

Real Business Cycle Theory

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- Introduction: Lucas' methodological proposal
- The application to the analysis of business cycle fluctuations: The Real Business Cycle Theory
 - ① Data: measuring the business cycle (Table 1)
 - ② The model economy: a rigorous description
 - ③ Table 2: matching moments
- The solution of DSGE models: the Blanchard-Khan method
- Evaluation of the RBC approach
- RBC and the labour market

- **TRADITIONAL BUSINESS CYCLE THEORY:** output trend \bar{Y}_t evolves smoothly over time, $\bar{Y}_t = a + bt$. Cycles are viewed as deviation from trends, i.e. $Y_t - \bar{Y}_t$
- **RBC THEORY:** cycles can be explained also assuming that \bar{Y}_t evolves according to a random walk, i.e., $\bar{Y}_t = b + \bar{Y}_{t-1} + u_t$. In this case much of the movements in Y_t are due to movements in \bar{Y}_t rather than to trend deviations $Y_t - \bar{Y}_t \Rightarrow$ Integrating growth and business cycle theory

The Lucas Methodological Proposal

- Lucas ('76,'77,'80,'87). Two important references:
 - ① Lucas, R.E., 1976, *Econometric Policy Evaluation: A Critique*, Carnegie-Rochester Conference Series on Public Policy, Vol. 1, pp. 19-46.
 - ② Lucas, R.E., 1987, *Models of Business Cycles*, 1985 Yrjö Jahnsson Lectures, Basil Blackwell, Oxford.

1. **The Lucas' critique:** Macroeconomists should build so-called structural models, i.e. models where the agents' behavior is invariant with respect to policy
 - Microeconomic foundations
 - General Equilibrium
 - **No distinction between micro and macro: Economic theory**

2. **Explicitly dynamic models** from the outset:

- Dynamic optimization
- Need for a theory of expectations formation => The Rational Expectations Revolution of the 1970s is the logical outcome of Lucas' research program

3. **The Methodological Proposal**

- New analytical and computational instruments (Lucas/Stokey and Prescott, Kydland and Prescott)
- A new equilibrium concept: recursive equilibrium and "from a point to a path"
- Importance of expectations in the design of policy experiments
- **Welfare analysis**

CONCLUSIONS

- Modern macroeconomics should employ dynamic general equilibrium models (DSGE), that is, a macroeconomic model should be the results of the solution of dynamic optimization problems under uncertainty by optimizing agents populating the model economy.
- Build a "laboratory economy": much more difficult task than old Keynesian theorizing
- Kydland & Prescott (1982) accepted the challenge posed by Lucas: they built the first Real Business Cycle (RBC) model.

The basic RBC model in a nutshell

- Outline of the RBC methodology:

a discrete-time stochastic model of the economy populated by maximizing households and firms

MAIN SOURCE OF FLUCTUATIONS:

- The erratic nature of technological progress

THE BASIC MODEL: AS SIMPLE AS POSSIBLE

- The Ramsey model of growth \Rightarrow integrating growth and cycle
 - All prices are flexible
 - All markets are walrasian
 - Rational expectations
 - No money
- Endogenous labour
- Technology shocks (and possibly other shocks) \Rightarrow stochastic steady state

MAIN RESULT AND FIRST INTUITION

- There is only one final good in the economy which is produced according to a constant return to scale (CRS) production function

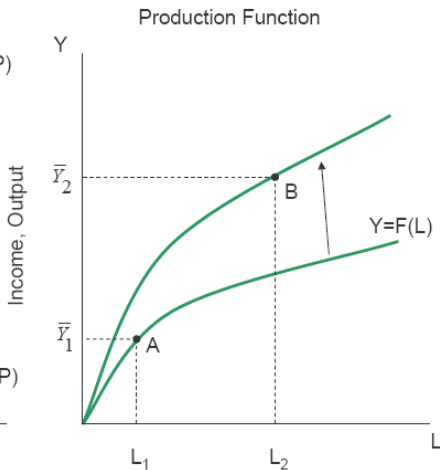
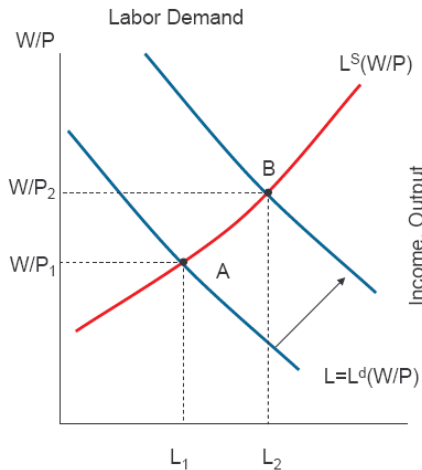
$$Y_t = A_t F(K_{t-1}, N_t)$$

where $\ln(A_t/A) = a_t$ is an exogenous process of technological progress (or total factor productivity TFP), which evolves according to:

$$a_t = \rho_a a_{t-1} + \hat{a}_t, \quad \hat{a}_t \sim N(0, \sigma_a^2) \quad i.i.d.$$

- A positive shock to the TFP shifts firms' labor demand and the AS curve
- Movements in employment and economic activity are seen as the efficient responses of a perfectly competitive economy to a productivity shock. \implies Stochastic path of Walrasian equilibria

POSITIVE TECHNOLOGY SHOCK



THE PLANNER PROBLEM

RBC models do not consider any distortion or market imperfection, therefore the welfare theorems apply to these models:

- 1) the competitive equilibrium is pareto-optimal
- 2) a pareto-optimal allocation can be decentralized as a competitive equilibrium

The social planner equilibrium and the competitive equilibrium are identical and admit a unique solution

THE BASIC MODEL: AS SIMPLE AS POSSIBLE

- Prescott did not think such a simple model could be of any use => surprising result!
- **Main policy conclusion:** fluctuations of all variable (output, consumption, employment, investment...) are the optimal responses to technology shocks exogenous changes in the economic environment.
- Shocks are not always desirable. But once they occur, this is the best possible outcome: business cycle fluctuations are the optimal response to technology shocks => no need for government interventions: it can be only deleterious

- Furious response from the "people from the Oceans" => Rogoff: "brilliant theories first look ridiculous then they become obvious".
- From mid'80s to mid'90s: ten years lost in useless ideological debates between the Oceans and the Lakes
- From mid'90s: convergence on methodology: "the RBC approach as the new orthodoxy in macroeconomics"

Data: Measuring the Business Cycle

H-P Filter

$$\text{Min}_{\{y_t^g\}_{t=0}^{\infty}} \sum_{t=1}^{\infty} \left\{ (y_t - y_t^g)^2 + \lambda [(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2 \right\}$$

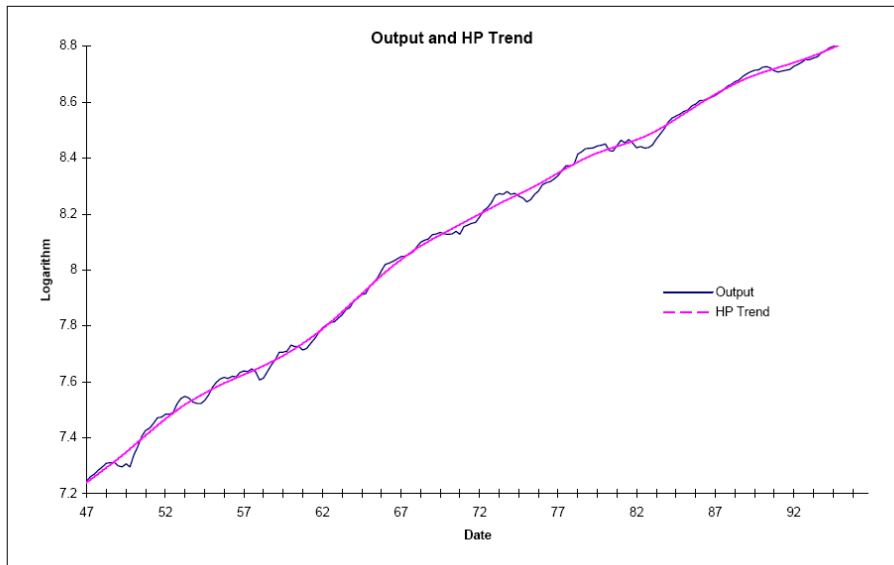
H-P filter suppresses the really low frequency fluctuations ≈ 8 years
quarterly data $\lambda = 1600$

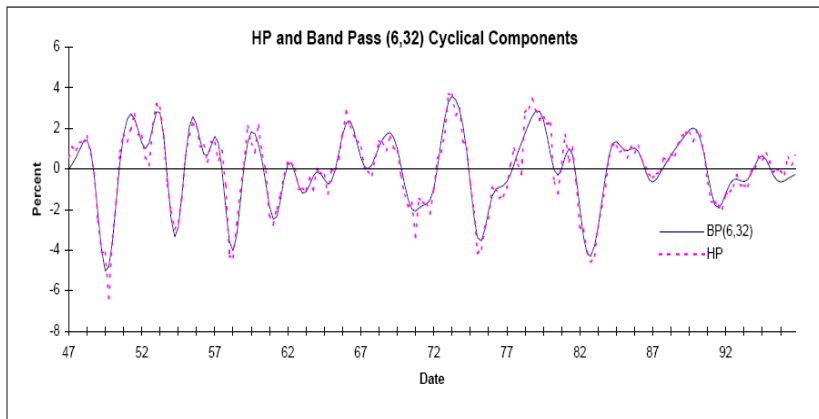
linear trend $\lambda \rightarrow \infty$

original series $\lambda = 0$

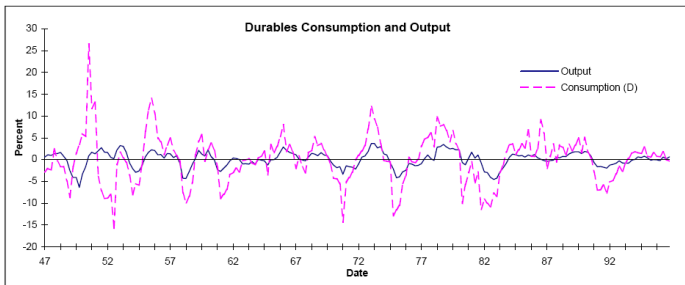
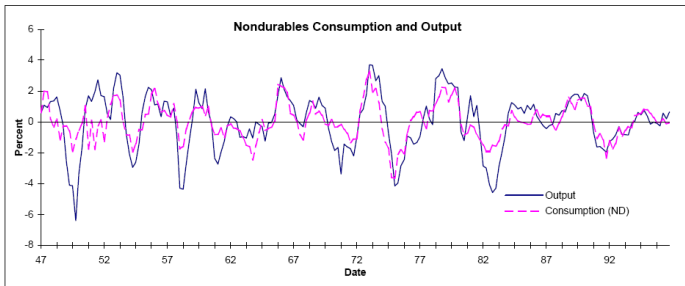
This makes the trend component a weighted average of past, present and future values \Rightarrow and the cyclical component is defined as

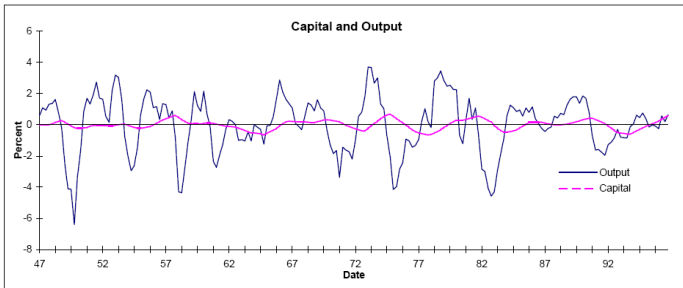
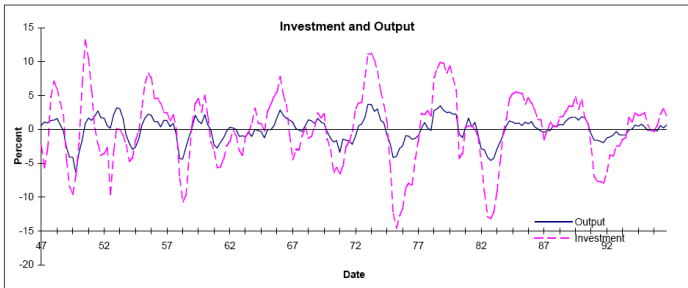
$$y_t^c = y_t - y_t^g = y_t - \sum_{j=-J}^J a_j y_{t-j}$$





Business cycles are all alike





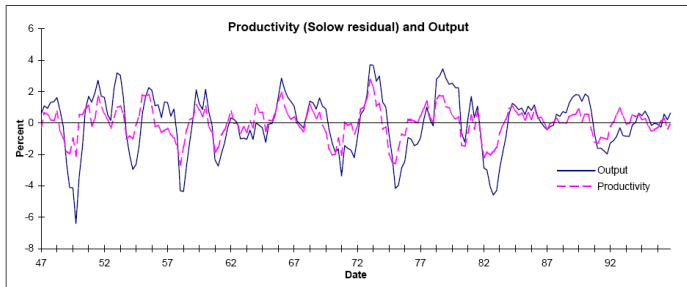
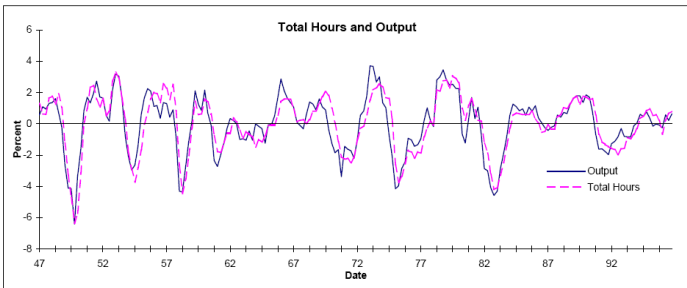
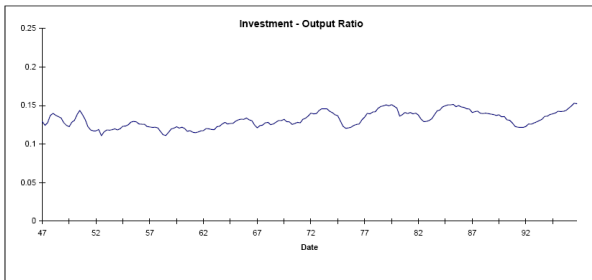
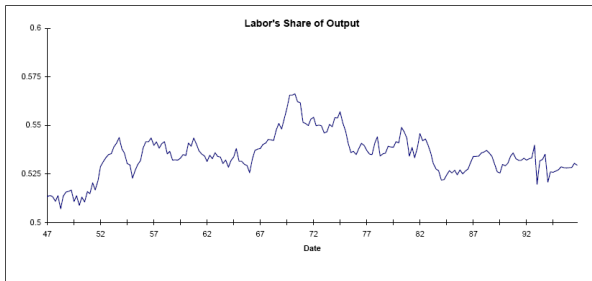


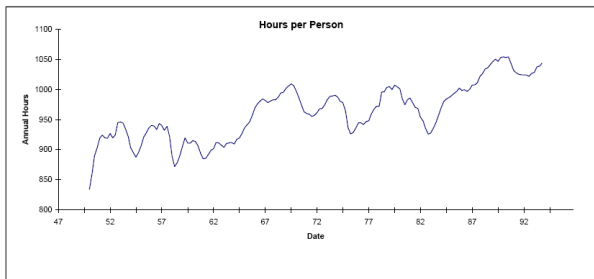
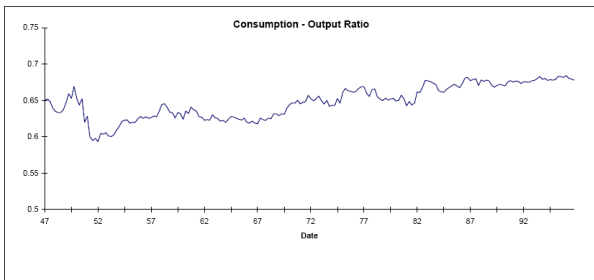
Table 1
Business Cycle Statistics for the U.S. Economy

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

Note: All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. Data sources are described in Stock and Watson [1998], who created the real rate using VAR inflation expectations. Our notation in this table corresponds to that in the text, so that Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity.

Some stylized facts about growth





The structure of a RBC model

- *Endowments.*

- One unit of time $l_t + h_t = 1$
- Initial stock of capital K_0
- Initial level of technology A_0

- *Preferences.*

- $\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t)$
- we will see restrictions on preferences

The structure of a RBC model

- *Technology.*
 - Only one final good
 - Production function: $Y_t = A_t F(K_t, H_t)$, where A_t is following a stochastic process. Agents must form expectations about it. They adopt the RE to do so.
 - Final output can be consumed or invested: $Y_t = C_t + I_t$
 - **Capital accumulation equation:** $K_{t+1} = (1 - \delta) K_t + I_t$

The structure of a RBC model

- *Markets and Ownership.*

- Households are the owners of labor h_t and capital k_t , which they rent to firms. Rental rates of labor and capital are: $w_t = \frac{W_t}{P_t}$ and r_t .
- Markets for inputs and output are competitive.
- The price of final output is normalized to 1.

The equilibrium is characterized by a sequences of $\{H_t, N_t, C_t, K_t, I_t, w_t, r_t\}$ such that

- Households solve their intertemporal problem, taking all prices w_t and r_t^k as given
- Competitive firms solve their profit maximization problem, taking all prices w_t and r_t as given
- All markets clear

$$\begin{aligned}k_t^S &= K_t^D \\h_t^S &= H_t^D \\C_t + I_t &= Y_t = A_t F(K_t, H_t)\end{aligned}$$

- Capital accumulation equation holds

This is the basic mechanism in the RBC model

- Intertemporal substitution in consumption \Rightarrow standard Euler Equation

$$\frac{u_c(c_t, 1 - h_t)}{\beta u(c_{t+1}, 1 - h_{t+1})} = 1 - \delta + r_{t+1}$$

- Intertemporal substitution in consumption \Rightarrow Lucas/Rapping (1969)

$$\frac{u_l(c_t, 1 - h_t)}{\beta u_l(c_{t+1}, 1 - h_{t+1})} = (1 - \delta + r_{t+1}) \frac{w_t}{w_{t+1}}$$

- Relative labour supply responds to relative wages between two different periods \Rightarrow households substitute labour intertemporally
- Also the interest rate matters for labour supply $\Rightarrow \uparrow r \Rightarrow \uparrow h^s$ today, because MPK is high \Rightarrow crucial channel for employment fluctuations
- What is the effect of $\uparrow w$ or $\uparrow r$?

- If you answered the previous question as it is, then go back to slide 5!!
- temporary $\uparrow w \Rightarrow$ substitution effect prevails $\uparrow h_t^s \Rightarrow \downarrow \left(\frac{c_t}{w_t} \right)$
 (given the intratemporal trade-off between consumption and labour:
 $\frac{u_l(c_t, 1-h_t)}{u_c(c_t, 1-h_t)} = w_t$)
- permanent $\uparrow w \Rightarrow$ income and substitution effects cancel out, no change in h_t^s and $\left(\frac{c_t}{w_t} \right)$
- Temporary increase in both w and $r \Rightarrow$ intertemporal substitution both in labour and consumption $\Rightarrow \uparrow\uparrow h_t^s$

The standard neoclassical intratemporal trade-off between consumption and labour

$$\frac{u_l(c_t, 1 - h_t)}{u_c(c_t, 1 - h_t)} = w_t$$

hence, for a given wage, C and H tend to move in the opposite direction
How one can get both C and H highly pro-cyclical?

- **Highly procyclical real wage (\Rightarrow productivity shocks!!)**

EXAMPLE: With $U(C_t, L_t) = \log C_t - \frac{N_t^{1+\phi}}{1+\phi}$, then, the labour supply becomes:

$$w_t = C_t N_t^\phi$$

or

$$N_t = \left(\frac{w_t}{C_t} \right)^{\frac{1}{\phi}}$$

NOTICE that:

$$\frac{\partial N_t}{\partial w_t} \frac{w_t}{N_t} = \underbrace{\frac{1}{\phi} w_t^{\frac{1}{\phi}-1} C_t^{-\frac{1}{\phi}}}_{\frac{\partial N_t}{\partial w_t}} \underbrace{\frac{w_t}{C_t^{-\frac{1}{\phi}}}}_{\frac{w_t}{N_t}} = \frac{1}{\phi}$$

is the elasticity of labor supply with respect to real wages (Frisch elasticity). The higher is ϕ the lower is the Frisch elasticity of labor supply.

- Next we will see **in details** an example taken from King, Plosser and Rebelo (JME, 1988)
- We will look at how to solve the model
 - from a theoretical perspective \Rightarrow Blanchard and Khan (Ecta, 1980)
 - from a numerical perspective \Rightarrow simulation codes

HOW TO SOLVE THE MODEL - SEVEN STEPS

- 1 Find all the first order necessary conditions
- 2 Calculate the economy steady state
- 3 Log-linearize the model around the steady state
- 4 Solve for the recursive law of motion
- 5 Calculate the IRFs in response to different shocks
- 6 Calculate the moments: correlations, and standard deviations for the different variables both for the artificial economy and for the actual economy.
- 7 Compare how well the model economy matches the actual economy's characteristics

We will see that

- Productivity shocks are central. They need to be
 - LARGE
 - PERSISTENT
 - OTHER SHOCKS?
- that's because the internal propagation mechanism of the basic model is weak (Cogley and Nason, AER, 1995)

Before going to the example, the solution algorithm and the codes, let's ask ourselves:

IS THIS MODEL PROMISING?

The (un)importance of capital accumulation

Solow (1956)

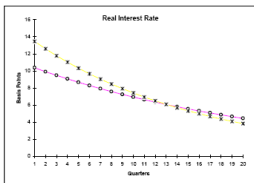
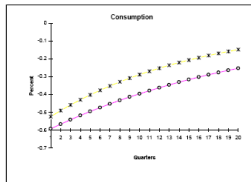
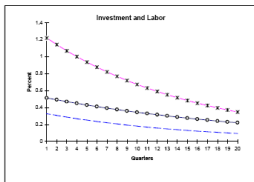
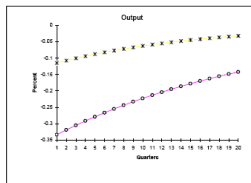
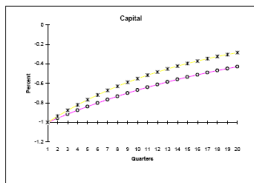
$$\begin{aligned}\log SR_t &= \log Y_t - \alpha \log H_t - (1 - \alpha) \log K_t = \\ &= \log \left(\frac{Y_t}{H_t} \right) - (1 - \alpha) \log \left(\frac{K_t}{H_t} \right)\end{aligned}$$

so the change in average productivity can be decomposed into

$$\Delta \log \left(\frac{Y_t}{H_t} \right) = \Delta \log \left(\frac{Y_t}{H_t} \right) - (1 - \alpha) \Delta \log \left(\frac{K_t}{H_t} \right)$$

It turns out that capital accumulation explains only 1/8 of the total change in average labour productivity => Romer: maybe we should look elsewhere

Transitional dynamics is not right

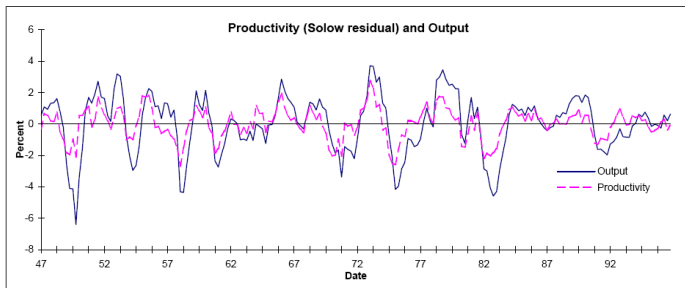


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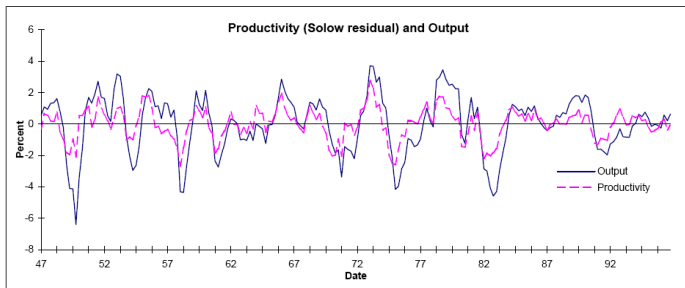
- notesRBC.pdf
- detailmodel_matlab.pdf

- **Empirical philosophy:** calibration vs. estimation
 - Too many parameters?
 - Too much flexibility in choosing parameters? "The book of Ed"
 - No possible statistical testing against alternatives
 - is matching moments a desirable feature? econometrician are suspicious of correlations

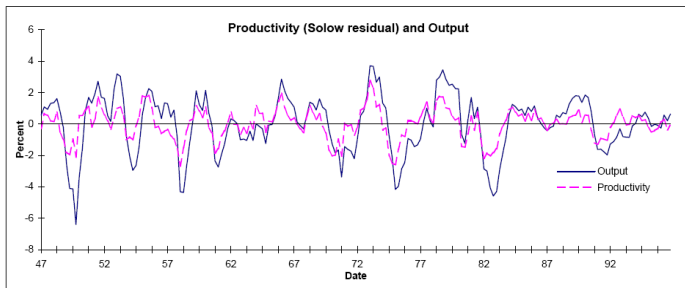
- **Solow residual and technology shocks**



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- Need for highly persistent technological shocks: why? and above all: what are they?



- **Solow residual and technology shocks**
- Need for highly persistent technological shocks: why? and above all: what are they?
- Solow residual is highly correlated with output



RBC View:

- RBC theory argue that the strong correlation between output growth and Solow residuals is the evidence that productivity shocks are an important source of economic fluctuations.

Critics:

- Are economic fluctuations really caused by productivity shocks?

Expansions arise because of increases in productivity!

... **What does that mean about recessions? (Summers 1986)**

- It means that recessions are periods of technical regress! Burnside et al. estimates the probability of technological regress implied by SR to be 37%!
- Less implausible if supply shock considered more broadly (OPEC, strikes etc.)

Real Achille's heel: "measure of our ignorance"

- Hall (1988): SR is useful to forecast military spending or other variables that should not be affected by technological shocks
- Hall (1990): SR is mis-measured if there is imperfect competition, IRTS, or labour hoarding, or variable capacity utilization
- Evans (1992):
 - money, interest rates and public spending Granger-cause SR
 - a substantial component of σ_{SR} seems to be caused by aggregate demand shocks

Bottom line: SR as measured out from a simple C-D production function is very spurious

The Labour Market

- Need for an highly elastic labour supply \Rightarrow labour market problems
The increase in productivity translates to an increase in hours worked and to an increase in real wages. The effect on the real wage is relatively stronger the lower is the elasticity of labor supply $\frac{1}{\phi}$. In the extreme case of fixed labor supply all the increase in labor demand would translate into an increase in the real wage.

- **CRITICS:**

- labor supply does not depend that much on the intertemporal real wage;
- high unemployment is mainly involuntary

Two problems:

- $\sigma_h > \sigma_w$ in the data, but not in the model
 - Fixer: Need for an highly elastic labour supply (Hansen, JME, 85)
- $\text{corr}(H, w) \simeq 0$ in the data, close to 1 in the model
 - Fixer: single shock problem (Christiano and Eichenbaum, AER, 92)

Table 3

Cyclical Properties of U.S. and Model-Generated Time Series

Type of Data or Model	% S.D. of Output σ_y	Variable vs. Output				Hours vs. Productivity	
		Consumption σ_c/σ_y	Investment σ_i/σ_y	Hours σ_h/σ_y	Productivity σ_w/σ_y	σ_h/σ_w	$\text{cor}(h,w)$
U.S. Time Series*							
Output	1.92	.45	2.78	—	—	—	—
<i>Hours Worked:</i>							
1. Household Survey (All Industries)	—	—	—	.78	.57	1.37	.07
2. Establishment Survey (Nonag. Industries)	—	—	—	.96	.45	2.15	-.14
Models**							
Standard	1.30	.31	3.15	.49	.53	.94	.93
Nonseparable Leisure	1.51	.29	3.23	.65	.40	1.63	.80
Indivisible Labor	1.73	.29	3.25	.76	.29	2.63	.76
Government Spending	1.24	.54	3.08	.55	.61	.90	.49
Home Production	1.71	.51	2.73	.75	.39	1.92	.49

*U.S. data here are the same as those in Table 2; they are for the longer time period: 1947:1–1991:3.

**The standard deviations and correlations computed from the models' artificial data are the sample means of statistics computed for each of 100 simulations. Each simulation has 179 periods, the number of quarters in the U.S. data.

Source: Citicorp's Citibase data bank

Fixing the first problem

Modelling the extensive margin

Most of movements in unemployment hours comes from movements from in and out of unemployment

Assume workers either work 0 hours or work h_0

Given total labour demand H_t , the numbers of workers needed are $E_t = \frac{H_t}{h_0}$

Rogerson's lotteries \Rightarrow Given the labour force N_t , the probability of getting a job is $\frac{E_t}{N_t} = \frac{H_t/h_0}{N_t}$

Expected utility from leisure

$$\begin{aligned} E(U) &= \frac{H_t/h_0}{N_t} U(c, 1 - h_0) + \frac{N_t - H_t/h_0}{N_t} U(c, 1) = \\ &= \frac{H_t/h_0}{N_t} [U(c, 1 - h_0) - U(c, 1)] + U(c, 1) \end{aligned}$$

LINEAR IN H_t !! \Rightarrow infinite elasticity

Fixing the second problem

Government spending shock

Charts 1–5

Hours Worked vs. Productivity in the Data and the Models

Percentage Deviations From Trend

Chart 1 The U.S. Data, 1947:1–1991:3
Based on the Household Survey

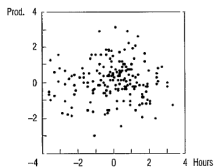


Chart 2 The Standard Model

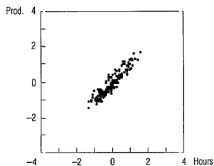


Chart 3 The Nonseparable Leisure Model

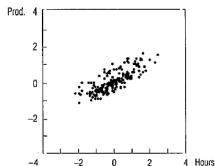
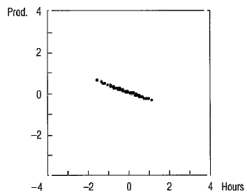
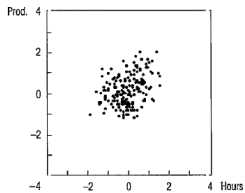


Chart 4 The Government Spending Model
Without Technology Shocks . . .



Source of basic data: Citicorp's Citibase data bank

Chart 5 . . . And With Technology Shocks



Are wages and prices flexible?

- RBC theory assumes that wages and prices are completely flexible, so markets always clear.
- RBC proponents argue that the degree of price stickiness occurring in the real world is not important for understanding economic fluctuations.
- They also assume flexible prices to be consistent with microeconomic theory.

Critics:

- Wage and price stickiness explains involuntary unemployment and the non-neutrality of money.

SUMMING UP

- Empirical significance of intertemporal labor substitution mechanism is doubtful.
- RBC theory implies that recessions are periods of technical regress!
- Money is neutral so does not explain positive correlation between prices and output; and that this can be rectified by endogenizing the money supply (Cooley and Hansen, AER, 1989)
- Wage and price rigidity can help to explain involuntary unemployment and non-neutrality of money