Trend Inflation, Taylor Principle and Indeterminacy*

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Abstract

Positive trend inflation shrinks the determinacy region of a basic new Keynesian DSGE model when monetary policy is conducted by a contemporaneous interest rate rule. Neither the Taylor principle, which requires the inflation coefficient to be greater than one, nor the generalized Taylor principle, which requires that the nominal interest rate should be raised by more than the increase in inflation in the long run, is a sufficient condition for local determinacy of equilibrium.

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This finding holds for different types of Taylor rules, inertial policy rules and price indexation schemes. Therefore, regardless of the theoretical set up, the monetary literature on interest rate rules cannot disregard average inflation in both theoretical and empirical analysis.

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1 Introduction

Average inflation in the post-war period in developed countries was moderately different from zero and varied across countries.[1] Nonetheless, much of the extensive literature on monetary policy rules employed models approximated around the zero inflation steady state (see e.g., Clarida et al. 1999; Galí 2003; Woodford 2003; or the book edited by Taylor 1999).

In this article we address this inconsistency by extending the basic small scale new Keynesian DSGE model to allow for positive trend inflation.[2] We add a Taylor rule to describe the monetary authority's behaviour and then examine to what extent the properties of the model economy change as trend inflation varies. We show that *even moderate* levels of trend inflation greatly modify the conditions under which the rational expectations equilibrium is determinate or unique.

Trend inflation has substantial effects on the well-known Taylor principle for the determinacy of the rational expectations equilibrium. This result is driven by the steady state relative price distortions induced by trend inflation in the staggered adjustment mechanism a la Calvo (1983). As shown by Ascari (2004) and Yun (2005), the steady state relation between output and inflation is highly nonlinear. The long-run Phillips curve is positively sloped around the zero inflation steady state; however, as soon as trend inflation takes up even moderate positive values, the long-run Phillips curve inverts and becomes negatively sloped reflecting the relative price distortions. In other words, the higher the trend inflation the lower the level of output in steady state. In this article, we demonstrate that this feature of the model has remarkable implications for the celebrated

Taylor principle. Therefore, a natural suspicion arises that many of the results in the literature are drawn from a case, namely the zero inflation steady state, which is both empirically unrealistic and theoretically special.

Our key result is generalized and proved to be qualitatively robust to a number of checks: (a) different types of Taylor rules (contemporaneous, backward-looking, forward-looking and hybrid nominal interest rate rules, see e.g., Clarida et al. 2000; Bullard and Mitra 2002); (b) inertial Taylor rules for all the cases listed in (a); (c) different price indexation schemes (see, e.g., Yun 1996 and Christiano et al. 2005); and (d) different calibration values.

In a nutshell, research in the field of monetary policy cannot neglect trend inflation, as both the theoretical model and determinacy properties of Taylor rules are sensitive to low and moderate levels of positive trend inflation, as generally observed in western countries.

The seminal analysis in Clarida et al. (2000) can be taken as an example. Clarida et al. (2000) were the first to estimate a Taylor rule on US data. They found that during the pre-Volcker period the nominal interest rate reacted less than one-to-one in response to variations in inflation, while afterwards the response of the policy rate was more than proportional. Strictly speaking, US monetary policy did not satisfy the Taylor principle in the first sub-sample, while it did in the second one. Thus, Clarida et al. (2000) interpreted this evidence as responsible for inflation getting out of control in the Seventies, while getting back on track later. However, the data set used in Clarida et al. (2000) features an average inflation of roughly 4 per cent (see Table II, p. 157, therein). Yet, their analysis is based on a theoretical model that assumes zero trend inflation. When appropriately taken into account, positive trend inflation substantially changes the model structural equations and the determinacy region, so that one needs to account for trend inflation in order to label the rational expectations equilibrium as determinate. Indeed, using our benchmark parameters calibration in the standard new Keynesian DSGE model, the Clarida et al. (2000) baseline estimates of the Taylor rule coefficients would deliver indeterminacy in the pre-Volcker as well as in the Volcker-Greenspan sample period.

Few articles in the literature investigate the effects of different levels of trend inflation in the standard new Keynesian model.[3] King and Wolman (1996) and Ascari (1998) are early papers that look at the effects of trend inflation on the properties of the steady state. Following these contributions, Karanassou et al. (2005) studies the long-run relationship between inflation and output in the New Keynesian framework, from both a theoretical and an empirical perspective. Ascari (2004) examines, instead, the effects of trend inflation on the dynamics of the standard new Keynesian model both with Calvo (1983) and Taylor (1979) price setting specification. Ascari (2004), however, assumes an autoregressive process for the money supply and thus the issue of determinacy under different policy rules remains unexplored. Amano et al. (2007) studies how the business cycle characteristics of the model (i.e., persistence, correlations, and volatilities) vary with trend inflation. Ascari and Ropele (2007) analyzes how optimal short-run monetary policy changes with trend inflation. Cogley and Sbordone (2005) estimates the New Keynesian Phillips Curve (NKPC, henceforth) allowing for trend inflation. The key finding by Cogley and Sbordone (2008) is that once shifts in trend inflation are properly taken into account, the NKPC is structural. In other words, a Calvo pricing model with constant parameters fits the data very well with no need for indexation or a backward-looking component.

Finally, Kiley (2007) is a paper closely related to ours. Kiley (2007) investigates how trend inflation influences the determinacy region and the unconditional variance of inflation in a model where prices are staggered a la Taylor (1979) and monetary policy is described by a Taylor rule. Moreover, Hornstein and Wolman (2005) looks at a model similar to Kiley (2007), but allow for firm-specific capital. The results in Kiley (2007) are qualitatively similar to ours, but we extend and complement his analysis in several ways. First, we embed trend inflation in the standard New Keynesian framework (see, e.g., Galí 2003 or Woodford 2003) using the more popular Calvo (1983) staggered pricing framework. While the model employed in Kiley (2007) is quite stylized (i.e., two-period Taylor-type staggering), the Calvo pricing scheme allows to vary the average price duration of price contracts. Second, we provide clear analytical results about how trend inflation affects the Taylor principle, while Kiley (2007) presents only numerical results. Third, we generalize the analysis to different kinds of price indexation schemes, different kinds of Taylor rules (contemporaneous, forward and backward looking) and different degrees of inertia in the rules. Indeed, a further contribution of this article is to provide a compact presentation of the basic New Keynesian DSGE model approximated around a general trend inflation level with price indexation. As such, we further generalize the model in Ascari and Ropele (2007) by allowing for different price indexation schemes.

The next section presents the model. Section 3 provides a general formulation of the NKPC allowing for trend inflation and different kinds and degrees of indexation. Section 4 discusses a series of analytical results concerning how trend inflation affects both the determinacy of the rational expectation equilibrium. Section 5 displays numerical results regarding indexation, different kinds of Taylor rules and parameter sensitivity analysis. Section 6 concludes.

2 The Model

In this section we extend the basic new Keynesian DSGE model of Clarida et al. (1999), Galí (2003) and Woodford (2003) to allow for positive trend inflation and price indexation.

Households. Households live forever and their expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \chi \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right), \tag{1}$$

where $\beta \in (0,1)$ is the subjective rate of time preference and E_0 is the expectation operator conditional on time $t = 0$ information. The instantaneous utility function is increasing in the consumption of a final good (C_t) and decreasing in labour (N_t). The parameter σ_n represents the inverse intertemporal elasticity of substitution in labour supply while χ is a positive constant. At a given period t, the representative household faces the following nominal flow budget constraint

$$P_t C_t + B_t \leq P_t w_t N_t + (1 + i_{t-1}) B_{t-1} + F_t \tag{2}$$

where P_t is the price of the final good, B_t represents holding of bonds offering a one-period nominal return i_t, w_t is the real wage, and F_t are firms' profits that are returned to households. The households' problem is to maximize (1) subject to the sequence of budget constraints (2), yielding the following first order conditions:

$$w_t = \chi N_t^{\sigma_n} C_t, \tag{3}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \frac{(1+i_t) P_t}{P_{t+1}} \right]. \tag{4}$$

Equations (3) and (4) have the usual economic interpretation.

Final Good Producers. In each period, a final good Y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $Y_{i,t}$ indexed by $i \in [0,1]$ and a standard CES production function $Y_t = \left[\int_0^1 Y_{i,t}^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}$, with $\theta > 1$. Taking prices as given, the final good producer chooses intermediate good quantities $Y_{i,t}$ to maximize profits, resulting in the usual demand schedule: $Y_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$. The zero profit condition of final good producers leads the aggregate price index $P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}$.

Intermediate Goods Producers. Intermediate inputs $Y_{i,t}$ are produced by a continuum of firms indexed by $i \in [0,1]$ with technology $Y_{i,t} = N_{i,t}$. Prices are sticky, with intermediate goods producers in monopolistic competition setting prices according to a generalized discrete-time version of the Calvo (1983) mechanism. In each period there is a fixed probability $1-\alpha$ that a firm can re-optimize its nominal price, i.e., $P_{i,t}^*$. With probability α, instead, the firm may either keep its nominal price unchanged or index it. In the latter case the firm may index its nominal price partly to steady state inflation (e.g., Yun 1996) and/or partly to past inflation rate (e.g., Christiano et al.

2005). The maximization problem of a price-resetting firm can be formulated as

$$\max_{P_{i,t}^*} \quad E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\frac{C_t}{C_{t+j}}\right) \left(\frac{P_{i,t}^*}{P_{t+j}} \Omega_{t,t+j-1} - w_{t+j}\right) Y_{i,t+j}, \tag{5}$$

subject to

$$Y_{i,t+j} = \left(\frac{P_{i,t}^*}{P_{t+j}} \Omega_{t,t+j-1}\right)^{-\theta} Y_{t+j},$$

where $\Omega_{t,t+j-1} \equiv \overline{\Pi}^{j(1-\omega)\varepsilon} \left(P_{t+j-1}/P_{t-1}\right)^{\omega\varepsilon}$ with $j = 0, 1, 2, \cdots$. In particular, $\Omega_{t,t+j-1}$ represents a general price indexation rule that allows for any degree of indexation to trend inflation and to past inflation. Furthermore, $\overline{\Pi}$ is the central bank's long-run inflation target or the level of trend inflation, the parameter $0 \leqslant \varepsilon \leqslant 1$ measures the overall degree of price indexation, while the parameter $0 \leqslant \omega \leqslant 1$ denotes the degree of (geometric) combination of indexation to trend or past inflation rate.[4] Moreover, the aggregate price level evolves as

$$P_t = \left[\alpha \left(P_{t-1}\Omega_{t-1,t-1}\right)^{1-\theta} + (1-\alpha) \left(P_{i,t}^*\right)^{1-\theta}\right]^{1/(1-\theta)}. \tag{6}$$

The solution to the profit maximization problem (5) returns a formula for the optimal relative price:

$$\frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\frac{C_t}{C_{t+j}}\right) \left(\frac{P_t}{P_{t+j}} \Omega_{t,t+j-1}\right)^{-\theta} Y_{t+j} w_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\frac{C_t}{C_{t+j}}\right) \left(\frac{P_t}{P_{t+j}} \Omega_{t,t+j-1}\right)^{1-\theta} Y_{t+j}}. \tag{7}$$

Assumption 1. *For given parameter values of* $0 \leqslant \alpha < 1$, $0 < \beta < 1$, $0 \leqslant \varepsilon \leqslant 1$ *and* $\theta > 1$, *the* positive *level of trend inflation fulfils the restrictions:* $1 \leqslant \overline{\Pi} < \left(\frac{1}{\alpha}\right)^{\frac{1}{(\theta-1)(1-\varepsilon)}}$ *and* $1 \leqslant \overline{\Pi} < \left(\frac{1}{\alpha\beta}\right)^{\frac{1}{\theta(1-\varepsilon)}}$.

In particular, Assumption 1 ensures that in the deterministic steady state the optimal relative price implied by (6) is strictly positive and that the infinite sums in (7) converge.

To fully understand the effects of trend inflation on the optimal reset price, it is insightful to look at the case of no indexation, i.e., $\varepsilon = 0$, for which equation (7) becomes

$$\frac{P_{i,t}^{*}}{P_t} = \frac{\theta}{\theta-1}\frac{E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left(\frac{C_t}{C_{t+j}}\right)\left(\frac{P_{t+j}}{P_t}\right)^{\theta}Y_{t+j}w_{t+j}}{E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left(\frac{C_t}{C_{t+j}}\right)\left(\frac{P_{t+j}}{P_t}\right)^{\theta-1}Y_{t+j}}, \tag{8}$$

and then focus on the steady state behaviour of (8). In the standard case of zero trend inflation, $\overline{\Pi} = 1$ and the factor (P_{t+j}/P_t) attached to future expected terms is equal to one at all times. Future expected terms are discounted by $\alpha\beta$. With positive trend inflation, $\overline{\Pi} > 1$ and two effects come into play. First, the effective discount factors in the numerator and denominator change and become $\alpha\beta\overline{\Pi}^{\theta}$ and $\alpha\beta\overline{\Pi}^{\theta-1}$, respectively. Accordingly, when intermediate firms are free to adjust, they will set higher prices to try to offset the erosion of relative prices and profits that trend inflation automatically creates. Second, future terms in (8) are progressively multiplied by larger discount factors. This means that optimal price-setting under trend inflation reflects future economic conditions more than short-run cyclical variations. Price-setting firms become more forward-looking. Extending the same reasoning to (7), it is easy to see that price indexation mitigates the two effects just described.

Relative price dispersion. At the level of intermediate firms, it holds true that $(P_{i,t}/P_t)^{-\theta}Y_t = N_{i,t}$. Integrating this expression over i yields $Y_t s_t = N_t$, where we define $s_t \equiv \int_0^1 (P_{i,t}/P_t)^{-\theta} di$ and $N_t \equiv \int_0^1 N_{i,t} di$. In other words, the variable s_t measures the relative price dispersion across intermediate firms. **?**) shows that the variable s_t is bounded below at one. s_t represents the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism: the higher s_t, the more labour is needed to produce a given level of output. It can be shown to evolve as

$$s_t = (1-\alpha)\left(\frac{P^*_{i,t}}{P_t}\right)^{-\theta} + \alpha\left(\frac{\Pi_t}{\Omega_{t-1,t-1}}\right)^{\theta} s_{t-1}, \tag{9}$$

where $\Pi_t = P_t/P_{t-1}$ denotes the gross inflation rate. The variable s_t directly affects the real wage via the labour supply equation (3): $w_t = \chi Y_t^{\sigma_n} s_t^{\sigma_n} C_t$.

Market clearing conditions. The market clearing conditions in the goods and labour markets are: $Y_t = C_t$; $Y^s_{i,t} = Y^D_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$, $\forall i$; and $N_t = \int_0^1 N_{i,t} di$.

3 A generalized New Keynesian Phillips Curve

Log-linearizing (3) and (4), and using the market clearing condition $\widehat{Y}_t = \widehat{C}_t$, yields

$$\widehat{w}_t = \sigma_n \widehat{N}_t + \widehat{Y}_t, \tag{10}$$

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \left(\widehat{i}_t - E_t \widehat{\Pi}_{t+1}\right), \tag{11}$$

where hatted variables denote percentage deviations from deterministic steady state.

The log-linearization of (6), (7) and (9) is more complicated and leads to the following system of difference equations characterizing the generalized NKPC under trend inflation and price indexation

$$\begin{cases} \Delta_t = \beta\overline{\Pi}^{1-\varepsilon} E_t \Delta_{t+1} + \lambda_{(\overline{\pi},\varepsilon)} \left[(1+\sigma_n)\widehat{Y}_t + \sigma_n \widehat{s}_t\right] + \eta_{(\overline{\pi},\varepsilon)} E_t \left[(\theta-1)\Delta_{t+1} + \widehat{\phi}_{t+1}\right] \\ \\ \widehat{\phi}_t = \alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)} E_t \left[(\theta-1)\Delta_{t+1} + \widehat{\phi}_{t+1}\right] \\ \\ \widehat{s}_t = \xi_{(\overline{\pi},\varepsilon)} \Delta_t + \alpha\overline{\Pi}^{\theta(1-\varepsilon)} \widehat{s}_{t-1} \end{cases} \tag{12}$$

where $\Delta_t \equiv \widehat{\Pi}_t - \varepsilon\omega\widehat{\Pi}_{t-1}$ defines the quasi-difference of inflation rate and $\widehat{\phi}_t$ is an auxiliary variable. The coefficients $\lambda_{(\overline{\pi},\varepsilon)}$, $\eta_{(\overline{\pi},\varepsilon)}$ and $\xi_{(\overline{\pi},\varepsilon)}$ are complicated convolutions of parameters that depend, *inter alia*, on trend inflation and price indexation (for their expressions see Table 1). Of course, our generalization (12) encompasses the standard NKPC:[5] with zero trend inflation $\overline{\Pi} = 1$ and $\eta = \xi = 0$. In this case, both the auxiliary variable and the measure of relative price dispersion become irrelevant for inflation dynamics. Thus, the system (12) is reduced to the standard specification: $\Delta_t = \beta E_t (\Delta_{t+1}) + \lambda (1 + \sigma_n) \widehat{Y}_t$. This is also true if there is full price indexation (i.e., $\varepsilon = 1$), whatever the value of $\overline{\Pi}$.

Several remarks are noteworthy. As stressed by Ascari and Ropele (2007), trend inflation sensibly alters the inflation dynamics compared to the usual Calvo model with $\overline{\Pi} = 1$ (or $\varepsilon = 1$). Firstly, trend inflation enriches the dynamic structure by adding two new endogenous variables: $\widehat{\phi}_t$, which is a forward-looking variable, and $\widehat{s}_t$, which is a predetermined variable. Secondly, trend inflation directly affects the coefficients of the NKPC. As price-setting becomes more "forward-looking", higher trend inflation leads to a smaller coefficient on current output and a larger coefficient on future expected inflation. Under Assumption 1 it holds true that: $\partial\lambda_{(\overline{\pi},\varepsilon)}/\partial\overline{\Pi} < 0$, $\partial\xi_{(\overline{\pi},\varepsilon)}/\partial\overline{\Pi} < 0$ and $\partial\eta_{(\overline{\pi},\varepsilon)}/\partial\overline{\Pi} > 0$ (see Appendix A.1). Consequently, as trend inflation increases, the short-run NKPC flattens when drawn in the plane $(\widehat{Y}_t, \widehat{\Pi}_t)$. Hence, the contemporaneous relation between $\widehat{\Pi}_t$ and $\widehat{Y}_t$ progressively weakens: the inflation rate becomes less sensitive to variations in current output and more forward-looking. Thirdly, trend inflation increases the autoregressive coefficient in the equation of relative price dispersion.

Other things being equal, higher trend inflation yields a more persistent adjustment of inflation. Finally, the effects of trend inflation on the NKPC coefficients are partly counterbalanced by the degree of price indexation. Indeed, one can show: $\partial\lambda_{(\overline{\pi},\varepsilon)}/\partial\varepsilon > 0$, $\partial\xi_{(\overline{\pi},\varepsilon)}/\partial\varepsilon > 0$ and $\partial\eta_{(\overline{\pi},\varepsilon)}/\partial\varepsilon < 0$. In case of full price indexation, the effects of trend inflation are completely neutralized.

To close the model we assume the central bank sets the short run nominal interest rate according to the classic contemporaneous Taylor rule

$$\widehat{i}_t = \phi_\Pi \widehat{\Pi}_t + \phi_Y \widehat{Y}_t, \tag{13}$$

where ϕ_Π and ϕ_Y represent the response coefficients to inflation and output, respectively.

In the following sections, to examine the determinacy of the REE we first substitute the policy rule (13) into (11) and then write the structural equations in canonical format $x_t = \mathbf{A}E_t x_{t+1}$, where the vector x_t includes the endogenous variables of the model and $\mathbf{A}$ is a conformable matrix.

4 Analytical results

This section presents the analytical derivation of our main results. To this end, we firstly state the following assumption.

Assumption 2. *Throughout this section, we set $\sigma_n = 0$ (i.e., indivisible labour) and $\omega = 0$ (i.e., no indexation to past inflation); moreover, $\phi_\Pi \geqslant 0$ and $\phi_Y \geqslant -1$ and at least one different from zero.*

To fully understand the effects of trend inflation on the determinacy properties of the New Keynesian model, it is useful to briefly recall the standard results obtained under zero trend inflation.

4.1 Determinacy condition under zero inflation steady state

Substituting $\overline{\Pi} = 1$ (or $\varepsilon = 1$) into (12), the model economy reduces to a bivariate dynamic system in the two forward-looking variables, namely $\widehat{Y}_t$ and $\widehat{\Pi}_t$. In this case, the necessary and sufficient conditions for determinacy of the rational expectation equilibrium (henceforth REE) can be written as

$$\phi_\Pi + \frac{1-\beta}{\lambda}\phi_Y > 1, \tag{14}$$

and

$$\phi_Y + \lambda\phi_\Pi > \beta - 1. \tag{15}$$

Figure 1 plots the determinacy region (shaded area) in the plane (ϕ_Π, ϕ_Y) under zero inflation steady state (or full indexation to trend inflation). This type of picture is very well-known in the literature, although only the positive orthant is usually displayed (e.g., Bullard and Mitra 2002, Woodford 2003). Evidently, condition (14) binds in the positive orthant of the plane (ϕ_Π, ϕ_Y) while condition (15) does not. As stressed by Bullard and Mitra (2002) and Woodford (2001, 2003, see Ch. 4.2.2) among others, condition (14) is a generalization of the standard Taylor principle: to ensure determinacy of the REE, the nominal interest rate should rise by more than the increase of inflation in the long run. Indeed, the term $(1-\beta)/\lambda$ represents the long run multiplier of inflation on output in a standard NKPC log-linearized around the zero inflation steady state. Hence, the left-hand side of (14) "represents the long-run increase in the nominal interest rate prescribed [...] for each unit of permanent increase in the inflation rate" (Woodford 2003, p. 254). Therefore, "The Taylor principle continues to be a crucial condition for determinacy, once understood to refer to the *cumulative* response to a *permanent* inflation increase" (Woodford 2003, p. 256, italics as in the original). In other words, the Taylor principle has to be understood as (where LR stands for long run),

$$\left.\frac{\partial \widehat{i}}{\partial \widehat{\Pi}}\right|_{LR} = \phi_{\Pi} + \phi_{Y} \left.\frac{\partial \widehat{Y}}{\partial \widehat{\Pi}}\right|_{LR} > 1. \tag{16}$$

Note that condition (14) has two main implications. Firstly, it implies a sort of trade-off between ϕ_{Π} and ϕ_{Y}: values of $\phi_{\Pi} < 1$ may still ensure determinacy provided the central bank responds more aggressively to output. Secondly, in reality this trade-off is very weak: as the subjective discount factor is calibrated very close to one, the coefficient $(1-\beta)/\lambda$ turns out to be roughly zero. Consequently, most researchers, particularly in the empirical monetary policy literature, have concentrated on condition $\phi_{\Pi} > 1$, while neglecting the role of ϕ_{Y} (see e.g., Clarida et al. 2000).

4.2 The Taylor principle under trend inflation

Here we extend the discussion in Woodford (2003) to the case of positive trend inflation. To begin with, we examine the long-run effects of inflation on output as trend inflation varies.

Proposition 1. Effect of trend inflation on the slope of the long-run Phillips

Curve. *The slope of the long-run Phillips Curve depends non linearly on trend inflation, that is*

$$\left.\frac{\partial \widehat{Y}}{\partial \widehat{\Pi}}\right|_{LR} \equiv \delta_{(\overline{\pi},\varepsilon)} = \frac{\left(1-\beta\overline{\Pi}^{1-\varepsilon}\right)\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]-\eta_{(\overline{\pi},\varepsilon)}(\theta-1)}{\lambda_{(\overline{\pi},\varepsilon)}\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]};$$

Moreover, under Assumptions 1 and 2, there exists a value of trend inflation $\overline{\Pi}^{*} \in \left(1, \beta^{\frac{1}{\varepsilon-1}}\right)$ *such that:*

$$\delta_{(\overline{\pi},\varepsilon)} > 0, \text{ for } \overline{\Pi} \in \left[1, \overline{\Pi}^{*}\right), \tag{17}$$

$$\delta_{(\overline{\pi},\varepsilon)} \leqslant 0, \text{ for } \overline{\Pi} \in \left[\overline{\Pi}^{*}, (\alpha\beta)^{\frac{1}{\theta(\varepsilon-1)}}\right). \tag{18}$$

Proof. See Appendix A.2.

Under positive trend inflation the coefficient $\delta_{(\overline{\pi},\varepsilon)}$, which represents the long-run elasticity of output to inflation, switches sign from positive to negative as soon as $\overline{\Pi}$ becomes larger than $\overline{\Pi}^{*}$. For standard calibration values, $\overline{\Pi}^{*}$ is very small.[6] The long-run NKPC is extremely non-linear around $\overline{\Pi} = 1$: it is positively sloped at $\overline{\Pi} = 1$ (because of a discounting effect), but then the slope turns negative, because of the relative price dispersion effect (see Ascari 2004, Yun 2005, Ascari and Ropele 2007).

The first two restrictions stated in Assumptions 2, namely, indivisible labour (see, Hansen 1985), i.e., $\sigma_n = 0$, together with price indexation to trend inflation, i.e., $\omega = 0$, greatly simplify the specification of the generalized NKPC. Firstly, under indivisible labour, the real wage in the labour supply equation (3) is independent from the measure of relative price dispersion: $w_t = \chi Y_t$. Then, the variable $\widehat{s}_t$ does not contribute to the joint dynamics of output and inflation, but only determines the path of labour.

Secondly, under price indexation to trend inflation, $\omega = 0$ and $\Delta_t \equiv \widehat{\Pi}_t$; so, the lagged inflation rate does not enter the system (12). Overall, these two restrictions remove the predetermined variables present in the model, i.e., $\widehat{\Pi}_{t-1}$ and $\widehat{s}_t$. In this case, the matrix representation of the model is: $x_t = \begin{bmatrix} \widehat{\Pi}_t & \widehat{\phi}_t & \widehat{Y}_t \end{bmatrix}'$ and

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -\lambda_{(\overline{\pi},\varepsilon)} \\ 0 & 1 & 0 \\ \phi_\Pi & 0 & 1+\phi_Y \end{bmatrix}^{-1} \begin{bmatrix} \beta\overline{\Pi}^{1-\varepsilon} + \eta_{(\overline{\pi},\varepsilon)}(\theta-1) & \eta_{(\overline{\pi},\varepsilon)} & 0 \\ (\theta-1)\,\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)} & \alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)} & 0 \\ 1 & 0 & 1 \end{bmatrix}. \tag{19}$$

As the vector x_t includes only non-predetermined variables, determinacy of the REE obtains if and only if all the eigenvalues of $\mathbf{A}$ lie inside the unit circle.[7]

Proposition 2. Necessary and sufficient conditions for determinacy of the REE. *Under Assumptions 1 and 2, determinacy of the REE under positive trend inflation obtains if and only if*

$$\phi_\Pi + \phi_Y \delta_{(\overline{\pi},\varepsilon)} > 1, \tag{20}$$

$$\phi_Y + \lambda_{(\overline{\pi},\varepsilon)}\phi_\Pi > \alpha\beta^2\overline{\Pi}^{\theta(1-\varepsilon)} - 1, \tag{21}$$

$$\mathrm{D}^2 - \mathrm{TD} + \mathrm{M} < 1, \tag{22}$$

where $\delta_{(\overline{\pi},\varepsilon)}$ is the long-run elasticity of output to inflation (see Proposition 1), and T,M and D are the trace, the sum of leading minors, and the determinant of $\mathbf{A}$, *respectively.*

Proof. See Appendix A.3.

Focus on condition (20). It is worth noting that also in the more general case of positive trend inflation condition (20) corresponds to (16). Therefore, the generalized Taylor principle continues to be a crucial condition for determinacy of the REE. What are then the effects of positive trend inflation on the Taylor principle? Clearly, these effects have to come through the coefficient $\delta_{(\overline{\pi},\varepsilon)}$, as described in Proposition 1. In the plane (ϕ_Π, ϕ_Y), as trend inflation increases, the upper determinacy frontier defined by $\phi_Y = (1 - \phi_\Pi) / \delta_{(\overline{\pi},\varepsilon)}$ progressively turns clockwise tilting around the point $(\phi_\Pi = 1, \phi_Y = 0)$. In light of Proposition 1, for $\overline{\Pi} < \overline{\Pi}^*$ the upper determinacy frontier is negatively sloping, while for $\overline{\Pi} > \overline{\Pi}^*$ it is positively sloping.

Figure 2 visualizes what happens to the Taylor principle (20) as trend inflation increases. The intuition is exactly as described above by Woodford (2003). One has to keep in mind that the Taylor principle relates to the long-run properties of the model, that is, to "*cumulative* responses to a *permanent* inflation increase". The fact that the long-run slope of the NKPC switches sign is evident in the first equation of the system (12), where the term $\beta\overline{\Pi}^{1-\varepsilon}$ becomes bigger than one for low levels of trend inflation.

Note that the presence of positive trend inflation overturns the two implications stemming from (14) under zero trend inflation. Firstly, even for low levels of trend inflation, the trade-off between ϕ_Π and ϕ_Y disappears as the slope of the upper determinacy frontier switches sign (from negative to positive). Along the upper determinacy frontier, a central bank that wanted to be less strict on inflation (i.e., a lower value of ϕ_Π) should be at the same time less aggressive towards output. Similarly, a central bank that wanted to be more aggressive towards output should also be tighter on inflation.

Secondly, the higher the level of trend inflation, the larger the absolute value of $\delta_{(\overline{\pi},\varepsilon)}$, hence the flatter the upper determinacy frontier. Now, this upgrades the role of the policy coefficient ϕ_Y. Indeed, given our postulated Taylor rule, the central bank has to be careful not to over-react to output. Why? Because under positive trend inflation, in the long-run attempts to decrease output via a contractionary monetary policy yields higher inflation.

Therefore, under positive trend inflation the Taylor principle remains valid in its more general formulation, however its implications are radically different. This in turn casts shadows on the results in most of the literature which are based on a particular case, i.e., $\overline{\Pi} = 1$, which is theoretically special as well as empirically unrealistic.

Focus on conditions (21) and (22). In general the Taylor principle does not suffice for determinacy of the REE. Also in the standard case of zero inflation steady state, a second condition needs to be fulfilled, namely (15). As shown in Figure 1, while this second condition does not bind in the positive orthant of the plane (ϕ_Π, ϕ_Y), generally it does for values of $\phi_Y < 0$.[8] In particular, given $\phi_\Pi > 1$ the determinacy of the REE still requires the central bank not to implement a policy that is excessively pro-cyclical.

Similarly, in the case of positive trend inflation, two more restrictions need to be fulfilled. Interestingly, both these conditions can be regarded as generalizations of (15) to the case of trend inflation. (15) and (21) simply correspond to the condition $|\mathrm{D}| < 1$ (see Appendix A.3), respectively in the bivariate and in the trivariate dynamic system, where in the latter $\mathrm{D}= \frac{\alpha\beta^2\overline{\Pi}^{\theta(1-\varepsilon)}}{1+\phi_Y+\lambda_{(\overline{\pi},\varepsilon)}\phi_\Pi}$. Furthermore, condition (22) also collapses to (15) when translated from a trivariate to a bivariate dynamic system.[9] Due to the obscure convolution of parameters in (22), it is hard to provide a readable expression for it and hence we resort to the numerical analysis (as discussed in section 5). Notwithstanding, we may suggest an intuition of what happens in the simulation. Appendix A.4 shows that, assuming that the coefficient $\eta_{(\overline{\pi},\varepsilon)}$ is small enough (which is quite likely under moderate trend inflation levels), then (22) holds if

$$\phi_Y + \lambda_{(\overline{\pi},\varepsilon)}\phi_\Pi > \beta\overline{\Pi}^{1-\varepsilon} - 1. \tag{23}$$

Note that condition (23) implies (21), which then becomes redundant. Moreover, it also yields (15) if $\overline{\Pi} = 1$. It is easy to see how trend inflation affects the line described by condition (23) in the plane (ϕ_Π, ϕ_Y). As visualized in Figure 3[10], trend inflation reduces $\lambda_{(\overline{\pi},\varepsilon)}$, and thus it flattens the line, and it increases the intercept, which become positive for values of $\overline{\Pi} > \beta^{\frac{1}{1-\varepsilon}}$. As trend inflation increases, therefore, the lower determinacy frontier progressively shifts upwards and eventually crosses the upper determinacy frontier for $\phi_Y > 0$. Trend inflation then implies the two determinacy frontiers may cross in the positive orthant. In other words, while most of the literature discarded the second determinacy condition (15) because it was satisfied for positive values of (ϕ_Π, ϕ_Y) in the case of zero inflation steady state, this is no longer true under positive trend inflation.

Condition (23) is however only necessary, but not sufficient for (22), and thus to investigate the relevance of this qualitative result we need to resort to numerical simulations. Figure 4 illustrates the numerical determinacy region in the plane (ϕ_Π, ϕ_Y) for different levels of annualized trend inflation, i.e., 0, 2, 4, 6 and 8 per cent, showing that the analytical insights of this section holds true.[11] As in Figure 1, the determinacy region lies at the right of the two frontiers, for each given level of trend inflation.

Result 1. Effect of trend inflation on condition (22). *As trend inflation increases, the* lower determinacy frontier *implicitly defined by* $\mathrm{D}^2 - \mathrm{TD} + \mathrm{M} = 1$ *progressively shifts upwards crossing the* upper determinacy frontier *in the positive orthant of the plane* (ϕ_Π, ϕ_Y)*.*

According to our calibration, the intersection in the positive orthant between the upper and lower determinacy frontiers happens for levels of annualized trend inflation greater than 2.42 per cent. For levels of annualized trend inflation greater than this value, not only does the smallest admissible value of ϕ_Π positively co-move with $\overline{\Pi}$ (because of the upper shift of the lower frontier), but also the central bank cannot always implement a strict inflation targeting policy. Moreover, Figure 4 visualizes the crucial role that the policy coefficient on output plays with positive trend inflation. As an example, in Figure 4 we highlight with a cross the classical Taylor rule specification, i.e., $\phi_\Pi = 1.5$ and $\phi_Y = 0.5$. As (annualized) trend inflation exceeds 2.4 per cent, the classical Taylor rule yields indeterminacy of the REE. Hence, in empirical applications for realistic values of trend inflation the value of ϕ_Y cannot be neglected.

4.2.1 The effects of price indexation

Proposition 3. Effects of price indexation to trend inflation on REE determinacy. *An increase in* ε*, that is, the degree of partial price indexation to trend inflation counteracts the effects trend inflation has on REE determinacy properties described above.*

Proof. Notice the indexation parameter only appears in the model coefficients as power to trend inflation, i.e., $\overline{\Pi}^{1-\varepsilon}$. Thus, increasing the value of indexation is equivalent to decrease the level of trend inflation.

So, the whole set of results discussed above carries on, although partial price indexation to trend inflation mitigates the effects of $\overline{\Pi}$ to some extent.

In summary, trend inflation unambiguously affects the determinacy properties of the REE: as $\overline{\Pi}$ increases, the determinacy region shrinks, increasing the possibility of sunspot fluctuations. As trend inflation rises, implementable monetary rules call for increasingly larger and positive coefficients on inflation and smaller coefficients on output. These outcomes are in agreement with the policy prescriptions suggested in Schmitt-Grohé and Uribe (2004, 2007) and in Bullard and Mitra (2002). Although dealing with different issues, these two articles robustly advocate a monetary policy rule characterized by a large response to current inflation and a close to zero coefficient on output. Allowing for positive trend inflation in a basic new Keynesian DSGE model casts some doubts on the *leaning against the wind* prescription in Clarida et al. (1999). As $\overline{\Pi}$ increases, the central bank cannot run the risk of stabilizing the output (in deviation from steady state) but should focus primarily on inflation.

5 Numerical results

In this section, we check the robustness of our analytical results to the simplifying assumptions introduced in Section 4. We remove the assumption of labour indivisibility, which implies that now the dispersion of relative prices contributes to explain the dynamics of inflation. We also consider both price indexation schemes to trend and the past inflation rate and varying degrees of overall indexation. Furthermore, we investigate the effects of changing the monetary policy rule, by introducing inertial or backward-looking and forward-looking components.

Assumption 3. Calibration of parameter values and further restrictions. *Throughout this section we follow (Galí 2003) and set:* $\sigma_n = 1$, $\alpha = 0.75$, $\beta = 0.99$ *and* $\theta = 11$; *furthermore, we restrict* $0 \leqslant \phi_{\Pi} \leqslant 5$ *and* $-1 \leqslant \phi_Y \leqslant 5$.

5.1 Price indexation

We begin our analysis by comparing the effects of price indexation to trend inflation, i.e., $\omega = 0$, *versus* past inflation, i.e., $\omega = 1$, when monetary policy follows the Taylor rule described in (13). In the case of past inflation indexation, the model is further complicated by the presence of another endogenous predetermined variable, namely $\widehat{\Pi}_{t-1}$. Figure 5 reports the determinacy regions for different levels of trend inflation, i.e. 0, 2, 4, 6 and 8 per cent, in the cases of partial indexation, i.e., $\varepsilon = 0.5$, and full indexation, i.e., $\varepsilon = 1$.

The overall results are in line with the findings presented in previous sections. Firstly, positive trend inflation shrinks the determinacy region. The upper determinacy frontier tilts clockwise, becoming positively sloping even for low levels of trend inflation, while the lower determinacy frontier shifts upwards. However, with respect to Figure 4, partial price indexation visibly counteracts the effects of $\overline{\Pi}$. For example, for $\varepsilon = 0.5$ the basic Taylor specification (marked with a cross in the three panels of Figure 5) ensures determinacy up to levels of trend inflation slightly below 6 per cent. Moreover, the lowest admissible value of ϕ_Π becomes relatively less sensitive to trend inflation. Secondly, for a given level of trend inflation, price indexation to past inflation (i.e., $\omega = 1$) yields a larger number of determinate interest rate rules than under price indexation to trend inflation. While the location of the upper determinacy frontier is similar under both price indexation schemes (see panels A and B in Figure 5), price indexation to past inflation has a different effect on the lower determinacy frontier, which is shifted further downwards. So, past inflation indexation enlarges the determinacy region in favour of more pro-cyclical monetary policy rules, i.e., more negative values of ϕ_Y. Finally, allowing for full price indexation, i.e., $\varepsilon = 1$, which neutralizes any effects of trend inflation, has different implications for the determinacy region. Full price indexation to trend inflation returns the determinacy region that would arise under zero inflation steady state; whereas, full price indexation to past inflation restores the original Taylor (1993) principle, i.e., $\phi_\Pi > 1$, making ϕ_Y completely irrelevant for determinacy.[12]

5.2 Interest rate rules

Inertial interest rate rules

Empirical works on Taylor rules report that central banks tend to adjust the nominal interest rate only gradually (see, e.g., Rudebusch 1995, Judd and Rudebusch 1998 or Clarida et al. 2000). Moreover, the recent optimal monetary literature emphasizes the benefit of inertial behavior in the conduct of monetary policy when private agents are forward-looking. So, here we consider specifications of the Taylor rule that allow the nominal interest rate to respond also to its own lagged values, that is $\hat{\imath}_t = \phi_\Pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_i \hat{\imath}_{t-1}$, where the degree of interest rate smoothing is measured by ϕ_i. More generally, cases where $0 \leqslant \phi_i \leqslant 1$ are referred to as partial adjustment; case $\phi_i = 1$ is labelled as a difference rule; cases where $\phi_i \geqslant 1$ represent instead superinertial behaviour (as in Rotemberg and Woodford 1999 and Woodford 2003).

Figure 6 illustrates the effects of trend inflation on determinacy when $\phi_i = 0.5$, 1, 2 and 5. To interpret the Figure, let us focus on panel A and recall that in the zero trend inflation case, condition (16) becomes $\phi_\Pi + \phi_Y \left.\frac{\partial \widehat{Y}}{\partial \widehat{\Pi}}\right|_{LR} > 1 - \phi_i$. From a graphical point of view, with respect to Figure 4, the inertia parameter ϕ_i shifts to the left, without affecting its slope, the upper determinacy frontier corresponding to the generalized Taylor principle (16), and makes steeper the lower determinacy frontier, corresponding to (15). As a consequence, the crossing point of the two frontiers defining the determinacy region moves leftward. The same happens in the positive trend inflation cases.

Moreover, in the zero trend inflation case, given $\phi_\Pi + \phi_Y \left.\frac{\partial \widehat{Y}}{\partial \widehat{\Pi}}\right|_{LR} > 1 - \phi_i$, $\phi_i \geq 1$ is a sufficient condition for a determinate equilibrium in the positive orthant, as evident from the other panels of Figure 6, where the upper determinacy frontier is shifted so much to the left that it does not appear anymore. In other words, a determinate REE necessarily exists for superinertial rules with zero trend inflation (see Woodford 2003, p. 256). Also in the case of positive trend inflation, inertia graphically has the same effect: it shifts leftward the crossing point of the two frontiers. Superinertial rules, however, do not rule out indeterminacy in the positive orthant when trend inflation is positive. Moreover, it is the value of ϕ_Y that actually matters for REE determinacy. Looking at panel B, it is evident that there is no longer a sufficient condition on ϕ_Π (provided that is positive) or on ϕ_i. On the contrary, for sufficiently high levels of trend inflation, we can eventually state a sufficient condition on ϕ_Y. As stressed in Section 4.2, this is due to the switch in the sign of $\delta_{(\overline{\pi},\varepsilon)}$ and to the fact that $\delta_{(\overline{\pi},\varepsilon)}$ is increasing with trend inflation in absolute value. For values of trend inflation at least as large as 6 per cent, the value of the parameter $\delta_{(\overline{\pi},\varepsilon)}$ becomes so high (in absolute value), that ϕ_Y becomes the crucial monetary policy parameter for condition (16) to be satisfied.

Overall, the figure confirms that interest rate inertia makes indeterminacy less likely, as in the basic New Keynesian model with zero inflation steady state. Moreover, the somewhat counterintuitive feature that explosive rules enlarge the determinacy region survives in the positive trend inflation case. As discussed in Rotemberg and Woodford (1997, p.100-101), it is exactly the possibility of the explosiveness of the nominal interest rate that keeps the model on track.

Other interest rate rules and sensitivity analysis

We further explore whether the results of the previous sections are robust to simple variants of the Taylor rule commonly used in the literature (i.e., forward-looking interest rate rule, backward-looking interest rate rule, and various kinds of hybrid interest rate rules) and to changes in the structural parameters of the model. In all these cases, the main result of the paper carries over: moderate levels of trend inflation substantially shrink the determinacy region.[13]

In this section, we just briefly report the results concerning the determinacy conditions in the case of the backward-looking interest rate rule, as for the other policy rules the results are very similar to those presented in previous sections.

When the monetary authority sets the nominal interest rate as a function of lagged values of inflation and output, i.e., $\hat{\imath}_t = \phi_\Pi \hat{\pi}_{t-1} + \phi_Y \hat{Y}_{t-1}$, positive levels of trend inflation have some peculiar effects on the determinacy regions. Panel A of Figure 7 illustrates the standard case of zero inflation steady state. Roughly speaking, there are two frontiers that divide the plane into four areas: one frontier is almost horizontal with the ϕ_Y-intercept at two; the other frontier corresponds to the equivalent of condition (16). Notice that, above the almost horizontal frontier, the determinacy region now lies *on the left hand side of condition (16) and not on its right*, where the instability region lies. Panels B, C and D of Figure 7 show the effects of positive trend inflation. Once again, the frontier corresponding to (16) again visibly rotates clockwise.[14] However, due to the fact that the determinacy region is partly on the left and partly on the right of this line, the effect of trend inflation is less clear-cut. Roughly speaking, as trend inflation increases: (i) above the almost horizontal frontier, the instability region progressively shrinks and gives way to new determinate equilibria; (ii) below the almost horizontal frontier, the indeterminacy region enlarges. While this latter implication parallels the effect analyzed in previous sections, the former effect is specific of the lagged interest rate rule. Moreover, as trend inflation rises a central bank that follows a backward-looking interest rate rule is progressively left with two options to ensure determinacy. It could respond relatively more to inflation and less to output, as in previous sections; or, alternatively, the central bank could just respond with a large coefficient to output, i.e. $\phi_Y > 2$, and discard ϕ_Π. Introducing inertial behavior in the backward-looking interest rate rule shifts upward the almost horizontal line in Figure 7. Consequently, the effect described in (i) becomes progressively less important and disappears for superinertial policies.

Finally, we check the robustness of our numerical findings to changes in the structural parametrization. A lower value of the Calvo parameter α mitigates the effects of trend inflation, and thus, in our case it makes the determinacy frontier close less rapidly compared with the baseline case. This leaves room for a relatively larger set of implementable policies for a given trend inflation, but it does not qualitatively change our main results, as evident from the analytical results in Section 4. Lowering the value of the elasticity of substitution across goods, i.e., θ, has a similar implication. Considering higher values of the inverse of the intertemporal elasticity of the labour supply has a negligible quantitative effect on the results presented above. In choosing a logarithmic utility function in consumption we considered a specification biased against our argument. The slope of the NKPC is quite sensitive to the values of the intertemporal elasticity of substitution in consumption (i.e., IESC). Setting higher values of IESC, as in Rotemberg and Woodford (1997) and Bullard and Mitra (2002), strongly strengthens our results from a quantitative point of view.[15]

6 Conclusions

Despite the fact that average inflation in the post-war period in developed countries was moderately different from zero, much of the vast literature on monetary policy rules worked with models approximated around the zero inflation steady state. In this article, we generalize the basic new Keynesian dynamic stochastic general equilibrium model with Calvo staggered prices by taking a log-linear approximation around a general level of trend inflation. Imposing the monetary authority follows a simple contemporaneous Taylor rule, we then show that moderate levels of trend inflation modify the parameter space that defines a determinate rational expectation equilibrium. Trend inflation substantially alters the Taylor principle. Our key result is then proved to be robust to: (a) Taylor type rules; (b) inertial Taylor rules ; (c) indexation schemes; (d) parameter values.

In summary, the literature on monetary policy rules is based on a case, i.e., the zero inflation steady state, that is both empirically unrealistic and theoretically special. In line with Ascari and Ropele (2007), this article shows that trend inflation greatly affects the previous results established in the monetary policy literature. Our analysis therefore shows the literature cannot neglect trend inflation in either empirical or theoretical investigation. As non-superneutrality is a basic feature of the standard model, future work should aim at integrating the long-run properties and the short-run dynamics into a fully non-linear analysis.

In future work, the relationship between price stickiness and trend inflation in this type of analysis should be embedded. In particular, one may argue that α is not a truly structural parameter, and it should decrease with trend inflation. As previously noted, the empirical work of Cogley and Sbordone (2008) justifies the analysis put forward in this work and supports the empirical relevance of the results. From a theoretical perspective, however, a possibility would be to employ the framework in Levin and Yun (2007) that features endogenous contract duration in this analysis. Given the findings in Levin and Yun (2007), our conjecture is that the results would not qualitatively change for a moderate rate of inflation, as considered in this paper, while they would for high levels of inflation, where the Calvo model is a poor approximation of price setting.

Notes

[1]For example, Schmitt-Grohe and Uribe (2004) using US data from 1960 to 1998 calibrate trend inflation at 4.2%. In the same period, Germany, Italy, Spain, and the UK experienced average inflation rates of 3.2%, 8.1%, 7.1% and 9.0% respectively (source: OECD).

[2]Throughout our analysis, we shall use indifferently trend inflation or long-run inflation to denote the inflation rate in the deterministic steady state. As our focus is on the effects of trend inflation, we abstract from other extensions of the model that may modify the key structural equations and thereafter the Taylor principle. For example, Kurozumi (2006) considers a non separable utility function between consumption and real money balances, while Surico (2008) introduces a cost channel.

[3]Several papers do allow for non-zero steady state inflation in their analysis, but they do not look at what happens when trend inflation changes. Khan et al. (2003) solves the optimal monetary policy problem and then investigates the dynamics of the economy around the given optimal steady state inflation level. Schmitt-Grohe and Uribe (2004) simulates the model under different Taylor-type rules calibrating average inflation on US data.

[4]For example: the case where $\varepsilon = 1$ and $\omega = 1$ represents full price indexation to the past inflation rate; the combination $\varepsilon = 0.5$ and $\omega = 0.5$ represents the case in which prices are indexed for 25% to trend inflation and for 25% to the past inflation rate; finally, when $\varepsilon = 0$ there is no price indexation (whatever the value of ω).

[5]In the case of zero trend inflation or full price indexation we omit the subscript

"$_{(\overline{\pi},\varepsilon)}$" when referring to the coefficients of the NKPC.

[6]Just to give an idea, with the parameter values used in Section 5 and assuming no indexation, i.e., $\varepsilon = 0$, it turns out that $\overline{\Pi}^* = 1.00098$; this corresponds to an annualized value of trend inflation equal to 0.39 per cent.

[7]See proposition 1 in Blanchard and Kahn (1980). As we work with a linearly approximated model, all our propositions and results relate to local properties of the rational expectations equilibrium.

[8]In the more general case in which both ϕ_Y or ϕ_Π may take negative values, restrictions (14) and (15) are only necesary but not sufficient conditions for determinacy.

[9]See Theorem 2 in (Brooks 2004). If an eigenvalue is equal zero, the set of inequalities (35)-(37) are the same as the stability ones for a bivariate system, where the sum of minors is replaced by the determinant.

[10]As in Figure 1, for each level of trend inflation, the determinacy region lies at the right of the two frontiers.

[11]As in Figure 3, for each level of trend inflation, the determinacy region lies at the right of the two frontiers. In drawing Figure 4, we set $\varepsilon = 0$ and the other free parameters as in Section 5, Assumption 3: $\sigma_n = 1$, $\alpha = 0.75$, $\beta = 0.99$, and $\theta = 11$.

[12]Ropele (2007) analytically shows that condition $\phi_\Pi > 1$ is indeed the necessary and sufficient condition for the determinacy of REE in this case.

[13]For a more detailed discussion of the results, the interested reader can look at the working paper version of this work, available at the authors' webpage.

[14]The other almost horizontal line is, in contrast, only slightly sensitive to changes

in trend inflation for our calibration values. Finally, note the presence also of the lower frontier that qualitatively moves as in previous cases, shifting upwards with trend inflation.

[15]Moreover, as already noted by Bullard and Mitra (2002), the value of the IESC turns out to be quite important for the backward-looking interest rate rule case, because it substantially affects the position of the almost horizontal line in Figure 7.

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A Appendix

A.1 The coefficients of the generalized NKPC

In this section of the Appendix, we show that $\frac{\partial\lambda_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} < 0$, $\frac{\partial\eta_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} > 0$, and $\frac{\partial\xi_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} > 0$.

The coefficient $\lambda_{(\overline{\pi},\varepsilon)}$ is decreasing in $\overline{\Pi}$, i.e., $\frac{\partial\lambda_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} < 0$. From the expression of $\lambda_{(\overline{\pi},\varepsilon)}$ reported in Table 1, compute the partial derivative with respect to $\overline{\Pi}$:

$$\frac{\partial\lambda_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} = -\frac{(1-\varepsilon)\left[\theta\left(\overline{\Pi}^{2\theta\varepsilon+1} - \alpha^2\beta\overline{\Pi}^{2\theta+\varepsilon}\right) - \overline{\Pi}^{2\theta\varepsilon+1} + \alpha\beta\overline{\Pi}^{\theta+\theta\varepsilon+1}\right]}{\alpha\overline{\Pi}^{(\theta+1)(\varepsilon+1)}}.$$

Note that the expression in square brackets can be factorized as follows

$$\begin{aligned}
& \theta\left(\overline{\Pi}^{2\theta\varepsilon+1} - \alpha^2\beta\overline{\Pi}^{2\theta+\varepsilon}\right) - \overline{\Pi}^{2\theta\varepsilon+1} + \alpha\beta\overline{\Pi}^{\theta+\theta\varepsilon+1} \\
= \; & \theta\overline{\Pi}^{2\theta\varepsilon+1}\left[1-\alpha^2\beta\overline{\Pi}^{\theta+\theta+\varepsilon-(2\theta\varepsilon+1)}\right] - \overline{\Pi}^{2\theta\varepsilon+1}\left[1-\alpha\beta\overline{\Pi}^{\theta+\theta\varepsilon+1-(2\theta\varepsilon+1)}\right] \\
= \; & \overline{\Pi}^{2\theta\varepsilon+1}\left\{\theta\left[1-\alpha^2\beta\overline{\Pi}^{\theta+\theta+\varepsilon-(2\theta\varepsilon+1)}\right] - \left[1-\alpha\beta\overline{\Pi}^{\theta+\theta\varepsilon+1-(2\theta\varepsilon+1)}\right]\right\}.
\end{aligned}$$

Moreover, the expression in curly brackets can be written as

$$\overline{\Pi}^{2\theta\varepsilon+1}\left\{\theta\left[1-\alpha^2\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\overline{\Pi}^{\theta(1-\varepsilon)}\right] - \left[1-\alpha\beta\overline{\Pi}^{\theta(1-\varepsilon)}\right]\right\}.$$

Under Assumption 1, i.e., $0 \leqslant \alpha\beta\overline{\Pi}^{\theta(1-\varepsilon)} < 1$ and $0 \leqslant \alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)} < 1$, the last expression is positive. Thus, $\frac{\partial\lambda_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} < 0$.

The coefficient $\eta_{(\overline{\pi},\varepsilon)}$ is increasing in $\overline{\Pi}$, i.e., $\frac{\partial\eta_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} > 0$. From the expression of $\eta_{(\overline{\pi},\varepsilon)}$ reported in Table 1, compute the partial derivative with respect to $\overline{\Pi}$,

$$\frac{\partial\eta_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} = (1-\varepsilon)\frac{\overline{\Pi}^{-\varepsilon}\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right] + \alpha(\theta-1)\overline{\Pi}^{(\theta-1)(1-\varepsilon)-1}\left(\overline{\Pi}^{1-\varepsilon}-1\right)}{\beta\left\{\left(\overline{\Pi}^{1-\varepsilon}-1\right)\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]\right\}^2} > 0,$$

which is positive under Assumption 1.

The coefficient $\xi_{(\overline{\pi},\varepsilon)}$ is increasing in $\overline{\Pi}$, i.e., $\frac{\partial \xi_{(\overline{\pi},\varepsilon)}}{\partial \overline{\Pi}} > 0$. From the expression of $\xi_{(\overline{\pi},\varepsilon)}$ reported in Table 1, compute the partial derivative with respect to $\overline{\Pi}$,

$$\begin{aligned}\frac{\partial \xi_{(\overline{\pi},\varepsilon)}}{\partial \overline{\Pi}} &= \frac{(1-\varepsilon)\,\theta\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\left[(\theta-1)\,\overline{\Pi}^{-1}\left(\overline{\Pi}^{1-\varepsilon}-1\right)+\overline{\Pi}^{-\varepsilon}\right]}{1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}} \\ &\quad + \frac{\alpha^2\theta\,(\theta-1)\,(1-\varepsilon)\,\overline{\Pi}^{2(\theta-1)(1-\varepsilon)-1}\left(\overline{\Pi}^{1-\varepsilon}-1\right)}{\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]^2}.\end{aligned}$$

Again, under Assumption 1 it follows that $\frac{\partial \xi_{(\overline{\pi},\varepsilon)}}{\partial \overline{\Pi}} > 0$.

A.2 The long-run multiplier of trend inflation on output

We derive the long-run multiplier of trend inflation on output, i.e., the partial derivative $\partial\widehat{Y}/\partial\Delta$, where $\Delta \equiv (1-\varepsilon\omega)\,\widehat{\Pi}$. To begin with, we eliminate from (12) all time subscripts and expectation operators, and then collect terms,

$$\left[1-\beta\overline{\Pi}^{1-\varepsilon}-\eta_{(\overline{\pi},\varepsilon)}\,(\theta-1)\right]\Delta \equiv (1+\sigma_n)\,\lambda_{(\overline{\pi},\varepsilon)}\widehat{Y}+\sigma_n\lambda_{(\overline{\pi},\varepsilon)}\widehat{s}+\eta_{(\overline{\pi},\varepsilon)}\widehat{\phi},$$

$$\left(1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right)\widehat{\phi} = \alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\,(\theta-1)\,\Delta, \tag{24}$$

$$\left(1-\alpha\overline{\Pi}^{\theta(1-\varepsilon)}\right)\widehat{s} = \xi_{(\overline{\pi},\varepsilon)}\Delta.$$

Then, we compute the derivatives:

$$\frac{\partial \widehat{Y}}{\partial \Delta} = \frac{1-\beta\overline{\Pi}^{1-\varepsilon}-\eta_{(\overline{\pi},\varepsilon)}(\theta-1)-\eta_{(\overline{\pi},\varepsilon)}\frac{\partial \widehat{\phi}}{\partial \Delta}-\lambda_{(\overline{\pi},\varepsilon)}\sigma_n\frac{\partial \widehat{s}}{\partial \Delta}}{(1+\sigma_n)\lambda_{(\overline{\pi},\varepsilon)}}, \tag{25}$$

$$\frac{\partial \widehat{\phi}}{\partial \Delta} = \frac{\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}(\theta-1)}{1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}, \tag{26}$$

$$\frac{\partial \widehat{s}}{\partial \Delta} = \frac{\xi_{(\overline{\pi},\varepsilon)}}{1-\alpha\overline{\Pi}^{\theta(1-\varepsilon)}}. \tag{27}$$

Therefore, substituting (26) and (27) into (25) yields

$$\frac{\partial \widehat{Y}}{\partial \Delta} = \frac{1}{(1+\sigma_n)\lambda_{(\overline{\pi},\varepsilon)}}\left[1-\beta\overline{\Pi}^{1-\varepsilon}-\frac{(\theta-1)\eta_{(\overline{\pi},\varepsilon)}}{1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}-\frac{\sigma_n\lambda_{(\overline{\pi},\varepsilon)}\xi_{(\overline{\pi},\varepsilon)}}{1-\alpha\overline{\Pi}^{\theta(1-\varepsilon)}}\right]. \tag{28}$$

Finally, setting $\sigma_n = 0$ yields,

$$\frac{\partial \widehat{Y}}{\partial \Delta} = \delta_{(\overline{\pi},\varepsilon)} \equiv \frac{\left(1-\beta\overline{\Pi}^{1-\varepsilon}\right)\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]-\eta_{(\overline{\pi},\varepsilon)}(\theta-1)}{\lambda_{(\overline{\pi},\varepsilon)}\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]}. \tag{29}$$

Next we prove there exists a value of trend inflation, denoted by $\overline{\Pi}^*$, such that $\delta_{(\overline{\pi}^*,\varepsilon)} = 0$. Note that at $\overline{\Pi} = 1$ the coefficient $\delta_{(\overline{\pi},\varepsilon)} = \frac{1-\beta}{\lambda_{(\overline{\pi},\varepsilon)}} > 0$ while at $\overline{\Pi} = \beta^{\frac{1}{\varepsilon-1}}$ the coefficient $\delta_{(\overline{\pi},\varepsilon)} = \frac{-\eta_{(\overline{\pi},\varepsilon)}(\theta-1)}{\lambda_{(\overline{\pi},\varepsilon)}\left[1-\alpha\beta^{2-\theta}\right]} < 0$. Therefore, as $\delta_{(\overline{\pi},\varepsilon)}$ is a continuous function there exists a value of trend inflation $1 \leqslant \overline{\Pi}^* \leqslant \beta^{\frac{1}{\varepsilon-1}}$ such at $\overline{\Pi} = \overline{\Pi}^*$it holds true $\delta_{(\overline{\pi},\varepsilon)} = 0$. Secondly, note that the sign of $\delta_{(\overline{\pi},\varepsilon)}$ depends only on the sign of its numerator, as the denominator is always positive. Given that the numerator of $\delta_{(\overline{\pi},\varepsilon)}$ monotonically decreases with $\overline{\Pi}$ it follows that $\overline{\Pi}^*$ is unique and therefore the proposition follows.

A.3 REE Determinacy Conditions

With price indexation to trend inflation, i.e., $\omega = 0$, and infinite labour supply elasticity, i.e., $\sigma_n = 0$, the dynamic system is trivariate and includes only non-predetermined variables, namely, $\widehat{\Pi}_t$, $\widehat{\phi}_t$ and $\widehat{Y}_t$. In this case, the matrix representation is given by

$$\begin{bmatrix} \pi_t \\ \widehat{\phi}_t \\ \widehat{Y}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -\lambda_{(\overline{\pi},\varepsilon)} \\ 0 & 1 & 0 \\ \phi_\Pi & 0 & 1+\phi_Y \end{bmatrix}^{-1} \begin{bmatrix} \beta\overline{\Pi}^{1-\varepsilon} + \eta_{(\overline{\pi},\varepsilon)}(\theta-1) & \eta_{(\overline{\pi},\varepsilon)} & 0 \\ (\theta-1)\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)} & \alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)} & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \pi_{t+1} \\ \widehat{\phi}_{t+1} \\ \widehat{Y}_{t+1} \end{bmatrix}. \tag{30}$$

In general, the characteristic polynomial associated with a cubic matrix can be written as

$$-z^3 + Tz^2 - Mz + D, \tag{31}$$

where T, M and D denote the trace, the sum of leading minors of order two and the determinant of matrix $\mathbf{A}$, respectively. In our case, it follows that:

$$T = \mu_{(\overline{\pi},\varepsilon)} + \frac{1 + \lambda_{(\overline{\pi},\varepsilon)} + \beta\overline{\Pi}^{1-\varepsilon}(1+\phi_Y) + \mu_{(\overline{\pi},\varepsilon)} q_{(\overline{\pi},\varepsilon)} (\theta-1)(1+\phi_Y)}{1+\phi_Y+\lambda_{(\overline{\pi},\varepsilon)}\phi_\Pi}, \tag{32}$$

$$M = \frac{\beta\overline{\Pi}^{1-\varepsilon} + \mu_{(\overline{\pi},\varepsilon)}\left[1 + \lambda_{(\overline{\pi},\varepsilon)} + q_{(\overline{\pi},\varepsilon)}(\theta-1) + \beta\overline{\Pi}^{1-\varepsilon}(1+\phi_Y)\right]}{1+\phi_Y+\lambda_{(\overline{\pi},\varepsilon)}\phi_\Pi}, \tag{33}$$

$$D = \frac{\beta\overline{\Pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}}{1+\phi_Y+\lambda_{(\overline{\pi},\varepsilon)}\phi_\Pi}, \tag{34}$$

where $\mu_{(\overline{\pi},\varepsilon)} \equiv \alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)} < 1$, and $q_{(\overline{\pi},\varepsilon)} \equiv \frac{\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]\left(\overline{\Pi}^{1-\varepsilon}-1\right)}{\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}$. Under Assumption 2: $T > 0$, $M > 0$ and $D > 0$.

Theorem 1 in (Brooks 2004) states the necessary and sufficient conditions for (3×3)

matrix to have all its eigenvalues inside the unit circle, which are

$$|D| < 1, \tag{35}$$

$$|T+D| < M+1, \tag{36}$$

$$D^2 - TD + M < 1. \tag{37}$$

Substituting the expressions for T, M and D in (35)-((37) returns the inequalities stated in Proposition 1 in the main text.

A.4 Factorization of (22)

Substituting the relevant terms into condition $1-D^2+TD-M>0$ yields the following expression,

$$\begin{aligned} 0 < & \left[1+\phi_Y+\lambda_{(\overline{\pi},\varepsilon)}\phi_\Pi - \beta\overline{\Pi}^{1-\varepsilon}\right] \\ & \left\{\lambda_{(\overline{\pi},\varepsilon)}\left(\phi_\Pi + \delta_{(\overline{\pi},\varepsilon)}\phi_Y - 1\right) + \eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left[\frac{\phi_Y}{1-\mu_{(\overline{\pi},\varepsilon)}} - 1\right]\right. \\ & \left. + \left(1-\mu_{(\overline{\pi},\varepsilon)}\right)\left[1-\beta\overline{\Pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)} + \lambda_{(\overline{\pi},\varepsilon)} + \phi_Y\beta\overline{\Pi}^{1-\varepsilon}\right]\right\} \\ & + \eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\beta\overline{\Pi}^{1-\varepsilon}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_Y\right)-1\right]. \end{aligned} \tag{38}$$

A necessary, but not sufficient condition for this last expression to hold is

$$0 < 1+\phi_Y+\lambda_{(\overline{\pi},\varepsilon)}\phi_\Pi - \beta\overline{\Pi}^{1-\varepsilon},$$

which is exactly condition (23) in the main text.

Moreover note all the terms and parentheses in (38) are positive, apart two ambiguous terms: (i) $\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left[\frac{\phi_Y}{1-\mu_{(\overline{\pi},\varepsilon)}} - 1\right]$ in the curly bracket; (ii) the last term

$\eta_{(\overline{\pi},\varepsilon)} (\theta - 1) \beta \overline{\Pi}^{1-\varepsilon} \left[\mu_{(\overline{\pi},\varepsilon)} (1 + \phi_Y) - 1\right]$. Both of them are multiplied by $\eta_{(\overline{\pi},\varepsilon)}$. So assuming $\eta_{(\overline{\pi},\varepsilon)}$ is small enough, then (23) is the relevant condition. In our numerical exercises, $\eta_{(\overline{\pi},\varepsilon)}$ is indeed very small ($2.081\,1 \times 10^{-3}$ is the highest value for the 8% annual inflation), so that the necessary condition (23) approximates quite well the necessary and sufficient condition.

Parameters	*Description*	*Reference*
β	Subjective discount factor	Eq. (1)
σ_n	Intertemporal elasticity of labour supply	Eq. (1)
θ	Dixit-Stiglitz elasticity of substitution	Eq. (5)
α	Calvo probability not to optimize prices	Eq. (5)
ε	Degree of overall indexation	Eq. (5)
ω	Extent of past inflation indexation	Eq. (5)
$1-\omega$	Extent of trend inflation indexation	Eq. (5)
$\overline{\Pi}$	Central Bank's inflation target (or trend inflation)	Eq. (5)
NKPC Coefficients		
Δ_t	$\widehat{\Pi}_t - \varepsilon\omega\widehat{\Pi}_{t-1}$	Eq. (12)
$\lambda_{(\overline{\pi},\varepsilon)}$	$\frac{\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]\left[1-\alpha\beta\overline{\Pi}^{\theta(1-\varepsilon)}\right]}{\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}$	Eq. (12)
$\eta_{(\overline{\pi},\varepsilon)}$	$\beta\left(\overline{\Pi}^{1-\varepsilon}-1\right)\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]$	Eq. (12)
$\xi_{(\overline{\pi},\varepsilon)}$	$\frac{\theta\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\left(\overline{\Pi}^{1-\varepsilon}-1\right)}{1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}$	Eq. (12)
Other coefficients		
$\delta_{(\overline{\pi},\varepsilon)}$	$\frac{\left(1-\beta\overline{\Pi}^{1-\varepsilon}\right)\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]-\eta_{(\overline{\pi},\varepsilon)}(\theta-1)}{\lambda_{(\overline{\pi},\varepsilon)}\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]}$	Proposition 1
T	trace of $\mathbf{A}$	Eq. (32)
M	sum of leading minors of $\mathbf{A}$	Eq. (33)
D	determinant of $\mathbf{A}$	Eq. (34)

Table 1: Parameters and basic symbols

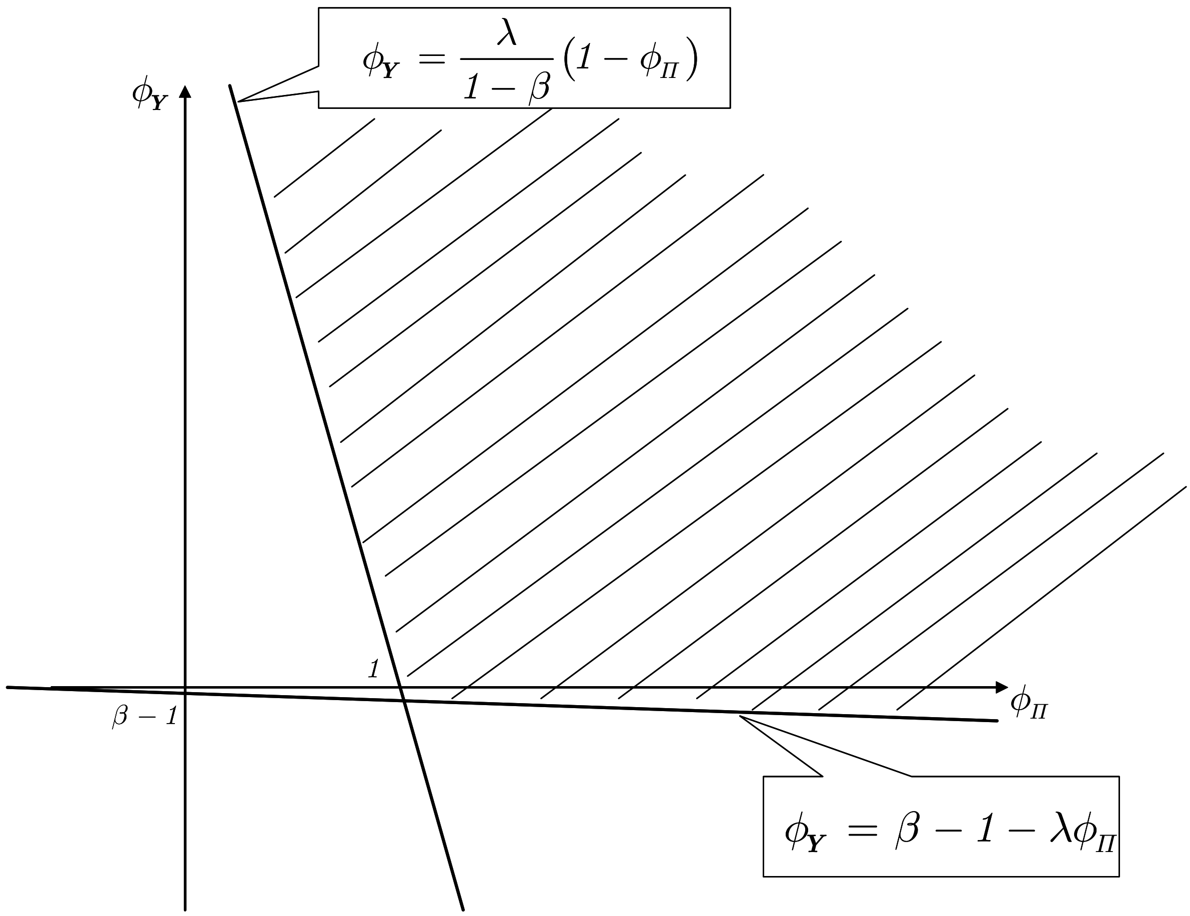

Figure 1: The determinacy region in the zero inflation steady state case.

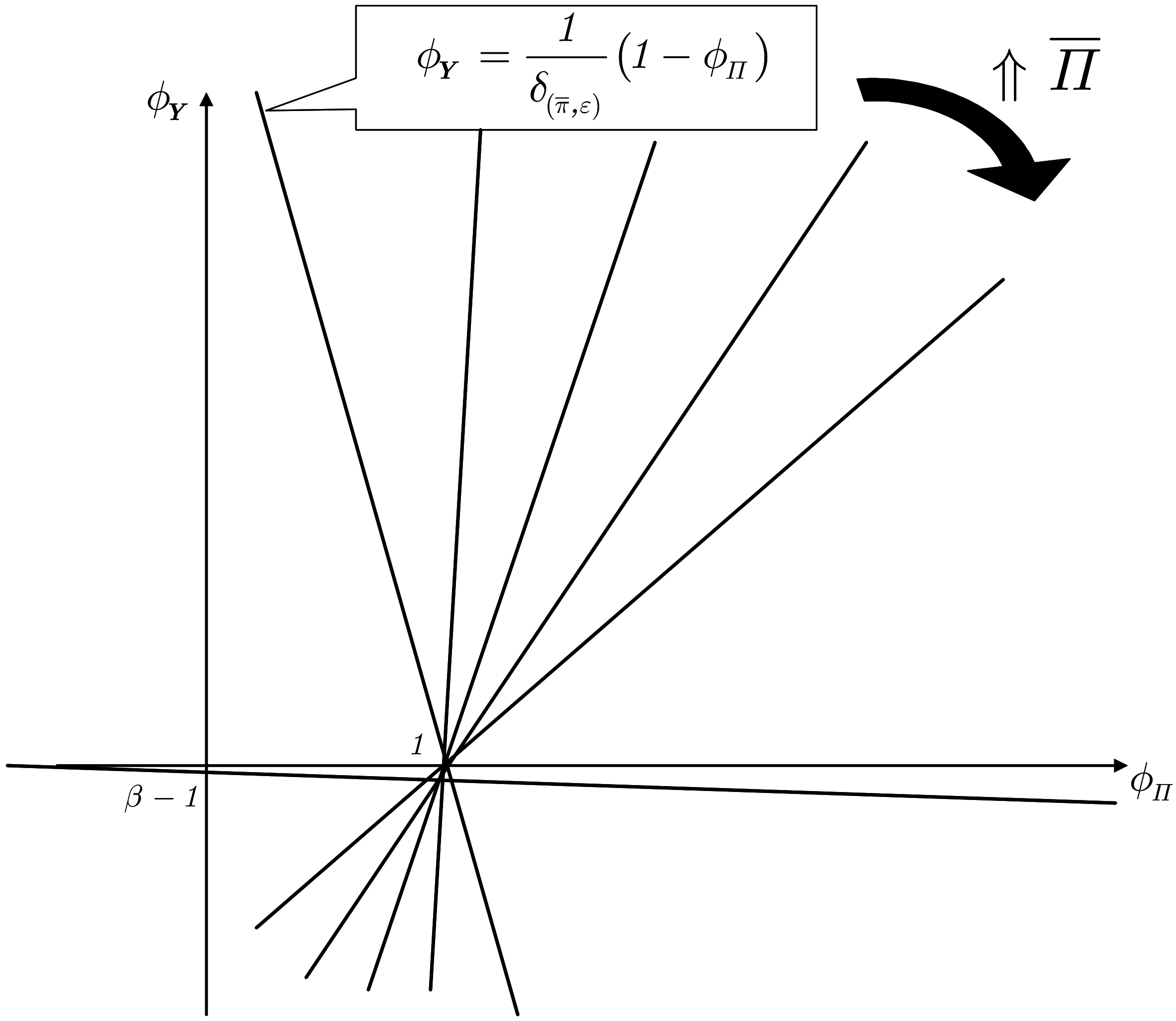

Figure 2: The effect of trend inflation on the Taylor principle.

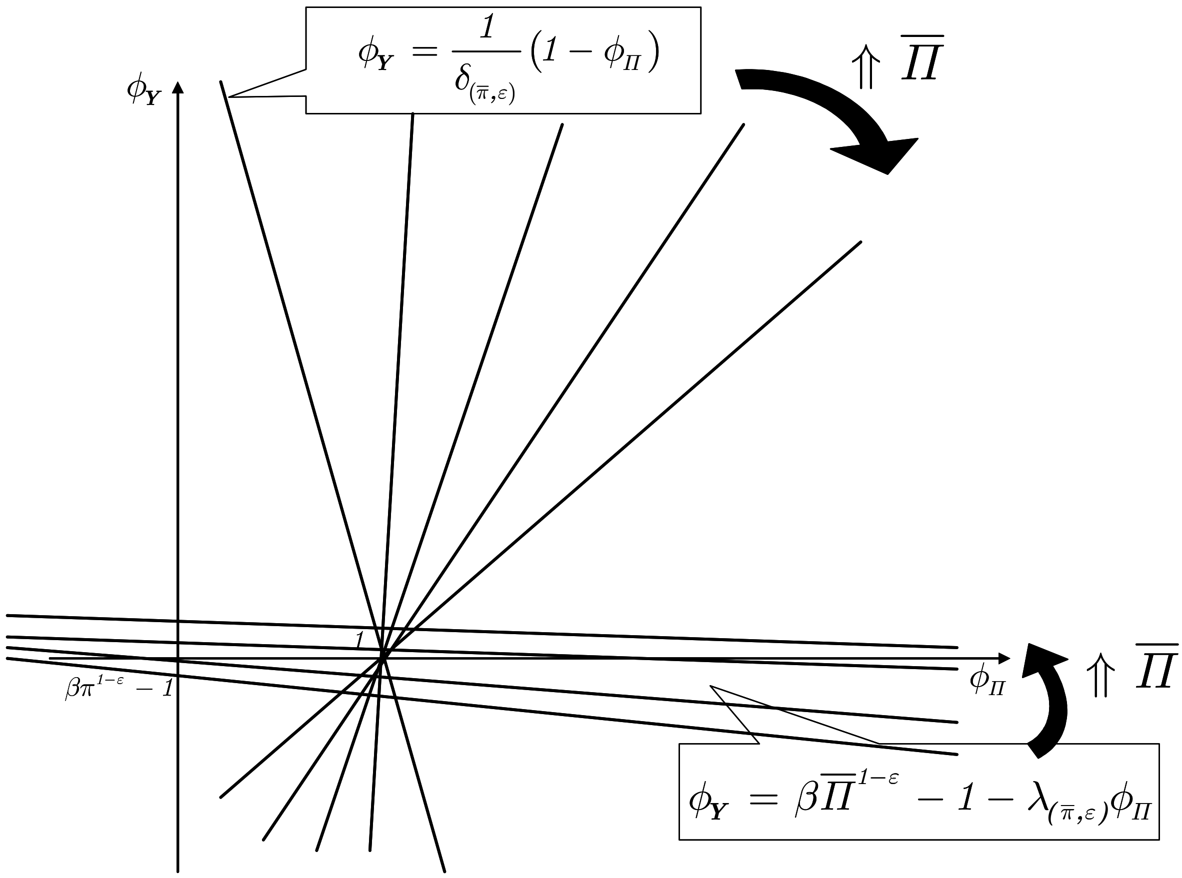

Figure 3: The effects of trend inflation on the determinacy conditions.

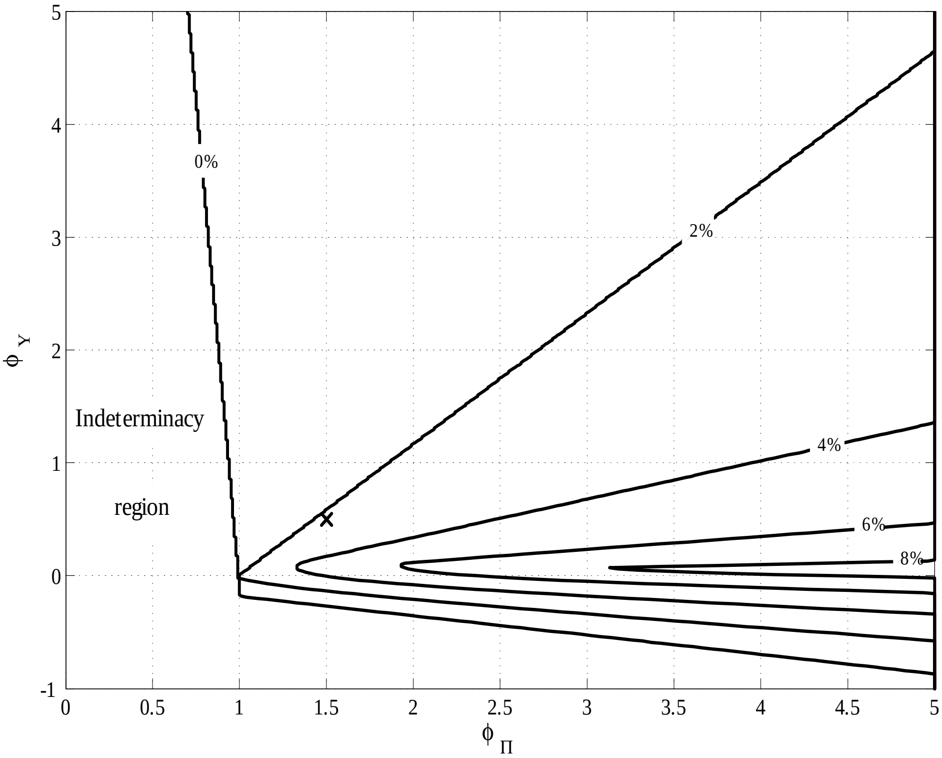

Figure 4: Contemporaneous nominal interest rate rule and the effects of trend inflation on REE determinacy. The cross marker identifies the classic Taylor rule specification, i.e. $\phi_\pi = 1.5$ and $\phi_Y = 0.5$.

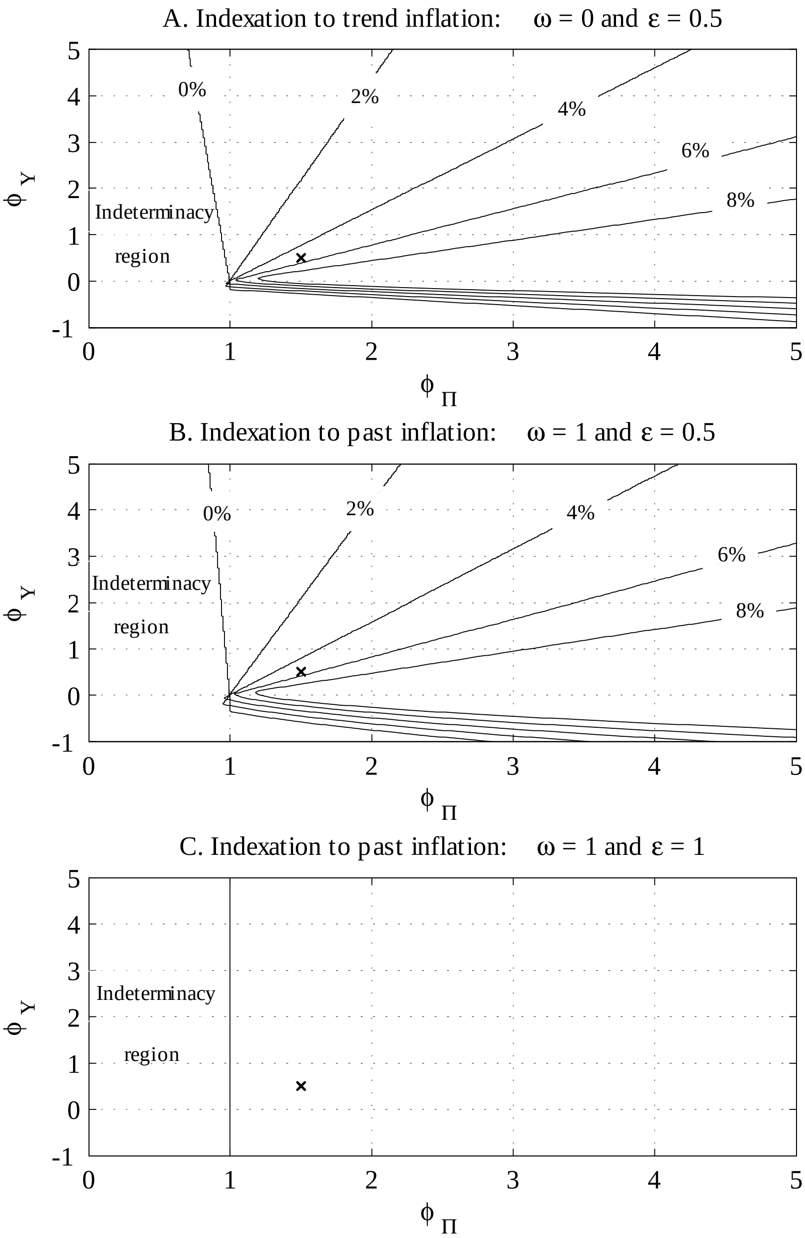

Figure 5: Contemporaneous interest rate rule, price indexation and the effects of trend inflation. The cross marker identifies the canonical Taylor rule, i.e. $\phi_\pi = 1.5$ and $\phi_Y = 0.5$.

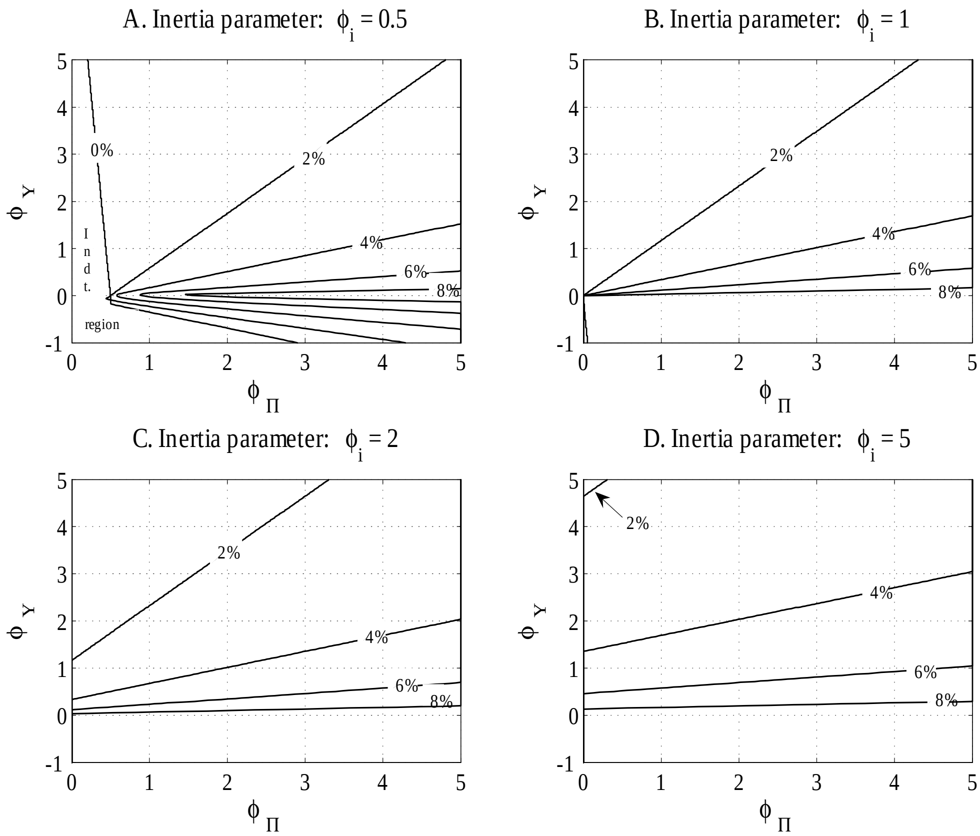

Figure 6: Inertial contemporaneous interest rate rule and the effects of trend inflation.

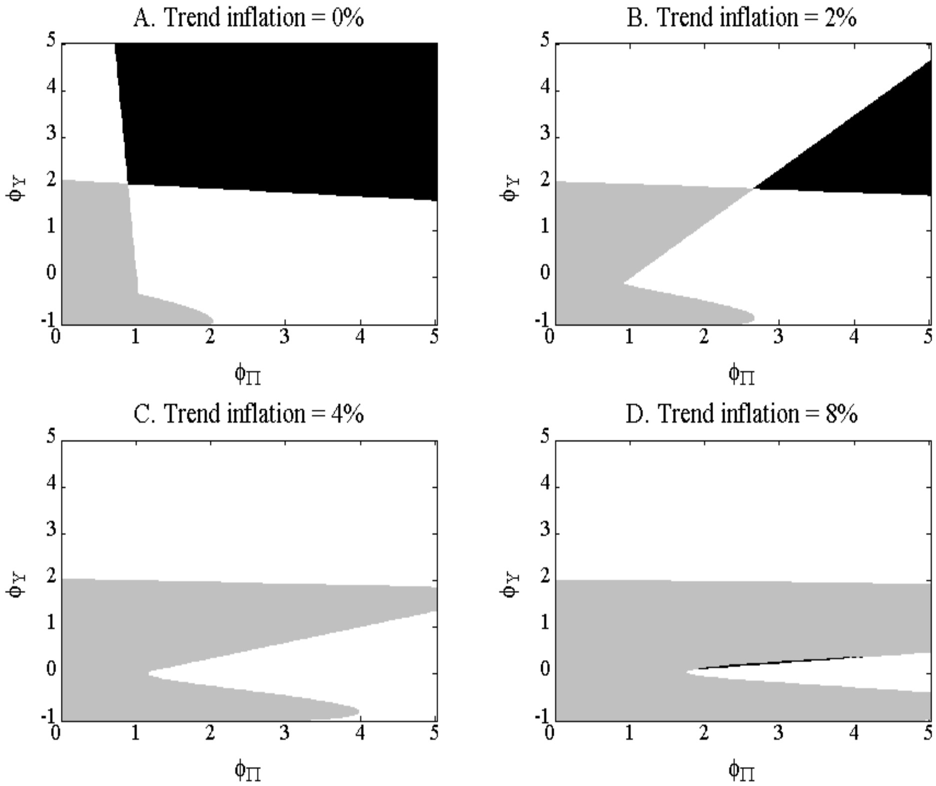

Figure 7: Backward looking interest rate rule and the effects of trend inflation (Black area = REE instability; Grey = REE indeterminacy; White = REE determinacy).