AIM

- Analyze the relationship between macroeconomics and financial markets, that is the bond and the stock market
- Introduce students to dynamic macroeconomics

MAIN MESSAGE

- Thinking in terms of dynamics
- The importance of expectations in macroeconomics
- Effects of policies depends on agents’ expectations, so they are difficult to analyze and to predict
To do that we use some model examples

1) Introduce into an IS-LM model:
   - Dynamics
   - The distinction between short-run interest rate and the long-run interest rate => The yield curve / term structure of interest rates
   - The stock market

2) Then we will analyze the dynamic theory of investment: the Abel (1982) model

Outline

- 1 Lecture: A dynamic IS-LM model with the yield curve
- 2 Lecture: A dynamic IS-LM model with the stock market
- Mentorium: Output, the Stock Market and Interest Rates: effects of Fiscal and Monetary policy
- 3 Lecture: The dynamic theory of investment: set up of the problem
- 4 Lecture: The dynamic theory of investment: solution of the Abel model
- Mentorium: Effects of taxes, news and uncertainty on investment

Slides available at: http://economia.unipv.it/pagp/pagine_personali/gascari/ascari_ff.htm
Main references

- Useful readings:

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Introduction

Pricing a bond

Pricing a share

How do economists price an asset?

**Price of an asset = present discounted value of expected returns**
The bonds market

**THE BONDS MARKET**

- **Bonds have a yield to maturity (Yield):**
  The yield is the interest rate of a bond given its price and maturity. *Yield-to-maturity* = average (annual/period) interest rate over the life of a bond.

- **Yield Curve or Term structure** => relationship between the yield-to-maturity and the maturity of a bond. The graphic representation of the yield relative to maturity on a given day (term structure of interest rates).
Let’s consider two types of bond:

1. a **1 year bond**, that pays 100€ after 1 year

2. a **2 year bond**, that pays 100€ after 2 years

Price of a bond = present discounted value of expected returns

- Price of a bond that pays 100€ next year
  \[ \mathbf{P}_{1t} = \frac{100}{1 + i_{1t}} \]

- Price of a bond that pays 100€ two years from now
  \[ \mathbf{P}_{2t} = \frac{100}{(1 + i_{1t})(1 + i_{t+1})} \]
ARBITRAGE

*Shall I buy a one-year or a two-year bond?*

If agents are only interested in the expected return

The two bonds should yield the same expected return in equilibrium

\[ 1 + i_{1t} = \frac{\mathcal{E}P_{1t+1}^e}{\mathcal{E}P_{2t}^e} \]
Hence

\[ 
\epsilon P_{2t} = \frac{\epsilon P_{1r+1}^e}{1+i_{1r}} 
\]

Today price of a 2-year bond is equal to the present discounted value of a 2-year bond price next year.

Sonext question: what is the expected price of 2-year bond tomorrow? ... easy:

\[ 
\epsilon P_{1r+1}^e = \frac{\epsilon 100}{1+i_{1r+1}^e} 
\]

But if

\[ 
\epsilon P_{2r} = \frac{\epsilon P_{1r+1}^e}{1+i_r} \quad \text{and} \quad \epsilon P_{1r+1}^e = \frac{\epsilon 100}{1+i_{1r+1}^e} 
\]

Then we recover the initial expression

\[ 
\epsilon P_{2r} = \frac{\$100}{(1+i_r)(1+i_{1r+1}^e)} 
\]

Arbitrage implies that the price of 2-year bonds is the present value of the payment in two years.
Yield to Maturity

The constant annual interest rate that makes the bond price today equal to the present value of future payments on the bond

\[
\frac{€100}{(1 + i_t)(1 + i_{t+1}^e)} = \frac{€100}{(1 + i_{2t})^2}
\]

Which is approximately equal to

\[i_{2t} \approx \frac{1}{2}(i_t + i_{t+1}^e)\]

Two-Year Rate = The average of the current one-year rate and next year’s expected one-year rate
Generalizing to a n-year bond

\[ i_{nt} \approx \frac{1}{n} (i_{lt} + i^e_{lt+1} + \ldots + i^e_{lt+n-1}) \]

Long-run interest rates are given by the average of the expected short-run interest rates.

The slope of yield curve indicates financial market expectations of future short-term rates. Upward sloping yield curve indicates that financial markets expect the short-term rates to increase in the future and vice versa.

A dynamic model of the yield curve
Assumptions

- Certainty => no risk-premia
- No transaction costs
- Continuous time
- Two bonds:
  - A short-run bonds (instantaneous) => yield = \( r(t) \)
  - A long-run bond (infinite maturity) that pays a constant coupon \( c \) each instant.

What’s the price \( Q \) of the LR bond?

- Price = \( Q = \frac{c}{R} \) where the yield = \( R = \frac{c}{Q} \)

What’s the instantaneous return of the LR bond?

- \( \dot{r}(t) = \frac{c}{Q(t)} + \frac{Q(t)}{Q(t)} \)

Arbitrage

- \( r(t) = \frac{c}{Q(t)} + \frac{Q(t)}{Q(t)} \)
Let's put $c=1 \Rightarrow Q = \frac{1}{R}$

so

$$\frac{\dot{Q}}{Q} = -\frac{R}{R^2} = -\frac{R}{R}$$

And

$$r = c - \frac{\dot{Q}}{Q} \Rightarrow \dot{R} = R(R - r)$$

Intuition: if $R > r$, then the yield on LR bond is bigger than the one on SR bond, hence it must be compensated by a **loss in capital gain** $\Rightarrow$ **expected decrease in the price** of the LR bond $\Rightarrow$ **$R$ is expected to increase** (and vice versa).

Only if $R = r$, no variation in the price of the LR bonds are needed to compensate the interest rate differentials.

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The LR interest rate is a forward-looking variable

- Linearize the last equation to get
  $$\dot{R} = R(R - r)$$

- This is a simple differential equation that can be solved as
  $$\dot{R} - RR = -Rr$$
  $$(\dot{R} - RR)e^{-\frac{R}{r}} = -Re^{-\frac{R}{r}}$$

  $$(R(T)e^{-\frac{R}{r}} - R(t_0)e^{-\frac{R}{r}}) = -\int_{t_0}^{T} Re^{-\frac{R}{r}} dt$$

  $$(R(T)e^{-\frac{R}{r}} - R(t_0)e^{-\frac{R}{r}}) = \int_{t_0}^{T} Re^{-\frac{R}{r}} dt$$

  $$(R(T)e^{-\frac{R}{r}} - R(t_0)e^{-\frac{R}{r}}) = R(T)e^{-\frac{R}{r}(T-t_0)} + \int_{t_0}^{T} Re^{-\frac{R}{r}(T-t)} dt$$
The LR interest rate is a forward-looking variable

Making $T$ tends to infinity: 

$$
\lim_{T \to \infty} R(T)e^{-R(T-t_0)} = 0
$$

and

$$
R(t_0) = R(T)e^{-R(T-t_0)} + \int_{t_0}^{T} R e^{-R(t-t_0)} dt
$$

becomes

$$
R(t_0) = \int_{t_0}^{\infty} r(t) \bar{R} e^{-R(t-t_0)} dt
$$

YIELD CURVE: Long-run interest rates are given by the weighted average of the expected short-run interest rates.

EXERCISE FOR THE MENTORIUM

Given 

$$
\dot{R} = \bar{R}(R - r)
$$

assume that the SR interest rate is following 

$$
\dot{r} = -\beta (r - \bar{r})
$$

so that it is converging at speed $\beta > 0$, to an equilibrium level: 

$$
\bar{r} = r = \bar{R} = R
$$

Derive the phase diagram in the space $(\bar{R}, r)$ and explain why the stable manifold is positively slope flatter than the 45° line.
A dynamic IS-LM model with the yield curve

- **LM** => $m(t) = k y(t) - \lambda r(t)$

- **AD** => $d(t) = \beta y(t) - \gamma R(t) + \delta g(t)$  \( \beta < 1 \)

- **Output Dynamics** => $y(t) = \sigma(d(t) - y(t))$

- **Yield Curve** => $\dot{R} = \bar{R}(R - r)$
A dynamic IS-LM model with the yield curve

- Substitute AD into output dynamics eq.
  \[ y = \sigma(\beta y - \gamma R + \delta g - y) \Rightarrow y = \sigma(\beta - 1)y - \sigma\gamma R + \sigma\delta g \]

- Substitute LM into the yield curve eq.
  \[ \dot{R} = \frac{1}{\sigma} \left( R - \frac{ky - m}{\lambda} \right) \Rightarrow \dot{R} = -\frac{ky}{\lambda} + \overline{R} \frac{R - \overline{R} + \lambda m}{\lambda} \]

- Two equations dynamic system in the space \((R,y)\) which are the two endogenous vbls, and two policy instruments \((g,m)\)

Building the phase diagram

- Find the **stationary loci**: lines so that the two vbls. do not move \(\Rightarrow y = 0\) and \(\dot{R} = 0\)

- **Equilibrium** = crossing of the two loci

- Describe qualitatively the dynamics outside the stationary loci

- Find the **stable manifold** of the **saddle point**: 1 forward-looking vbl and 1 backward-looking vbl.
Stationary loci

- **IS**

  \[ y = 0 \Rightarrow R = \frac{1}{\gamma}[(\beta - 1)y + \delta g] \quad \frac{dR}{dy} = \frac{\beta - 1}{\gamma} < 0 \]

- **LM**

  \[ R = 0 \Rightarrow R = \frac{1}{\lambda}(ky - m) \quad \frac{dR}{dy} = \lambda^{-1}k > 0 \]

The Phase Diagram

Which dynamics off the loci?
The Phase Diagram

\[ \dot{R} = \bar{R} \left( R - \frac{ky - m}{\lambda} \right) \]

LM (\( \dot{R} = 0 \))

\[ y = \sigma(\beta - 1)y - \sigma yR + \sigma \delta g \]

IS (\( y = 0 \))

Monetary and Fiscal policy effects

- Standard IS-LM analysis:
  - What if \( m \) increases (expansionary monetary policy)?
  - What if \( g \) increases (expansionary fiscal policy)?

- Is this a correct question “thinking in terms of dynamics”?
Monetary and Fiscal policy effects

We need to distinguish between

1) permanent vs. temporary

2) anticipated vs. not anticipated
Restrictive Monetary Policy: anticipated and permanent

Restrictive Fiscal Policy: anticipated and permanent

ANTI-KEYNESIAN EFFECTS OF FISCAL POLICY