Exercise 1

Monetary Policy in the Classical Model. Consider a classic economy with flexible prices and wages and perfect competition in all markets. The representative consumer maximises $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$ where $\beta \equiv \frac{1}{1 + \rho}$, subject to the sequence of budget constraints

$$P_t C_t + B_t = W_t N_t + (1 + i_{t-1})B_{t-1} + D_t$$

for $t = 0, 1, 2, \ldots$. Assume the utility function takes the form

$$U(C_t, N_t) = C_t - \frac{N_t^{1+\varphi}}{1+\varphi}$$

(note: utility is linear in consumption). Additionally, the firms have a production function $Y_t = A_t N_t$. The central bank follows the rule:

$$i_t = \rho + \phi \pi_t$$

where $\phi > 1$.

1. Find the optimality conditions for consumers and firms and express them in log-linear form.

2. Determine the equilibrium level of production and employment in terms of the exogenous variable $a_t \equiv \log A_t$. What is the role of monetary policy in determining these variables?

3. Demonstrate that inflation is constant and equal to zero in equilibrium. Would you have obtained the same result if the utility function were not linear? Why (not)?

Exercise 2

Optimal Monetary Policy and simple rules in the New-Keynesian Model. Consider the model with sticky prices described by the equations:

$$\tilde{y}_t = E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho_t)$$

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + s \tilde{y}_t$$

where $\tilde{y}_t$ represents the output gap, $i_t$ is the nominal interest rate, $\pi_t$ is the inflation level and $\rho_t$ is the discount rate of consumers. Assume that $\{ \rho_t \}$ is a random variable distributed independently over time, with mean $\rho$ and variance $\sigma^2$. We can rewrite $\rho_t$ as $\rho_t = \rho + u_t$ where $u_t$ is a random variable distributed independently over time, with mean 0 and variance $\sigma^2$. 
1. Describe (in words) where equations (1) and (2) come from. What implicit assumption did we make about the natural level of production?

2. Suppose that the central bank follows the simple rule for the interest rate

\[ i_t = \rho + \phi \pi_t \tag{3} \]

Determine the behavior of \( \pi_t \) and \( \tilde{y}_t \) in equilibrium (hint: assume that \( \tilde{y}_t = au_t \) and \( \pi_t = bu_t \), and determine \( a \) and \( b \) using the method of undetermined coefficients, recalling that \( \{u_t\} \) is white noise and therefore \( E_t\{u_{t+1}\} = 0 \).

3. Assume that the central bank's loss function is given by:

\[ \text{var}(\pi_t) + \alpha \text{var}(\tilde{y}_t) \]

and determine its value as a function of \( \phi \).

4. What is the value of \( \phi \) that minimizes the losses? Determine the output gap, the inflation rate and nominal interest rate under the optimal policy.

5. Analyse the equilibrium when the central bank follows the rule

\[ i_t = \rho_t + \phi \pi_t \]

where the coefficient \( \phi \) is finite and compare the results with the previous case. What advantages and disadvantages are there in both cases?

**Exercise 3**

*Monetary Policy in the New-Keynesian model.* Consider the New-Keynesian model as analysed in class and described by the following equations:

\[ x_t = E_t \{x_{t+1}\} - \frac{1}{\sigma}(i_t - E_t \{\pi_{t+1}\} - \rho) \tag{1} \]

\[ \pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t \tag{2} \]

where \( x_t \) represents the output gap (with respect to the level of efficient production), \( i_t \) is the nominal interest rate, \( \pi_t \) is the level of inflation and \( u_t \) is an exogeneous shock. Assume that \( \{u_t\} \) is a random variable distributed independently over time and with mean zero (that is, a white noise process).

1. Describe briefly where equations (1) and (2) come from and give an interpretation for the origin of the shock \( u_t \).

2. Assume the central bank follows the simple interest rate rule

\[ i_t = \rho + \phi \pi_t \tag{3} \]

Determine the behavior of \( \pi_t \) and \( x_t \) in equilibrium (hint: assume that \( x_t = au_t \) and \( \pi_t = bu_t \), and determine \( a \) and \( b \) using the method of undetermined coefficients).

3. Determine the optimal value of the coefficient \( \phi \) when the central bank intents to minimise the loss function

\[ \text{var}(\pi_t) + \alpha \text{var}(x_t) \]

and comment on how this coefficient depends on the parameter \( \alpha \).
Exercise 4

Extreme Monetary Policy and optimality. The losses of consumer welfare are given by:

$$\alpha \text{var}(x_t) + \text{var}(\pi_t)$$

Inflation and the output gap are related by the equation:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t$$

where \(\{u_t\}\) follows an exogenous white noise process with mean zero and variance \(\sigma_u^2\).

1. Explain where the shock \(\{u_t\}\) comes from and what its implications are for monetary policy.
2. Determine the equilibrium behavior of the output gap \(x_t\) when the central bank fully stabilises inflation, which means \(\pi_t = 0\) for every \(t\). Calculate the welfare losses.
3. Determine the equilibrium behavior of inflation \(\pi_t\) when the central bank fully stabilises the output gap, which means \(x_t = 0\) for every \(t\). Calculate the welfare losses.
4. Suppose that the central bank follows an interest rate rule

$$i_t = \rho + \phi \pi_t$$

Given the dynamic IS equation

$$x_t = E_t \{x_{t+1}\} - \frac{1}{\sigma}(i_t - E_t \{\pi_{t+1}\} - \rho)$$

determine the equilibrium behavior of inflation and the output gap (hint: assume the solution takes the form \(\pi_t = \psi \pi u_t\) and \(x_t = \psi x u_t\) and solve using the method of undetermined coefficients)

5. Determine the value of \(\phi\) that minimizes the welfare losses and explain how it depends on the parameter \(\alpha\).
Exercise 5

Preference shocks and Monetary Policy. The representative consumer maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; X_t) \right\}$$

where the utility function is given by

$$U(C_t, N_t; X_t) = X_t \left( \ln C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

where $C_t$ represents consumption, $N_t$ is employment and $X_t$ is an exogenous preference shock. Its logarithm $x_t \equiv \ln X_t$ follows a white noise process with mean zero and variance $\sigma_x^2$.

All of the production is consumed. The labour market is competitive. The production function for firms is given by

$$Y_t = N_t$$

where $Y_t$ represents the production.

Every period a fraction of firms adjust their prices. The resulting New-Keynesian Phillips curve is:

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t$$

where $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation between $t - 1$ and $t$ and $\tilde{y}_t \equiv y_t - y^*_{t-1}$ is the output gap. We assume that when prices are flexible, the firms maintain a constant margin $\mu \equiv \ln M$ (in logarithms) above the marginal cost.

1. Derive the intratemporal optimality condition for the consumer (in log-linear form).
2. Derive the intertemporal optimality condition for the consumer (in log-linear form).
3. Determine the equilibrium level of output under the assumption of flexible prices and check that is constant (ie, not affected by the shock $x_t$).
4. Determine the equilibrium level of output and inflation with sticky prices when the central bank adopts an interest rate rule:

$$i_t = \rho + \phi_\pi \pi_t$$

where $\rho \equiv -\ln \beta$. Hint: use the method of undetermined coefficients.
5. What would be the value of $\phi_\pi$ that maximises consumer welfare? Why do central banks not choose this value in practice?
6. Assume the central bank follows the rule

$$i_t = \rho + \pi^* + \phi_\pi (\pi_t - \pi^*)$$

where $\pi^*$ is the target inflation rate and the values of $\rho$ and $\phi_\pi$ are given (the central bank cannot influence them). Suppose that the shock $x_t$ is normal\(^1\), how do you calculate the minimum value that $\pi^*$ has to take such that the probability of hitting the “Zero Lower Bound” (that is a nominal interest rate smaller than 0) generated by the rule above is smaller than 5%?

\(^1\)That is, $x_t \sim N(\mu_x, \sigma_x^2)$
Exercise 6

True, False or Uncertain? Discuss the following statements.

1. The massive purchase of public debt by the ECB during the crisis has distorted the money supply and has resulted in not reaching the inflation target during the last two years.
2. Central banks should try to minimize the volatility of production and employment.
3. In the classical monetary model equilibrium prices depend only on the current money supply, but not on the expected money supply or on real shocks.
5. A simple Taylor rule achieves the optimal allocation to the extent that the central bank responds with sufficient force to GDP.
6. If monetary policy were neutral we would observe a zero correlation between real interest rates and production.
7. Expectations of future monetary policy can affect inflation and the current production only in the presence of nominal rigidities.
8. The ability of central banks to influence the level of activity requires that expected inflation responds proportionally to changes in the nominal interest rate, so that the real rate is not changed.
9. The observed short-term variations of the real exchange rate are explained mainly by the inflation differential between two countries.
10. Price stability should be, in any circumstance, the central objective of monetary policy.
11. The macroeconomic instability of the 1970s in the United States was due to a policy of too high interest rates by the Federal Reserve.
12. In the New-Keynesian model monetary policy influences the level of activity since the changes in money supply affect household wealth and therefore their level of consumption.
13. When prices are flexible enough, there will be the so-called liquidity effect.
14. Empirical evidence shows that the price level responds quickly to monetary policy shocks. This phenomenon is known as the liquidity effect.
15. In an economy with flexible prices, inflation is constant, regardless of monetary policy.
16. Fluctuations in inflation levels are always suboptimal. It is therefore desirable, that central banks try to stabilize this variable as much as possible.
17. According to the New-Keynesian Phillips curve it is not possible that a country has a high level of inflation and simultaneously experiences a drop in GDP.