the effects of technology shocks on employment.

Exercise 4

Preference shocks in a simple RBC model. Consider a real business cycle model without capital as discussed in class, but where the utility function is given by

$$U(C_t, N_t) = \log C_t - \chi_t N_t^{1+\phi}$$

where $\chi_t = \exp\{\xi_t\}$ is a stochastic parameter that represents shocks to preferences and follows an AR(1)-process:

$$\xi_t = \rho \xi_{t-1} + \varepsilon_t$$

where $\rho \in [0, 1)$ and $\{\varepsilon_t\}$ is white noise.

The technology is given by a production function

$$Y_t = A_t N_t^{1-\alpha}$$

1. Determine the equation that describes labor supply and express it in log-linear form.
2. Determine the equation that describes labor demand and express it in log-linear form.
3. Determine the equilibrium values of production, employment and real wages as a functions of shocks to technology $a_t$ and preferences $\xi_t$.
4. Determine the cyclic behavior of labor productivity and show how it depends on the relative importance of the shocks (hint: determine the sign of the covariance between wages $w_t$ and production $y_t$). Comment on the result given the empirical evidence.
5. Explain using a graph of the labor market how a shock to preferences $\xi_t$ is transmitted to the different variables.
Exercise 5

**Effects of government spending.** Consider an economy where consumer utility is given by

\[ U(C_t, N_t) = C_t - \frac{1}{2} N_t^2 \]

where \( C_t \) represents consumption and \( N_t \) are hours worked. The budget constraint in each period is:

\[ P_tC_t = \tilde{W}_tN_t - \tilde{T}_t \]

where \( P_t \) is the price, \( \tilde{W}_t \) represents the wage and \( \tilde{T}_t \) is the level of lump-sum taxes. The firms operate under perfect competition with a production function \( Y_t = N_t \). The equilibrium condition in the goods market is \( Y_t = C_t + G_t \), where \( G_t \) is government spending.

1. Derive the intratemporal optimality condition for the consumer (labor supply) and the optimality condition for the firm (profit maximisation).

2. Determine how the equilibrium levels of production \( Y_t \) and aggregate consumption \( C_t \) change when there is an increase in government spending \( G_t \). Compare the predictions of the model with empirical evidence.

3. How would your previous answer change if the utility function were instead \( U(C_t, N_t) = \log C_t - \frac{1}{2} N_t^2 \)? What is the economic intuition that explains the difference in the answers?

Exercise 6

**True, False or Uncertain?** Discuss the following statements.

1. An increase in government spending always has an expansionary effect on economic activity, independent of how it is financed.

2. According to real business cycle models, an increase in government spending increases the demand for labor and hence wages.

3. Real business cycle models can explain why the real interest rate increases when inflation does.

4. Empirical evidence on the effects of technological shocks is consistent with Prescott’s model of real business cycles.

5. In real business cycle models fiscal policy is neutral, i.e. changes in government spending have no effect on output or employment.

6. The fact that employment is highly pro cycle shows that technology shocks are the dominant cause of business cycles.

7. In a real business cycle model increased government spending generates higher wages and, as a result, an increase in consumption.

8. Both in theory and in practice a positive technology shocks always generates a rise in employment.

9. Neither theory nor empirical evidence justify allowing the use of fiscal policy as a countercyclical instrument.