Advanced Macroeconomics II
Monetary Models (I): the Classical Monetary Model

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TODAY

- **The classical Monetary Model**: Main Reference: Gali (2008), chp. 1 and 2.
  - Derivation of the competitive equilibrium
  - Log-linearization
  - Model Evaluation
THE CLASSICAL MONETARY MODEL

- A simple microfounded discrete time monetary model, with:
  1. Two agents: Households and Firms
  2. All markets are perfectly competitive
  3. All agents are *price-taker*
  4. In the simple version of the classical monetary model money is considered a unit of account $\Rightarrow$ *cashless economy*
  5. No capital accumulation.
HOUSEHOLDS PROBLEM

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \] with \( 0 < \beta < 1 \), \( t = 0, 1, 2... \) (1)

\[ P_t C_t + Q_t B_t = B_{t-1} + W_t N_t + D_t \]

- \( P_t \) is the price of good
- \( W_t \): nominal wage;
- \( B_t \): quantity of one-period, nominally riskless bonds purchased in \( t \) and maturing in \( t + 1 \).
- Each bond pays 1 unit of money at maturity. Its price is \( Q_t = \frac{1}{1+i_t} = \frac{1}{R_t} \), \( R_t \): gross nominal interest rate
- \( D_t \): profits
HOUSEHOLDS

- Solving the Langrangean:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t, N_t) - \lambda_t \left[ P_tC_t + R_t^{-1}B_t - B_{t-1} - W_tN_t - T_t \right] \right\}
\]

Labor supply:

\[
\frac{W_t}{P_t} = - \frac{U_N(t)}{U_C(t)}
\]

Euler equation:

\[
R_t^{-1} = \beta E_t \left[ \left( \frac{P_t}{P_{t+1}} \right) \frac{U_C(t+1)}{U_C(t)} \right]
\]
The period utility is:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi}$$

then optimality conditions imply

$$\frac{W_t}{P_t} = N_t^\varphi C_t^\sigma$$  \hspace{1cm} (2)

$$R_t^{-1} = \beta E_t \left[ \left( \frac{1}{\Pi_{t+1}} \right) \left( \frac{C_{t+1}^{-\sigma}}{C_t^\sigma} \right) \right]$$  \hspace{1cm} (3)

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$.
FIRMS

- We abstract from capital accumulation and assume the following DRS technology:

\[ Y_t = A_t N_t^{1-\alpha}. \]  \hspace{1cm} (4)

where \( \ln(A_t/A) = a_t \) is an exogenous process of technological progress (or total factor productivity TFP), which evolves according to:

\[ a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim WN(0, \sigma_a^2) \quad i.i.d. \]
FIRMS PROFITS MAXIMIZATION

\[ \max_{N_t} D_t = P_t Y_t - W_t N_t \]

s.t. \( Y_t = A_t N_t^{1-\alpha} \)

Foc:

\[ \frac{W_t}{P_t} = (1 - \alpha) N_t^{-\alpha} A_t \] \hspace{1cm} (5)

or marginal costs = Price, i.e. \( \frac{W_t}{(1-\alpha) N_t^{-\alpha} A_t} = P_t \). Notice:

\[ CT_t = W_t N_t (Y_t) = W_t \left( \frac{Y_t}{A_t} \right)^{1-\alpha} \]

then the marginal costs \( \Psi_t \) is:

\[ \Psi_t = \frac{dCT_t}{dY_t} = \frac{W_t Y_t^{\frac{\alpha}{1-\alpha}}}{(1 - \alpha) A_t^{\frac{1}{1-\alpha}}} = \frac{W_t Y_t^{\frac{\alpha}{1-\alpha}}}{(1 - \alpha) A_t^{\frac{\alpha}{1-\alpha}} A_t} = \frac{W_t}{(1 - \alpha) N_t^{-\alpha} A_t} \]
MARKET CLEARINGS

\[ C_t = Y_t \]  \hspace{1cm} (6)

\[ N^s = N^d \]
\[ N_t^q C_t^\sigma = (1 - \alpha) N_t^{-\alpha} A_t \]  \hspace{1cm} (7)

\[ B_t = 0 \]  \hspace{1cm} (8)

\[ R_t^{-1} = \beta E_t \left[ \left( \frac{1}{\Pi_{t+1}} \right) \left( \frac{C_{t+1}^{-\sigma}}{C_t^\sigma} \right) \right] \]  \hspace{1cm} (9)
Steady state I

Euler \[ \Pi = 1 \implies R = \frac{1}{\beta} \]

RC \[ C = Y = AN^{(1-\alpha)} \implies N = \left( \frac{Y}{A} \right)^{\frac{1}{1-\alpha}} \]

LS \[ \frac{W}{P} = N^\varphi Y^\sigma = \left( \frac{Y}{A} \right)^{\frac{\varphi}{1-\alpha}} Y^\sigma = \frac{Y^{\frac{\varphi+(1-\alpha)\sigma}{1-\alpha}}}{A^{\frac{\varphi}{1-\alpha}}} \]

LD \[ \frac{W}{P} = (1-\alpha) AN^{-\alpha} = (1-\alpha) A \frac{Y^{-\frac{\alpha}{1-\alpha}}}{A^{-\frac{\alpha}{1-\alpha}}} \]
Steady State II
From Labor market equilibrium

\[
\frac{Y^{\frac{\phi+(1-\alpha)\sigma}{1-\alpha}}}{A^{\frac{\phi}{1-\alpha}}} = (1 - \alpha) \frac{Y^{-\frac{\alpha}{1-\alpha}}}{A^{-\frac{\alpha}{1-\alpha}}}
\]

then

\[
Y = (1 - \alpha) \frac{1-\alpha}{\phi+(1-\alpha)\sigma+\alpha} A^{\frac{\phi+(1-\alpha)}{\phi+(1-\alpha)\sigma+\alpha}} = (1 - \alpha) \frac{1-\alpha}{\phi+(1-\alpha)\sigma+\alpha}
\]

NOTE with \( A = 1 \). Then

\[
C = Y = (1 - \alpha) \frac{1-\alpha}{\phi+(1-\alpha)\sigma+\alpha}
\]

\[
N = (1 - \alpha) \frac{1}{(\phi+(1-\alpha)\sigma+\alpha)}
\]

\[
\frac{W}{P} = (1 - \alpha) \frac{\phi+(1-\alpha)\sigma}{\phi+(1-\alpha)\sigma+\alpha}
\]

RESULT 1: in the long-run, real variables do not depend on monetary variables \( \implies \) MONEY IS NEUTRAL in the LONG-RUN!!
The Log-linearized model

Households labor supply (2) in log-terms: $\hat{c}_t \hat{n}_t$

$$\hat{w}_t - \hat{p}_t = \sigma \hat{c}_t + \varphi \hat{n}_t$$ \hspace{1cm} (10)

Log-linearizing the Euler equation (3) around the steady state:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma} (\hat{i}_t - E_t \pi_{t+1})$$ \hspace{1cm} (11)

with $\hat{i}_t = \ln \left( \frac{R_t}{\check{R}} \right)$. Firms labor demand (5) in log-terms:

$$\hat{w}_t - \hat{p}_t = a_t - \alpha \hat{n}_t$$ \hspace{1cm} (12)

Firms production function (4) in log-terms:

$$\hat{y}_t = a_t + (1 - \alpha) \hat{n}_t$$ \hspace{1cm} (13)
**Equilibrium I (in log-deviations from the SS)**

1. The feasibility constraint

\[ \hat{c}_t = \hat{y}_t \]  

2. Labor Market equilibrium and (14)

\[ \sigma \hat{y}_t + \phi \hat{n}_t = a_t - \alpha \hat{n}_t \]  

3. Assets

\[ \hat{b}_t = 0 \]  

4. Interest rate

\[ \hat{r}_t = \sigma E_t \{ \Delta \hat{y}_{t+1} \} \]  

Remember that in log terms:

\[ r_t = \rho + \sigma E_t \{ \Delta y_{t+1} \} \]

5. Production Function

\[ \hat{y}_t = a_t + (1 - \alpha) \hat{n}_t \]
Equilibrium I

The equilibrium level of output using (13) and solving for $\hat{y}_t$

$$\hat{y}_t = \frac{1 + \phi}{\sigma (1 - \alpha) + \varphi + \alpha} a_t$$  \hfill (18)

Notice, with CRS, i.e. with $\alpha = 0$, then

$$\hat{y}_t = \frac{1 + \phi}{\sigma + \phi} a_t$$  \hfill (19)
Equilibrium II

The equilibrium level of employment
From the production function:

\[ \hat{n}_t = \frac{(1 - \sigma)}{(\sigma (1 - \alpha) + \varphi + \alpha)} a_t \] (20)

The equilibrium level of the real interest rate
Define \( \hat{r}_t \equiv \hat{i}_t - E_t \pi_{t+1} \).
From the log-linear Euler Equation + (14):

\[ \hat{r}_t = \sigma E_t \{ \Delta \hat{y}_{t+1} \} = \sigma \psi_{ya} E_t \{ \Delta a_{t+1} \}, \] (21)

with \( \psi_{ya} = \frac{1 + \varphi}{\sigma (1 - \alpha) + \varphi + \alpha} \).

The equilibrium level of real wages
Define \( \hat{\omega}_t \equiv \hat{\omega}_t - \hat{p}_t \). From the labor demand:

\[ \hat{\omega}_t = \frac{\sigma + \varphi}{\sigma (1 - \alpha) + \varphi + \alpha} a_t. \] (22)
RESULT II: in the short-run, real variables do not depend on monetary variables $\iff$ MONEY IS NEUTRAL also in the SHORT-RUN!!

What happens to nominal variables?

Nominal variables are: $\hat{i}_t, \hat{p}_t, \hat{m}_t, \hat{w}_t$
What happens to the nominal variables?
Remember the Fisher equation (in logs):

\[ i_t = E_t \pi_{t+1} + r_t \]  \hspace{1cm} (23)

In order to find their equilibrium behavior we need to specify the behavior of the monetary authority.

1) Exogenous interest rate rule: \( i_t = \rho \). Then

\[ E_t \pi_{t+1} = \rho - r_t \]  \hspace{1cm} (24)

Expected inflation is determined by \( \rho \) and by \( \hat{r}_t \) which is a function of \( a_t \) (real variable) and is independent from monetary policy. Actual inflation is:

\[ p_{t+1} - p_t = \rho - r_t - \zeta_{t+1} \]  \hspace{1cm} (25)

where \( \zeta_{t+1} \) is a sunspot shock. The price level may be caused by non-fundamental factors and therefore there is price level indeterminacy. The money supply and nominal wages are indeterminate. \( \implies \) The equilibrium is indeterminate (multipla equilibria).
2) A Simple feedback rule:

\[ \hat{r}_t = \phi_\pi \pi_t \]

substituting in the Fisher equation:

\[ \pi_t = \frac{1}{\phi_\pi} E_t \pi_{t+1} + \frac{1}{\phi_\pi} \hat{r}_t \]

(26)

with \( \hat{r}_t \equiv r_t - \rho \). If \( \phi_\pi > 1 \) then \( \mu_\pi = \frac{1}{\phi_\pi} < 1 \). Iterating forward (26) and excluding bubbles:

\[ \pi_t = \sum_{i=0}^{\infty} \mu_\pi^{i+1} E_t \{ \hat{r}_{t+i} \} \]

Remember: \( \hat{r}_t = -\sigma \psi_{ya} (1 - \rho_a) a_t \implies E_t \{ \hat{r}_{t+i} \} = -\sigma \psi_{ya} (1 - \rho_a) \rho_a^i a_t \), and

\[ \pi_t = -\sigma \psi_{ya} (1 - \rho_a) \sum_{i=0}^{\infty} \mu_\pi (\mu_\pi \rho_a)^i a_t \]

\[ = - \frac{\sigma \psi_{ya} (1 - \rho_a)}{\phi_\pi - \rho_a} a_t \]

The equilibrium is unique.
GENERALIZING
Consider the **Interest Rate Rule**:

\[ i_t = \rho + \phi_\pi \pi_t + \nu_t \]

with \{\nu_t\} ∼ \text{AR}(1) exogenous monetary policy shock

\[ \nu_t = \rho_v \nu_{t-1} + \varepsilon_t^\nu \]

**Fisher Equation**: \( i_t = r_t + E_t\{\pi_{t+1}\} \).

Combining the two equations and defining as usual \( \hat{r}_t \equiv r_t - \rho \):

\[ \phi_\pi \pi_t = E_t\{\pi_{t+1}\} + \hat{r}_t - \nu_t \]

Assuming again that \( \phi_\pi > 1 \), the solution for \( \pi_t \) now is:

\[
\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\hat{r}_{t+k} - \nu_{t+k}\}
\]

\[ = -\frac{\psi_r}{\phi_\pi - \rho_a} a_t - \frac{1}{\phi_\pi - \rho_v} \nu_t \]

A positive monetary shock, which increases the nominal interest rate will decrease inflation.
3) An exogenous money supply

- If the money supply is exogenous, then the nominal interest rate is endogenous. As:

\[ \Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \]  
(27)

- To eliminate \( i_t \) we need to specify a money demand. As:

\[ m_t - p_t = y_t - \eta i_t \]  
(28)

- Combining together (28) and the Fisher equation

\[ \rho_t = \frac{\eta}{1+\eta} E_t \rho_{t+1} + \frac{1}{1+\eta} m_t + u_t \]  
(29)

where \( u_t = \frac{1}{1+\eta} (\eta r_t - y_t) \).
3) An exogenous money supply II

Iterating forward (29):

\[
p_t = \frac{1}{1 + \eta} \sum_{i=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^i m_{t+i} + \sum_{i=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^i u_{t+i}
\]

\[
= m_t + \sum_{i=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^i \Delta m_{t+i} - \Delta m_t + ...
\]

(30)

\[
\Delta m_{t+i} = \rho^i m \Delta m_t \implies p_t = m_t + \sum_{i=0}^{\infty} \left( \frac{\eta \rho_m}{1 + \eta} \right)^i \Delta m_t - \Delta m_t + ...
\]

\[
p_t = m_t + \frac{\eta \rho_m}{1 + \eta (1 - \rho_m)} \Delta m_t + ...
\]

\[
= m_{t-1} + \left( 1 + \frac{\eta \rho_m}{1 + \eta (1 - \rho_m)} \right) \Delta m_t + ...
\]

\[
\frac{\partial p_t}{\partial \Delta m_t} = 1 + \frac{\eta \rho_m}{1 + \eta (1 - \rho_m)} > 1.
\]
3) An exogenous money supply III

From the money demand:

\[ i_t = \frac{1}{\eta} y_t - \frac{1}{\eta} (m_t - \rho_t) \]

\[ = \frac{\rho_m}{1 + \eta (1 - \rho_m)} \Delta m_t + \frac{1}{\eta} y_t + ... \]

then \( \frac{\partial i_t}{\partial \Delta m_t} = \frac{\rho_m}{1 + \eta (1 - \rho_m)} > 0 \) An increase in money supply is followed by an increase in the nominal interest rate \( \implies \text{NO LIQUIDITY EFFECT!!} \)
MAIN RESULTS

- Real variables do not depend on monetary policy.
- Money is neutral both in the long and in the short-run.
- The price level responds more than one for one with the increase in the money supply.
- **ABSENCE of a LIQUIDITY EFFECT!!!**
EMPIRICAL EVIDENCE: CEE (1999)

IRFS to an EXOGENOUS TIGHTENING of MONETARY POLICY