Disinflation effects in a medium-scale New Keynesian model: money supply rule *versus* interest rate rule

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May 12, 2011

Abstract

Empirical studies show that successful disinflations entail a period of output contraction. Using a medium-scale New Keynesian model, we compare the effects of disinflations of different speed and timing, implemented through either a money supply or an interest rate rule. In terms of transitional output loss, cold-turkey disinflations under an interest rate rule are less costly than those under a money supply rule and are accomplished more rapidly. Furthermore, gradual or anticipated disinflations deliver lower sacrifice ratios. From a welfare perspective, despite the transitional economic contraction, disinflations are overall welfare-improving. Interestingly, the overall welfare gain is not affected by how the disinflation is actually implemented: what really matters is the achievement of a permanently lower inflation rate.

JEL classification: E31, E5.

Keywords: Disinflation, Sacrifice ratio, Nonlinearities

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1 Introduction

The analysis of disinflation and how to implement a permanent reduction in inflation have been topical issues in economics at least since the 1970s. Their relevance has attracted mounting interest as the monetary policy literature has largely emphasized the benefits of achieving and maintaining price stability and many central banks worldwide have committed to low inflation targets.

The empirical literature on disinflations makes it quite clear that successful disinflations are accompanied by temporary economic downturns (e.g., Gordon and King, 1982; Ball, 1994b; Cecchetti and Rich, 2001). Estimates of the so-called sacrifice ratio (SR), which measures the cumulative output loss for each percentage point of reduction in inflation, vary considerably depending on the country, the historical episode and the econometric technique used. In general, from the available empirical evidence on the real costs of disinflations, a plausible range for the SR is between 0.5 and 3. Institutional factors, such as the monetary policy framework, have been shown to affect the disinflation costs. For the famous Volcker disinflation in the United States, often referred to as a monetarist experiment, Mankiw (1999) estimates an SR of 2.8. For a broad group of inflation-targeting countries, Corbo et al. (2001) find a lower average SR, 0.6.

Most of the disinflations considered in the empirical literature took place at times when the monetarist school held sway. Indeed, the most closely studies episode in history, the Volcker disinflation, is often called a monetarist experiment, after the celebrated monetary policy reform of October 1979 that abandoned federal funds targeting in favour of nonborrowed reserves targeting to control the money supply.[1] Sims and Zha (2006) econometrically identify a Volcker reserve-targeting period that "shows clearly the targeting of monetary aggregates, rather than interest rates, in that regime" (p. 55). Since then the theory and practice of monetary policy have changed radically. Nowadays, it is standard in theoretical models to assume an inflation targeting framework, where monetary policy is conducted through a simple nominal interest rate rule. In

[1] The extent to which the Volcker disinflation can actually be considered as a monetarist experiment is discussed at length in Lindsey et al. (2005) (see also the other papers in the same *Fed of St. Louis Review* issue) and Goodfriend and King (2005).

light of these considerations, it is of paramount importance to assess the implications of these two different monetary policy frameworks for disinflation dynamics.

In discussing conditions for a successful disinflation without too much output loss, several authors have emphasized the role played by the speed and timing of disinflation. Taylor (1983) has argued that a gradual approach to disinflation entails less output cost since inertial prices and wages take time to adjust after the monetary tightening. On the other hand, Sargent (1983) and Ball (1994b) have advocated a quick ("cold-turkey") disinflation on the grounds that a rapid disinflation enhances credibility and thus the shift in expectations.

In this paper, we address these issues using a medium-scale New Keynesian model with nominal and real frictions à la Christiano et al. (2005). As in Ascari and Ropele (2010), where it is shown that a theoretical model of this kind successfully accounts for the main stylized facts of disinflations without resorting to imperfect credibility or irrational expectations, we develop our analysis focusing on fully credible disinflationary monetary policy.[2]

Our main contributions are threefold. First, we examine to what extent the costs of disinflation depend on the monetary policy framework, i.e. money supply rule (MSR) versus interest rate rule (IRR), and on the operational procedure, i.e. cold-turkey, gradualism or anticipation. Our results show that the monetary policy strategy for disinflation significantly affects the SR and the dynamics of the model. On the transitional output costs of disinflation we find that: (*i*) disinflations under MSR or IRR involve a long-lasting decline in output; (*ii*) the theoretical SR values are in line with empirical estimates, with those under MSR generally larger than those under IRR; (*iii*) gradual or announced cold-turkey disinflations yield even lower SR values; and (*iv*) the theoretical SR values decrease with average inflation.

Second, we supplement the study of the transitional output costs of disinflation with a rigorous welfare analysis. Despite the prolonged output downturn, we find that disinflations are overall welfare-improving. The long-run welfare gain of a permanently

[2] Credibility is certainly an important aspect of a policy change towards disinflation. Several recent studies address this issue by assuming a learning behaviour on the part of private agents; see e.g. Erceg and Levine (2003), Goodfriend and King (2005) and Cogley et al. (2010).

lower inflation rate outweighs the transitional welfare cost. Still, given our benchmark parameters' calibration, the magnitude of these welfare effects is rather small. In terms of consumption equivalent units, each percentage point of diminished inflation increases the representative household's initial steady-state consumption by about 0.07% each period. Interestingly, this finding is quite robust with regards to the practical implementation of disinflation. Although alternative disinflation strategies or procedures involve different effects on the transitional dynamics of output and on the SR, from a welfare perspective there are no sizable differences. So, at least from a welfare perspective what really matters is achieving a permanent lower inflation rate, and it matters less how this goal is achieved in practice.

Third, our analysis also makes a methodological contribution. As in Ascari and Ropele (2010), we do not linearize the structural equations of the model, but simulate the non-linear first order conditions. We do this for the following reasons. Ascari and Merkl (2009) show that the use of linear approximations to study disinflations may lead to misleading results, as a disinflation entails a transition from one steady state to another. Furthermore, the standard approach of taking linear approximations may rule out some important transmission mechanisms. Yun (2005), for instance, emphasizes the role of relative price dispersion, often neglected in linear models, in driving his results for optimal monetary policy.

2 A brief empirical review of disinflation costs and dynamics

In this section we briefly review the basic empirical regularities that characterize disinflationary monetary policies. More specifically, we first examine the real output cost of disinflations and survey how that cost has been measured in empirical analyses. Then, we review the transmission mechanism of disinflation, stressing in particular the dynamic adjustment of output and inflation.

Transitional costs of disinflations. Most of the empirical studies on the costs of disinflation have used the SR indicator, calculated as the ratio of the cumulative percentage output loss to the disinflation size.

In broad terms, three approaches have been used to estimate the SR. One approach is based on estimating the slope of simple linear Phillips curve regressions. Using this strategy, Gordon and King (1982) estimate SR values ranging from 0 to 8 for the US economy. More recently, Andersen and Wascher (1999) offer a comprehensive analysis for 19 industrialized countries and show that SR estimates are sensitive to the particular Phillips curve specification used in the estimation and to the particular historical period considered. They also report that the average SR rise from 1.5 to 2.5 as average inflation decreased throughout 1980s and 1990s and with the flattening of the aggregate supply curve. Filardo (1998) highlights the non-linearities in the costs of disinflation, as the estimated slope of the Phillips curve is different in periods of sustained economic growth compared with periods of weak economic activity. Finally, using data for euro-area countries in the period 1960-2001, Cuñado and Gracia (2003) estimate SRs between 0.6 and 2.0 and, as in Andersen and Wascher (1999), find an inverse relation between average inflation and the SR.

Ball (1994a) popularized another approach to gauge the costs of disinflation. In essence, Ball's strategy relies on the analysis of single disinflation episodes, identified by locating peaks and troughs in trend inflation.[3] He examines 19 moderate-inflation OECD countries in the period 1960-1991 and reports SR estimates between 1.8 and 3.3. In particular, for the Volcker disinflation of 1982-1985 he finds an SR of 1.8, which is close to the estimate of 1.7 reported by Erceg and Levin (2003) but somewhat smaller than the estimate of 2.8 reported in Mankiw (1999). More recently, Zhang (2005) has generalized Ball's approach, demonstrating that the SR estimates are larger when long-lived effects on output are taken into account. Furthermore, Zhang confirms the evidence of an inverse relation between average inflation and the SR.

Andersen and Wascher (1999) and Cecchetti and Rich (2001) have criticized Ball's (1994a) approach and have advocated the use of still another methodology based on structural VAR models. Cecchetti and Rich use a structural VAR model estimated on

[3] Ball (1994a) defines trend inflation as the centered nine-quarter moving average of actual inflation.

US data during the period 1959-1997, testing different identification schemes, and find SRs ranging from 1 to nearly 10. More recently, Durand et al. (2008) perform a study similar to Cecchetti and Rich's for twelve euro-area countries and find much lower SRs, in the range 0.2 to 0.8. Interestingly, they also find evidence in favour of an inverse relation between average inflation and the SR. Finally, Collard, Fève, and Matheron (2007) and Fève et al. (2010) use a structural VAR model for euro-area countries and find an average SR of 4.3.

Dynamic adjustment after a disinflation. The use of structural VAR models has also made it possible to illustrate the dynamic adjustment of several macroeconomic variables throughout a disinflation. In general, as one might expect, in response to a disinflationary monetary policy shock output declines temporarily while inflation decreases and eventually reaches a new lower level. But having said this, different structural VAR specifications, different identification schemes and different data have been shown to imply, to some extent, different transmission mechanisms. Using US data, Cecchetti and Rich (2001) (in one of their model's specifications) and Collard et al. (2007) find a markedly sluggish adjustment pattern of inflation, which increases on impact and then slowly declines towards the new lower level. Meanwhile, output falls, leading to a severe and protracted recession. Instead, Cecchetti and Rich (2001) in their benchmark model specification for the US economy and Fève et al. (2010) for euro-area data find that after a disinflationary shock inflation abruptly falls on impact, then picks up mildly and eventually converges to its lower level through fluctuations. The dynamic adjustment of output is similar to the one described above, although the economic downturn is smaller and less prolonged.

3 A medium-scale New Keynesian model

To examine the macroeconomic effects of disinflation we use a medium-scale New Keynesian model, in many regards similar to the one in Christiano et al. (2005), Smets and Wouters (2003) and Schmitt-Grohé and Uribe (2004). In particular, our theoretical model extends the basic three-equation New Keynesian model à la Clarida et al. (1999)

by adding a broader set of real and nominal frictions and by including endogenous capital accumulation. Real frictions include: monopolistic competition in goods and labour markets, internal habit in consumption, variable capital utilization and adjustment costs in investment decisions. As for nominal frictions: prices and wages are sticky according to a staggered adjustment mechanism à la Calvo. Finally, money balances enter the model in two ways: households derive direct utility from holding real money balances (i.e. assumption of money-in-the-utility function) and firms hold nominal money balances to pay wages before production (i.e. assumption of working capital).[4]

Instead of dwelling on the details the model, here we highlight some of its features, Appendix A contains the analytical description of the structural equations and the calibration of parameters.[5]

First, as regards the conduct of monetary policy we assume two possible frameworks: the central bank uses a nominal money supply rule (MSR) or a simple interest rate rule (IRR). Under an MSR framework, the central bank controls the growth rate of the nominal money supply (π_t^*), and thus the stock of nominal money (M_t) evolves according to the following law of motion

$$M_t = (1 + \pi_t^*) M_{t-1}. \tag{1}$$

Clearly, in this framework the steady-state inflation rate is pinned down by π^*.

Under an IRR framework the monetary policy instrument is the nominal interest rate. Thus, the central bank sets the nominal interest rate (i_t) after observing the inflation rate (π_t) and in accordance with the following rule

$$\frac{1 + i_t}{1 + i_t^*} = \left(\frac{1 + \pi_t}{1 + \pi_t^*} \right)^{\phi} \tag{2}$$

where π_t^* and i_t^* represent the inflation and the nominal interest rate targets, respectively. The parameter ϕ is assumed to be greater that one.[6] Unlike more conventional

[4]Christiano et al. (2005) for the US economy and Smets and Wouters (2003) for the euro-area document the importance of these rigidities to match the business cycle empirical regularities.

[5]For further details, see Schmitt-Grohé and Uribe (2004).

[6]In particular, the nominal interest rate target is given by $1 + i_t^* = (1 + \pi_t^*)/\beta$, where β is the

Taylor-type rules, according to our postulated rule the central bank does not respond to the output gap or/and to the lagged nominal interest rate. The rationale underlying our choice is that we see a disinflation as a situation in which the monetary policy is primarily, if not entirely, concerned with the ultimate goal of lowering inflation. Accordingly, we disregard the other terms that typically enter the standard nominal interest rate rule and that are instead useful to describe the conduct of monetary policy *in normal times*.

Second, with regards to nominal price and wage rigidity we employ a variant of the standard Calvo (1983) staggered adjustment mechanism. As in Smets and Wouters (2003), we assume that prices and wages that cannot be optimally adjusted are indexed *partially* to past inflation (with a weight equal to χ for prices and $\tilde{\chi}$ for wages) and *partially* to long-run inflation (with a weight equal to $1 - \chi$ for prices and $(1 - \tilde{\chi})$ for wages).[7] This formulation, which implies that in the long-run all nominal prices and wages are adjusted one-to-one with steady-state inflation, is advantageous because regardless of the level of χ there will not be any price and wage dispersion in steady state. In other words, higher levels of positive steady-state inflation imply no inefficiency loss in the resource allocation, which otherwise would be reflected, in general, in a lower level of steady-state output.

Third, although prices and wages are in effect fully indexed as in Smets and Wouters (2003), money in our model is non-superneutral. This is due to the working capital assumption, which makes the firms pay the wage bill before production and the real marginal costs to depend positively on the nominal interest rate. Although this feature helps to increase the empirical fit of the model (see Christiano et al., 2005), it also affects the steady-state relationship between output and inflation. The higher the level of steady-state inflation, the greater the labour costs for firms; hence, *ceteris paribus*, the lower the wage paid to workers. In response, households reduce their labour supply and employment falls. Firms in turn decrease their capital stock, because labor and capital are complements in the production function. Eventually, the level of output decreases. The long-run Phillips Curve is not vertical. Using the calibration reported in

representative household's subjective discount factor.

[7]Note that this formulation nests the price and wage indexation rule used in Christiano et al. (2005) when $\chi = \widetilde{\chi} = 1$ (i.e., full indexation to past inflation).

Schmitt-Grohé and Uribe (2004): a permanent 1% reduction in inflation implies roughly a 0.1% increase in steady-state output.[8]

Fourth, as our main focus here is on the effects of disinflationary monetary policies we deliberately refrain from addressing the potential role of fiscal policy. Under the MSR we only make the technical assumption that seigniorage revenues are returned to households via a lump-sum transfer.

3.1 The speed and timing of disinflation

Throughout our analysis, a disinflationary monetary policy is implemented by means of a permanent reduction of the target π_t^*. In particular, we assume that before the disinflation is actually carried out, say for $t = -\infty, \cdots, -2, -1$, the economy has been in a steady state characterized by a positive long-run inflation π_{old}^*. At $t = 0$, the central bank decides to disinflate the economy by lowering the target from π_{old}^* to π_{new}^*. Furthermore, we assume that the shift in the target is permanent, and that agents, once the disinflation is fully completed, do not expect any other policy change. From a methodological perspective we simulate the model using a non-linear solution method.[9]

As is widely discussed in the literature, disinflation programs can be designed in several ways depending on the speed and/or timing of reduction of π_t^*. A so-called *cold-turkey disinflation* entails an immediate reduction of the policy target, that is

$$\pi_t^* = \begin{cases} \pi_{\text{old}}^* & t = -\infty, ..., -2, -1 \\ \pi_{\text{new}}^* & t = 0, 1, ..., \infty \end{cases} .$$

A *gradual disinflation* entails instead a steady reduction of the policy target, completed

[8] The assumption of 100% price and wage indexation to past inflation rules out any potential real effect originating from the Calvo nominal friction. With partial indexation, a positive level of steady-state inflation would increase price and wage dispersion, yielding an inefficiency output loss (e.g. Ascari, 2004, Schmitt-Grohé and Uribe, 2004). In this case, the real effects of steady-state inflation would be significantly larger.

[9] In particular, we use the DYNARE package to numerically simulate the disinflation (see the web-page: http://www.cepremap.cnrs.fr/dynare/).

in k periods, that is

$$\pi_t^* = \begin{cases} \pi_{\text{old}}^* & t = -\infty, ..., -2, -1 \\ \pi_{t-1}^* - k^{-1}\left(\pi_{\text{old}}^* - \pi_{\text{new}}^*\right), \text{ with } k \geqslant 1 & t = 0, 1, ..., k-1 \\ \pi_{\text{new}}^* & t = k, k+1, ..., \infty \end{cases}.$$

Thus, the parameter k controls the disinflation speed: the lower is k, the faster is the reduction of the policy target. Clearly, the cold-turkey disinflation pertains to $k = 1$.

Another interesting case to consider is *announced disinflation*. In this event, the central bank at $t = 0$ declares the intention to disinflate the economy after k periods. In what follows we will only consider the case of announced cold-turkey disinflations, that is

$$\pi_t^* = \begin{cases} \pi_{\text{old}}^* & t = -\infty, ..., -2, -1 \\ \text{Announcement} & t = 0 \\ \pi_{\text{old}}^* & t = 0, 1, ..., k-1 \\ \pi_{\text{new}}^* & t = k, k+1, ..., \infty \end{cases}.$$

4 Measuring the transitional output cost of disinflation

In this section we measure the transitional output cost of disinflation. To this end, we follow the empirical literature and calculate a model-consistent SR, using the following formula

$$\text{SR} = -\frac{1}{\pi_{\text{old}}^* - \pi_{\text{new}}^*} \sum_{t=0}^{T} \left(\frac{Y_t - Y_{\text{new}}}{Y_{\text{new}}} \right), \tag{3}$$

where Y_{new} represents the steady-state level of output at π_{new}^*. Our model-consistent SR effectively cumulates the percentage output losses the economy has to sacrifice for each percentage point of permanent reduction in steady-state inflation. It is worth emphasizing two features of (3). First, we calculate the SR by considering output in deviation from its new steady-state level (Y_{new}). Second, we truncate to the first T periods the cumulation of the percentage output losses, where T indicates the number of periods inflation takes to settle down to the new steady-state level.[10]

In the following subsections, we shall examine how well different disinflation programs replicate the stylized facts reviewed in section 2 and we compare the outcomes in the two monetary policy frameworks, namely MSR and IRR. More specifically, we answer the following questions: (*i*) Do cold-turkey, gradual and announced disinflations have recessionary effects? (*ii*) How large are the SRs? (*iii*) Does the disinflation size, i.e. the difference between π^*_{old} and π^*_{new}, matter for the SR? (*iv*) Do initial and final values of steady-state inflation matter for the SR?

4.1 The effects of cold-turkey disinflations

In this section we compare the effects of cold-turkey disinflationary monetary policies under MSR and IRR. In both cases, we consider disinflations that start from moderately inflated steady-states, i.e., $\pi^*_{\text{old}} = \{2\%, 4\%, 6\%, 8\%\}$, and aim to achieve full price stability, i.e., $\pi^*_{\text{new}} = 0\%$.

4.1.1 Under MSR ...

As shown in Figure 1, under MSR cold-turkey disinflations come with a considerable recession. Output decreases following a hump-shaped pattern and eventually converges to the new steady-state through diminishing fluctuations. Inflation immediately falls, yielding a long-lasting deflation. Real money balances gradually build up while the nominal interest rate decreases. The ex-ante real interest rate rises and then reverts to steady state.

To better understand the mechanism underlying these adjustment paths, let us take as an example the disinflation from $\pi^*_{\text{old}} = 2\%$.

At time $t = 0$ when the central bank stops printing money (recall that $\pi^*_{\text{new}} = 0$), only a random share of firms optimize prices: aware of the new inflation target and the ensuing output contraction (necessary to curb inflation), these firms lower prices. The remaining firms mechanically adjust their prices one-to-one to past inflation, raising

[10] In particular, the value of T is chosen such that for $t > T$ it holds that $|\widetilde{\pi}_t - \pi^*_{\text{new}}| < 10^{-5}$, where $\widetilde{\pi}_t$ denotes the local minimia/maxima of π_t.

prices by $1+\pi^*_{\text{old}}$. As shown in Figure 1, the former pricing decision prevails on the latter, with the result that the aggregate price index decreases. The ensuing deflation boosts real money balances and drives down the nominal interest rate. The ex-ante real interest rate rises significantly, mainly reflecting the long-lasting future deflation, leading to a gradual reduction in consumption and investment spending. Output falls.

At time $t = 1$, optimizing as well as non-optimizing firms lower prices. The former do so as they anticipate a hump-shaped decline in aggregate demand, driven by habit in consumption and investment adjustment costs. The latter instead lower prices because of indexation to a negative inflation rate. As a result, deflation intensifies: the ex-ante real interest rate peaks and output reaches the trough. From then on the ex-ante real interest rate slowly reverts and as it stays below steady-state the economy experiences a temporary and mild expansion of output. Finally, the cold-turkey disinflation is completed in about 28 quarters.

Cold-turkey disinflations from higher inflation rates, i.e., $\pi^*_{\text{old}} = \{4\%, 6\%, 8\%\}$, exhibits qualitatively similar dynamics. Neither the transmission mechanism nor the timing of turning points in output and inflation are affected. Yet, higher initial levels of steady-state inflation have strong effects on the amplitude of output and inflation declines. In percentage deviations from the new steady-state, the fall in output at the trough, which occurs after two quarters following the disinflation, is nearly 2% for $\pi^*_{\text{old}} = 2\%$, then nearly doubles for $\pi^*_{\text{old}} = 4\%$, and becomes 6% for $\pi^*_{\text{old}} = 8\%$. Likewise, the decline in inflation at the trough, which occurs after four quarters, intensifies as π^*_{old} increases. Intuitively, as the initial level of steady-state inflation rises, optimizing firms lower prices to a greater degree, leading to a deeper deflation and a larger rise in the ex-ante real interest rate.

So, MSR cold-turkey disinflations are accompanied by a decline in output, but how costly is the disinflation? The top panel of Table 2 reports the theoretical SRs, calculated with $T = 28$. Several results are noteworthy. First, the SRs are roughly equal to 2.8, thus in line with the empirical evidence. So, in economic terms, this means that to achieve a permanent reduction of steady-state inflation, say, from 4% to zero, the economy has to incur a cumulative output loss of 5.7%. Second, varying the disinflation size has minor affects on the SR.

With regards to the robust empirical finding on the inverse relation between the SR and the average inflation, we consider fixed size cold-turkey disinflation experiments, from $\pi^*_{\text{old}} = \{4\%, 6\%, 8\%\}$ to $\pi^*_{\text{new}} \equiv \pi^*_{\text{old}} - 2\%$. As shown in Table 2, the SRs decrease as π^*_{old} rises. Disinflating from 4 to 2% entails an SR of 2.2, i.e., 0.8 percentage points lower than from 2 to 0%. The SRs decrease even more for $\pi^*_{\text{old}} = 6\%$ and $\pi^*_{\text{old}} = 8\%$, to 1.8 and 1.6, respectively. Fixed-size cold-turkey disinflations therefore appear to have notable and non-linear effects on the SR. Figure 2 shows disinflation paths in these cases.

4.1.2 ... and under IRR

We now repeat the same disinflation experiments assuming the central bank operates under a IRR framework. As in Taylor (1993) we set $\phi = 1.5$. As shown in Figure 3, in this case cold-turkey disinflations again come with a considerable recession. Although the transmission mechanism is broadly similar to that under MSR, a number of qualitative and quantitative differences stand out. First, under IRR cold-turkey disinflations involve an immediate rise in the nominal interest rate.[11] The prolonged period in which the ex-ante real interest rate stays above its steady-state level lowers aggregate demand and yields a decline in output. As inflation starts steadily to decline, the central bank cuts the policy rate. Second, inflation converges to the new steady-state level through a gradual adjustment path. This stands in stark contrast with the deflation that characterizes the cold-turkey disinflations under MSR (see Figure 1). Third, for a given disinflation size, cold-turkey disinflations under IRR yield less macro volatility than under MSR. At the trough the fall in output under MSR is approximately three times greater than under IRR. Fourth, under IRR cold-turkey disinflations are accomplished in 15 quarters, roughly half the time it takes under MSR.

[11] The reduction of the policy target π^* has two opposite effects on the nominal interest rate under IRR. On the one hand, the permanent decline in π^* opens up a temporary inflation gap (given that the inflation does not adjust immediately) and this calls for an increase in the policy rate. On the other hand, it leads to a permanent decline in the nominal interest rate target, i.e., i^*. The latter effect calls for a cut in the policy rate. *Ceteris paribus*, which of the two forces prevails crucially depends on the policy parameter ϕ. For low values of ϕ, the initial increase in the nominal interest rate would be absent. Nonetheless, we think that in the context of disinflation it is realistic to assume a substantial value for ϕ.

In Figure 3 we also report the time-varying growth rate of the money supply implied by the interest rate rule.[12] It turns out that the growth rate of the nominal money supply declines suddenly, then increases, overshooting the initial growth rate, and thereafter gradually converges to the new steady-state level. The immediate response of m_t reflects the initial rise in the nominal interest rate, which temporarily depresses demand for real money balances.

The bottom panel of Table 2 reports the theoretical SRs, calculated for $T = 15$. Not surprisingly, they are substantially lower (approximately equal to 1) than those under MSR and still in line with the estimates founded in several empirical studies. Moreover, it turns out that they are quite insensitive to different sizes or fixed-size disinflations.

4.1.3 Discussion: the role of monetary policy framework

Inflation dynamics are rather different in the two monetary policy frameworks. Under MSR, the central bank freezes the nominal money supply; however, real money balances have to increase to reach the new steady-state level. The only possible way these patterns can square is for inflation to decrease more than the growth rate of nominal money supply, which is zero in Figure 1 and positive in Figure 2. Thus, the requisite aggregate price dynamics are brought about by a long-lasting output contraction, which induces firms to lower prices producing a decrease in inflation. This, in turn, rationalizes the lack of inflation persistence under MSR, despite the assumption of full price indexation to past inflation. Although, full price indexation to past inflation does cause a relatively more sluggish adjustment of inflation and a deeper economic downturn, it is not the fundamental driver of the recession. Even with prices fully indexed to steady-state inflation, the same qualitative dynamic adjustment illustrated in Figures 1 and 2 would carry on.

Under IRR, by contrast, if prices are fully indexed to steady-state inflation there will be no recession. This is because under a Taylor rule the money supply is endogenous and can thus adjust freely to satisfy the increase in the demand for real money balances.

[12] The time-varying growth rate of the nominal money supply is computed as $\pi_t^* = (m_t/m_{t-1})\,\pi_t$, where m_t denotes holding of real money balances.

In this case, monetary policy would increase the money supply initially to meet the increased money demand, and since inflation is a pure forward-looking variable there would not be the need for a recession to occur. It follows that under IRR the assumption of price (and wage) indexation to past inflation is the main cause of the recession. In fact, in Figure 3 the growth rate of the nominal money supply jumps, after an initial sharp fall, and diminishes only gradually. Monetary policy has to fight the inertia in indexation in the initial period, and it slashes the rate of growth of the money supply, leading to an increase in the nominal interest rate. In the following period, however, it accommodates money demand, since a relatively high rate of growth of money supply would be absorbed by money demand without inflationary pressure (and is actually needed to avoid a deflationary period).

All in all, the key message here is that the monetary policy framework matters for disinflation. Our results show that an IRR disinflation generally entails lower output costs than those attainable under MSR, reflecting the different nominal money supply paths in the two monetary policy frameworks. Furthermore, price indexation to past inflation causes a recession only under IRR. If the Volcker disinflation can truly be deemed a "monetarist experiment", then indexation to past inflation is not really needed to explain the high cost of a disinflation.[13]

4.2 The effects of gradual/announced disinflation

In this section we examine gradual and announced disinflations. To save space we report disinflations from $\pi^*_{\text{old}} = \{2\%, 4\%, 8\%\}$ to $\pi^*_{\text{new}} = 0\%$, and only show the adjustment paths of output and inflation.

4.2.1 Gradual *versus* cold-turkey disinflation

Figures 4 and 5 illustrate the effects of gradual disinflations for $k = \{4, 8, 12\}$ under MSR and IRR respectively. In general, gradual disinflations are accompanied by a hump-shaped decline in output, though for a given π^*_{old} a slower reduction of the policy

[13] This obviously does not rule out the possibility that the lack of credibility could have added significant costs to the Volcker disinflation.

target implies less output volatility, for as k increases optimizing firms lower prices less aggressively and the ex-ante real interest rate rises less.

The data reported in Table 2 confirm that gradualism unambiguously reduces real output costs. This is particularly evident under MSR, whereby a more gradual disinflation reduces the SR monotonically. For example, a three-year disinflation, i.e., $k = 12$, yields an SR that is roughly half the one that would arise under a cold-turkey policy. Also, for given disinflation speed, it is less costly to disinflate from higher levels of steady-state inflation. With regards to IRR, a gradual approach to disinflation still delivers lower SRs than those under a cold-turkey, though in this case the difference is somewhat less pronounced. Furthermore, under IRR the reduction of the SR due to a more gradual disinflation does not appear to be monotonic in k: the SR decreases until $k = 8$ and then starts to rise. Finally, with regards to fixed-size disinflations, for a given k the SR decreases with higher levels of initial steady-state inflation, especially under MSR.

4.2.2 Announced *versus* cold-turkey disinflation

Figures 6 and 7 illustrate the effects of announced cold-turkey disinflation policies under MSR and IRR, respectively. In each case we consider disinflations announced 1, 2 and 4 quarters in advance. The main result here is that anticipated cold-turkey disinflations entail long-lasting output downturns. Under MSR, the effects of announcing a future cold-turkey disinflation reflect upon output and inflation dynamics. Regardless of the disinflation size, output contraction gets smaller (see for instance the percentage fall of output at the trough) and inflation may even converge smoothly to steady-state without any deflation (see for instance the case with $\pi^*_{\text{old}} = 8\%$ and a 1-year anticipated cold-turkey disinflation). In general, the top entries of Table 2 confirm that the anticipation of future cold-turkey disinflation brings about monotonic declines of the SR values, regardless of the disinflation size.[14]

[14] In other disinflation experiments not reported here, we found that for announced cold-turkey disinflation longer than two years the SR starts to increase. This naturally raises the question for the optimal design of fully credible anticipated disinflation policies. The answer to this question is beyond the scope of this paper and is the subject of ongoing research.

Under IRR, the announcement of future cold-turkey disinflations has stabilizing effects on output, but – and this is in contrast with MSR – barely any effect on inflation (see Table 2). As shown in Figure 8, these policies tend to de-stabilize the nominal interest rate as well as the ex-ante real interest rate. This is actually an artifact of the specific experiment we are considering since the central banker keeps targeting the old inflation target π^*_{old} until the disinflation is truly implemented. Right after the central bank announces it intends to disinflate, optimizing firms lower prices. As inflation falls moderately relative to the old target, the central bank reduces the nominal interest rate. The ex-ante real interest rate increases slightly leading to a more muted output contraction. When the central bank actually executes the reduction of the inflation target, the nominal as well as the ex-ante real interest rates peak and thereafter decrease monotonically towards their respective steady-state.[15]

To summarize our results, both MSR and IRR gradual or announced disinflationary monetary policies deliver lower SRs than under cold-turkey. However, the relation between the SR and the speed of disinflation is not necessarily monotonic.

5 Measuring the welfare effects of disinflation

As remarked by Gordon and King (1982), the mere existence of output losses following a disinflation does not by itself have any policy implications. A thorough balance should be drawn of the welfare cost of forgone output and the welfare benefits of lower inflation. On the later point, the recent new Keynesian monetary policy literature has largely emphasized under which conditions and why the achievement of full price stability is socially desirable (see Woodford, 2003 and the references therein).

In this section we tackle this issue and follow Ascari and Ropele (2010) to calculate an indicator that measures the total welfare effect that arises during a disinflation. As detailed in Appendix B, our proposed indicator of the *total welfare effect* of disinflation

[15] The sudden reversal and zigzag behaviour of the nominal interest rate may lead to peculiar adjustment dynamics, especially for announcement experiments longer than one year. In a 2-year announcement case, on impact the real interest rate may even decrease, thereby yielding an expansion in output.

is given by

$$\mathcal{W} = \frac{1 - \exp\left[(1-\beta)(\mathbb{V}_1 - \mathbb{V}_{\text{old}})\right]}{\pi^*_{\text{old}} - \pi^*_{\text{new}}} \tag{4}$$

while the *transitional welfare effect* is computed as

$$\widetilde{\mathcal{W}} = \frac{\exp\left[(1-\beta)\left(\mathbb{V}_{\text{new}} - \mathbb{V}_{\text{old}}\right)\right] - \exp\left[(1-\beta)(\mathbb{V}_1 - \mathbb{V}_{\text{old}})\right]}{\pi^*_{\text{old}} - \pi^*_{\text{new}}} \tag{5}$$

where $\mathbb{V}_{\text{old}}$ and $\mathbb{V}_{\text{new}}$ denote the representative household's value function in the old and in the new steady states; and $\mathbb{V}_1$ denotes the representative household's value function in the first period after the central bank implements the disinflation.

Two remarks are in order. First, for the sake of interpretation, our welfare results are expressed in terms of consumption equivalent units. In practice, the consumption equivalent measure is defined as the constant fraction of the initial consumption level that the representative household has to give away each period in order to obtain the same level of value function it would obtain if the disinflationary policy were implemented. Note that this is an accurate measure of the costs of disinflation in terms of consumption: it measures how much the representative household suffers in terms of forgone consumption in exchange for a permanent reduction in inflation. Second, our welfare-based indicators are computed echoing the construction of the SR, so a positive (negative) value of the welfare-based indicator has to be interpreted as a welfare loss (gain).

5.1 Welfare results

Table 3 reports the results for cold-turkey disinflations under MSR and IRR. For all disinflation experiments, the total welfare indicator is negative, signifying that *disinflations are welfare improving.*[16] We think this is an interesting result. Most empirical studies focus only on the transitional costs of disinflation in terms of forgone output, but neglect, often by construction, the potential long-run benefits of a lower inflation

[16] Qualitatively, this result does not depend on the inclusion of real money balances in the utility function. We have also calculated welfare-based measures without accounting for the utility gain coming from the long-run increase in real money balances. In this case, the welfare results would be smaller than the ones reported in the table by 30%.

rate. We show that in a medium-scale New Keynesian model a cold-turkey disinflation policy is overall welfare improving. Still, these welfare gains are rather small, amounting to an extra 0.06 per cent of consumption each period.

These results are even more striking when the total welfare gain is decomposed between transitional and long-run effects. We have shown in previous sections that cold-turkey disinflations under MSR or IRR entail a large and lasting output decline and that the theoretical SRs tally with empirical estimates. From a welfare perspective, however, it turns out that, as indicated in Table 3, the transitional welfare loss is quantitatively negligible and equal to a fall of 0.01% in initial consumption for each following period. Inspection of the transitional welfare losses also shows that disinflations under IRR are less welfare decreasing than those under MSR, though the differences are quantitatively very small. Disinflation size appears to matter only for the long-run welfare gain, which almost linearly improves as π^*_{old} rises. For fixed-size disinflation experiments, both the transitional welfare loss and the long-run welfare gain improve for higher levels of π^*_{old}. Lastly, Table 4 reports the welfare effects for gradual and announced disinflations. In general, the more gradual the disinflation or the longer is anticipated, the larger the welfare gain. In any case, these effects are quantitatively very small.

The key result here is that no matter how a disinflation is implemented (under MSR or IRR; cold-turkey, gradually or announced), the total effect on the representative household's value function is to improve welfare. Yet, the order of magnitude of welfare gains is quite small and corresponds for each percentage point of diminished inflation to an increase of about 0.06% in initial consumption in each period. We next illustrate the intuition for this finding by taking as an example a cold-turkey disinflation under MSR. Figure 9 displays adjustment paths of consumption, employment and the utility function, with and without real money balances. The cold-turkey disinflation produces a prolonged recession, induced by a lasting decrease in consumption and employment. The levels of consumption and employment, however, have opposite effects on the representative household's utility function. Thus, the net effect on the representative agent's utility function is ambiguous. On impact, the fall in consumption dominates, dragging down utility. However, already in the second quarter the effect of falling employment takes over and drive utility above the new steady-state level. Moreover, utility will remain above its steady-state level throughout the recession, mainly because the decline in employment is larger in percentage terms and relatively more sluggish, so the positive effect of employment is quite effective in counterbalancing the negative effect of lower consumption. Overall, the transition entails a short-run cost, as shown above, but of a negligible order of magnitude in terms of utility. Figure 9 also illustrates the adjustment path of the utility function net of real money balances to make it clear that our results do not depend on the dynamics of real money balances.

The above analysis shows that the result that disinflations are welfare improving hinges on the representative agent framework, which cannot account for the fact that some individuals may experience sharp drops in utility during recessions as they are thrown out of work. Nonetheless, our results show two important aspects. On the one hand, they cast a shadow on the use of DSGE models for welfare evaluation without "inspecting the mechanism". In particular, the ranking across different monetary policy rules or the optimal policy problems are bound to be based on mechanism similar to ours. On the other hand, if markets were complete (and agents the same ex-ante), then all agents could have the same marginal utility from consumption. Therefore, our results simply show once again that if the economy could provide efficient risk-sharing across agents (either through capital markets or some public welfare system), then disinflation in particular and recession in general could be less of a problem than they are normally thought to be.

6 Robustness analysis

Table 5 shows the results of a comprehensive robustness analysis on the parameters of the model, in order to investigate the different channels through which a disinflation operates. Whenever possible we refer again to Christiano et al. (2005), experimenting the values of the parameters implied by the upper and lower limits of the 95% confidence band reported in their estimation.

Unsurprisingly, the most important friction is price and wage stickiness. For example, regardless of the policy rule, the SR is roughly doubled with respect to our benchmark case when the degree of price rigidity is increased to 0.77, and it is about halved when α is lowered to 0.44. The sensitivity of the SR to the degree of wage stickiness is somewhat smaller, but still significant. Reducing both of them would therefore result in a substantial reduction of the SR. Note, however, that the welfare-based measure of the cost (gain) of a disinflation is only marginally affected by the change in the parameter values, as we should expect in light of the discussion in the previous section.

Another important source of nominal rigidity is the degree of indexation to past inflation. Section 4.1.3 discussed why indexation is particularly important for disinflation dynamics under IRR. Table 5 confirms the same implications: the SR is quite sensitive to the degree of indexation to past inflation in prices and wages. Under IRR, looking at the transitional dynamics, indexation makes inflation adjustment more sluggish and the recession deeper. Under MSR, instead, indexation has only marginal effects on the SR. According to our results, moreover, our welfare indicator is practically insensitive to changes in the degree of price and wage indexation.

So while nominal frictions do affect the SR, real rigidities appear to have weaker consequences. In general, real frictions have a bearing on the dynamics of the model inducing more inertia and less volatility in the adjustment path of variables. Consequently, the larger are the real frictions, the more muted are the peaks and troughs throughout the adjustment after the disinflation. Thus, a lower degree of habit persistence or the degree of capital adjustment costs leads, under MSR or IRR, to increases in the SR.

For similar reasons, softening firms' cash-in-advance constraint tends to increase the SR, as it increases the volatility of output response. Note, however, that, contrary to the other parameters, a decrease in ν substantially diminishes the welfare indicator. This simply reflects the fact that the long-run welfare gain depends on the cash-in-advance constraint on firms (see Section 3). Without the working capital assumption (i.e. $\nu = 0$), money will be superneutral and the long-run welfare gain will be due solely to the real money balance term in the utility function (see footnote 16).

Finally, in the case of MSR, money demand may play an important role. Increasing σ_m (i.e. the inverse of the elasticity of households' money demand) diminishes the SRs under MSR, but not under IRR (where policy basically accommodates money demand to target a given interest rate). Intuitively, increasing σ_m decreases the elasticity of households money demand: real money balances will be lower in equilibrium and less responsive to changes in the equilibrium interest rate. Therefore, the change in real money balances from one steady state to the other one is lower. It follows that the prices should adjust less in the case of MSR, because the required adjustment in real money balance is lower.

This section highlights the role of the different frictions and the different channels operating along the adjustment dynamics after a disinflation. Except for the degree of wage and price stickiness, changes in the other structural parameters of the model do not appear to modify significantly either the SR or the total welfare effects.

7 Conclusions

Aboundant empirical evidence indicates that successful disinflations in actual economies entail a sustained period of economic downturn. A classical policy issue regards the disinflation design to minimize the output loss associated with a period of disinflation. On the one hand, Taylor (1983) argued that a gradual disinflation is less expensive as it allows wages and prices enough time to adjust to the new policy target. Likewise, disinflations announced farther in advance may deliver even lower costs. On the other hand, Sargent (1983) contended that a fast disinflation, the so-called cold-turkey approach, is more desirable because expectations adjust faster.

In this paper we revisited the widely debated issue in monetary economics of the effects of different speed and timing of disinflations by means of a medium-scale New Keynesian dynamic general equilibrium model. In particular, we investigated which disinflation approach is less costly when the monetary policy is implemented either through a nominal money supply rule or an interest rate rule. Our comparative analysis on the costs of disinflation offered two perspectives. First, we evaluated the real costs of disinflation by constructing a theoretical sacrifice ratio that measures the cumulative output loss for each percentage point permanent reduction in inflation. Second, we used a novel metric based on the representative agent's welfare function. Such an indicator in practice balances the short-run welfare losses from the economic contractions and the long-run welfare gains, deriving from the fact that a lower steady-state inflation rate increases the levels of real variables.

Our results can be summarized as follows. On the short-run costs of disinflation, we found that cold-turkey disinflations implemented through an interest rate rule are in general less costly, in terms of the sacrifice ratio, than those achieved by means of a money supply rule. Furthermore, in the former case, the permanent reduction in inflation is accomplished more rapidly. Under both rules, gradual and anticipated disinflations deliver even lower sacrifice ratios, though in the case of an interest rate rule the relation between the sacrifice ratio and the speed of disinflation is not monotonically decreasing. On the welfare analysis our results showed that despite the substantial output contraction disinflations are overall welfare enhancing. The long-run welfare gains of permanently lower inflation outweighs the short-run welfare costs. Still, given our benchmark parameters calibration of the model, the welfare effects are quantitatively rather small. In terms of consumption equivalent units, each percentage point of diminished inflation increases the representative household's initial steady-state consumption by about 0.07% each period. Interestingly, this finding holds up quite well however disinflation is implemented.

The two main results of the paper offers a useful benchmark for future research.

First, the result that disinflating by controlling the money supply is more costly than disinflating by changing the inflation target hinges partly on the way money demand is modelled. So to address this issue further one must think carefully about the money and the financial markets. This is surely a promising avenue for future research, especially seeing that the financial crisis has stimulated developments of DSGE macromodels with a banking sector and financial frictions.

Second, the result that a disinflation is welfare improving despite the short-run recession needs to be taken with caution. The last section of the paper clarifies that an heterogeneous agent framework that can account for different costs of the recession across agents can overturn the result, which is basically another side of the coin of the

Lucas' finding of the low cost of business cycles in a representative agent framework. Research in this direction is therefore important for evaluating not only the cost of disinflation but also the cost of business cycles and thus the optimal policy literature.

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A A medium-scale New Keynesian model

In this appendix we describe the medium-scale New Keynesian model, following closely the outline in Schmitt-Grohé and Uribe (2004).

Households

There is a continuum of infinitely-lived households whose expected intertemporal utility function is given by

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(c_t - bc_{t-1}; h_t^s; m_t^h \right) \right\} \tag{6}$$

where E_0 defines the mathematical expectation operator conditional on the information set available at time 0, β is the subjective discount factor, function $u \left(c_t - bc_{t-1}; h_t^s; m_t^h \right)$ is well-behaved and increasing in consumption c_t and money holdings m_t^h, while decreasing in hours worked h_t^s. Preferences display habit in consumption levels, measured by the parameter b.

There is a continuum of final goods. indexed by $i \in [0, 1]$, which are aggregated in the usual CES consumption bundle c_t

$$c_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{7}$$

where the parameter η indicates the elasticity of substitution between different varieties of goods. The standard household problem defines the optimal demand of good i, given by $c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} c_t$, where P_t is the general price index given by $P_t = \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}$.

There is a continuum of labour services h_{jt}, $j \in [0, 1]$, which are combined according to the following technology

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj \right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}}, \tag{8}$$

where $\tilde{\eta}$ is the elasticity of substitutions of labour types. The standard cost minimization problem for firms yields the labour-specific demand function given by $h_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\tilde{\eta}} h_t^d$, where W_{jt} is the wage paid to labour type j and W_t is a wage index defined as $W_t = \left[\int_0^1 W_{it}^{1-\tilde{\eta}} di \right]^{\frac{1}{1-\tilde{\eta}}}$. The total labour supply is found by integrating labour-specific demand functions, to obtain h_t^s

$$h_t^s \equiv \int_0^1 h_{jt} dj = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj. \tag{9}$$

Agents own physical capital k_t that depreciates at rate δ. The capital accumulation equation is

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right], \tag{10}$$

where the function S introduces the adjustment cost on investment and satisfies the properties that $S(1) = S'(1) = 0$, $S''(1) > 0$. The model also features variable capacity utilization of physical capital, denoted by u_t. The cost of capital then depends on the degree of utilization and it is given by $a(u_t)$. Agents rent capital to firms at a real interest rate r_t^k and also decide on the utilization rate. There are complete markets for state contingent assets, such that all agents choose the same level of consumption.

Household first order conditions are thus given by

$$u_{c_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + u_{c_t}\left(c_{t+1} - bc_t; h_{t+1}^s; m_{t+1}^h\right) = \lambda_t \tag{11}$$

$$u_{h_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) = -\lambda_t \frac{w_t}{\tilde{\mu}_t} \tag{12}$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t}\left[q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - a(u_{t+1})\right] \tag{13}$$

$$q_t \lambda_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left[S_i\left(\frac{i_t}{i_{t-1}}\right)\right] i_t\right] - \beta q_{t+1}\lambda_{t+1} S_i\left(\frac{i_{t+1}}{i_t}\right) i_{t+1} = \lambda_t \tag{14}$$

$$a_{u_t}(u_t) = r_t^k \tag{15}$$

$$u_{m_t^h}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + \beta \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t. \tag{16}$$

Wages are sticky à la Calvo, and $1 - \tilde{\alpha}$ is the probability of being able to reset wages next period. If wages can not be re-optimized, they are automatically updated according to the following indexation rule: $w_{j,t+1} = w_{j,t} \pi_t^{\tilde{\chi}} \overline{\pi}^{1-\tilde{\chi}}$, where $\tilde{\chi}$ is the degree of indexation to past inflation and $\overline{\pi}$ is the level of steady-state inflation. Define $\tilde{w}_t$ as the optimal wage set every period t. The union chooses the optimal wage maximizing its utility function given by equation (7), subject to demand for labour in the specific market $h_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} h_t^d$ and the probability of not being able to re-optimize in future periods. The resulting first-order condition is

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}} \overline{\pi}^{1-\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}} \overline{\pi}^{1-\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right] = 0. \tag{17}$$

All the reset optimal wages are identical in all labour markets.

Firms

Each good is produced by a firm that monopolistically supply its own variety using a production technology of the form

$$z_t F(k_{it}, h_{it}) - \psi,$$

where z_t is an aggregate technology factor common across firm, and ψ represents a fixed cost of production. The production function $F(k_{it}, h_{it})$ is well-behaved and the same across firms. Final goods can be used for consumption, investment, public expenditure and to pay cost of capital utilization. Each firm faces the following demand function

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t, \tag{18}$$

where

$$y_t = c_t + i_t + g_t + a(u_t) k_t. \tag{19}$$

Firms rent capital from households in a competitive market, and must pay a fraction ν of wages in cash at the beginning of the period by cash. Therefore their money demand function is

$$m_{it}^f = \nu w_t h_{it} \tag{20}$$

The firms' problem is then to maximize the expected value of future profits, under their demand function (18) and the cash-in-advance constraint (20). The first-order conditions with respect to capital and labour services are

$$mc_{it} z_t F_{k_{it}} (k_{it}, h_{it}) = r_t^k \quad (21)$$

$$mc_{it} z_t F_{h_{it}} (k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t} \right]. \quad (22)$$

Since F is homogeneous of degree one, equation (21) and equation (22) imply that all firms have the same marginal costs and aggregation across firms is straightforward.

Prices are sticky à la Calvo. Every period each firm can choose a new price of its own good with a probability $1-\alpha$. Like wages, prices that cannot be reset optimally are automatically updated according to the following indexation rule: $P_{it} = P_{it-1} \pi_{t-1}^{\chi} \overline{\pi}^{1-\chi}$, where χ is the degree of indexation to past inflation. The first-order condition for the optimal price is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t} \right)^{-\eta} y_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi} \overline{\pi}^{1-\chi}} \right)^{\eta} \left[\frac{\eta - 1}{\eta} \frac{\tilde{P}_t}{P_t} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi} \overline{\pi}^{1-\chi}}{\pi_{t+k}} \right) - mc_{i,t+s} \right] = 0. \quad (23)$$

Again, all the reset optimal prices are identical for all goods.

Government

Government expenditure is financed through lump-sum taxes and seigniorage

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t}, \quad (24)$$

where m_t denotes real money balances and $\pi_t \equiv P_t / P_{t-1}$ is the (gross) inflation rate at time t. Government minimizes the costs of acquiring the composite good; hence, given public expenditure, government's absorption of a single type of good is $g_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} g_t$. To close the model we postulate that the monetary policy uses the simple non-linear nominal interest rate rule as described in the paper.

Equilibrium

The model equilibrium conditions are

$$
\begin{array}{ll}
\text{Money market:} & m_t = m_t^h + m_t^f \\
\text{Labour market:} & h_t^s = \int_0^1 h_{it}^d di \\
\text{Capital market:} & \int_0^1 k_{it} di = u_t k_t \\
\text{Good } i \text{ market:} & z_t F\left(k_{it}, h_{it}\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \left(\frac{P_{it}}{P_t}\right)^{-\eta} \\
\begin{array}{c}\text{Aggregate}\\ \text{Goods market}\end{array} : & z_t h_t^d F\left(\frac{u_t k_t}{h_t^d}, 1\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di.
\end{array}
$$

$s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta}$ is the price dispersion generated by price staggering, causing a wedge between aggregate supply and aggregate absorption; similarly, wage staggering gives rise to wage dispersion, given by $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$; see (9).

Functional forms and calibration

As in Schmitt-Grohé and Uribe (2004), we assume the following functional forms:

$$
\begin{aligned}
u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) &= \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2} h_t^2 + \phi_1 \frac{\left(m_t^h\right)^{1-\sigma_m}}{1-\sigma_m} \\
F\left(u_t k_t, h_t^d\right) &= \left(u_t k_t\right)^{\theta} \left(h_t^d\right)^{1-\theta} \\
S\left(\frac{i_t}{i_{t-1}}\right) &= \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 \\
a\left(u_t\right) &= \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2.
\end{aligned}
$$

Calibration is also as in Schmitt-Grohé and Uribe (2004) and Christiano et al. (2005). The parameters values are listed in the Table 1.

B A welfare-based measure of disinflation costs

In this appendix we closely follow Ascari and Ropele (2010) to derive the indicator of the welfare effect of disinflation. One notable advantage of working with a structural model is that it provides a natural metric to evaluate the welfare implication of disinflation:

Parameter	Value	Description
β	$1.03^{-0.25}$	Time discount rate
θ	0.36	Share of capital
ψ	0.5827	Fixed cost (guarantee zero profits in steady-state)
δ	0.025	Depreciation of capital
ν	1	Fraction of wage bill subject to CIA constraint
η	6	Elasticity of substitution of different varieties of goods
$\tilde{\eta}$	21	Elasticity of substitution of labour services
α	0.6	Probability of not setting a new price each period
$\tilde{\alpha}$	0.64	Probability of not setting a new wage each period
b	0.65	Degree of habit persistence
ϕ_0	1.1196	Preference parameter
ϕ_1	0.5393	Preference parameter
σ_m	10.62	Intertemporal elasticity of money
κ	2.48	Investment adjustment cost parameter
χ	1	Price indexation
$\tilde{\chi}$	1	Wage indexation
γ_1	0.0324	Capital utilization cost function parameter
γ_2	0.000324	Capital utilization cost function parameter
z	1	Steady-state value of technology shock

Table 1: Calibration of parameters in Christiano, Eichenbaum and Evans (2005).

this is the representative household's value function. Mimicking the construction of the sacrifice ratio in the main text, a measure of the welfare effect due to a disinflation is given by the difference between the level of the value function in period 1 ($\mathbb{V}_1$), i.e., value function the first period after the implementation of the disinflationary policy, and the level of the value function if the policy were not implemented, which corresponds to the value function in the old steady state ($\mathbb{V}_{\text{old}}$). So our total welfare effect indicator can be calculated as

$$\textbf{Welfare} = -\frac{\mathbb{V}_1 - \mathbb{V}_{\text{old}}}{\pi^*_{\text{old}} - \pi^*_{\text{new}}} \tag{25}$$

It is important to note that $\mathbb{V}_1$ includes both the transition dynamics and the long-run effects implied by the disinflation. Furthermore, note that if the disinflation brings about a total welfare gain then $V_1 - V_{\text{old}} > 0$ and our indicator takes on a negative value, i.e., $\textbf{Welfare} < \mathbf{0}$.

The above welfare indicator is not suitable to make cardinal comparisons of different disinflation policies, because it is being based on the utility of the representative agent. One way to overcome this problem is to express the term $(\mathbb{V}_1 - \mathbb{V}_{\text{old}})$ in terms of consumption equivalent units.[17]

The value function in the old (or initial) steady state is simply given by

$$\mathbb{V}_{\text{old}} = \frac{1}{1-\beta}\left[\ln\left((1-b)c_{\text{old}}\right) - \frac{\phi_0}{2}h_{\text{old}}^2 + \frac{\left(m_{\text{old}}^h\right)^{1-\sigma_m}}{1-\sigma_m}\right], \tag{26}$$

where c_{old}, h_{old} and m_{old}^h denote the values of consumption, hours worked and households' real money balances in the old steady state. Given the value of $\mathbb{V}_1$ that we retrieve when numerically simulating the disinflation, we just need to solve for the constant fraction $(\lambda\ )$ of steady-state consumption households have to give away in each period in the starting steady-state to equate the value of $\mathbb{V}_1$. In other words, we need to solve the following equation for λ:

$$\mathbb{V}_1 = \frac{1}{1-\beta}\left[\ln\left((1-b)c_{\text{old}}(1-\lambda)\right) - \frac{\phi_0}{2}h_{\text{old}}^2 + \frac{\left(m_{\text{old}}^h\right)^{1-\sigma_m}}{1-\sigma_m}\right]. \tag{27}$$

Consequently, the consumption equivalent measure is given by $\lambda = 1-\exp\left[(1-\beta)(\mathbb{V}_1 - \mathbb{V}_{\text{old}})\right]$. Finally, our proposed *total welfare effect* indicator is simply given by $\mathcal{W} = \lambda/\left(\pi_{\text{old}}^* - \pi_{\text{new}}^*\right)$. Note that if $\mathbb{V}_1 - \mathbb{V}_{\text{old}}$ is negative (positive), then $\mathcal{W}$ is positive (negative), meaning that the disinflation is welfare worsening (improving).

To disentangle the long-run and the transitional welfare effects of disinflation, we proceed as follows. First, we compute the long-run welfare effect (in terms of consumption equivalent units), that is $\lambda_\infty = 1-\exp\left[(1-\beta)\left(\mathbb{V}_{\text{new}} - \mathbb{V}_{\text{old}}\right)\right]$, where $\mathbb{V}_{\text{new}}$ and $\mathbb{V}_{\text{old}}$ denote the value function in the new and old steady state. The *long-run welfare effect indicator* is then simply given by $\mathcal{W}_\infty = \lambda_\infty/\left(\pi_{\text{old}}^* - \pi_{\text{new}}^*\right)$. Finally, the *transitional welfare effect indicator* can be simply calculated as: $\widetilde{\mathcal{W}} = \mathcal{W} - \mathcal{W}_\infty$.

[17] The consumption equivalent measure is defined as the constant fraction of consumption that households need to give away in each period in the starting steady-state, in order to obtain the same level of value function that households would get if the disinflationary policy is implemented. Note that this is a true measure of the costs of disinflation in terms of consumption: it measures how much households have to suffer in terms of consumption loss in order to reduce the inflation rate permanently by a certain amount.

		Money Supply Rule (T=28)						
$\pi^*_{\text{old}} - \pi^*_{\text{new}}$		2%-0	4%-0	6%-0	8%-0	4%-2%	6%-4%	8%-6%
Cold-turkey	k=0	2.94	2.85	2.78	2.73	2.22	1.89	1.69
	k=4	2.32	2.12	1.96	1.83	1.63	1.35	1.19
Gradualism	k=8	1.78	1.52	1.34	1.21	1.22	1.03	0.93
	k=12	1.55	1.32	1.16	1.06	1.16	1.06	1.00
	j=1	2.49	2.31	2.18	2.08	1.80	1.49	1.31
Announcement	j=2	2.09	1.86	1.70	1.82	1.45	1.16	1.00
	j=4	1.50	1.24	1.07	1.15	0.98	0.77	0.67
		Interest Rate Rule (T=15)						
$\pi^*_{\text{old}} - \pi^*_{\text{new}}$		2%-0	4%-0	6%-0	8%-0	4%-2%	6%-4%	8%-6%
Cold-turkey	k=0	1.06	1.05	1.04	1.03	1.03	1.01	0.99
	k=4	0.95	0.93	0.92	0.91	0.91	0.90	0.88
Gradualism	k=8	0.86	0.85	0.84	0.83	0.83	0.82	0.81
	k=12	1.05	1.04	1.03	1.02	1.02	1.01	1.00
	j=1	1.00	0.98	0.98	0.97	0.98	0.98	0.95
Announcement	j=2	0.90	0.89	0.88	0.87	0.88	0.87	0.86
	j=4	0.71	0.71	0.71	0.70	0.69	0.69	0.66

Table 2: Theoretical sacrifice ratio.

π^*_{old}	π^*_{new}	Money supply rule ($\times 10^{-2}$)			Interest rate rule ($\times 10^{-2}$)		
		Total	Long	Transitional	Total	Long	Transitional
2%	0	-6.40	-7.40	1.00	-6.59	-7.40	0.81
4%	0	-6.29	-7.30	1.00	-6.49	-7.30	0.81
6%	0	-6.20	-7.20	1.00	-6.39	-7.20	0.81
8%	0	-6.12	-7.11	1.00	-6.31	-7.11	0.80
4%	2%	-6.26	-7.18	0.91	-6.39	-7.18	0.79
6%	4%	-6.12	-7.00	0.87	-6.23	-7.00	0.76
8%	6%	-5.99	-6.82	0.83	-6.08	-6.82	0.74

Table 3: Welfare-based indicator of the effects of cold-turkey disinflations.

		Money supply rule ($\times 10^{-2}$)						
Disinflation		2%-0	4%-0	6%-0	8%-0	4%-2%	6%-4%	8%-6%
	k=4	-6.42	-6.36	-6.28	-6.20	-6.32	-6.18	-6.04
Gradualism	k=8	-6.51	-6.42	-6.35	-6.27	-6.36	-6.21	-6.07
	k=12	-6.53	-6.45	-6.37	-6.30	-6.37	-6.21	-6.07
	j=1	-6.44	-6.34	-6.26	-6.18	-6.30	-6.16	-6.02
Anticipation	j=2	-6.48	-6.38	-6.30	-6.23	-6.33	-6.19	-6.05
	j=4	-6.53	-6.45	-6.37	-6.30	-6.38	-6.23	-6.09
		Interest rate rule ($\times 10^{-2}$)						
Disinflation		2%-0	4%-0	6%-0	8%-0	4%-2%	6%-4%	8%-6%
	k=4	-6.61	-6.50	-6.41	-6.33	-6.41	-6.24	-6.09
Gradualism	k=8	-6.62	-6.51	-6.42	-6.34	-6.42	-6.25	-6.10
	k=12	-6.61	-6.50	-6.41	-6.33	-6.41	-6.24	-6.09
	j=1	-6.60	-6.49	-6.40	-6.31	-6.40	-6.23	-6.08
Anticipation	j=2	-6.61	-6.50	-6.41	-6.33	-6.41	-6.24	-6.09
	j=4	-6.63	-6.51	-6.43	-6.35	-6.43	-6.26	-6.11

Table 4: Welfare analysis: gradual and announced disinflationary monetary policies.

	From π_{old}=4% to π_{new}=0				From π_{old}=4% to π_{new}=2%			
	MSR		IRR		MSR		IRR	
Parameter	SR	$\mathcal{W}$	SR	$\mathcal{W}$	SR	$\mathcal{W}$	SR	$\mathcal{W}$
Benchmark	2.85	-6.29	1.05	-6.49	2.21	-6.26	1.03	-6.39
α=0.77*	6.18	-6.07	2.48	-6.44	4.80	-6.12	2.45	-6.36
α=0.44*	1.77	-6.37	0.62	-6.50	1.37	-6.32	0.61	-6.40
α=0	0.77	-6.41	ind.	ind.	0.60	-6.33	ind.	ind.
$\tilde{\alpha}$ =0.7*	3.26	-6.28	1.13	-6.48	2.52	-6.25	1.11	-6.39
$\tilde{\alpha}$=0.58*	2.56	-6.29	0.99	-6.49	2.01	-6.26	0.98	-6.39
$\tilde{\alpha}$ =0	1.69	-6.26	0.88	-6.48	1.49	-6.25	0.88	-6.38
χ=0.5	2.51	-6.35	0.33	-6.55	1.99	-6.31	0.33	-6.45
χ =0.25	2.53	-6.35	0.2	-6.57	1.99	-6.32	0.2	-6.46
χ =0	2.61	-6.35	0.15	-6.58	2.04	-6.32	0.15	-6.48
$\tilde{\chi}$ =0.58	2.57	-6.30	0.70	-6.49	2.14	-6.27	0.68	-6.39
$\tilde{\chi}$ =0.5	2.56	-6.30	0.64	-6.49	2.14	-6.27	0.63	-6.39
$\tilde{\chi}$ =0.25	2.55	-6.30	0.47	-6.49	2.17	-6.27	0.46	-6.39
$\tilde{\chi}$ =0	2.59	-6.31	0.34	-6.49	2.21	-6.27	0.33	-6.40
b =0.73*	2.73	-6.34	1.04	-6.50	2.13	-6.30	1.02	-6.40
b =0.57*	2.96	-6.24	1.06	-6.47	2.29	-6.22	1.04	-6.38
b =0	3.49	-6.04	1.18	-6.41	2.70	-6.06	1.16	-6.32
κ =5.74	2.35	-6.13	0.95	-6.39	1.89	-6.11	0.93	-6.29
κ =3.34*	2.64	-6.22	1.01	-6.45	2.08	-6.20	1.00	-6.35
κ =1.62*	3.21	-6.40	1.10	-6.54	2.43	-6.36	1.09	-6.45
ν =0.5	3.55	-4.04	0.99	-4.28	2.68	-4.05	0.98	-4.20
ν =0	n.a.	n.a.	0.95	-2.10	3.90	-1.80	0.95	-2.04
σ_m =11.96*	2.58	-5.98	1.05	-6.16	2.06	-5.96	1.03	-6.08
σ_m =9.28*	3.21	-6.72	1.05	-6.95	2.42	-6.68	1.03	-6.83

Table 5: Cold-turkey disinflation: Robustness analysis. Note. As in previous tables, the welfare indicator is multiplied by 100. The asterisx indicates parameter values computed using the estimated standard errors reported in Christiano et al. (2005). *ind* indicates equilibrium indeterminacy; *n.a.* not applicable because the solution of the model does not converge.

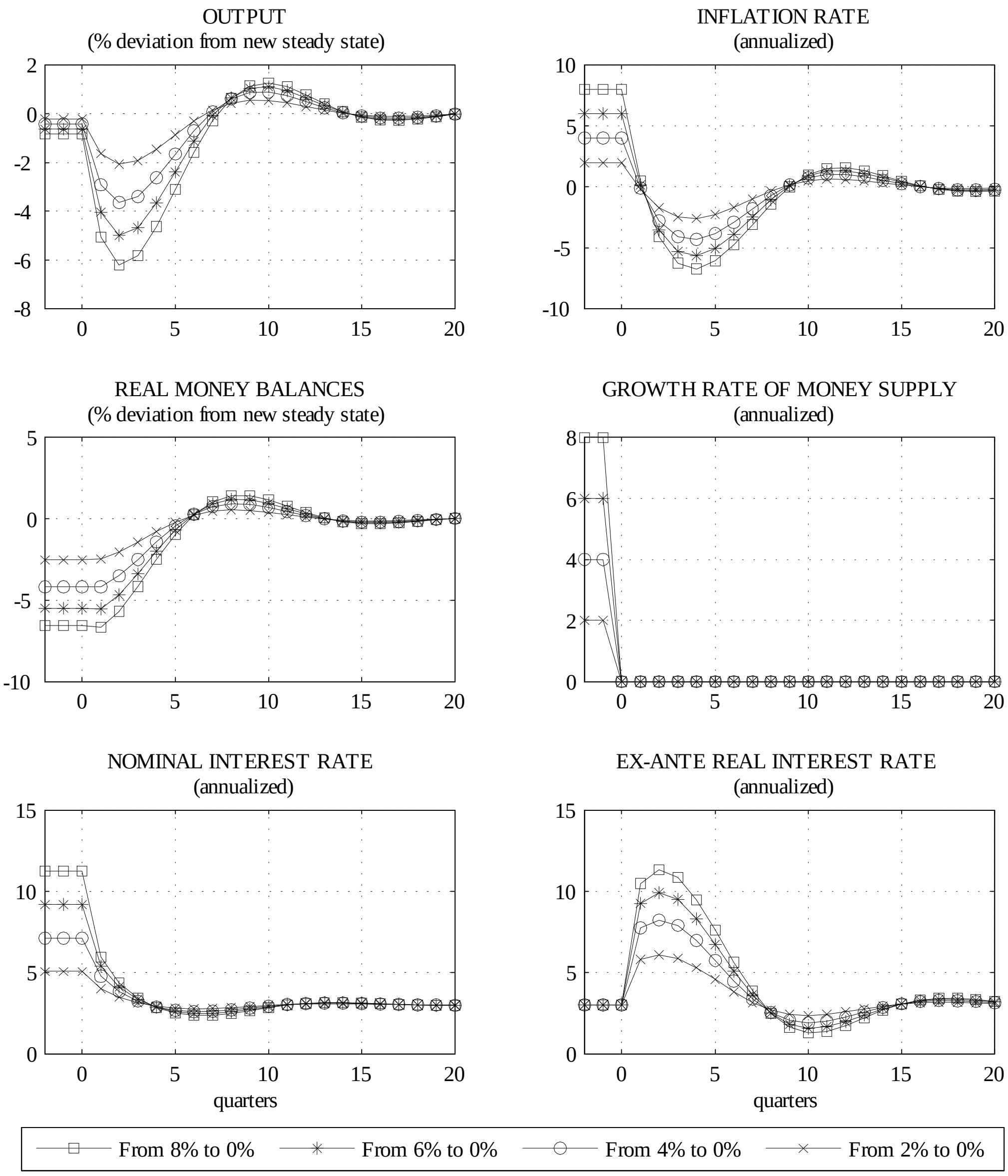

Figure 1: Cold-turkey disinflation under money supply rule.

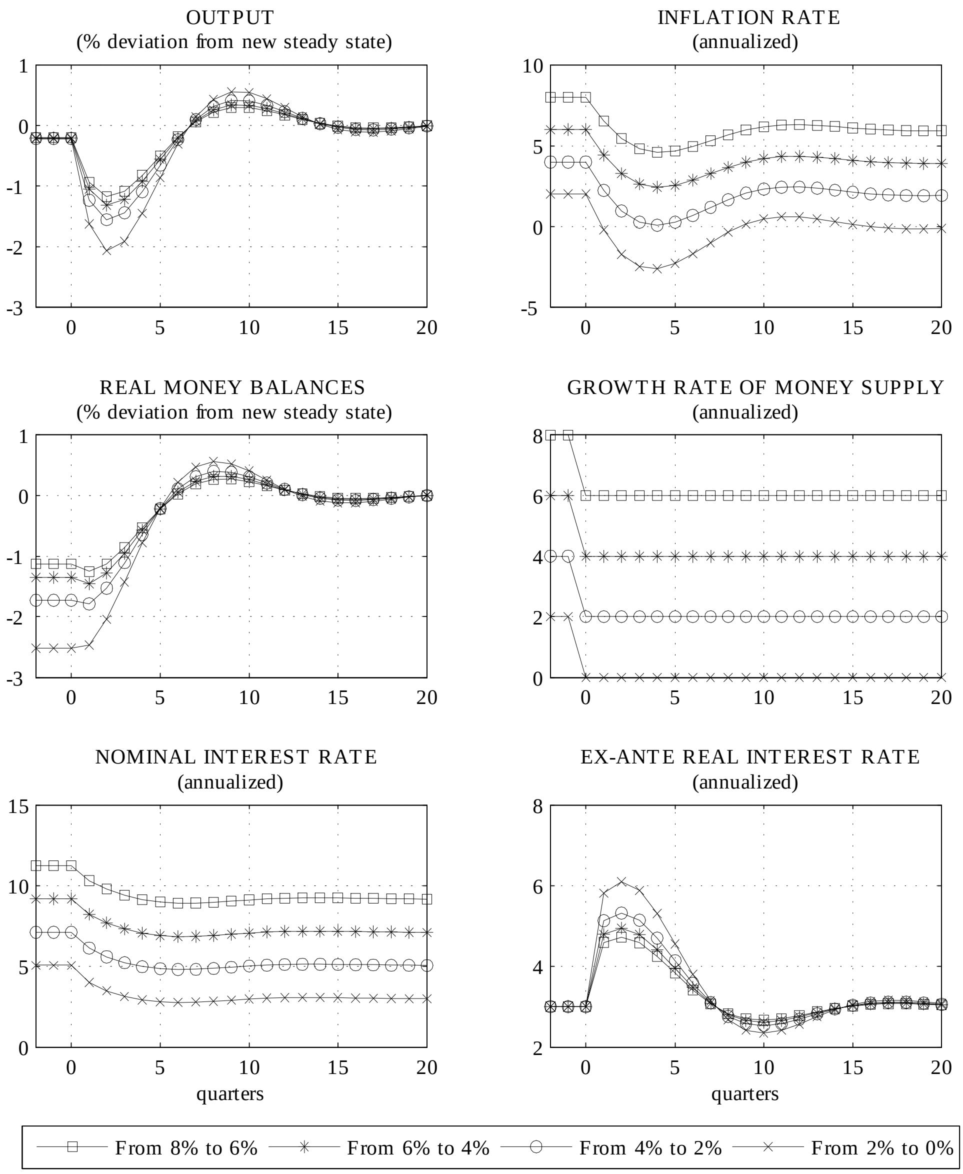

Figure 2: Cold-turkey disinflation under money supply rule for a fixed disinflation size: $\pi^*_{\text{old}} - \pi^*_{\text{new}} = 2\%$.

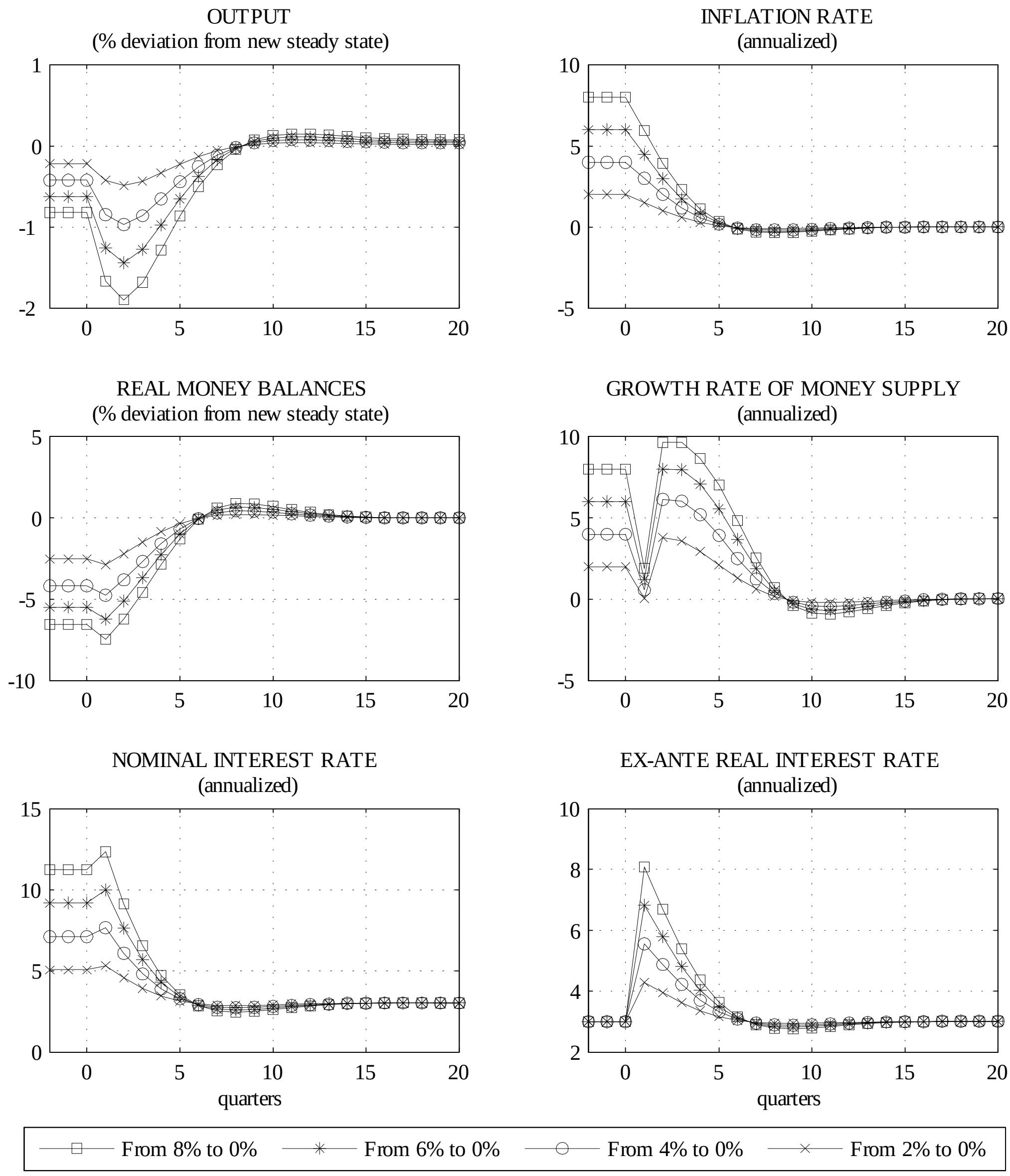

Figure 3: Cold-turkey disinflation under interest rate rule.

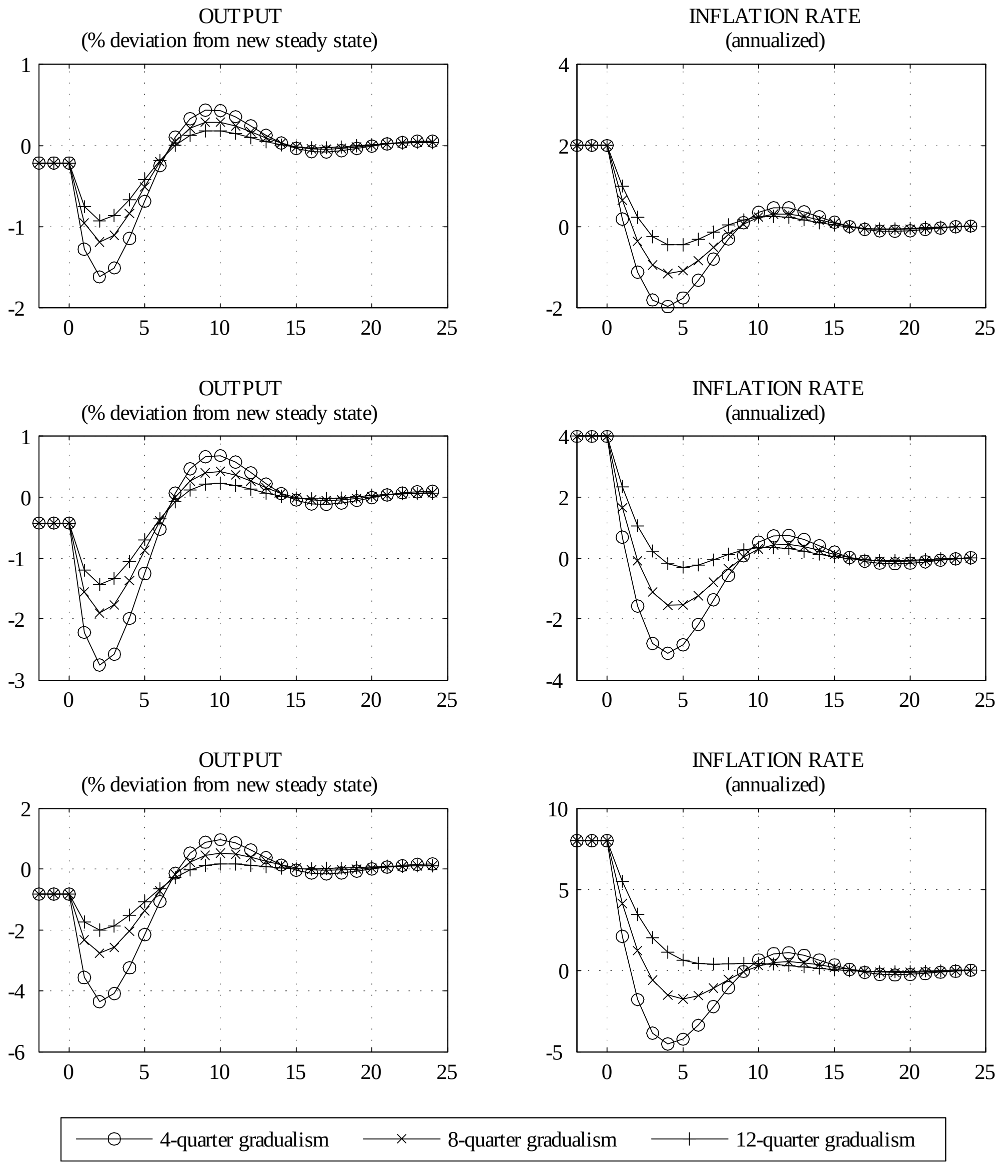

Figure 4: Gradual disinflation under money supply rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; bottom panels: $\pi^*_{\text{old}} = 8\%$.

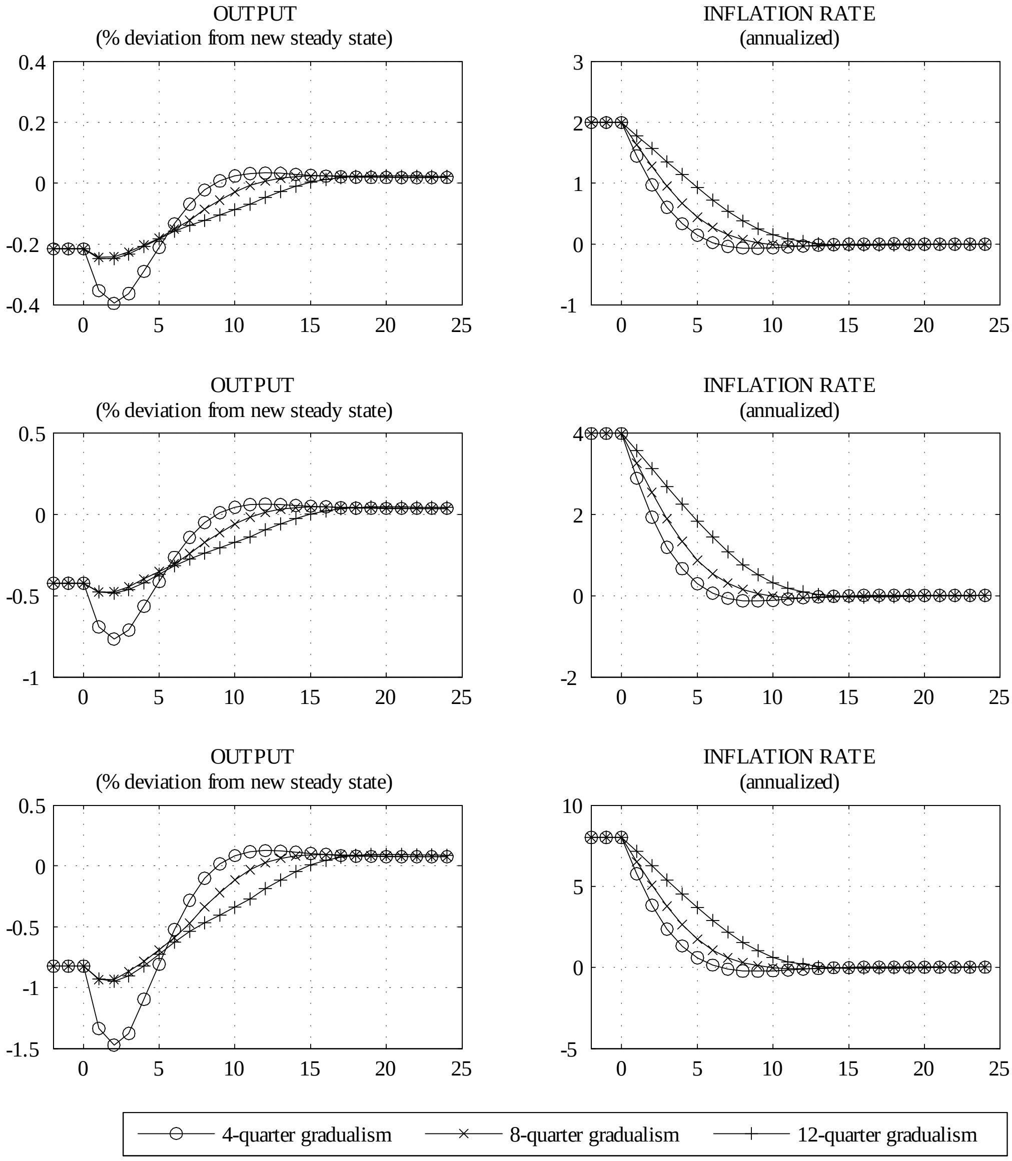

Figure 5: Gradual disinflation under interest rate rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; bottom panels: $\pi^*_{\text{old}} = 8\%$. Transition paths are expressed in percentage deviations from the new steady state.

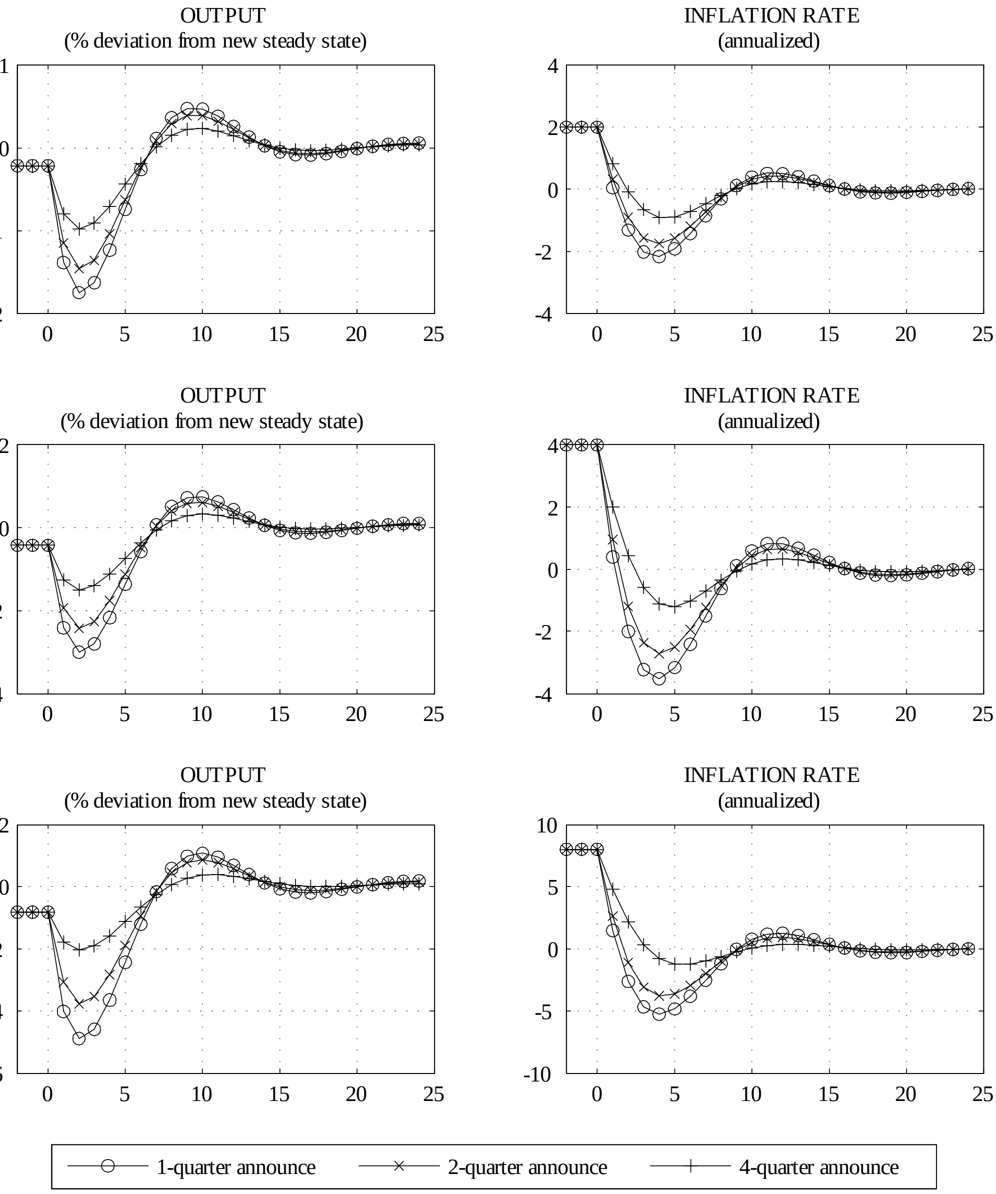

Figure 6: The effects of announcing future (cold-turkey) disinflations under money supply rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; bottom panels: $\pi^*_{\text{old}} = 8\%$.

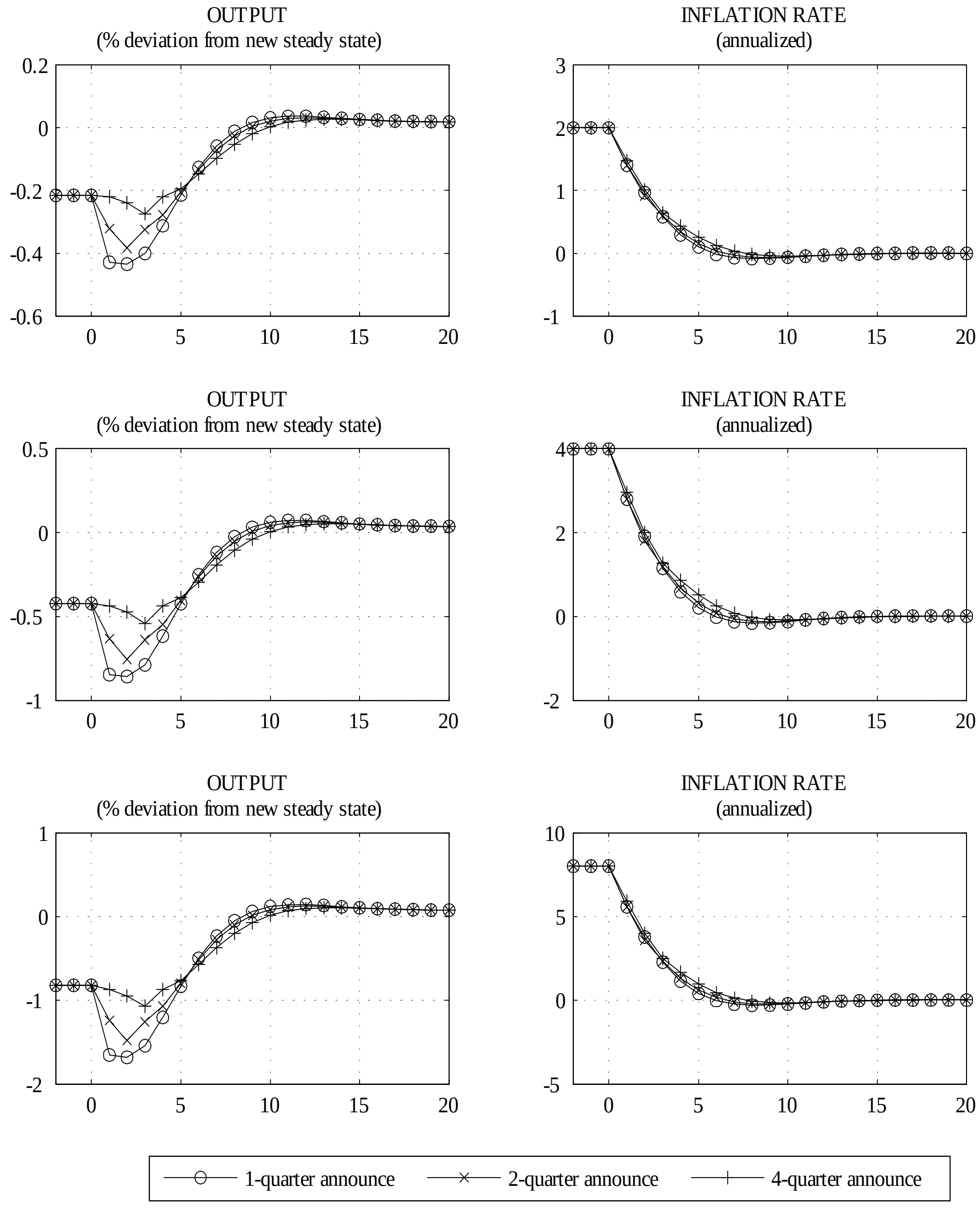

Figure 7: The effects of announcing future (cold-turkey) disinflations under interest rate rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; bottom panels: $\pi^*_{\text{old}} = 8\%$.

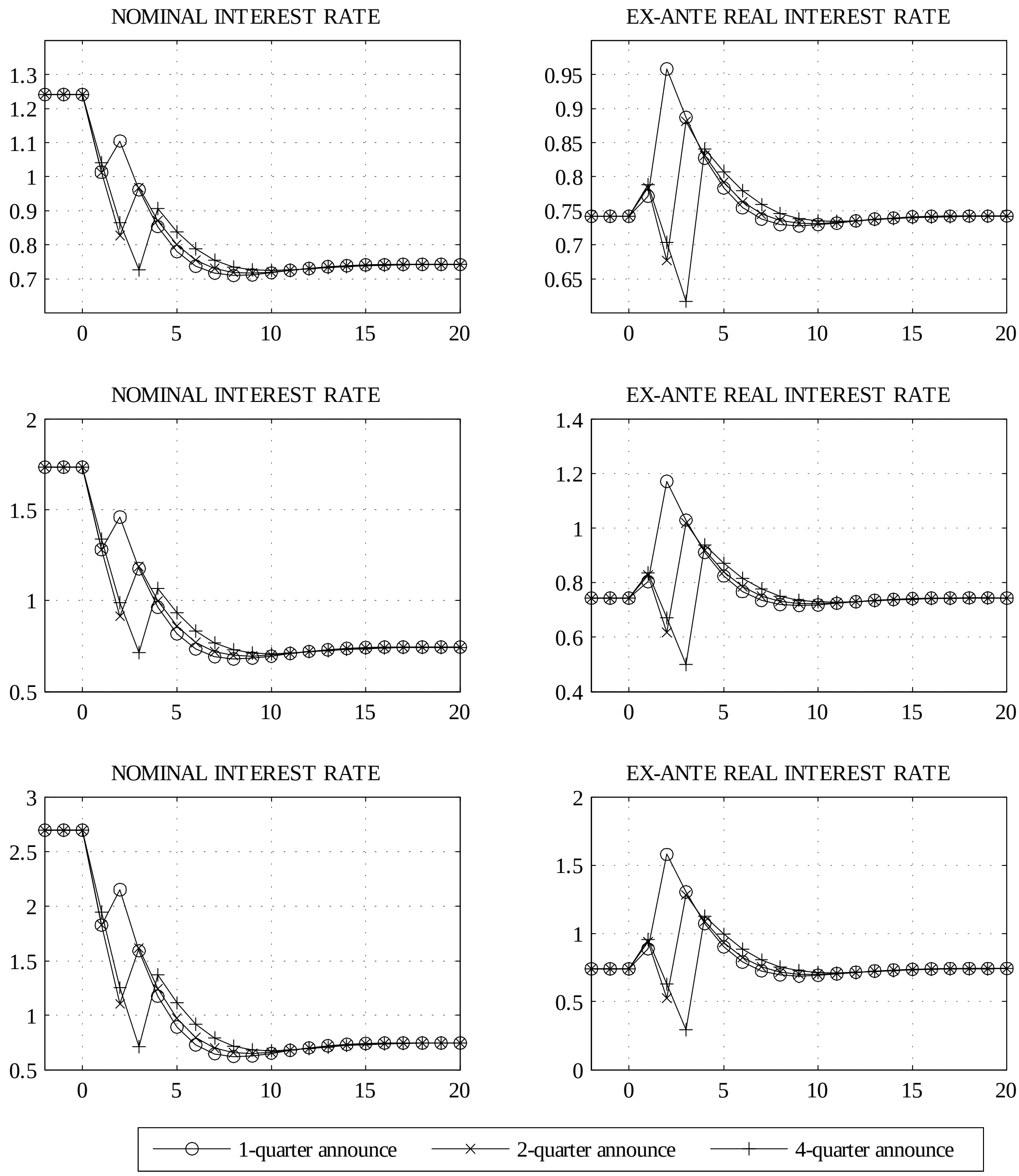

Figure 8: Anticipated disinflation under interest rate rule. Top panels: $\pi^*_{\text{old}} = 2\%$; middle panels: $\pi^*_{\text{old}} = 4\%$; bottom panels: $\pi^*_{\text{old}} = 8\%$.

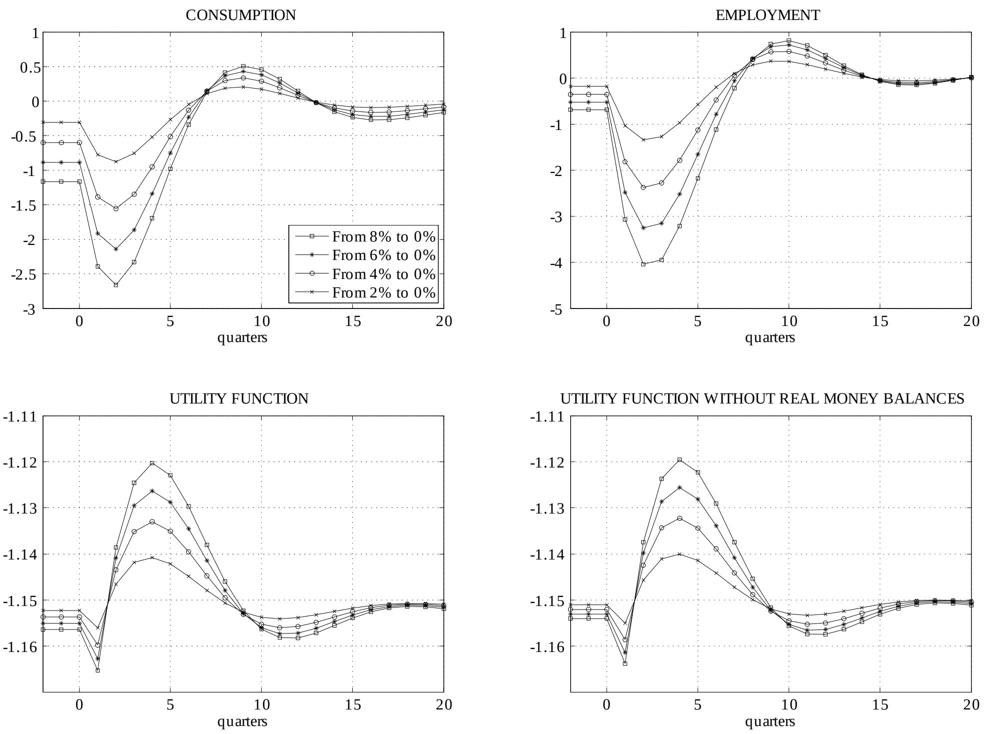

Figure 9: Cold-turkey disinflation under money supply rule. Consumption and employment paths are expressed in percentage deviations from the new steady state. Utility function are expressed in levels.