

Technical Appendix to “Trend Inflation, Taylor Principle and Indeterminacy”

Guido Ascari

University of Pavia

Tiziano Ropele

Bank of Italy

June 9, 2007

A Derivation of the NKPC

In this section we show the steps to derive the generalized new Keynesian Phillips curve. For this purpose we need two structural equations, namely the one for the optimal price chosen by adjusting firms, equation (1), and the aggregate price dynamics expression under the Calvo mechanism, equation (2):

$$P_{i,t}^* = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ P_{t+j}^\theta Y_{t+j} \Gamma'_{t+j} (\bar{\pi}^{-\theta \varepsilon j})^{1-\mathfrak{S}} \left(\Pi_{t,t+j-1}^{-\theta \varepsilon} \right)^{\mathfrak{S}} \right\}}{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ P_{t+j}^{\theta-1} Y_{t+j} [\bar{\pi}^{(1-\theta)\varepsilon j}]^{1-\mathfrak{S}} \left[\Pi_{t,t+j-1}^{(1-\theta)\varepsilon} \right]^{\mathfrak{S}} \right\}}, \quad (1)$$

and

$$P_t = \left\{ \alpha \left[\bar{\pi}^{(1-\theta)\varepsilon} \right]^{1-\mathfrak{S}} \left[\pi_{t-1}^{(1-\theta)\varepsilon} \right]^{\mathfrak{S}} P_{t-1}^{1-\theta} + (1-\alpha) (P_{i,t}^*)^{1-\theta} \right\}^{\frac{1}{1-\theta}}, \quad (2)$$

where $\mathcal{D}_{t,t+j} \equiv \beta^j \frac{Y_t}{Y_{t+j}}$ is the stochastic discount factor, Γ'_t is the real marginal cost function, $\bar{\pi}$ is the steady state (gross) inflation rate. Moreover, $\Pi_{t,t+j-1}$ represents the cumulative gross inflation rate and is given by

$$\Pi_{t,t+j-1} = \begin{cases} \left(\frac{P_t}{P_{t-1}} \right) \left(\frac{P_{t+1}}{P_t} \right) \times \dots \times \left(\frac{P_{t+j-1}}{P_{t+j-2}} \right) & \text{for } j = 1, 2, \dots \\ 1 & \text{for } j = 0. \end{cases} \quad (3)$$

Recall from main text that $\mathfrak{S} \in [0, 1]$ and $\varepsilon \in [0, 1]$ where the first parameter refers to the type of assumed indexation and the second to the degree of indexation. $\varepsilon = 0$ yields the case of no price indexation. Finally note that $\Gamma'_{t+j} = w_{t+j}$, given the simple linear production function of intermediate goods producers.

As we deal with a deterministic steady state characterized by positive trend inflation, whereby nominal variables grow at the same rate, we firstly transform all nominal variables so to make them stationary. In particular, divide both sides of (1) and (2) by P_t and rearrange to obtain:

$$p_{i,t}^* = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ \Pi_{t+1,t+j}^{\theta} Y_{t+j} w_{t+j} (\bar{\pi}^{-\theta \varepsilon j})^{1-\mathfrak{S}} \left(\Pi_{t,t+j-1}^{-\theta \varepsilon} \right)^{\mathfrak{S}} \right\}}{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ \Pi_{t+1,t+j}^{\theta-1} Y_{t+j} [\bar{\pi}^{(1-\theta)\varepsilon j}]^{1-\mathfrak{S}} \left[\Pi_{t,t+j-1}^{(1-\theta)\varepsilon} \right]^{\mathfrak{S}} \right\}}, \quad (4)$$

and

$$1 = \left\{ \alpha \left[\bar{\pi}^{(1-\theta)\varepsilon} \right]^{1-\mathfrak{S}} \left[\pi_{t-1}^{(1-\theta)\varepsilon} \right]^{\mathfrak{S}} \pi_t^{\theta-1} + (1-\alpha) (p_{i,t}^*)^{1-\theta} \right\}^{\frac{1}{1-\theta}}, \quad (5)$$

where $p_{i,t}^* = P_{i,t}^*/P_t$.

STEP 1. Quasi-differentiate the optimal relative price. Write the optimal relative price in the following format

$$\frac{P_{i,t}^*}{P_t} = \left(\frac{\theta}{\theta - 1} \right) \frac{\psi_t}{\phi_t}, \quad (6)$$

where

$$\psi_t \equiv E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \Pi_{t+1,t+j}^{\theta} w_{t+j} \left(\bar{\pi}^{-\theta \varepsilon j} \right)^{1-\mathfrak{S}} \left(\Pi_{t,t+j-1}^{-\theta \varepsilon} \right)^{\mathfrak{S}}, \quad (7)$$

$$\phi_t \equiv E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \Pi_{t+1,t+j}^{\theta-1} \left[\bar{\pi}^{(1-\theta)\varepsilon j} \right]^{1-\mathfrak{S}} \left[\Pi_{t,t+j-1}^{(1-\theta)\varepsilon} \right]^{\mathfrak{S}}. \quad (8)$$

Notice that in defining these two auxiliary variables, i.e. ψ_t and ϕ_t , we used the definition of the stochastic discount factor (defined above). Interestingly, we can do away with the infinite summations in (7) and (8) and rewrite the two expressions *recursively*.

Take for instance equation (7) and write it extensively as:

$$\begin{aligned} \psi_t \equiv & w_t + \alpha\beta E_t \left[\Pi_{t+1,t+1}^{\theta} w_{t+1} \left(\bar{\pi}^{-\theta \varepsilon} \right)^{1-\mathfrak{S}} \left(\Pi_{t,t}^{-\theta \varepsilon} \right)^{\mathfrak{S}} \right] + \\ & + E_t \left[(\alpha\beta)^2 \Pi_{t+1,t+2}^{\theta} w_{t+2} \left(\bar{\pi}^{-2\theta \varepsilon} \right)^{1-\mathfrak{S}} \left(\Pi_{t,t+1}^{-\theta \varepsilon} \right)^{\mathfrak{S}} \right] + \dots \end{aligned}$$

The above expression can be re-adjusted as

$$\begin{aligned} \psi_t \equiv & w_t + \alpha\beta E_t \left\{ \Pi_{t+1,t+1}^{\theta} \left(\bar{\pi}^{-\theta \varepsilon} \right)^{1-\mathfrak{S}} \left(\Pi_{t,t}^{-\theta \varepsilon} \right)^{\mathfrak{S}} \times \right. \\ & \left. \times \left[w_{t+1} + \alpha\beta \Pi_{t+2,t+2}^{\theta} w_{t+2} \left(\bar{\pi}^{-\theta \varepsilon} \right)^{1-\mathfrak{S}} \left(\Pi_{t+1,t+1}^{-\theta \varepsilon} \right)^{\mathfrak{S}} + \dots \right] \right\} \end{aligned}$$

Notice that the expression the square brackets is exactly the definition for $E_t\psi_{t+1}$, thus

$$\psi_t \equiv w_t + \alpha\beta \left(\bar{\pi}^{-\theta\varepsilon}\right)^{1-\mathfrak{S}} \left(\pi_t^{-\theta\varepsilon}\right)^{\mathfrak{S}} E_t \left[\pi_{t+1}^{\theta}\psi_{t+1}\right]. \quad (9)$$

Following the same procedure, it is possible to quasi-differentiate equation (8) and obtain:

$$\phi_t \equiv 1 + \alpha\beta \left[\bar{\pi}^{(1-\theta)\varepsilon}\right]^{1-\mathfrak{S}} \left[\pi_t^{(1-\theta)\varepsilon}\right]^{\mathfrak{S}} E_t \left[\pi_{t+1}^{\theta-1}\phi_{t+1}\right] \quad (10)$$

Note that the household first order condition on labor supply: $\chi_n N_t^{\sigma_n} C_t = w_t$ and $Y_t s_t = \int_0^1 N_{i,t} di = N_t$, yield the following expression for the real marginal cost $\Gamma'_t = w_t = \chi_n Y_t^{1+\sigma_n} s_t^{\sigma_n}$.

STEP 2. Deterministic steady state. Next, we evaluate equations (6), (9), (10) and (2) at the deterministic steady state characterized by positive trend inflation equal to $\bar{\pi}$:

$$p_i^* = \left(\frac{\theta}{\theta-1}\right) \frac{\psi}{\phi}, \quad (11)$$

$$\psi \equiv \frac{w}{1 - \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)}}, \quad (12)$$

$$\phi \equiv \frac{1}{1 - \alpha\beta\bar{\pi}^{(\theta-1)(1-\varepsilon)}}, \quad (13)$$

$$1 = \left[\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)} + (1-\alpha)(p_i^*)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \quad (14)$$

Moreover, we can write $w = \chi_n Y^{1+\sigma_n} s^{\sigma_n}$ (the steady state for the relative price dispersion is defined in the next section).

STEP 3. Log linear approximation around the deterministic steady state.

We now take a (log) linear approximation to equations (6), (9), (10) and (2) around the deterministic steady state defined in STEP 2.

Log-linearization of equation (6):

$$\widehat{p}_{i,t}^* = \widehat{\psi}_t - \widehat{\phi}_t. \quad (15)$$

Log-linearization of equation (7):

$$\widehat{\psi}_t \equiv \left[1 - \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)}\right] \widehat{\Gamma}'_t + \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)} \left(\theta E_t \Delta_{t+1} + E_t \widehat{\psi}_{t+1}\right), \quad (16)$$

where $\Delta_{t+1} \equiv \widehat{\pi}_{t+1} - \mathfrak{S}(\varepsilon\widehat{\pi}_t)$.

Log-linearization with respect to equation (10):

$$\widehat{\phi}_t \equiv \alpha\beta\bar{\pi}^{(\theta-1)(1-\varepsilon)} \left[(\theta-1) E_t \Delta_{t+1} + E_t \widehat{\phi}_{t+1} \right]. \quad (17)$$

Notice that equation (17) exactly corresponds to the second equation that appears in the system of the generalized New Keynesian Phillips curve.

Log-linearization of equation (2):

$$\widehat{p}_{i,t}^* = \frac{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} \Delta_t. \quad (18)$$

STEP 4. Reduction of the system (final step). In this final step we rewrite equations (15), (16), (17) and (18) as a system of only two expectational difference equations.

Combing equations (15) and (18) yields an expression for $\widehat{\psi}_t$, i.e.

$$\widehat{\psi}_t = \widehat{\phi}_t + \frac{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} \Delta_t. \quad (19)$$

Next, we substitute the latter expression in (16) to obtain:

$$\widehat{\phi}_t = \left[1 - \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)} \right] \widehat{\Gamma}'_t - \frac{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} \Delta_t \quad (20)$$

$$+ \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)} \left[\theta E_t \Delta_{t+1} + E_t \widehat{\phi}_{t+1} + \frac{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} E_t \Delta_{t+1} \right] \quad (21)$$

In the above expression, we substitute out for $\widehat{\phi}_t$, using its definition given by expression (17),

$$\begin{aligned} \Delta_t &= \frac{\alpha\beta\bar{\pi}^{\theta(1-\varepsilon)}}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} E_t \Delta_{t+1} + \frac{[1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}] [1 - \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)}]}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} \widehat{\Gamma}'_t \\ &+ \frac{[\alpha\beta\bar{\pi} - \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)}] [1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}]}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} \left[(\theta-1) E_t \Delta_{t+1} + E_t \widehat{\phi}_{t+1} \right]. \end{aligned}$$

Collecting terms, then yields

$$\begin{aligned} \Delta_t &= \beta\bar{\pi}^{1-\varepsilon} E_t \Delta_{t+1} + \frac{[1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}] [1 - \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)}]}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} \widehat{\Gamma}'_t \\ &+ \beta (\bar{\pi}^{1-\varepsilon} - 1) \left[1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)} \right] \left[(\theta-1) E_t \Delta_{t+1} + E_t \widehat{\phi}_{t+1} \right]. \end{aligned}$$

Using the fact that $\widehat{\Gamma}'_t = \widehat{w}_t = (1 + \sigma_n) \widehat{Y}_t + \sigma_n \widehat{s}_t$, then the above expression coincides with the first equation that describes the generalized New Keynesian Phillips curve, for which

we also defined $\kappa \equiv \frac{[1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}] [1 - \alpha\beta\bar{\pi}^{\theta(1-\varepsilon)}]}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}} (1 + \sigma_n)$ and $\eta \equiv \beta (\bar{\pi}^{1-\varepsilon} - 1) [1 - \alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}]$.

We still need to derive the approximated expression for the relative price dispersion. This is what we move to in the next section.

B The relative price dispersion measure

In the paper we define the relative price dispersion measure as:

$$s_t = \int_0^1 (P_{i,t}/P_t)^{-\theta} di. \quad (22)$$

Under the Calvo price mechanism, the above expression can be rewritten as:

$$\begin{aligned} s_t = & (1 - \alpha) \left[\frac{P_{i,t}^*}{P_t} \right]^{-\theta} + \alpha(1 - \alpha) \left[\frac{P_{i,t-1}^* (\bar{\pi}^\varepsilon)^{1-\mathfrak{S}} (\pi_{t-1}^\varepsilon)^{\mathfrak{S}}}{P_t} \right]^{-\theta} + \\ & + \alpha^2(1 - \alpha) \left[\frac{P_{i,t-2}^* (\bar{\pi}^{2\varepsilon})^{1-\mathfrak{S}} (\pi_{t-1}^\varepsilon \pi_{t-2}^\varepsilon)^{\mathfrak{S}}}{P_t} \right]^{-\theta} + \dots \end{aligned}$$

Collecting terms then yields:

$$\begin{aligned} s_t = & (1 - \alpha) \left[\frac{P_{i,t}^*}{P_t} \right]^{-\theta} + \alpha (\bar{\pi}^{-\theta\varepsilon})^{1-\mathfrak{S}} (\pi_{t-1}^{-\theta\varepsilon})^{\mathfrak{S}} \pi_t^\theta \times \\ & \times \left\{ (1 - \alpha) \left[\frac{P_{i,t-1}^*}{P_{t-1}} \right]^{-\theta} + \alpha^2(1 - \alpha) \left[\frac{P_{i,t-2}^* (\bar{\pi}^\varepsilon)^{1-\mathfrak{S}} (\pi_{t-2}^\varepsilon)^{\mathfrak{S}}}{P_{t-1}} \right]^{-\theta} + \dots \right\}. \end{aligned}$$

Notice that the expression in the curly brackets is exactly the definition of s_{t-1} . Thus, it follows that the equation for s_t can be written recursively as:

$$s_t = (1 - \alpha) \left[\frac{P_{i,t}^*}{P_t} \right]^{-\theta} + \alpha (\bar{\pi}^{-\theta\varepsilon})^{1-\mathfrak{S}} (\pi_{t-1}^{-\theta\varepsilon})^{\mathfrak{S}} \pi_t^\theta s_{t-1}. \quad (23)$$

Likewise we did for the optimal relative price, we now take a log-linear approximation to (23) around the deterministic steady state, yielding:

$$\hat{s}_t = \xi \Delta_t + \alpha \bar{\pi}^{\theta(1-\varepsilon)} \hat{s}_{t-1} \quad (24)$$

where $\xi = \theta \frac{\alpha \bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1 - \alpha \bar{\pi}^{(\theta-1)(1-\varepsilon)}} (\bar{\pi}^{1-\varepsilon} - 1)$ and we used the fact that $\hat{p}_{i,t}^* = \frac{\alpha \bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1 - \alpha \bar{\pi}^{(\theta-1)(1-\varepsilon)}} \Delta_t$.