Price Indexation and Optimal Simple Rules in a Medium Scale New Keynesian Model

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January 21, 2010

Abstract

Indexation was the subject of a substantial literature in the era of high inflation, but it was then neglected. Recently, it is becoming standard to assume backward looking indexation in prices in the New Keynesian DSGE models. Given the backward looking indexation scheme, we study how the *degree* of indexation affects the optimal simple policy rule in a medium-scale model. We show that both the shape and the parameters of the simple policy rule are affected by the degree of indexation. In particular, indexation stabilizes price dispersion, thus optimal monetary policy does not need to focus on stabilizing inflation. We also show that full indexation is a very peculiar assumption, eliminating the price dispersion mechanism in the model.

JEL classification: E31, E52.

Keywords: Indexation, Trend Inflation, New Keynesian model

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1 Introduction

Indexation of wages and prices was the subject of substantial literature in macro in the era of high inflation (see the seminal paper by Gray, 1976). However, after the disinflation in the '80s, indexation vanished from the policy and scientific debates. Quite recently many authors reintroduced indexation in the New Keynesian models, within the Calvo price staggering framework (e.g., Yun, 1996, Christiano et al., 2005, CEE henceforth). These models are then used for both positive and normative policy analysis, that is, to describe the functioning of the monetary transmission mechanism and to investigate the optimal monetary policy. However, it should be clear that the results of both the analysis do depend on the assumed indexation scheme.

In this branch of the literature indexation is introduced in a completely *ad hoc* manner. When indexation is introduced into a model, two issues arise: the *type* and the *degree* of indexation. With respect to the first issue, the nowadays most popular form of indexation embedded in these models is the so-called backward-looking indexation, firstly suggested by CEE. The main reason is empirical in order to have a lagged term in the New Keynesian Phillips Curve (NKPC henceforth) to match the inflation persistence found in the data (see e.g., Galí and Gertler, 1999, Mankiw, 2001, Rudd and Whelan, 2007). With respect to the second issue, indexation is most of the times assumed to be full (e.g., CEE and Altig et al., 2004).

This paper looks at the second issue. So we do not address the normative question about which is the optimal indexation scheme.[1] We rather take the by now standard backward-looking indexation scheme proposed by CEE and ask the following question: given the backward looking indexation scheme, how the *degree* of indexation affects the optimal policy rule.

We think this is an interesting question to answer for both empirical and theoretical reasons.

From an empirical perspective, past inflation indexation does not seem a structural parameter and it is estimated to be partial. First, in a recent paper Benati (2008) strongly criticizes the hardwiring of a backward-looking component in the NKPC, because the original finding of inflation persistence in the literature is due to a failure to

[1] This is an obviously important question that we leave to further research. A first interesting attempt in this direction is Mash (2007).

control for structural breaks in trend inflation due to change in monetary policy regimes. In other words, the inflation persistence found in the data for developed countries is not structural in the sense of Lucas. It follows that also backward-looking indexation, as a mechanism to insert an intrinsic inflation persistence term in the NKPC, is not invariant across monetary policy regime. Moreover, under the more recent stable inflation regimes, indexation is estimated to be zero (see Benati, 2009). A similar message is suggested by Cogley and Sbordone (2008), that estimates an NKPC with no intrinsic inflation persistence, but allowing for shifts in trend inflation. They find that this model successfully describes US inflation dynamics. Second, in the empirical estimates of medium-scale New Keynesian models, indexation is usually found to be only partial, and the estimates vary across papers, countries and samples.[2] Note that, given the wide range of estimates, even if one is willing to consider indexation as structural parameter, it seems interesting to evaluate the sensitivity of the normative conclusions in the literature to that estimate.[3]

From a theoretical perspective, indexation has many, and potentially conflicting, effects both on the dynamics and the long-run properties of the model (see Section 3). Backward-looking indexation changes the trade-offs monetary policy is facing. Woodford (2003) already shows how the degree of backward-looking indexation affects the appropriate microfounded loss function and the dynamics implied by the NKPC in a basic log-linear model. Moreover, in a non-linear model with positive trend inflation, price dispersion is another important channel through which backward-looking indexation affects the dynamics of the model and optimal policy (see Schmitt-Grohé and Uribe, various papers).

As a consequence, from a normative perspective, backward looking indexation, through price dispersion, strongly impacts on the optimal policy and welfare. To our knowledge,

[2] See, e.g., Sbordone (2005) , Levin et al. (2006), Del Negro et al. (2005) and Smets and Wouters (2007) for the U.S., and Smets and Wouters (2003) for the Euro area.

[3] Various empirical papers assume partial indexation to both trend inflation and past inflation, to achieve full indexation in the long-run (Smets and Wouters, 2003, 2007). This specification guarantees money superneutrality, avoiding the problems arising with positive inflation in steady state (see Ascari, 2004). The general idea is that the Calvo pricing scheme should be thought as a reduced form of more "deep" nominal rigidity mechanisms (e.g., rational inattention), useful to deliver non-neutralities in the short-run, but not in the long-run. This specification will be investigated in Section 5.2.

the literature on optimal policy in a non-linear context mainly concentrates on extreme values of the degree of backward-looking indexation, either zero (Schmitt-Grohé and Uribe, 2006, 2007) or one (Schmitt-Grohé and Uribe, 2004a). However, full indexation is not optimal in many settings (e.g., Ascari and Branzoli, 2007a). Moreover, as we will see, the extreme values of such parameters may induce quite particular features of the model.

In this paper we aim to contribute to this literature with a thorough investigation of the effects of the *degree* of backward-looking indexation on the optimal monetary policy and welfare.[4] Despite this kind of indexation scheme is very commonly used, to the best of our knowledge no paper so far assesses the implications of different degrees of indexation for the shape of the optimal monetary policy rule. Indeed, as discussed above, there are neither positive nor normative compelling reasons for assuming either zero or full backward-looking indexation. It seems therefore important to assess how backward looking indexation affects price dispersion, and hence how it impacts on the optimal policy and welfare. This is what this paper does.[5]

We need to make several modelling choices to answer our research question. First, we use an "operational" medium-scale model, more precisely the model in Schmitt-Grohé and Uribe (2004a) (SGU henceforth). Once backward-looking indexation is introduced in a model, the order of the dynamic system increases such that no analytical results are possible even in the simplest New Keynesian model. Given that we need to resort to simulation, we decide to use a model with a rich set of both nominal and real frictions. This model is indeed becoming a sort of benchmark model for the literature and CEE show it to be able to replicate sufficiently well the behavior of the US business cycle.

Second, we confine our analysis to optimal, "simple and implementable" monetary policy rules, following closely SGU.[6] This insightful paper characterizes the optimal

[4] The degree of indexation is an important parameter, but it is obviously not a policy variable. The sense of this exercise thus should not be confused as looking for the "optimal" degree of indexation in any sense.

[5] On the contrary, many papers in the New Keynesian literature focus on the effects on the transmission mechanism, and hence on optimal policy and welfare, of another central and "not structural" parameter as the Calvo degree of price stickiness parameter (e.g., Woodford 2003). In Section 5.1, we will investigate how the two key reduced form parameters of the New Keynesian model (i.e., the degree of price stickiness and of backward-looking indexation) interact in shaping the optimal simple rule.

[6] Following SGU, we assume commitment to the rules. Results will be generally different under

coefficients in simple "operational" Taylor rules. For a rule to be "operational", it has to satisfy the following three properties: (i) it must be a function of a few readily observable macroeconomic variables; (ii) it must deliver an unique rational expectation equilibrium; (iii) it must induce an equilibrium that satisfy a constraint on the lower bound on the nominal interest rates.

Third, methodologically, we decide for a painstaking grid-search algorithm for the results in the main Section of the paper. As in SGU, we consider nine different cases, combining on the one hand backward-looking, current-looking and forward-looking Taylor rules and, on the other hand, no inertia, inertial and superinertial Taylor rules. This allows us to focus on the implications of the degree of backward-looking indexation for both the shape and the coefficients of the optimal simple rule from the point of view of the stochastic steady state. The grid search method allows us to find the global maximum in our parameter grid, but it is computationally very costly. We therefore switch to a numerical maximization algorithm, as in Schmitt-Grohé and Uribe (2006, 2007), in the robustness section of the paper, i.e., Section 5. Therefore, this section will also indirectly provide a robustness check of our results over the employed methodology.

Fourth, we just focus on the *degree of indexation of prices*, and not of wages. The main reason is that, given our research strategy, the curse of dimensionality is very high, so we could not perform both the analysis.[7] Moreover assuming full wage indexation seems more compelling from both an empirical and theoretical point of view,[8] and also from anecdotal evidence.

Our results should be first compared with the analysis in SGU. Their main findings are: (i) the optimal simple monetary policy rule is a forward-looking one with no inertia; (ii) the optimal and simple rule takes the form of a real interest rate targeting rule; (iii) the optimal rule should not respond to output: providing that the response to output is mute, there is actually not much difference in terms of welfare evaluation across the

discretion. We leave this interesting question to further research.

[7] For the main Section of the paper, we performed 677448 simulations, each of one take 90 seconds (on a standard Pentium IV (R) 3GHz), reaching a total of 16936.2 computer hours (almost 2 years).

[8] For example, Levin et al. (2006) estimates the average price indexation between 0.11 and 0.19 while the average wage indexation between 0.77 and 0.86 (see also Smets and Wouters, 2003, 2007). Furthermore Ascari and Branzoli (2007a) shows that full wage indexation maximizes the steady state welfare for every level of price indexation.

optimal rules for the different cases considered; (iv) inflation volatility under the optimal rule is significant if there is full indexation, while near zero if there is no indexation.

We complement their analysis focusing on the role of the degree of price indexation, and its link with price dispersion. Some of the results in SGU are confirmed. In particular, none of the several optimal policies in the various cases features a substantial reaction to output.[9]

Our main results confirm that the shape of the optimal policy is mainly driven by the degree of price dispersion in the economy. This becomes very clear in our analysis, because the degree of indexation is the key parameter that governs price dispersion. We were thus able to clarify the rationale for the prescription regarding the optimal volatility of inflation. The lower the degree of indexation, the higher is price dispersion, and the associated costs for the economy. The optimal response of the policy is to stabilize inflation in order to contain price dispersion. Indeed, we show that the variance of price dispersion decreases, while the one of inflation increases with indexation. Moreover, a general prescription for monetary policy is that the lower is the degree of indexation, the larger should be the response of the monetary policy to inflation gap.

Price dispersion, however, feeds back into the dynamics of inflation: low indexation induces price dispersion inertia that makes inflation more difficult to control. The lower is indexation, therefore, the more important is the ability to exploit the expectational channel of monetary policy in order to stabilize inflation and price dispersion. This task is better accomplished by inertial policies. Indeed, a robust finding is that the optimal rule is no inertial for high levels of indexation, while it is inertial or superinertial for low levels of indexation. In general, hence, the degree of price indexation changes the trade-offs monetary policy is facing in a non-obvious way.

Another important, and not much debated, issue regards the first order effects of changing the degree of indexation.[10] Our results show that the difference in conditional welfare across the various cases is mainly driven by the first order steady state effects,

[9] With the exception of the rules maximizing unconditional welfare, see Section 5.4.

[10] First order effects derive from a change in the steady state of the model, while second order effects derive from changes that do not influence the steady state (but only the dynamics around the steady state). In this sense, a change of the degree of indexation causes first-order effects because it affects the steady state, while a change in the policy rule parameters cause only second-order effects because it does not affect the steady state of the model.

suggesting that the literature often ends focusing on second-order analysis (as the shape of optimal policy in an approximated model), overlooking important first-order aspects (as the calibration of the degree of indexation) that strongly influence the shape of the optimal policy and have important welfare effects.

Finally, one of the main message of this paper is that full indexation is a very special assumption, because it nullifies the effects of price dispersion, which is one of the key variables in the Calvo type price staggering models. When prices that are not repotimized are fully anchored to inflation, price dispersion become irrelevant not only from a long-run perspective (as it is obvious), but also in the dynamics of the model. Full indexation is a very special case, almost like a discontinuity point, in this sense, because it cancels one of the main mechanism of the model.

This paper adds to the existing literature a thorough investigation of the link between the degree of indexation and price dispersion, and the effects of this interrelation on optimal policy and welfare. Another way of interpreting the results in this paper, however, is that one should be very careful in introducing *ad hoc* features, as backward-looking indexation, in a microfounded model to match some empirical evidence and hence use these models for normative analysis. The results of such an analysis are going to depend on the *degree* of indexation, and thus one needs to think deeply about the calibration of such a parameter, before drawing policy conclusions.[11]

The next Section will briefly sketch the main properties of the CEE model, that is formally described in the Appendix. Section 3 analyzes the effects of the degree of price indexation on the dynamics of the model. Section 4 presents the main results of the paper. Section 5 checks the robustness of the main results to different degrees of price stickiness, type of indexation, trend inflation level and measure of welfare. The last Section concludes.

2 The Model

The basic setup is a medium-scale macroeconomic model, obtained by augmenting the standard New Keynesian model with nominal and real frictions that are proved to be crucial in replicating the dynamics of US business cycle. Since the model is exactly the

[11] Leaving aside the important question of the optimal type of indexation scheme. This is an important question, that is beyond the scope of this paper. See footnote 1.

one described in many papers such as SGU and CEE[12], we will briefly introduce here the key elements, leaving to the Appendix all the details about the model and calibration.

The model displays both real and nominal frictions. The real features of the model are monopolistic competition, habit persistence in consumption, fixed cost in an otherwise standard Cobb-Douglas production function that generates increasing return to scale and guarantees zero profit in equilibrium, variable capacity utilization and adjustment costs in investment. Money is introduced into the model via real balances in the utility function and cash-in-advance constraint on wage payments of firms. Wages and prices are sticky à la Calvo. Furthermore, as in CEE, prices and wages, that are not reoptimized each period, are indexed to past inflation. CEE argues that all these frictions appear to be crucial in replicating the dynamics of macroeconomic variables along the business cycle.

As in SGU, the long-run level of inflation is set equal to the average inflation of the US in the post World-War II period, i.e. 4.2%. While this complicates considerably the analytics of the model, it is an important feature of the data and will be also crucial for our results, since our analysis needs a non zero steady state inflation.

Finally, monetary policy follows a Taylor-type rule, expressed in log deviation from steady state's values (denoted with a hat):

$$\hat{R}_t = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R E_t \hat{R}_{t-i}, \tag{1}$$

where $i \in \{-1, 0, 1\}$ indicates respectively forward, current and backward looking policies. We assume the existence of a full commitment technology. This specification considers only policies that are simple in the sense that they depend only on a few simple and readable macroeconomics variables.

3 The Macroeconomic Effects of Price Indexation

Before looking in depth at the various results of the simulations, it is useful to think about the effects of indexation on the equilibrium of the model. Price indexation reduces the degree of price dispersion in the economy. Indeed, in a standard Calvo pricing scheme without indexation, a positive trend inflation implies that there will be some firms with

[12] Other empirical papers use analogous set-up such as Smets and Wouters (2003) and Altig et al. (2004).

very low relative prices (i.e., the ones that set the price long ago in the past) and firms with high relative prices (i.e., the ones that instead just reset their prices). The degree of price dispersion is thus increasing in the level of trend inflation. It is well known that price dispersion is inefficient because it leads to a misallocation of resources across different firms and it acts as a negative shift in productivity (see e.g., Schmitt-Grohé and Uribe, 2004a, 2006 and Ascari, 2004). Indexation counteracts this effect, because it allows also the non price-resetting firms to keep up with the pace of inflation. Partial indexation, thus, decreases price dispersion for any given trend inflation level. Both CEE and SGU assume that both wages and prices are fully indexed, that is indexation is 100%, so that there is no price dispersion in steady state.

Ascari and Branzoli (2007a) shows that full indexation does not maximize the steady state welfare of the representative household in the SGU medium-scale model. In particular, they find the maximum steady state welfare level to be attained at $\tilde{\chi} = 1$ and $\chi = 0.8788$, that is, full wage indexation, but only partial price indexation (where $\tilde{\chi}$ and χ measure the degree of wage and price indexation respectively, see Appendix). The reason why full price indexation does not maximize steady state welfare lies in the effect of price indexation on the average mark-up in the economy. On the one hand, the lower χ, the lower the ratio of the general price level to the price charged by resetting firms, and the lower the average mark-up in the economy. On the other hand, the lower χ, the higher the mark-up of the resetting firms, and the higher the average mark-up in the economy. These two counteracting effects compensate exactly at $\chi = 0.8788$, while for lower levels of χ, the second effect dominates such that the average mark-up decreases with χ, and vice versa for higher levels of χ. There are thus neither normative, nor positive reasons (see footnote 2) to assume full indexation in prices. As a result, and given the computational burden, in this paper we decide to focus on the effects of price indexation and to assume full indexation in wages.

It is important to stress that partial price indexation has first order effects on the model. In other words, assuming full rather than partial indexation substantially affects welfare, and generally will do it more than other features of the model that have second order effects, like demand and supply shocks under optimized policy rules. We will indeed show that the degree of indexation is a key and important variable in determining the welfare ranking of different policies.

The degree of indexation strongly affects also the dynamics of NK models. The lower the degree of indexation, the more trend inflation would impact on the adjustment of aggregate variables and on the forward-looking decision of agents.[13]

First, the dynamics of price dispersion would not affect the log-linearized version of the model if one assumes full indexation, while it matters at first-order with partial indexation. Partial indexation introduces thus a further backward-looking variable in the model that would tend to increase the inertia in the dynamic adjustment. Loglinearizing (27) and (34) in Appendix, and then substituting the term referring to the newly reset price, it yields the following expression for the log-linearized dynamics of price dispersion, i.e., s :[14]

$$\hat{s}_t = \left[\frac{\eta\alpha\bar{\pi}^{(\eta-1)(1-\chi)}\left(\bar{\pi}^{1-\chi}-1\right)}{1-\alpha\bar{\pi}^{(\eta-1)(1-\chi)}} \right] \left(\hat{\pi}_t - \chi\hat{\pi}_{t-1}\right) + \alpha\bar{\pi}^{\eta(1-\chi)}\hat{s}_{t-1}. \tag{2}$$

Note that the lower the level of χ, the higher is the inertia in the price dispersion term. To visualize the intuition behind the effect of indexation on price dispersion and inflation, Figure 1 and 2 plot the impulse responses of price dispersion to a positive productivity shock for three different levels of indexation.[15] These figures show that indexation influences the dynamics of price dispersion both qualitatively and quantitatively. Indeed comparing Figure 1 and 2 shows that partial indexation induces a negative rather than a positive response of price dispersion. Moreover, the response is of a different order of magnitude with partial indexation (such that it seems that there is no response with full indexation in Figure 1). Finally, also the degree of indexation matters: moving from $\chi = 0.8788$ to $\chi = 0$ results in a much larger response of price dispersion and a greater inertia.[16] It is clear from eq.(2) that lower levels of indexation increase the effect of inflation on price dispersion changing both qualitatively and quantitatively the dynamics of the model.

Second, the degree of indexation affects the first-order condition of the price resetting

[13] See Ascari and Ropele (2007) for a thourough discussions of the effects of trend inflation on optimal policy in New Keynesian models.

[14] In (2), $\hat{\pi}_t$ is the log-deviation of inflation, χ is the degree of price indexation, $\bar{\pi}$ is trend inflation, α is the degree of price stickiness and η is the elasticity of substitution across goods.

[15] We shock the model with a 1% increase in aggregate productivity under a standard current looking Taylor Rule with $\alpha_\pi = 1.5$, $\alpha_y = \alpha_R = 0$.

[16] Note that price dispersion decreases after such a shock. Clearly this is true after shocks that reduce the inflation rate, as shown in Figure 3.

firms. Similarly to the analysis of the effects of trend inflation in Ascari and Ropele (2007), a lower degree of indexation makes the optimal decision of price setters more forward-looking, because they will give higher weights to future economic conditions, trying to keep up with the trend, rather than responding to current transitory level of demand and costs.

Third, also the dynamics of the aggregate price level depends upon the degree of indexation. In particular, past inflation indexation, by construction, induce inertial adjustment in the price level, by making the inflation rate in the log-linearized version of (34) a function of its own lagged value. The higher the level of indexation, the more backward-looking is the inflation rate.

The second and third effects point in the same direction regarding the effect of the level of indexation on the inertia of the inflation rate. Indeed it is easy to show (see Sahuc, 2006) that the higher the degree of past inflation indexation, the higher, quite intuitively, is the inertia in the NKPC. The first effect through price dispersion, instead, works in the opposite direction: the lower the level of indexation, the more price dispersion is biting in the model and the more inertial are its dynamics. Figure 3 clarifies how the net result of these opposite effects shapes the response of inflation: positive indexation induce the hump-shaped response of inflation that we observe in the data and delivers a more inertial adjustment.

In sum, it is clear that the dynamics of the model, and hence the trade-offs monetary policy is facing, are quite different depending on the level of indexation. The two main effects of indexation on the dynamics of the model goes through price dispersion and inflation. Partial indexation affects both qualitatively and quantitatively the dynamics of price dispersion, which is a quite important variable because it induces very high output costs in these kind of models. Partial indexation also affects the degree of persistence of inflation. It is not clear what are the implications for welfare and hence for the optimal monetary policy design. From the point of view of dynamics, the higher the degree of indexation, the lower and the less responsive and inertial is price dispersion, and the more persistent is inflation. From the point of view of welfare, other things equal, the higher the degree of indexation, the lower is the level of price dispersion, and the lower the welfare costs induced by price dispersion and its dynamics. In what follows we assess the importance of these arguments in shaping optimal operational monetary policies.

4 Optimal Operational Monetary Rules

As stated in the introduction, the aim is to analyze simple and implementable Taylor rules, such that these policies can be actually "operational" for policy makers. Simple rules, as the ones considered here, are very easy to communicate and to be understood by the public, helping the transparency of central bank behavior, and easy to implement since they are functions of few readable economic variables. An operational rule should also be implementable in the sense that should both deliver a unique rational expectation equilibrium and satisfy the lower bound on the nominal interest rate.[17] As in SGU, we looked for the optimal monetary policy numerically discretizing the support $[-3, 3]$ in intervals of length 0.0625 for α_π and α_y in the particular classes of rule of the form (1). Moreover, in (1): (i) i can take three different values, i.e., $i \in \{-1, 0, 1\}$ corresponding to forward- looking, current-looking and backward-looking policies respectively; (ii) $\alpha_R \in \{0, 1, 2\}$, corresponding to no inertial, inertial and super inertial rules respectively. On top of that, we also allow the price indexation parameter to vary across the following levels of indexation: $\chi \in \{0, .75, .80, .85, .8788, .90, .95, 1\}$. Given the curse of dimensionality of our grid-search method, we investigate only 8 levels of indexation. We hence performed 677448 simulations, each of one take 90 seconds (on a standard Pentium IV (R) 3GHz), so it is about 16936.2 computer hours (almost 2 years). This was made possible by optimizing the functioning of MATLAB symbolic toolbox, and clustering 30 computers. Given the great number of cases to look at, we organize the presentation of results in this Section as follows. First, we illustrate how indexation changes the optimal rule and the dynamic response of the economy *across all the different types of rules considered.* Second, we analyze in detail how indexation changes the optimal rule and the dynamic response of the economy in the case of *one particular rule*, i.e., forward-looking no inertia (FLNI). Third, we will illustrate how the optimal rule changes with indexation *within each class of policy rules*.

[17] Following SGU, formally, we require the logarithm of the equilibrium nominal interest rate not to be lower than two times the variance of the nominal interest rate, i.e., $ln(R^*) \geq 2\sigma_{\hat{R}_t}$. If the equilibrium nominal interest rate was normally distributed around its target value, then this constraint would ensure a positive nominal interest rate 98 percent of the time. However, both SGU (2007, 2008) and Ascari and Branzoli (2007b) show that this constraint is not important in shaping the results in models as the one considered here.

4.1 Price Indexation and the Optimal Simple Rule

In this section we investigate the effect of indexation on the overall optimal simple monetary policy rules. Table 1 shows the type of policy rule, the optimal values of the coefficients and the corresponding welfare levels, for different values of the degree of indexation. Table 2 displays the corresponding unconditional moments for some variables of interest: consumption, output, price dispersion and inflation.

Welfare

Table 1 shows two different welfare measures: steady state welfare and conditional welfare. Our operational rules are expressed in deviations from the steady state, thus the welfare level of the deterministic steady state does not depend on the policy rule. The steady state welfare instead do depend on the degree of indexation, since the value of χ determines the price dispersion and thus affects the variables of the model. The different levels of steady state welfare, therefore, can be thought as a measure of the magnitude of the first-order effects of changing the degree of indexation.

The conditional welfare instead takes into account the stochastic steady state of the economy, and therefore the second-order effects deriving from the volatility of the shocks. It follows that the stochastic steady state depends on the particular monetary policy rule considered.[18]

Table 1 shows that the conditional welfare is always lower than the correspondent steady state welfare, since both the transitional dynamics (from the deterministic to the stochastic steady state) and the second order effects due to the volatility of the variables are taken into account. The difference between the two, however, is tiny and basically invariant across indexation levels. The loss across optimal policies is hence basically determined by the steady state one. The optimal simple rules maintain the conditional welfare very close to the steady state one. For example, given our calibration, the best indexation degree is 0.8788. If instead, the economy features full indexation the steady state welfare loss amounts to 0.0023%, while the loss in terms of conditional welfare amounts to 0.0020%. If instead, the economy features no indexation the steady state

[18] The conditional measure of welfare assumes a initial state of the economy and takes into consideration the transitional dynamics from that initial condition to the stochastic steady state implied by the policy rule. We will assume that the initial condition is always the deterministic steady state (recall that the deterministic steady state varies with the degree of indexation).

welfare loss amounts to 0.14%, and the loss measured in terms of conditional welfare is basically the same. Hence, the first order effects of indexation are much more important than the second-order ones. This suggests that an optimal simple monetary policy does a good job in stabilizing the cycle around the deterministic steady state, regardless the degree of indexation, but cannot do much in compensating the first order effects deriving from different degrees of indexation.

The same point is visualized in Figure 4, that displays the percentage welfare gain of the different indexation levels with respect to zero indexation. Each bar displays the steady state welfare gain and the overall conditional welfare gain net of the former.[19] The graph shows that an increase in the level of indexation reduces both the steady state losses and the losses associated with the stochastic steady state under the optimal rule, since indexation acts as a partial correcting mechanism for those firms that can not optimize their price. However, the effects of indexation on losses due to the stochastic volatility of the model are very small, compared with the losses induced in the deterministic steady state. In other words, indexation has a first order effect on steady state because it affects price dispersion. Moreover, it has also effect on the ability of the economy to respond to shocks. The optimal rule can partly offset the latter, but it can not improve on the former, which, for realistic calibration, is much larger in magnitude.

Furthermore this key result is robust across types of policies. SGU shows that in the full indexation case, the difference between the conditional welfare levels associated to the optimal rules in the different cases is very small. That is, conditional on choosing the optimal rule for each one of the different class of policies,[20] the differences in conditional welfare levels are extremely tiny. This result holds also for any given level of χ considered.[21] Therefore, *conditional on choosing the optimal policy, indexation matters much more for welfare than the actual form (current, backward, inertial etc..) of the*

[19] That is, define ssw_χ and cw_χ the steady state welfare and the conditional welfare, respectively, associated with a given value of χ. Then, for all levels of χ analyzed, the percentage conditional welfare gain is defined as: $\frac{cw_\chi - cw_0}{cw_0}$, and the percentage steady state gain (normalized over the conditional one) as: $\frac{ssw_\chi - ssw_0}{cw_0}$. Then the black area in the graph is $\frac{cw_\chi - cw_0}{cw_0} - \frac{ssw_\chi - ssw_0}{cw_0}$.

[20] Recall that we are considering 9 different classes of policies, depending both on the inertia (no inertial, inertial and super inertial) and on the "lookingness" (forward- looking, current-looking and backward-looking policies).

[21] For a given level of χ, conditional on choosing the optimal rule for each one of the different class of policies, the differences in conditional welfare levels are low. Results are available upon requests.

monetary policy rule. The literature, however, focuses more on the second aspect (i.e., the optimal policy type), while it is rather careless when it comes to define the calibrated value for the degree indexation or to develop normative analysis on the indexation level. However, that is proven to be much more important in terms of welfare effects.

Policy Rule

Table 1 shows that the degree indexation changes also the type of optimal operational policy. While the forward looking rule with no inertia is found to be optimal for the highest level of indexation, it turns out that lowering the degree of indexation leads the forward-looking rule to be substituted by the current-looking one. The backward-looking policy is never optimal. When there is no indexation then the forward looking inertial policy is optimal.

The Importance of Price Dispersion

Table 2 has one clear message: the lower is indexation, the higher is price dispersion, and the lower is the variance of inflation. First, the column $E(s)$ reports the expected deviation of s from steady state. This is very low, meaning that the mean value of price dispersion is its steady state value, that we know decreases with indexation. Second, the unconditional variance, σ_s, do not change very much across different degrees of indexation (except for full indexation) and it is very small. *This means that the main task of the optimal operational rule is to stabilize the degree of price dispersion* around the steady state value. Price dispersion is the main inefficiency associated with inflation in New Keynesian models, because it acts like a negative productivity shift in this economy, and thus the optimal policy response calls for its stabilization.

A temporary surge in inflation generates an increase in price dispersion, that needs to be stabilized by monetary policy. As we argued above, moving away from full indexation increases significantly the inertia in price dispersion. Furthermore the lower the degree of indexation, the more current inflation is going to affect current price dispersion. It follows that the lower the degree of indexation, the more important is to stabilize inflation. As a matter of fact under optimal rules the variance of inflation reduces as the degree of indexation decreases.[22]

Table 2 also shows that full indexation is a very special case. Indeed, under full

[22] See section 4.2.3 for a further discussion of this point.

indexation the cost of price dispersion is of second order magnitude.[23] Indeed, despite the rather high volatility in inflation, the volatility of price dispersion is infinitesimal, that is price dispersion is almost always zero. It is interesting to note that, even moving away only slightly from full indexation, i.e. $\chi = 0.95$, considerably worsens the trade-offs monetary policy is facing. Indeed, the volatility of inflation drops by roughly a half, while the one of output increases by one third. Despite the lower volatility of inflation induced by a higher α_π, the volatility of price dispersion is higher by a factor 10^{10}! For the other values of indexation we analyze, instead, the volatility of price dispersion are of similar order of magnitude.[24] This indeed signals that full indexation is a quite special case. Unfortunately, this is the case the literature often focus on, just to avoid any long-run effect of inflation. Assuming full indexation, however, undoes the role of price dispersion, which is the main mechanism in New Keynesian models. The full indexation assumption, hence, strongly affects the functioning of the economy, making the task of monetary policy easier.

As SGU shows, full backward-looking indexation, in fact, does not imply price stability, since the variance of inflation is about 2 per cent per annum, that is, half of its steady state value. However, as said above, the degree of indexation turns out to be very important in affecting the optimal operational policy, because it affects the long-run and short-run properties of the model. In particular, partial indexation calls for a tighter control of inflation, as a way to stabilize price dispersion. This important point is already noted by Schmitt-Grohé and Uribe (2007) where price indexation is calibrated to be zero in a model similar to the one in this paper. They show that the variance of inflation is virtually zero, such that the optimal policy keeps inflation constant at all times. More generally, without indexation, price dispersion is so costly that a minimum amount of price stickiness suffices to make price stability the central goal of optimal policy.[25]

[23] If $\chi = 1$, there is no first order effect of current inflation on price dispersion, see (2). In this case, the dynamic equation of price dispersion is autonomous from the model and does not influence it.

[24] Note that the volatility of price dispersion in the no indexation case is roughly one hundred times bigger than when $\chi = 0.95$. In this sense also $\chi = 0$ is an extreme case. But while the no indexation case is changing the dynamics of the model quantitatively (i.e., strengthening the effects of price dispersion), the full indexation case is changing the dynamics of the model also qualitatively (i.e., cancelling the price dispersion mechanism).

[25] This turns out to be true also in presence of other public finance effects calling for an increase in

How does the monetary policy stabilize inflation, and thus, price dispersion? By increasing α_π for lower indexation levels, that is, getting tougher on inflation deviations from target, as showed by the first 4 forward-looking policies in Table 1. The increase in α_π is only modest, possibly due to the fact that a lower indexation makes inflation more forward looking (Sahuc, 2005 and Ascari and Ropele, 2007), and, thus, easier to control by a credible forward-looking rule. Moreover, despite the increase in α_π, the ability of the optimal policy to stabilize price dispersion worsen as indexation decreases. For values of χ lower than 0.8788, the optimal policy becomes first current-looking and then forward-looking inertial, so we can not just compare the coefficients. It may surprise that this last inertial policy features a very low α_π, but an inertial policy means a permanent change in the nominal interest rate in response to inflation. Indeed, it is interesting to note that, for the inertial forward looking optimal policy when $\chi = 0$, the sum of α_π and α_R is the same as the value of α_π for the forward looking no inertial optimal policy for high values of χ. Moreover, in the no indexation case, inflation is basically kept fix at the steady state level. Note that this would be the case also if the optimal policy when $\chi = 0$ (i.e., forward looking, $\alpha_\pi = 0.1875, \alpha_y = 0$ and $\alpha_R = 1$) is implemented in the full indexation case. σ_π would then be very small and equal to 0.2416.[26] This is exactly the task accomplished by the inertial policy: stabilize inflation. However, such a policy is not chosen in the full indexation case, because there is no need to stabilize price dispersion: full indexation keeps it basically at zero. In other words, stabilizing inflation is no more a fundamental issue for monetary policy, because full indexation takes care of the problem of stabilizing price dispersion.

On the one hand, inertial policy stabilizes inflation because the unit root in the policy affects expectations of future inflation and the long-run interest rate, helping monetary policy to stabilize inflation more efficiently. On the other hand, an inertial policy lacks flexibility and therefore would also entail some welfare costs. For high degree of indexation, these costs are higher than the gains in controlling the variance of price dispersion through inflation, while when indexation is null (or very low) the gains in stabilizing inflation overcome the costs of an inertial policy rule. This may explain why in this case inertial policy is the best choice. Table 3 provides some further evidence in

inflation volatility (see Schmitt-Grohé and Uribe, 2006, 2008).

[26] In this case the conditional welfare is equal to -156.7254 and $\sigma_s = 4.8039\text{e-}016$.

this direction, showing how the optimal value of α_π in a forward-looking rule changes with the value of α_R and χ : α_π decreases with indexation, unless α_R assumes values close to 1. In other words, an increase in the inertia of the policy can work as a sort of substitute for an increase in α_π, because, as said above, inertia keeps inflation under control through the expectation channel.

A very robust feature of the optimal policies is that α_y is zero. We thus confirm the well-known results of the various Schmitt-Grohé and Uribe's papers that responding to output can entail very high welfare costs. Therefore, the result that the optimal policy response to output is mute is independent from the degree of indexation. In what follows, we will also see that this feature of optimal policy is very robust, unless unconditional welfare is considered.

4.2 Price indexation and the forward-looking rule

In this section we concentrate on a particular rule: the forward-looking rule with no inertia (FLNI), i.e., $i = -1$ and $\alpha_R = 0$. We look at this particular rule because it turns out to be the globally optimal one both in SGU and in our simulations. In doing so, we can focus on the effects of indexation on the parameters of the policy rule and on the dynamics of our economy within a single policy rule, leaving the comparison across rules to the next sections.

4.2.1 Implementability

Figure 5 shows how changes in the level of indexation can affect the determinacy and implementability regions for the FLNI. The graphs visibly display an increase in both the determinacy and implementability areas with indexation. Low levels of indexation tend thus to reduce the parameter space available for policy options. In particular, if indexation is not full then the Taylor principle ($\alpha_\pi > 1$) does not define a condition for determinacy, and also the value of α_y is very much restricted to be around zero. The main effect of a lower indexation, thus, is to increase price dispersion in the economy, which in turn increases the likely of sunspots fluctuations. *Ceteris paribus*, in fact, an increase in inflation leads to an increase in price dispersion, which in turn rises the marginal costs, and hence inflation. This mechanism gets stronger the lower is the indexation, and therefore the policy response needs to be tougher to induce determinacy of the rational expectation equilibrium.

4.2.2 Welfare Surfaces

As in SGU, we can plot the level of conditional welfare as a function of α_y and α_π for those policies that implies a welfare lower than 1% from the one attained by the best operational policy within FLNI policies. We can do that for different values of the indexation parameter. Figure 6, however, does not look very revealing. The welfare surfaces look piecewise smooth in α_π and α_y, but they are not, and this is the reason to cut them focusing only to the policies with small deviation from the best. These surfaces display multiple local maxima and minima. The shape of the welfare surfaces motivates the particular painstaking grid search method that SGU and we followed in the main Section of the paper. A clever alternative would have been to use an optimization algorithm that maximizes the objective function over the parameter space (e.g., Schmitt-Grohé ad Uribe, 2006). Given computer-time constraint, we will follow this approach in the Section 5, on robustness.

4.2.3 Indexation, Optimal Policy and Unconditional Moments

Table 4 and 5 are equivalent to Table 1 and 2 for the FLNI policy rule. They show the optimal values of the coefficients of the FLNI policy, the corresponding welfare levels and unconditional moments for some variables of interest. The results are qualitatively the same but much more clear-cut. A lower degree of indexation calls for a policy that further reduce the variance of inflation. The optimal policy does it in a straightforward way: by increasing the response to inflation, i.e., α_π, from 1.125 to 2.6875. If the policy $\alpha_\pi = 2.6875$ and $\alpha_y = 0.1875$ is implemented in the full indexation case, then $\sigma_\pi = 1.03$. Again in the full indexation case monetary policy could stabilizes inflation through a higher α_π, but it chooses not to do so, because price dispersion is zero thanks to full indexation. Note, however, that, when $\chi = 0$, the variance of inflation is even higher than the one in $\chi = 0.85$,despite the value of α_π that is twofold. This signals that price dispersion inertia induced by lower indexation makes inflation more difficult to control. As said above, this may explain why for sufficiently low levels of indexation, the inertial policy rule may be preferred. Furthermore, optimal policies are not responding to the output gap. Under full indexation, the optimal policy rule resembles a real interest rate targeting rule, while, as indexation decreases, the optimal policy rule shift to a pure inflation targeting rule with a stronger reaction to inflation deviation from target. Moreover, both the steady state and the conditional welfare levels are maximized when $\chi = 0.8788$, and again the difference across welfare levels is mainly driven by the the steady state effect.

Finally, as Table 2, Table 5 again shows that full indexation is a rather special case. While the partial indexation cases are all similar in terms of order of magnitude of the second moments of the variables, the full indexation tends to cancel the effects of price dispersion, as evident from the variances of s, π and y.

4.3 Price Indexation and the simple optimal rule within different class of policies

In this section we present how indexation affects the optimal operational rule also for each of the other policy class: current looking, backward looking and inertial policies.

Tables 6 to 8 display the results for the optimal operational simple policy rules within each different class of policies. Given the large number of policies we analyzed, Tables 6 to 8 show the optimal policy rules for just 3 levels of indexation: full indexation (i.e., $\chi = 1$), no indexation (i.e., $\chi = 0$) and the optimal steady state indexation level (i.e., $\chi = 0.8788$)[27]. Moreover, Table 6 presents the no inertial policies, Table 7 the inertial ones (i.e., $\alpha_R = 1$) and Table 8 the super-inertial ones (i.e., $\alpha_R = 2$).

The no inertial policy rules exhibit the same features explained above. The main message is that the degree of indexation modifies the trade-off monetary policy is facing, due to the interaction between price dispersion and inflation. The lower is indexation: (i) the higher is price dispersion and its costs; (ii) the higher price dispersion inertia and its variance; (ii) the less persistent is inflation, but the more it impacts on price dispersion. Therefore Table 6 confirm the following facts (i) the variance of price dispersion decreases, while the one of inflation increases, with indexation; (ii) the difference in conditional welfare across the various cases is mainly driven by the first order steady state effects; (iii) the case $\chi = 1$ eradicates the effects of price dispersion from the model (iv) the optimal rule is not responding to output; (v) the lower the degree of indexation, the larger α_π. The points (i)-(iv) basically hold true also for the inertial and super iner-

[27] For the other level of indexation analyzed results are available upon requests.

tial policies.[28] The inertial and super inertial policies, instead, exhibit quite a different pattern regarding the parameter α_π. In particular, α_π is surprisingly decreasing with the degree of indexation. In the case of super inertial policy rules and no indexation it even becomes substantially negative. Since the value of α_R is different for inertial and super inertial policy rules, it may not surprise to find different values for α_π and α_y, but actually we do not have an intuition of the effects of indexation on α_π in these cases.[29]

Regarding conditional welfare, the no inertial policy rules always perform the best for high degree of indexation, while the inertial ones generally perform the best when indexation is zero. This suggests that the results in SGU are not robust to a change in the indexation parameter.

5 Robustness

In this Section we check the robustness of our results along four main dimensions:[30] (i) the interrelation between the Calvo parameter and the degree of indexation; (ii) the type of indexation; (iii) the level of trend inflation; (iv) the welfare measure.

In order to do this exercise, we need to perform another large number of simulations, and we could not rely any longer on the grid-search method we used in the previous section. We therefore employ an optimization algorithm, as in Schmitt-Grohé and Uribe (2006, 2007). Both methods have advantages and disadvantages. The grid-search will always find the global maximum, but it discretizes the parameter space. The optimization algorithm, instead, will always find a local maximum, but it does not guarantees global convergence. Therefore, this section will provide an analysis of the different performance of the two algorithms and hence an indirect check of the robustness of our results over the employed methodology.

[28] There are two exceptions among the superinertial policy rules with full indexation: the current looking and forward looking policy rules, where α_y is equal to 0.3125 and 0.625, respectively.

[29] Moreover, while the policy rules in the Tables satisfy the requirements for an operational policy, some are quite close to the boundaries of the determinacy frontiers. In particular, it is clear that a combinations of value for $\{\alpha_\pi, \alpha_y, \alpha_R\}$ as {0,0,1} or {0,0,2} would immediately lead to an explosive path for the nominal interest rate.

[30] We thank the two referees for suggesting us to look at these issues.

5.1 The Calvo parameter

In a similar model, Schmitt-Grohé and Uribe (2007) shows that the optimal rate of inflation is very sensitive to the Calvo parameter, α, that measures the degree of price stickiness, which is given by the inverse of the probability that a firm can change its price in a particular period (1/(1-α)). Moreover, optimal inflation volatility is high under full indexation or under flexible prices (see SGU), while it is low if there is no indexation (see Schmitt-Grohé and Uribe, 2008). Here we complement the analysis in those papers by looking at the effect of the interrelation between these two parameters on the optimal simple rules. This analysis will shed further light on the mechanism described above.

Table 9a,b,c shows the optimal forward looking no inertia (FLNI) policies for different values of the degree of both price stickiness and indexation. We take six values of α between 0.55 and 0.8,[31] and three values of $\chi = 0, 0.5, 1$. The results confirm that the shape of the optimal policy is mainly driven by the degree of price dispersion in the economy. Indeed, given a positive level of trend inflation, α and χ are the two main parameters in determining the degree of price dispersion. Looking at Table 9a, i.e., $\chi = 0$, it is evident that the higher is α, the higher the price dispersion, the more the optimal policy need to be aggressive on inflation in order to stabilize it. Moreover: (i) the value of α has a strong impact on conditional welfare; (ii) within the class of policies considered (FLNI), the response to output increases as α decreases. Note, however, how all these three main features depend on the degree of indexation.

As χ increases, the effects of varying α becomes less pronounced, and eventually disappear when $\chi = 1$, as shown by Table 9c. Indeed, under full indexation, varying the value of α has only marginal effects on the optimal policy parameters and on conditional welfare. As argued above, full indexation is a very special case: it eradicates the effects of price dispersion, and hence it makes the value of price stickiness basically unimportant for optimal policy and welfare. The effects of varying α actually are surprisingly reversed under full indexation, even if only marginally: (i) the value of α_π can decrease with price

[31] This interval is suggestd by Schmitt-Grohè and Uribe (2008). The most recent evidence on the micro data suggests that prices change on average approximately between 7 (see Klenow and Kryvstov, 2008) and 8 to 11 months (see Nakamura and Steinsson, 2008), implying a value of α around 0.5. The estimates of macroeconomic models, instead, are usually higher: CEE estimates α to be 0.6, Altig et al. (2005) to be 0.8. The 90-percent posterior probability interval for α estimated in Del Negro et al. (2005) is (0.51, 0.83).

stickiness; (ii) conditional welfare is increasing in price stickiness. Both these results confirm that full indexation is surely a very special case for this class of models.

Finally it is worth noting that, for low values of α and χ, the value of α_y is low but far from zero and increasing as price stickiness decreases. This suggests that the SGU's results about the importance of not responding to output may not be robust to low degree of price stickiness.

5.2 Type of Indexation

We argued above that a different value of the indexation parameter has strong impact on the shape of the optimal simple rules and on conditional welfare. In a New Keynesian model with trend inflation, in fact, partial indexation induces first-order effects on real variables and welfare, because price dispersion causes long-run non-neutralities. We also argued that one way to get rid of these non-superneutralities is to assume full indexation. Full indexation to past inflation, however, alters the dynamics of the model in a substantial way, inducing a much higher inflation inertia and variability.

Another common assumption in the literature to induce long-run neutrality is to assume that the prices that can not be changed are automatically indexed to trend inflation. Table 10a and b show the optimal policies, across all type of policies considered, for five different levels of the degree of indexation from 0 to 1, under the two different indexation schemes. When there is full indexation to trend inflation the dynamics of the model is similar to the one of a model approximated around a zero inflation steady state, and the result of the optimality of price stability is restored, as already stressed by SGU. Thus in this case, even if there are no long-run effects, the optimal inflation volatility is very low. The latter then decreases even further when the steady state level of price dispersion increases because of partial indexation. Contrary to the case of backward-looking indexation, the optimal policy changes very little with the degree of indexation. The forward-looking superinertial policy is always optimal, simply because inertial policies are the most effective in stabilizing inflation. The effects on conditional welfare are very similar in the two cases.

It is also often assumed an hybrid indexation scheme, where fixed prices are indexed both to past inflation and to trend inflation. Such assumption implies full indexation in the long-run, and, hence, long-run super-neutrality. Table 10c shows the optimal policies in this case. Two main points are worth stressing. First, the welfare effects are obviously very small, because there are no long-run effects irrespective of the value of indexation to backward-looking indexation, since the latter simply changes the relative weight between the two types of indexation. Second, Table 10c shows once again how full indexation to past inflation is a very special case: inflation volatility is roughly 6 times higher with respect to all the other considered cases. The intuition is clear: while full indexation to trend inflation cancels the effects of price dispersion in the long-run, full indexation to past inflation reduces also the effects of price dispersion in the short-run. Price dispersion is smaller under past inflation indexation than under trend inflation indexation. Nonetheless, and maybe surprisingly, the two types of indexation schemes deliver almost identical welfare levels under optimal policies.

5.3 Trend inflation

Table 11 shows the optimal policies across the 9 types of policies considered here, when steady state inflation is reduced to 2%, instead of 4.2%. Results are qualitatively very similar (also in terms of moments, not shown) to the benchmark case. Clearly, the welfare effects of varying the degree of indexation are smaller, because the level of price dispersion is lower, being lower trend inflation. Thus also the first order effects due to the non-neutralities induced by price dispersion are lower.

5.4 Unconditional Welfare

Tables 12 and 13 also display the unconditional welfare levels implied by the policies. The unconditional welfare measure is the very often employed in the literature and is the expected value of welfare given the unconditional distribution of the variables, i.e. it is independent of the initial conditions of the state vector. In other words, one can see it as the weighted average of the conditional welfare levels associated with all possible values of the initial state vector, with weights given by their unconditional probabilities. Hence it should be clear that the unconditional welfare may imply different optimal policies from the ones obtained using conditional welfare as the ranking measure. As stressed in SGU, the different ranking implied by the two measures demonstrates the importance of considering the transitional dynamics and the initial condition and it indicates the fact that the optimal operational rule lacks time consistency.

For the sake of completeness, Table 12 presents the optimal policy for each level of indexation using unconditional welfare.[32] A first result is that unconditional welfare is substantially higher than steady state welfare. Moreover, unconditional welfare is always increasing with indexation. Figure 7 replicates Figure 1 for the unconditional welfare ranking. It shows that the unconditional welfare gains, net of the steady state ones, are quite sizeable although still lower than their long-run counterpart. Furthermore, the optimal policies are different from the ones presented in Table 1, not only quantitatively, but also qualitatively. The current looking no inertial policy is optimal for high level of indexation, while the backward looking no inertia is optimal for low ones. Recall that the backward looking policy was never optimal according to conditional welfare. Besides, all the optimal policies are very close to the upper bound for α_π in our set of values for the grid search (i.e., $\alpha_\pi \in [-3, 3]$). It is very likely that the optimal policy would have implied an higher level of α_π. Furthermore, the value of α_y is actually sizably different zero. As a result, the volatility of inflation implied by these optimal policies is higher than the one implied by the optimal policies under conditional welfare (the same is true for the other variables, not shown).

Finally, since the optimal policies are close to the upper bound of our grid-search interval, we calculate the optimal unconstrained parameter values for α_π and α_y using the same optimization algorithm of the previous subsections and for the usual five values of χ between 0 and 1. Table 13 shows that the results are indeed quite different with respect to the previous Table: (i) the values of α_π and α_y are extremely high; (ii) the unconditional welfare is the lowest welfare measure across the Tables of the paper; (iii) the forward-looking super inertia policy is always optimal. The implied optimal inflation volatility is, however, very similar across methods.

The results in this subsection show, once again, that the type of optimal policies depends very much on the welfare measure one wants to optimize. Indeed, maximizing conditional rather than unconditional welfare deliver very different insights, in terms both of optimal simple rules and of implied volatilities of the variables.

[32] These results are based on the grid-search method used in the main Section of the paper.

6 Conclusions

In this paper we analyze how the optimal, simple and implementable monetary policy rule changes with the degree of price indexation in a standard medium-scale New Keynesian model. The effects of the assumed degree of price indexation is a quite neglected issue in the literature, and we provide a thorough robustness analysis of how the degree of indexation affects the dynamics of the model, the optimal policy rules and the welfare. Many papers in the literature focus on full indexation. However, full indexation is a very special case, because it eradicates the effects of price dispersion from the model. Monetary policy hence faces very different trade-offs with respect to a world with partial indexation. Moreover, the degree of indexation has also important first order welfare effects.

We show that indexation strongly affects monetary policy: both the kind of policy rule and its parameters do depend on the degree of indexation. In particular, we show that indexation stabilizes price dispersion. As a consequence, lower degree of indexation increases the size and the variance of price dispersion, calling for monetary policy to control it by stabilizing inflation, which indeed is not a task of monetary policy when indexation is full.

One central message is that when *ad hoc* features are introduced in microfounded models, one should be careful in drawing normative implications, without assessing the effects of such a feature on the mechanics of the model, on welfare and hence on the shaping of the optimal policy.

In this respect the natural way ahead would be to build a model where indexation arises endogenously. This paper shows that this should be an urgent task for the optimal policy literature, given the sensitivity of the normative prescriptions to the indexation parameter.

References

Altig, D., L. J. Christiano, M. Eichenbaum, and J. Linde (2004). Firm-specific capital, nominal rigidities, and the business cycle. Technical Report wp 04-16, Federal Reserve Bank of Clevelend.

Ascari, G. (2004). Staggered prices and trend inflation: Some nuisances. *Review of Economic Dynamics 7*, 642–667.

Ascari, G. and N. Branzoli (2007a). The long-run optimal degree of indexation in the new keynesian model. Quaderni di Dipartimento No. 190 (02-07), Dipartimento di economia politica e metodi quantitativi Università degli studi di Pavia, forthcoming in *Economics Bulletin*.

Ascari, G. and N. Branzoli (2007b). Optimal simple rules and the lower bound on the nominal interest rate in the Christiano-Eichenbaum-Evans model of the US business cycle. *Portoguese Economic Journal 6*, 117–131.

Ascari, G. and T. Ropele (2007). Optimal monetary policy under low trend inflation. *The Journal of Monetary Economics 54*, 2568–2583.

Benati, L. (2008). Investigating inflation persistence across monetary regimes. *The Quarterly Journal of Economics 123(3)*, 1005–1060.

Benati, L. (2009). Are "intrinsic inflation persistence" models structural in the sense of Lucas (1976)? ECB working paper No. 1038.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Cogley, T. and A. Sbordone (2008). Trend inflation, indexation and inflation persistence in the new keynesian phillips curve. *American Economic Review 98(5)*, 2101–2126.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2005). On the fit and forecasting performance of New-Keynesian models. ECB working paper No. 491.

Galí, J. and M. Gertler (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics 44*, 195–222.

Gray, J. A. (1976). Wage indexation: a macroeconomic approach. *Journal of Monetary Economics 2*, 221–235.

Klenow, P. J. and O. Kryvtsov (2008). State-dependent or time-dependent pricing: Does it matter for recent U.S. inflation? *The Quarterly Journal of Economics 123*(3), 863–904.

Levin, A. T., A. Onatski, J. Williams, and N. Williams (2006). Monetary policy uncertainty in microfounded macroeconometric models. In M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomic Annual*, pp. 229–287. Cambridge, MA: MIT Press.

Mankiw, G. N. (2001). The inexorable and mysterious tradeoff between inflation and unemployment. *The Economic Journal*, C45–C61.

Mash, R. (2007). Endogenous indexing and monetary policy models. Economics Discussion Papers, No 2007-36. http://www.economics-ejournal.org/economics/discussionpapers/2007-36.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics 123*(4), 1415–1464.

Rudd, J. and K. Whelan (2007). Modeling inflation dynamics: A critical review of recent research. *Journal of Money, Credit and Banking 39(1)*, 155–170.

Sahuc, J.-G. (2006). Partial indexation, trend inflation, and the hybrid Phillips curve. *Economics Letters 90*, 42–50.

Sbordone, A. (2005). Do expected future marginal costs drive inflation dynamics? *Journal of Monetary Economics 52*, 1183–1197.

Schmitt-Grohé, S. and M. Uribe (2004a). Optimal operational monetary policy in the Christiano-Eichenbaum-Evans model of the U.S. business cycle. NBER wp No. 10724.

Schmitt-Grohé, S. and M. Uribe (2004b). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control 28*, 755–775.

Schmitt-Grohé, S. and M. Uribe (2006). Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomics Annual*, pp. 383–425. Cambridge, MA: MIT Press.

Schmitt-Grohé, S. and M. Uribe (2007). Optimal inflation stabilization in a medium-scale macroeconomic model. In K. Schmidt-Hebbel and R. Mishkin (Eds.), *Monetary Policy Under Inflation Targeting*, pp. 125–186. Santiago, Chile: Central Bank of Chile.

Schmitt-Grohé, S. and M. Uribe (2008). Policy implications of the New Keynesian Phillips Curve. *Federal Reserve Bank of Richmond Economic Quarterly 94*(4), 435–465.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association 1*, 1123–1175.

Smets, F. and R. Wouters (2007). Shocks and frictions in US business cycles: A bayesian DSGE approach. *American Economic Review 97*(3), 586–606.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

Yun, T. (1996). Nominal price rigidity, money supply endogeneity and business cycle. *Journal of Monetary Economics 37*, 345–370.

7 Appendix: The Christiano-Eichenbaum-Evans Model for US business Cycle

7.A Households

Agents live forever and discount future at a rate β. There is a continuum of households that seek to maximize their expected utility function, given by:

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(c_t - bc_{t-1}; h_t^s; m_t^h \right) \right\}. \tag{3}$$

E_0 defines the mathematical expectation operator conditional on the information set available at time 0. The function $u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right)$ is well-behaved and increasing in consumption c_t and money holdings m_t^h, decreasing in hours supplied h_t^s. Preferences display habit in consumption levels, measured by the parameter b.

There is a continuum of final goods, indexed by $i \in [0,1]$, that enter in the consumption bundle c_t through the usual Dixit-Stiglitz aggregator:

$$c_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{4}$$

where the parameter η indicates the elasticity of substitution between different varieties of goods. The standard household problem defines the optimal demand of good i, given by $c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t$, where P_t is the general price index given by $P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$.

We assume a continuum of labour services h_{jt}, $j \in [0,1]$, that are combined according to the following technology

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj \right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}},$$

where $\tilde{\eta}$ is the elasticity of substitutions of labour types. Given production plans, a firm that minimizes costs has a labour-specific demand function given by $h_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\tilde{\eta}} h_t^d$, where W_{jt} is the wage prevailing in labour market j and W_t is a wage index defined as $W_t = \left[\int_0^1 W_{it}^{1-\tilde{\eta}} di\right]^{\frac{1}{1-\tilde{\eta}}}$. Integrating labour-specific demand functions one obtains h_t defined as

$$h_t \equiv \int_0^1 h_{jt} dj = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t} \right)^{-\tilde{\eta}} dj. \tag{5}$$

Agents owns physical capital k_t that depreciates at rate δ. The capital accumulation equation is:

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right], \tag{6}$$

where the function S introduce a cost of varying the level of investment and satisfies the properties that $S(1) = S'(1) = 0$, $S''(1) > 0$.

Variable capacity utilization of physical capital is denoted by u_t, with an associated cost implicitly defined by $a(u_t)$. Agents owns firms and rent capital at a real interest rate r_t^k, earn profits and decide also over the utilization rate. Money is injected via lump-sum transfer τ_t. Finally the existence of complete markets on state contingent assets x_t^h assure that all agents choose the same level of consumption independently of the hours supplied. The budget constraint is then:

$$E_t r_{t,t+1} x_{t+1}^h + c_t + i_t + m_t^h + a(u_t) k_t = \frac{x_t^h}{\pi_t} + h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj + r_t^k u_t k_t + \phi_t - \tau_t \frac{m_{t-1}^h}{\pi_t} \quad (7)$$

Given wage stickiness à la Calvo wage dispersion $\int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$ can be written as:

$$w_t = \tilde{\alpha} w_{t-1} \frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t} + (1 - \tilde{\alpha}) \tilde{w}_t \quad (8)$$

where $\tilde{w}_t$ is the optimal wage set at time t.

The problem is to maximize (3) under eqs.(5)-(8). Household's first order conditions are hence given by

$$u_{c_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + u_{c_t}\left(c_{t+1} - bc_t; h_{t+1}^s; m_{t+1}^h\right) = \lambda_t \quad (9)$$

$$u_{h_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) = -\lambda_t \frac{w_t}{\tilde{\mu}_t} \quad (10)$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t}\left[q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - a(u_{t+1})\right] \quad (11)$$

$$q_t \lambda_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left[S_i\left(\frac{i_t}{i_{t-1}}\right)\right] i_t\right] - \beta q_{t+1} \lambda_{t+1} S_i\left(\frac{i_{t+1}}{i_t}\right) i_{t+1} = \lambda_t \quad (12)$$

$$a_{u_t}(u_t) = r_t^k \quad (13)$$

$$u_{m_t^h}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + \beta \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t. \quad (14)$$

Wages are sticky à la Calvo, and $1 - \tilde{\alpha}$ is the probability of being able to reset wages next period. With probability $\tilde{\alpha}$ wages can not be re-optimized, and thus they are updated with past inflation, more precisely they vary according to $w_{j,t+1} = w_{j,t}\pi_t^{\tilde{\chi}}$ where $\tilde{\chi}$ is the degree of indexation to past inflation. Define $\tilde{w}_t$ as the optimal real wage set every period t. A union chooses the optimal wage maximizing its the utility function given by equation (3), subject to demand of labour in the specific market $h_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} h_t^d$ and the probability of not being able to re-optimize in future periods. The resulting first order condition is:

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right] = 0. \tag{15}$$

Equation (15) states that optimal real wage must equate the future stream of marginal revenues from working to the expected sum of marginal cost of suppling labour. Given the structure of the model, all the reset optimal wages are identical in all labour markets in which the household can re-optimize. SGU shows how to write condition (15) in recursive form.

7.B Firms

Each good is produced by a firm which monopolistically supplies its own variety using a production technology of the form:

$$z_t F(k_{it}, h_{it}) - \psi,$$

where z_t is an aggregate exogenous technology factor that follow an AR(1) process.ψ represent a fixed cost of production that generates increasing return to scale and guarantees zero profits in equilibrium. The production function $F(k_{it}, h_{it})$ is well-behaved and the same for all firms. Final goods can be used for consumption, investment, public expenditure and to pay cost of capital utilization. Each firm faces the following demand function:

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t, \tag{16}$$

where:

$$y_t = c_t + i_t + g_t + a(u_t) k_t. \tag{17}$$

We assume that firms can access a centralized market for capital, and must pay a fraction ν of wages at the beginning of the period by cash. Therefore their money demand function is:

$$m_{it}^f = \nu w_t h_{it} \tag{18}$$

Firms maximizes the expected value of future profits, under their demand function (16) and the cash-in-advance constraint (18). Firms' first order condition with respect to capital and labour services are:

$$mc_{it} z_t F_{k_{it}}(k_{it}, h_{it}) = r_t^k \tag{19}$$

$$mc_{it} z_t F_{h_{it}}(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. \tag{20}$$

If we assume that all firms have access to the same factor markets and F is homogeneous of degree one, equation (19) and equation (20) imply that all firms have the same marginal costs and aggregation across firms is straightforward.

Prices are sticky a la Calvo. Every period each firm can choose a new price of its own good with a probability $1 - \alpha$. Those firms who can not reset their price update their price according to past inflation. Specifically their new price is $P_{it} = P_{it-1}\pi_{t-1}^{\chi}$ where χ is the degree of price indexation. The optimal price solve the first order condition:

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} y_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}}\right)^{\eta} \left[\frac{\eta - 1}{\eta} \frac{\tilde{P}_t}{P_t} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{i,t+s}\right] = 0 \tag{21}$$

These expression states that optimizing firms choose a price $\tilde{P}_t$ that equates the expected sum of future marginal costs with the expected sum of marginal revenues, conditional on not being able to re-optimize in the future. Given the structure of the model, all the reset optimal prices are identical in all good markets in which firms can re-optimize. SGU shows how to write condition (21) in recursive form.

7.C The Government

The government has two policy instruments: public expenditure and the nominal interest rate. Government expenditure is financed through lump-sum taxes and seigniorage:

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t}. \tag{22}$$

We assume an optimizing government that minimizes costs of acquiring the composite good, hence given public expenditure, government's absorption of a single variety of goods is $g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} g_t$.

7.D Equilibrium

Equilibrium on money market is simply: $m_t = m_t^h + m_t^f$. Equilibria in labour and capital markets imply that:

$$\int_0^1 h_{it}^d di = h_t^d \tag{23}$$

$$\int_0^1 k_{it} di = u_t k_t. \tag{24}$$

Consider equilibrium in the final goods' markets. The assumption that both government and agents minimize their expenditure choosing the optimal quantity of each variety of good implies the following condition:

$$z_t F\left(k_{it}, h_{it}\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \left(\frac{P_{it}}{P_t}\right)^{-\eta}. \tag{25}$$

Integrating the right side of the equation considering that the capital-labour ratio is the same among firms and imposing equations (23) and (24), yields:

$$z_t h_t^d F\left(\frac{u_t k_t}{h_t^d}, 1\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di, \tag{26}$$

where $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta}$ constitutes a wedge between aggregate supply and aggregate absorption and represents the price dispersion generated by price staggering. Then:

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{-\eta} s_{t-1} \tag{27}$$

and the equilibrium on final goods' markets is given by:

$$z_t F\left(u_t k_t, h_t^d\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) s_t. \tag{28}$$

Using the same properties, we aggregate equations (19) and (20) obtaining:

$$mc_t z_t F_{k_t}\left(u_t k_t, h_t^d\right) = r_t^k \tag{29}$$

$$mc_t z_t F_{h_t^d}\left(u_t k_t, h_t^d\right) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. \tag{30}$$

Finally, the expression for wage dispersion closely follows its price counterpart. Using labour-specific demand function we can write:

$$h_{jt}^d = \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} h_t^d \tag{31}$$

and integrating both sides and using equation (23) yields:

$$h_t = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj \qquad (32)$$

Then defining $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$, it is easy to show that:

$$\tilde{s}_t = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\eta}} \tilde{s}_{t-1} \qquad (33)$$

Finally the definition of the price and the wage index generates a low of motion for the aggregate price and wage levels:

$$P_t^{1-\eta} = \alpha\left(P_{t-1}\pi_{t-1}^{\chi}\right)^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta} \qquad (34)$$

$$w_t^{1-\tilde{\eta}} = \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\eta}} + (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\eta}} \qquad (35)$$

7.E Functional forms

As in SGU we assume the following functional forms:

$$\begin{aligned}
u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) &= \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2}h_t^2 + \phi_1 \frac{\left(m_t^h\right)^{1-\sigma_m}}{1-\sigma_m} \\
F\left(u_t k_t, h_t^d\right) &= (u_t k_t)^{\theta}\left(h_t^d\right)^{1-\theta} \\
S\left(\frac{i_t}{i_{t-1}}\right) &= \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 \\
a(u_t) &= \gamma_1 (u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2 .
\end{aligned}$$

The calibration is as in SGU and follows CEE's estimation results. One period in the model is interpreted as a quarter. The steady state displays full capital utilization. Furthermore the quantities of money held by households and firms are set to match the empirical distribution[33], likewise for long run inflation, 4.2% annual, and public expenditure, 52.44%. The rest of the calibration is listed in Table 14: the reader is referred to SGU and CEE for a discussion of exogenous processes and values use therein.

We solve the model with the perturbation method developed in Schmitt-Grohé and Uribe (2004b) and we rank policies using a measure of welfare based on second order approximation of the model around the non-stochastic steady state. We used both an unconditional and a conditional welfare measure, the latter to take into account of transitional dynamics.

[33] That is households hold 44% of money in steady state while fims hold the remaining 56%.

8 Tables

Table 1. Optimal Operational Monetary Policies Rules						
χ	Policy Class	α_R	α_π	α_y	SS Welf.	Conditional Welf.
1	Forward looking	0	1.1250	-0.0625	-156.7143	-156.7220
.95	Forward looking	0	1.1875	0	-156.7119	-156.7199
.90	Forward looking	0	1.1875	0	-156.7107	-156.7189
.8788	Forward looking	0	1.1875	0	-156.7106	-156.7188
.85	Current looking	0	1.0625	0	-156.7108	-156.7191
.80	Current looking	0	1.0625	0	-156.7122	-156.7205
.75	Current looking	0	1.0625	0	-156.7149	-156.7232
0	Forward looking	1	0.1875	0	-156.9351	-156.9428

Table 2. Unconditional Moments under Optimal Operational Rules (x10^{-2})								
χ	$E(c)$	$E(y)$	$E(s)$	$E(\pi)$	σ_c	σ_y	σ_s	σ_π
1	0.13	-0.19	3.9926(x10^{-6})	4.06	1.6722	3.7602	1.22(x10^{-15})	2.0483
.95	0.17	-0.38	5.9022(x10^{-6})	4.20	1.9147	5.1102	6.9(x10^{-4})	1.2012
.90	0.17	-0.38	1.2596(x10^{-5})	4.19	1.9127	5.0978	1.7(x10^{-3})	1.2808
.8788	0.17	-0.38	7.9551(x10^{-6})	4.20	1.9080	5.0661	2.0(x10^{-3})	1.2635
.85	0.16	-0.36	3.5347(x10^{-6})	4.19	1.9016	4.9235	1.6(x10^{-3})	0.7360
.80	0.16	-0.36	4.1604(x10^{-6})	4.19	1.8989	4.9050	2.4(x10^{-3})	0.7371
.75	0.16	-0.37	4.9830(x10^{-6})	4.19	1.8964	4.8863	3.3(x10^{-3})	0.7371
0	0.15	-0.34	2.9616(x10^{-6})	4.20	1.8065	4.7709	9.6(x10^{-3})	0.1716

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 3. Optimal Monetary Policies for different values of α_R Forward Looking No Inertia						
	α_R= 0.1		α_R= 0.5		α_R= 0.9	
χ	α_π	α_y	α_π	α_y	α_π	α_y
1	1	0.0625	0.5	0.0625	0.6875	0.1875
.8788	1.0625	0.0625	0.5625	0.0625	0.5625	0.125
0	2.1875	0.1875	1	0.0625	0.3125	0.0625

Table 4. Optimal Monetary Policies - Forward Looking No Inertia				
χ	α_π	α_y	SS Welf.	Conditional Welf.
1	1.1250	-0.0625	-156.7143	-156.7220
.95	1.1875	0	-156.7119	-156.7199
.90	1.1875	0	-156.7107	-156.7189
.8788	1.1875	0	-156.7106	-156.7188
.85	1.2500	0	-156.7108	-156.7193
.80	1.4375	0	-156.7122	-156.7216
.75	1.6250	0	-156.7149	-156.7267
0	2.6875	0.1875	-156.9351	-156.9767

Table 5. Unconditional Means under Optimal Forward Looking No Inertia Rule (x10^{-2})								
χ	$E(c)$	$E(y)$	$E(s)$	$E(\pi)$	σ_c	σ_y	σ_s	σ_π
1	0.13	-0.19	3.99(x10^{-6})	4.06	1.67	3.76	1.2195(x10^{-15})	2.05
.95	0.17	-0.38	5.90(x10^{-6})	4.20	1.91	5.11	6.8987(x10^{-4})	1.20
.90	0.17	-0.38	1.26(x10^{-6})	4.19	1.91	5.10	1.7(x10^{-3})	1.28
.8788	0.17	-0.38	7.96(x10^{-6})	4.20	1.91	5.07	2.0(x10^{-3})	1.26
.85	0.16	-0.37	6.40(x10^{-6})	4.20	1.89	5.01	2.2(x10^{-3})	1.04
.80	0.16	-0.37	4.29(x10^{-6})	4.20	1.87	4.96	2.4(x10^{-3})	0.75
.75	0.17	-0.45	1.18(x10^{-5})	4.26	1.93	5.4	4.8(x10^{-3})	1.16
0	0.13	-0.56	2.05(x10^{-4})	4.28	1.95	5.73	7.78(x10^{-2})	1.16

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 6. Optimal Operational Monetary Policies - No Inertia						
χ	α_π	α_y	σ_s	σ_π	Conditional Welf.	Unconditional Welf.
Forward Looking						
1	1.1250	-0.0625	1.22(x10^{-15})	2.0483	-156.7220	-156.5252
0.8788	1.1875	0	0.0020	1.2635	-156.7188	-156.4169
0	2.6875	0.1875	0.0778	1.1574	-156.9767	-156.6381
Current Looking						
1	1.0625	0	2.77(x10^{-15})	0.7278	-156.7227	-156.4289
0.8788	1.0625	0	0.0014	0.7645	-156.7189	-156.4299
0	1.625	0	0.0212	0.3353	-156.9456	-156.6679
Backward Looking						
1	1.3125	0.0625	1.83(x10^{-15})	1.5648	-156.7233	-156.3729
0.8788	1.375	0.0625	0.0015	1.3164	-156.7203	-156.3810
0	1.3125	0	0.0180	0.2919	-156.9451	-156.4442

Table 7. Optimal Operational Monetary Policies - Inertia, $\alpha_R = 1$						
χ	α_π	α_y	σ_s	σ_π	Conditional Welf	Unconditional Welf.
Forward Looking						
1	0.8125	0.1875	3.11(x10^{-15})	1.9911	-156.7232	-156.3456
0.8788	0.5	0.0625	0.0012	1.0058	-156.7199	-156.3975
0	0.1875	0	0.0096	0.1716	-156.9428	-156.6751
Current Looking						
1	0.4375	0.0625	1.16(x10^{-15})	1.6442	-156.7237	-156.3639
0.8788	0.4375	0.0625	0.0013	1.1334	-156.7203	-156.3911
0	0.0625	0	0.0076	0.1390	-156.9431	-156.6733
Backward Looking						
1	0.75	0.1250	1.54(x10^{-15})	1.3195	-156.7243	-156.3790
0.8788	0.5	0.0625	0.0011	0.9584	-156.7209	-156.3975
0	0.0625	0	0.0074	0.1355	-156.9432	-156.6730

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 8. Optimal Operational Monetary Policies - Super Inertia, $\alpha_R = 2$						
χ	α_π	α_y	σ_s	σ_π	Conditional Welf	Unconditional Welf.
Forward Looking						
1	1.6875	0.625	1.74(x10^{-15})	1.9939	-156.7237	-156.3435
0.8788	0.5625	0.125	8.40(x10^{-4})	0.6401	-156.7203	-156.4155
0	-0.9375	0	0.0207	0.3095	-156.9426	-156.6931
Current Looking						
1	0.8125	0.3125	1.03(x10^{-15})	1.3833	-156.7248	-156.3753
0.8788	0.25	0.0625	6.48(x10^{-4})	0.4206	-156.7210	-156.4254
0	-0.8750	0	0.0214	0.3206	-156.9450	-156.6871
Backward Looking						
1	-0.75	-0.0625	8.1(x10^{-16})	1.5262	-156.7250	-156.5191
0.8788	-0.75	-0.0625	0.0018	1.5077	-156.7203	-156.5230
0	-0.6875	0	0.0182	0.2738	-156.9453	-156.6809

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 9a. Optimal Forward Looking Policies No Inertia - $\chi = 0$			
Calvo Parameter (α)	α_π	α_y	Conditional Welfare
0.55	2.3015	0.2094	-156.9321
0.60	2.5157	0.1629	-156.9749
0.65	2.7819	0.1211	-157.0614
0.70	3.1083	0.0845	-157.2192
0.75	3.5048	0.0538	-157.5175
0.80	3.9931	0.0302	-158.1520

Table 9b. Optimal Forward Looking Policies No Inertia - $\chi = 0.5$			
Calvo Parameter (α)	α_π	α_y	Conditional Welfare
0.55	1.7507	0.1583	156.7694
0.60	1.8710	0.1100	-156.7681
0.65	2.2079	0.0842	-156.7778
0.70	2.6194	0.0258	-156.7976
0.75	2.6716	0.0173	-156.8388
0.80	2.8260	0.0007	-156.9195

Table 9c. Optimal Forward Looking Policies No Inertia - $\chi = 1$			
Calvo Parameter (α)	α_π	α_y	Conditional Welfare
0.55	1.1442	-0.0239	-156.7227
0.60	1.0888	-0.0336	-156.7213
0.65	1.0542	-0.0329	-156.7206
0.70	1.0538	-0.0225	-156.7202
0.75	1.0628	-0.0132	-156.7197
0.80	1.0920	-0.0034	-156.7192

Table 10a. Optimal Monetary Policies, Backward-looking indexation						
χ	Policy Class	α_R	α_π	α_y	Conditional Welf.	σ_π
1	Forward Looking	0	1.0888	-0.0336	-156.7214	1.83
.75	Forward Looking	2	0.1286	0.0199	-156.7240	0.32
.50	Forward Looking	2	0.0478	0.0063	-156.7576	0.28
.25	Forward Looking	2	0.4965	0.0054	-156.8289	0.25
0	Forward Looking	2	-0.0712	-0.0049	-156.9425	0.23

Table 10b. Optimal Monetary Policies, Trend inflation indexation						
χ	Policy Class	α_R	α_π	α_y	Conditional Welf.	σ_π
1	Forward Looking	2	0.0522	0.0051	-156.7223	0.27
.75	Forward Looking	2	0.0531	0.0052	-156.7227	0.26
.50	Forward Looking	2	0.0546	0.0049	-156.7569	0.24
.25	Forward Looking	2	0.0547	0.0049	-156.8287	0.24
0	Forward Looking	2	0.0712	0.0049	-156.9425	0.23

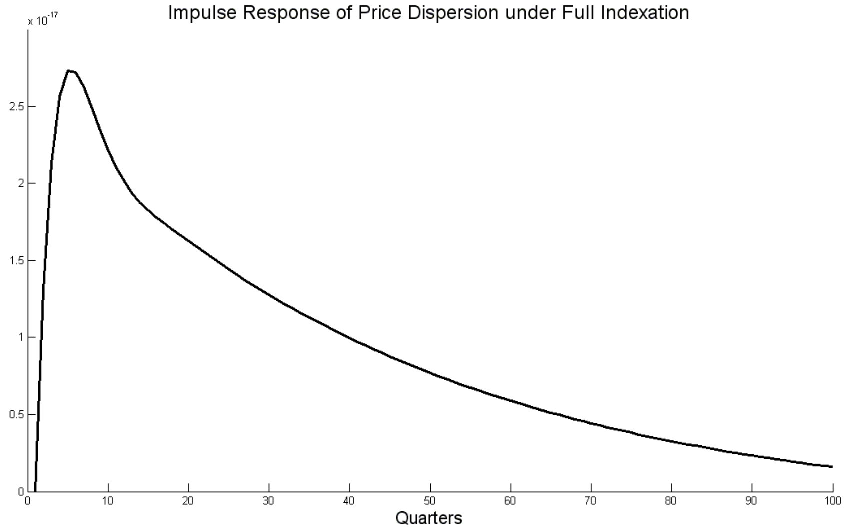
Table 10c. Optimal Monetary Policies, Hybrid indexation $\chi =$ degree of backward-looking indexation						
χ	Policy Class	α_R	α_π	α_y	Conditional Welf.	σ_π
1	Forward Looking	0	1.0888	-0.0336	-156.7214	1.83
.75	Forward Looking	1	0.2450	0.0109	-156.7235	0.46
.50	Forward Looking	1	0.1394	0.0021	-156.7230	0.33
.25	Forward Looking	2	0.0478	0.0060	-156.7226	0.30
0	Forward Looking	2	0.0521	0.0051	-156.7224	0.27

Table 11. Optimal Monetary Policies, Trend Inflation = 2%					
χ	Policy Class	α_R	α_π	α_y	Conditional Welfare
1	Forward Looking	0	1.0927	-0.0340	-156.5008878
.75	Forward Looking	0	1.4070	0.0241	-156.4974982
.50	Forward Looking	0	1.4921	0.1136	-156.5049626
.25	Forward Looking	2	0.0473	-0.0074	-156.5196983
0	Forward Looking	2	0.0437	0.0047	-156.5390798

Table 12. Optimal Monetary Policies ranked by unconditional welfare							
χ	Policy Class	α_R	α_π	α_y	Unconditional Welf.	Conditional Welf.	σ_π
1	Current Looking	0	2.8750	0.6250	-156.2653	-156.7267	3.26
.95	Current Looking	0	2.8750	0.6250	-156.2654	-156.7256	3.27
.90	Current Looking	0	2.8750	0.6250	-156.2669	-156.7276	3.27
.8788	Current Looking	0	2.8750	0.6250	-156.2681	-156.7277	3.28
.85	Current Looking	0	2.8750	0.6250	-156.2703	-156.7298	3.28
.80	Backward Looking	0	3	0.6875	-156.2778	-156.7319	3.35
.75	Backward Looking	0	3	0.6875	-156.2857	-156.7398	3.36
0	Backward Looking	0	3	0.2500	-156.6265	-156.9858	1.13

Table 13. Optimal Monetary Policies ranked by unconditional welfare							
χ	Policy Class	α_R	α_π	α_y	Unconditional Welf.	Conditional Welf.	σ_π
1	Forward Looking	2	8.0957	4.1546	-155.9998	-156.8276	3.33
.75	Forward Looking	2	8.7518	3.8128	-156.0332	-156.7935	3.30
.50	Forward Looking	2	10.8519	3.7786	-156.1456	-156.8252	3.16
.25	Forward Looking	2	11.9724	2.5497	-156.2991	-156.8945	1.76
0	Forward Looking	2	13.0999	1.7394	-156.4398	-156.9948	1.06

Table 14. Calibration		
β	$1.03^{-0.25}$	Time discount rate
θ	0.36	Share of capital
ψ	0.5827	Fixed cost (guarantee zero profits in steady state)
δ	0.025	Depreciation of capital
υ	1	Fraction of wage bill subject to CIA constraint
η	6	Elasticity of substitution of different varieties of goods
$\tilde{\eta}$	21	Elasticity of substitution of labour services
α	0.6	Probability of not setting a new price each period
$\tilde{\alpha}$	0.64	Probability of not setting a new wage each period
b	0.65	Degree of habit persistence
ϕ_0	1.1196	Preference parameter
ϕ_1	0.5393	Preference parameter
σ_m	10.62	Intertemporal elasticity of money
κ	2.48	Investment adjustment cost parameter
$\tilde{\chi}$	1	Wage indexation
γ_1	0.0324	Capital utilization cost function parameter
γ_2	0.000324	Capital utilization cost function parameter
z	1	Steady state value of technology shock
λ_z	0.979	Serial correlation of technology shock (in log-levels)
η_z	0.0072	Standard deviation of technology shock
λ_g	0.96	Serial correlation of demand shock (in log-levels)
η_g	0.02	Standard deviation of demand shock
σ	0.18	Parameter scaling all exogenous shocks



9 Figures

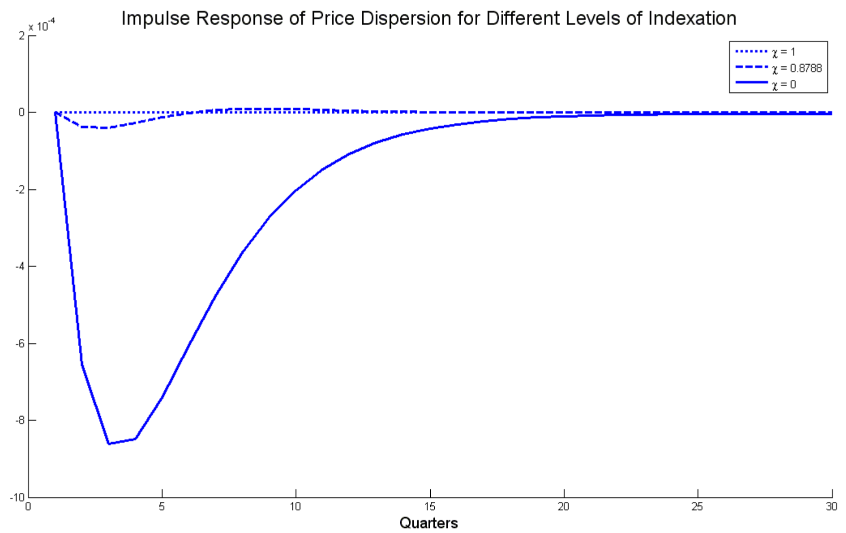


Figure 1. Impulse Response Functions of Price Dispersion after a 1% increase in the TFP for different levels of indexation; $\alpha_\pi = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ in the Taylor Rule (1)

Figure 2. Impulse Response Functions of Price Dispersion after a 1% increase in the TFP in the case of full indexation; $\alpha_\pi = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ in the Taylor Rule (1)

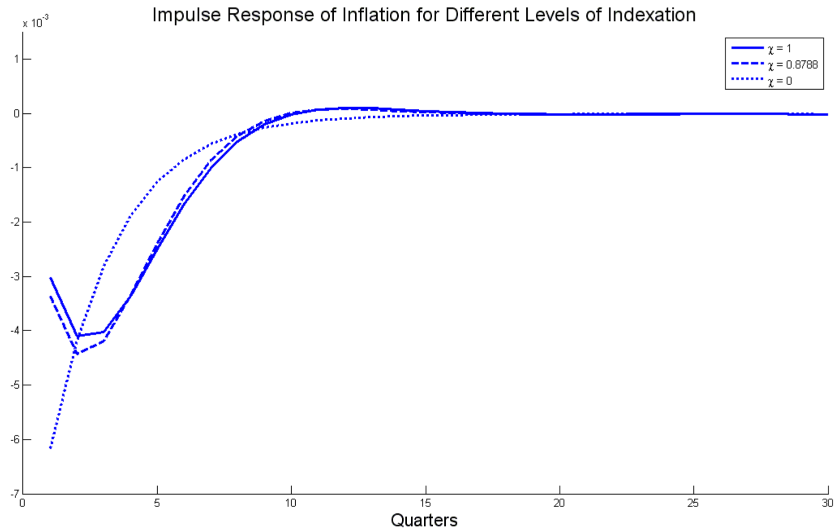


Figure 3. Impulse Response Functions of Inflation after a 1% increase in the TFP for different level of indexation; $\alpha_\pi = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ in the Taylor Rule (1)

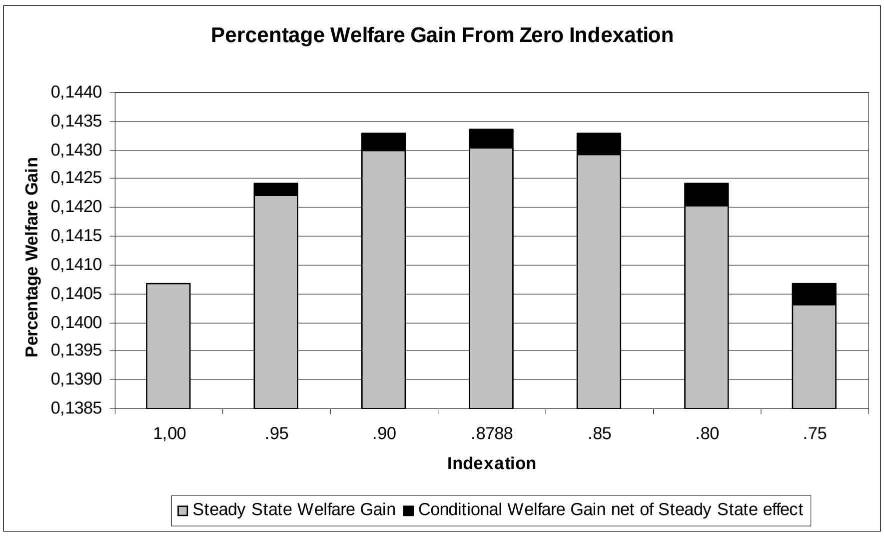


Figure 4. steady state and conditional percentage gain with respect to the 0 indexation case for the best policies ranked according to conditional welfare.

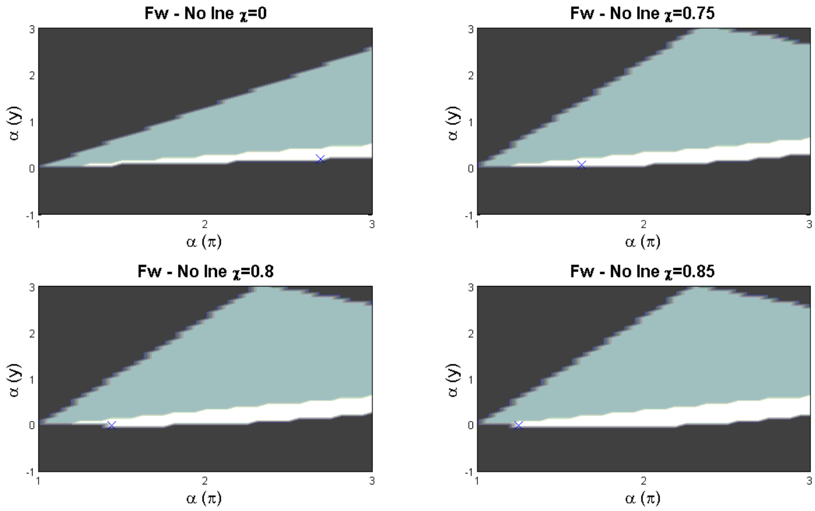


Figure 5. Indeterminacy regions

Note: Each panel shows three regions: the white one displays the values of α_y and α_π that deliver determinate rational expectation equilibria, the grey one signals that the equilibrium is not implementable in the sense described in footnote 17, and the black region represents indeterminate rational expectation equilibria. All the values of both $\alpha_y < -1$ and $\alpha_\pi < 1$ yield indeterminacy and are not shown in the figure.

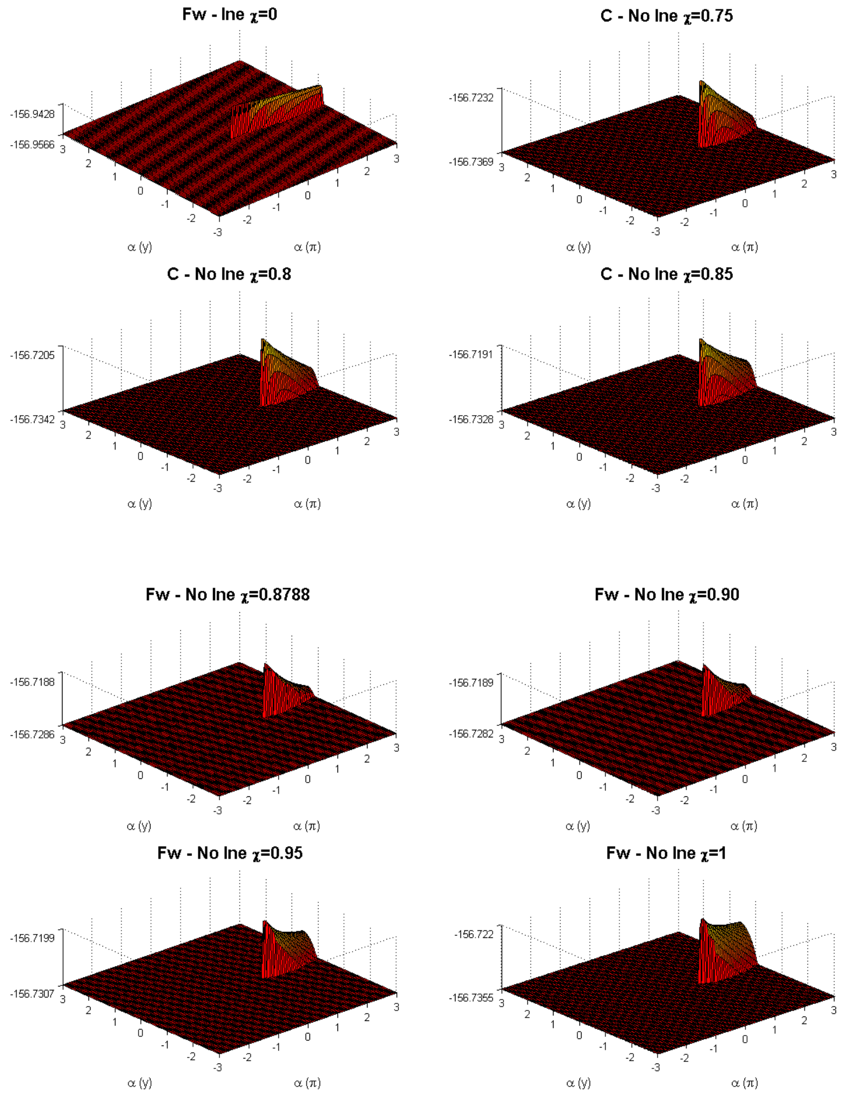


Figure 6. conditional welfare for those policies that imply less than 1% loss with respect to the best policy within the class of FLNI.

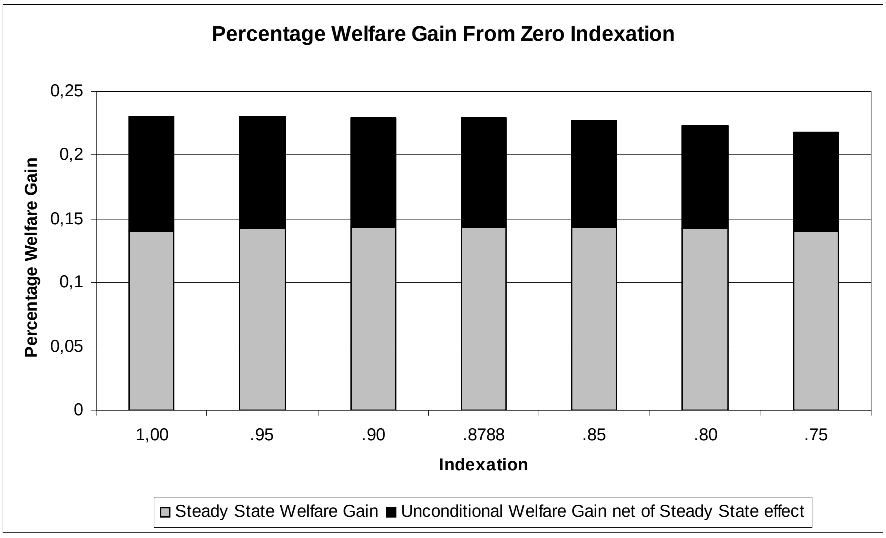


Figure 7. steady state and unconditional percentage gain with respect to the 0 indexation case for the best policies ranked according to unconditional welfare.