

Technical Appendix. Limited Asset Market Participation, Sticky Wages and Monetary Policy

(Not Intended for Publication)

Guido Ascari, Andrea Colciago and Lorenza Rossi

1 Introduction

As explained in footnote 7, p. 4, in the main text, in the baseline model union pools the income of both agents who, as a result, work for the same amount of time. For this reason our baseline framework does not fully nest Bilbiie (2008), where Ricardian and non-Ricardian agents are free to make different labor choices.

As a consequence, under flexible wages, the slope of the IS curve in our baseline model and in Bilbiie's model invert for different threshold values of λ . To see this, consider the case of log-utility in consumption, i.e. $\sigma = 1$, together with an infinitely elastic labor supply, i.e. $\phi = 0$. Under flexible wages and sticky prices, the IS curve derived in the main text collapses to:

$$x_t = E_t x_{t+1} - \left(1 - \frac{\lambda}{1 - \lambda}\right)^{-1} E_t \left(i_t - \pi_{t+1} - r_t^{Eff}\right) \quad (1)$$

the interest rate elasticity of aggregate demand turns positive if

$$\lambda > \bar{\lambda}^{fw} = 0.5 \quad (2)$$

In contrast, under the same preferences specification, Bilbiie's model yields

$$x_t = E_t x_{t+1} - E_t \left(i_t - \pi_{t+1} - r_t^{Eff}\right) \quad (3)$$

The IS-curve in (3) does not depend on the share of non-Ricardian agents and, thus, never inverts its slope. The reason is that in a perfectly competitive labor market, as that considered by Bilbiie, when labor supply is infinitely elastic agents have a constant-consumption labor supply function. For infinitely elastic labor this amounts to equating consumption with the real wage; consumption becomes independent of non-human

wealth and whether people hold assets or not is irrelevant. This mechanism is, instead, prevented in our baseline model, where the union forces agents to work for the same amount of time and, thus, to have consumption levels which depend on non-human wealth even under the preference specification described above.

From the point of view of microfoundations, however, we need to assume a union to introduce wage staggering. Since all labour types are uniformly spread across all households, both Ricardian and non-Ricardian, it seems natural to assume that the union groups the households supplying a particular labor type, regardless of the fact they are Ricardians or not. To have a model that nest Bilbiie’s one under flexible wage, we need to think about a different setting in which only Ricardians are unionized, while non-Ricardians supply their labour taking as given the wage fixed by Ricardian unions. We believe the microfoundations of this model to be much less palatable than the one in the main text. This Appendix, however, characterizes this alternative labor market framework, so to yield a model that under flexible wages leads to the same IADL-region as that in Bilbiie (2008). We show that in the sticky wages case the inversion of the slope of the IS curve requires a higher share of non-Ricardian agents with respect to our baseline model. Thus, the adoption of a labor market which, under flexible wages, nests that in Bilbiie’s reinforces our baseline result, namely that nominal wage rigidity leads to the inversion of the slope of the IS curve just for values of the share of non-Ricardian agents which are empirically implausible. In other words, the more coherent setup we adopt in the main text, is actually biased against our results.

2 Labor Market

We assume a continuum of differentiated labor inputs indexed by $j \in [0, 1]$. Each Ricardian household provides the differentiated labor type j , and acts as a monopolist on labor market j . On the contrary, non-Ricardian households do not perceive their power in the labor market and supply each labor type taking as given the wage fixed by Ricardian households. Under these assumptions Ricardian households stand ready to supply at the fixed wage W^j as many hours to the labor market j , L_t^j , as required by firms, taking into account that part of the demand is satisfied by the supply of non-Ricardian agents. The variable $L_t = \left[\int_0^1 (L_t^j(i))^{\frac{\theta_w-1}{\theta_w}} dj \right]^{\frac{\theta_w}{\theta_w-1}}$ represents the labor input used in the production process by intermediate goods producers and $\theta_w > 1$ is the elasticity of substitution between labor inputs.

The supply side of the economy is modelled as in Bilbiie.

2.1 Households

There is a continuum of households indexed by $i \in [0, 1]$. Households in the interval $(\lambda, 1]$ have the following period utility

$$\log(C_{S,t}(i)) - \chi \frac{L_{S,t}^j(i)^{1+\phi}}{1+\phi}, \quad (4)$$

where $C_{S,t}(i)$ is Ricardian agent i 's consumption, $L_{S,t}^j(i)$ are hours worked on market j by Ricardian agent i , and Ψ_t is an aggregate taste shock. Asset holders hold assets, smooth consumption and face the following flow budget constraint in nominal terms :

$$E_t \Lambda_{t,t+1} X_{t+1} + \Omega_{S,t+1} V_t \leq X_t + W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} (L_t^d - L_{H,t}) + \Omega_{S,t} (V_t + P_t D_t) - P_t C_{S,t}. \quad (5)$$

where we considered that hours worked by Ricardian agent i are given by $\left(\frac{W_t^j}{W_t} \right)^{-\theta_w} (L_t^d - L_{H,t})$. The variable L_t^d represents the total demand of the labor bundle by firms, while $L_{H,t}$ represents total supply of the bundle by non-Ricardian agents, which we define below. In each period t , asset holders can purchase any desired state-contingent nominal payment X_{t+1} in period $t+1$ at the dollar cost $E_t \Lambda_{t,t+1} X_{t+1}$. The variable $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t+1$ and t . Notice that we wrote the budget constraint imposing symmetry across agents. The problem of the household reads as

$$\max_{C_{S,t}, W_t^j, \Omega_{S,t+k}, X_{t+k}} E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \left\{ \log C_{S,t+k} - \chi \frac{(L_{S,t+k}^j)^{1+\phi}}{1+\phi} \right\} \quad (6)$$

s.t. (5) and $L_{S,t}^j = L_t^j - L_{H,t}^j = \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} (L_t^d - L_{H,t})$.¹

Let $\lambda_{S,t}$ and $\phi_{S,t}$ be the Lagrange multipliers on the first and the second constraint respectively. The FOCs with respect to $C_{S,t}$ and $L_{S,t+k}^j$ read respectively as

$$\frac{1}{C_{S,t+k}} = \lambda_{S,t} P_t \quad (7)$$

$$\chi (L_{S,t})^\phi = \phi_{S,t} \quad (8)$$

¹Below we show that labor supply of labor type j by non ricardian agents is equal to $\left(\frac{W_t^j}{W_t} \right)^{-\theta_w} L_{H,t}$, where $L_{H,t}$ is a constant.

which implies that

$$mrs_{S,t} = \chi C_{S,t+k} (L_{S,t})^\phi = \frac{\phi_{S,t}}{\lambda_{S,t} P_t}. \quad (9)$$

Combining the FOCs with respect to $C_{S,t}$, $\Omega_{S,t}$ and X_{t+1} together with the arbitrage condition on asset markets, i.e. $E_t \Lambda_{t,t+1} \equiv (1 + i_t)^{-1}$ we find the Euler equation for Ricardian agents:

$$\frac{1}{1 + i_t} = E_t \left\{ \beta \frac{\Psi_{t+1} u_c(C_{S,t+1})}{\Psi_t u_c(C_{S,t})} \frac{P_t}{P_{t+1}} \right\}. \quad (10)$$

We now reproduce below the parts of the Lagrangian of the household which are relevant for wage setting:

$$E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \left\{ \begin{array}{l} \lambda_{t+k} \left[W_{t+k}^j \left(\frac{W_t^j}{W_{t+k}} \right)^{-\theta_w} (L_{t+k}^d - L_{H,t+k}) \right] + \\ + \phi_{t+k} \left[L_{S,t+k} - \left(\frac{W_t^j}{W_{t+k}} \right)^{-\theta_w} (L_{t+k}^d - L_{H,t+k}) \right] \end{array} \right\} \quad (11)$$

The FOC for wage setting implies

$$\sum_{k=0}^{\infty} (\xi_w \beta)^k W_{t+k}^{\theta_w} (L_{t+k}^d - L_{H,t+k}) \left[W_t^j - \frac{\theta_w}{\theta_w - 1} \frac{\phi_{t+k}}{\lambda_{t+k}} \right] = 0 \quad (12)$$

given $\frac{\phi_{t+k}}{\lambda_{t+k}} = mrs_{S,t+k} P_{t+k}$ it follows

$$\sum_{k=0}^{\infty} (\xi_w \beta)^k \Phi_{t+k} \left[W_t^j - \frac{\theta_w}{\theta_w - 1} mrs_{S,t+k} P_{t+k} \right] = 0 \quad (13)$$

where $\Phi_{t+k} = W_{t+k}^{\theta_w} (L_{t+k}^d - L_{H,t+k})$ and where $L_{H,t+k}$ as it will shown is a constant.

Households in the interval $(\lambda, 1]$ do not hold asset and do not smooth consumption over time. They solve the following problem

$$\max_{C_{H,t}, L_{H,t}} \log(C_{H,t}(i)) - \chi \frac{L_{H,t}^{1+\phi}(i)}{1 + \phi} \quad (14)$$

s.t.

$$P_t C_{H,t}(i) = \int_0^1 L_{H,t}^j(i) W_t^j dj \quad (15)$$

where $L_{H,t}(i) = \left(\int_0^1 (L_{H,t}^j(i))^{\frac{\theta_w-1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w-1}}$. Given that the cost of one unit of the bundle is $W_t = \left(\int_0^1 (W_t^j)^{1-\theta_w} dj \right)^{1/(1-\theta_w)}$ the budget constraint can be written as

$$P_t C_{H,t}(i) = L_{H,t}(i) W_t \quad (16)$$

The first order condition with respect to $C_{H,t}(i)$ is

$$\frac{1}{C_{H,t}(i)} = \lambda_t P_t \quad (17)$$

that with respect to $L_{H,t}(i)$ is

$$\chi L_{H,t}^\phi(i) = \lambda_t W_t \quad (18)$$

where λ_t is the LM of the budget constraint. Combining the latter two equations with the budget constraint delivers

$$L_{H,t}(i) = \left(\frac{1}{\chi} \right)^{\frac{1}{1+\phi}} \quad (19)$$

Thus, non-Ricardian agents provide a constant amount of units of the bundle, and the total supply of the bundle by non-Ricardian agents is $\lambda \left(\frac{1}{\chi} \right)^{\frac{1}{1+\phi}}$. Next consider equation (15), the Lagrangian for non-Ricardian households is

$$\mathbb{L} = \log(C_{H,t}(i)) - \chi \frac{L_{H,t}^{1+\phi}(i)}{1+\phi} + \lambda_t \left[\int_0^1 L_{H,t}^j(i) W_t^j dj - P_t C_{H,t}(i) \right] \quad (20)$$

The choice of a non-Ricardian household must in fact also solve this problem. The first order condition with respect to the number of hours to provide on each market is²

$$\chi L_{H,t}^\phi(i) \frac{\partial L_{H,t}(i)}{L_{H,t}^j(i)} = \lambda_t W_t^j \quad (21)$$

or

$$\chi L_{H,t}^\phi(i) \left[\frac{\theta_w}{\theta_w-1} \left(\int_0^1 (L_{H,t}^j(i))^{\frac{\theta_w-1}{\theta_w}} dj \right)^{\frac{1}{\theta_w-1}} \frac{\theta_w-1}{\theta_w} (L_{H,t}^j(i))^{-\frac{1}{\theta_w}} \right] = \lambda_t W_t^j \quad (22)$$

²Recall that non ricardian agents take the wage as given and choose labor supply on each market.

which can be further reduced to

$$\chi L_{H,t}^\phi(i) \left(\frac{L_{H,t}(i)}{L_{H,t}^j(i)} \right)^{\frac{1}{\theta_w}} = \lambda_t W_t^j \quad (23)$$

Using the FOCs for the problem in the first stage, i.e. $\chi L_{H,t}^\phi(i) = \lambda_t W_t$

$$\lambda_t W_t \left(\frac{L_{H,t}(i)}{L_{H,t}^j(i)} \right)^{\frac{1}{\theta_w}} = \lambda_t W_t^j \quad (24)$$

and thus total supply of labor type j by non-Ricardian agents is

$$L_{H,t}^j = \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} L_{H,t}(i) = \lambda \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} \left(\frac{1}{\chi} \right)^{\frac{1}{1+\phi}}. \quad (25)$$

2.2 Aggregations and Market Clearings

Aggregate consumption is given by

$$C_t = \lambda C_{H,t} + (1 - \lambda) C_{S,t}. \quad (26)$$

The variable $\Omega_t = (1 - \lambda) \Omega_{S,t}$ represents aggregate asset holdings. In equilibrium $\Omega_t = 1$, thus each Ricardian agent has asset holdings equal to $\frac{1}{1-\lambda}$. The clearing of good and labor markets requires:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta_p} Y_t^d \forall z \quad Y_t^d = Y_t; \quad (27)$$

$$L_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} L_t^d \forall j \quad L_t = \int_0^1 L_t^j dj \quad (28)$$

where $Y_t^d = C_t$ represents aggregate demand, $L_t^j = \int_0^1 L_t^j(z) dz$ is total aggregate demand of labor input j and $L_t^d = \int_0^1 L_t(z) dz$ denotes firms' aggregate demand of the composite labor input L_t .

We assume the same production function as in Bilbiie, i.e.

$$Y_t(i) = A_t L_t(i) - F \quad (29)$$

where F is fixed cost common to all firms, where $\frac{F}{Y} = \mu$.

Aggregation implies

$$\int_0^1 Y_t(i) di = A_t \int_0^1 L_t(i) di - F = Y_t \underbrace{\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di}_{D_t} = A_t L_t - F$$

where D_t is a measure of price dispersion.

2.3 The Natural output and wage gap

We start our analysis of the model's log-linear equilibrium by deriving the natural output which is defined as in Galì (ch. 6). The concept of natural output is to be understood as referring to the equilibrium level of output in the absence of both price and wage rigidities. We also introduce a new variable, the real wage gap, denoted by $\tilde{\omega}_t$ defined as usual

$$\tilde{\omega}_t = \omega_t - a_t \quad (30)$$

which the gap between the log-deviations of actual real wage ω_t from its steady state and the flexible price/wage real wage a_t . Notice that under flexible wages the wage schedule of Ricardian agents is

$$\frac{W_t}{P_t} = \frac{\theta_w}{\theta_w - 1} C_{S,t} L_{S,t}^\phi \quad (31)$$

In log-deviations

$$\omega_t = c_{s,t} + \phi l_{s,t} \quad (32)$$

similarly the wage schedule of non-Ricardian is

$$\frac{W_t}{P_t} = C_{H,t} L_{H,t}^\phi \quad (33)$$

in log-deviations

$$\omega_t = c_{H,t} + \phi l_{H,t} \quad (34)$$

Aggregating (and as in Bilbiie assuming that $L_H = L_S$ in steady state) we get³:

$$\omega_t = c_t + \phi l_t \quad (35)$$

Using (38) solved for $l_t = \frac{y_t}{(1+\mu)} - a_t$, putting into the aggregate wage schedule and using the aggregate resource constraint $c_t = y_t$, we can write:

$$\begin{aligned} \omega_t &= c_t + \phi l_t = y_t \left(1 + \frac{\phi}{1+\mu} \right) - \phi a_t \\ &= \chi y_t - \phi a_t \end{aligned} \quad (36)$$

where $\chi = \left(1 + \frac{\phi}{1+\mu} \right) = \left(1 + \frac{1+\mu+\phi}{1+\mu} \right)$.

Aggregate labor demand is:

$$\widehat{m}c_t = \omega_t - a_t = \tilde{\omega}_t \quad (37)$$

³An alternative could be to introduce a employment subsidy/tax tax to get the same steady state result.

which states that real marginal costs coincides with the real wage gap. Notice that log-linearizing the aggregate production function, and considering that D_t is of second order, implies that in log-deviations from the flexible price steady state

$$y_t = (1 + \mu) a_t + (1 + \mu) l_t \quad (38)$$

Using the production function real marginal costs can be rewritten as follows:

$$\widehat{mc}_t = \left(1 + \frac{\phi}{1 + \mu}\right) y_t - (1 + \phi) a_t = \chi y_t - (1 + \phi) a_t \quad (39)$$

With flexible prices $\widehat{mc}_t = 0$ and the natural reads as:

$$y_t^n = \frac{1 + \mu}{(1 + \mu) + \phi} (1 + \phi) a_t = \chi^{-1} (1 + \phi) a_t \quad (40)$$

which is identical to that in Bilbiie. Further, notice that using the economy production function (38) we can find a relationship between the output gap $x_t = y_t - y_t^n$ and labor hours l_t which is given by:

$$x_t = (1 + \mu) l_t - \frac{\mu\phi}{(1 + \mu) + \phi} a_t \quad (41)$$

With $\mu = 0$ the latter collapses to

$$x_t = l_t. \quad (42)$$

2.4 Wage Inflation

Next we characterize the IS curve and the slope of the wage schedule under sticky wages. To this end we first log-linearize equation (13), i.e. the condition for optimal wage setting. Consider

$$\underbrace{W_t^j E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \Phi_{t+k}}_{LHS} = \underbrace{\frac{\theta_w}{\theta_w - 1} E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \Phi_{t+k} mrs_{S,t+k} P_{t+k}}_{RHS} \quad (43)$$

log-linearization of the LHS is, (we rename w_t^* as the log-deviation from the SS of the optimal wage W_t^j):

$$\frac{w_t^*}{1 - \xi_w \beta} \Phi W + E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \Phi W (1 + \phi_{t+k}) \quad (44)$$

log-linearization of the RHS is

$$\frac{\theta_w}{\theta_w - 1} E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \Phi mrs_P (1 + \phi_{t+k} + \widehat{mrs}_{S,t+k} + p_{t+k}) \quad (45)$$

Remember that in the steady state $\frac{W}{P} = \frac{\theta_w}{\theta_w - 1} mrs_S$.

Collecting LHS and RHS:

$$\begin{aligned} & \frac{w_t^*}{1 - \xi_w \beta} \Phi W + E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \Phi W (1 + \phi_{t+k}) \\ &= \frac{\theta_w}{\theta_w - 1} E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \Phi mrs_S P (1 + \phi_{t+k} + \widehat{mrs}_{S,t+k} + p_{t+k}) \end{aligned} \quad (46)$$

dividing by ΦW and simplifying for steady state values

$$\frac{w_t^*}{1 - \xi_w \beta} + E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \phi_{t+k} = E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \phi_{t+k} + E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k (\widehat{mrs}_{S,t+k} + p_{t+k}) \quad (47)$$

which can be simplified further so that we get

$$w_t^* = (1 - \xi_w \beta) (\widehat{mrs}_{S,t} + p_t) + (1 - \xi_w \beta) E_t \sum_{k=1}^{\infty} (\xi_w \beta)^k (\widehat{mrs}_{S,t+k} + p_{t+k}) \quad (48)$$

It is easy to show that

$$(1 - \xi_w \beta) E_t \sum_{k=1}^{\infty} (\xi_w \beta)^k (\widehat{mrs}_{S,t+k} + p_{t+k}) = \xi_w \beta E_t w_{t+1}^* \quad (49)$$

and rewrite the equation as

$$w_t^* = (1 - \xi_w \beta) (\widehat{mrs}_t + p_t) + \xi_w \beta E_t w_{t+1}^*. \quad (50)$$

We now consider the log-linearization of the equation of aggregate wage index, which reads as

$$w_t = (1 - \xi_w) w_t^* + \xi_w w_{t-1} \quad (51)$$

solving for w_t^* and substituting in equation (50) we get

$$\frac{w_t - \xi_w w_{t-1}}{1 - \xi_w} = (1 - \xi_w \beta) (\widehat{mrs}_{S,t} + p_t) + \xi_w \beta E_t \left(\frac{w_{t+1} - \xi_w w_t}{1 - \xi_w} \right). \quad (52)$$

multiplying by $(1 - \xi_w)$, adding and subtracting $\xi_w w_t$ on the LHS and $\xi_w \beta w_t$ on the RHS, after some algebra we get,

$$\xi_w (w_t - w_{t-1}) = (1 - \xi_w) (1 - \xi_w \beta) (\widehat{mrs}_{S,t} + p_t - w_t) + \xi_w \beta E_t (w_{t+1} - w_t) \quad (53)$$

Now remember that $\pi_t^w = w_t - w_{t-1}$ and $E_t \pi_{t+1}^w = E_t (w_{t+1} - w_t)$ and dividing by ξ_w we get

$$\pi_t^w = \beta E_t \pi_{t+1}^w + (1 - \xi_w) (1 - \xi_w \beta) (\widehat{mrs}_{S,t} + p_t - w_t) \quad (54)$$

where $(\widehat{mrs}_{S,t} + p_t - w_t) = \widehat{mrs}_{S,t} - \omega_t$. Considering that $c_t = y_t$ and $l_t = \frac{y_t}{(1+\mu)} - a_t$

$$\begin{aligned} & \widehat{mrs}_{S,t} - \omega_t \\ &= [c_{s,t} + \phi l_{s,t}] - \omega_t \\ &= \left[\frac{c_t}{1-\lambda} - \frac{\lambda}{1-\lambda} c_{H,t} + \phi \frac{l_t}{(1-\lambda)} \right] - \omega_t \\ &= \left[\frac{y_t}{1-\lambda} - \frac{\lambda}{1-\lambda} \omega_t + \phi \frac{l_t}{(1-\lambda)} \right] - \omega_t \\ &= \left[\frac{y_t}{1-\lambda} - \frac{\lambda}{1-\lambda} \omega_t + \phi \frac{y_t}{(1-\lambda)(1+\mu)} - \phi \frac{a_t}{(1-\lambda)} \right] - \omega_t \\ &= \left[\frac{y_t}{1-\lambda} + \phi \frac{y_t}{(1-\lambda)(1+\mu)} - \phi \frac{a_t}{(1-\lambda)} \right] - \frac{\lambda}{1-\lambda} \omega_t - \omega_t \\ &= \frac{1}{1-\lambda} \left[y_t \left(1 + \frac{\phi}{(1+\mu)} \right) - (1+\phi) a_t \right] - \frac{1}{1-\lambda} (\omega_t - a_t) \quad (55) \end{aligned}$$

Given equation (40) and defining $x_t = y_t - y_t^n$, we can rewrite the latter as

$$\widehat{mrs}_{S,t} - \omega_t = \frac{1}{1-\lambda} [\chi x_t - \tilde{\omega}_t], \quad (56)$$

where $\chi = \left(1 + \frac{\phi}{(1+\mu)} \right)$, then (54) becomes

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{(1 - \xi_w) (1 - \xi_w \beta)}{\xi_w (1 - \lambda)} [\chi x_t - \tilde{\omega}_t]. \quad (57)$$

2.5 The IS curve under flexible wages

Log-linearizing the Euler of Ricardian, as in the main text delivers

$$c_{s,t} = E_t c_{s,t+1} - (i_t - E_t \pi_{t+1}) \quad (58)$$

which is identical to that in Bilbiie (2008). Since labor supply of non-Ricardian agents is constant it follows that $l_{h,t} = 0$ and aggregate hours are

$$l_t = (1 - \lambda) l_{s,t} \quad (59)$$

Consumption of non-Ricardian reads as

$$c_{h,t} = \omega_t \quad (60)$$

while the log deviations of aggregate consumption are given by:

$$c_{s,t} = \frac{1}{1-\lambda}c_t - \frac{\lambda}{1-\lambda}c_{h,t} \quad (61)$$

Using (60) and (61) the Euler becomes

$$c_t = E_t c_{t+1} - \lambda \Delta \omega_{t+1} - (1-\lambda)(i_t - E_t \pi_{t+1}) \quad (62)$$

Since $c_t = y_t$, the IS curve reads as

$$y_t = E_t y_{t+1} - \lambda \Delta \omega_{t+1} - (1-\lambda)(i_t - E_t \pi_{t+1}). \quad (63)$$

Rewriting equation (63) in terms of output gap with respect to the natural output:

$$x_t = E_t x_{t+1} + \Delta y_{t+1}^n - \lambda \Delta \omega_{t+1} + (1-\lambda)(i_t - E_t \pi_{t+1}) \quad (64)$$

where $x_t = y_t - y_t^n$. Given $\tilde{\omega}_t = \omega_t - a_t$, adding and subtracting $\lambda E_t \Delta a_{t+1}$ leads to

$$x_t = E_t x_{t+1} + \Delta y_{t+1}^n - \lambda E_t \Delta \omega_{t+1} + \lambda E_t \Delta a_{t+1} - \lambda E_t \Delta a_{t+1} + (1-\lambda)(i_t - E_t \pi_{t+1}) \quad (65)$$

and finally to

$$x_t = E_t x_{t+1} - \lambda E_t \Delta \tilde{\omega}_{t+1} + (1-\lambda)(i_t - E_t \pi_{t+1} - r_t^n) - \lambda E_t \Delta a_{t+1}. \quad (66)$$

Using the log-linear aggregate wage schedule, (35), the production function, (38), and the log-linear aggregate resource constraint, we get

$$\omega_t = \left(1 + \frac{\phi}{1+\mu}\right) y_t - \phi a_t, \quad (67)$$

Substituting equation(67) into equation (63) delivers, after collecting terms, the IS curve

$$y_t = E_t y_{t+1} + (1+\mu)(1-\delta^{-1})E_t \Delta a_{t+1} - \delta^{-1}(i_t - E_t \pi_{t+1}) \quad (68)$$

where $\delta = 1 - \phi \frac{\lambda}{1-\lambda} \frac{1}{1+\mu}$. To rewrite the IS in terms of output gap with respect to the natural output y_t^n , we add and subtract $E_t \Delta y_{t+1}^n$

$$x_t = E_t x_{t+1} + [\chi^{-1}(1+\phi) + (1+\mu)(1-\delta^{-1})]E_t \Delta a_{t+1} - \delta^{-1}(i_t - E_t \pi_{t+1})$$

or

$$x_t = E_t x_{t+1} - \delta^{-1}(i_t - E_t \pi_{t+1} - r_t^n) \quad (69)$$

where r_t^n is the flexible price interest rate. We stress that equation (69) is identical to that in Bilbiie (2008). As a result the region where the IADL holds will be identical to that identified by the aforementioned author.

2.6 The IS curve and the IADL region under sticky wages

We now want to characterize the IS and the slope of the wage schedule under sticky wages and to study the IADL region. Remember that the wage inflation curve reads as

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{\kappa_w}{1-\lambda} [\chi x_t - \tilde{\omega}_t] \quad (70)$$

which can be rewritten as

$$w_t - w_{t-1} = \beta E_t w_{t+1} - \beta w_t - \frac{\kappa_w}{1-\lambda} w_t + \frac{\kappa_w}{1-\lambda} p_t + \frac{\kappa_w}{1-\lambda} [\chi x_t + a_t] \quad (71)$$

after some algebra, solving for $\omega_t = w_t - p_t$ yields

$$\begin{aligned} \omega_t = & \frac{1}{1 + \beta + \frac{\kappa_w}{1-\lambda}} [w_{t-1} - p_t] + \frac{\beta}{1 + \beta + \frac{\kappa_w}{1-\lambda}} E_t (w_{t+1} - p_t) + \\ & + \frac{\kappa_w}{(1 + \beta + \frac{\kappa_w}{1-\lambda})(1-\lambda)} (\chi x_t + a_t) \end{aligned} \quad (72)$$

As in the baseline model, this is a weighted average between: (i) the past nominal wage at current prices; (ii) the future nominal wage at current prices; (iii) $\chi x_t + a_t$. Note that as $\xi_w \rightarrow 0$, then $\kappa_w \rightarrow \infty$, and this expression collapses to the usual flexible wage case. Equation (72) also implies that:

$$\Delta \tilde{\omega}_{t+1} = F + \frac{\kappa_w}{(1 + \beta + \frac{\kappa_w}{1-\lambda})(1-\lambda)} (\chi \Delta x_{t+1} + \Delta a_{t+1}) - \Delta a_{t+1} \quad (73)$$

where $F = \frac{(1-\lambda)}{(1-\lambda)(1+\beta)+\kappa_w} [\Delta w_t - \Delta p_{t+1}] + \frac{(1-\lambda)\beta}{(1-\lambda)(1+\beta)+\kappa_w} E_t (\Delta w_{t+2} - \Delta p_{t+1})$. Substituting (73) into (66) and solving for x_t the IS curve can be written as

$$\begin{aligned} x_t = & E_t x_{t+1} - (\delta^{sw})^{-1} (1-\lambda) E_t \left(i_t - \pi_{t+1} - r_t^{flex} \right) - (\delta^{sw})^{-1} \lambda E_t F \\ & - \frac{\kappa_w}{(1 + \beta + \frac{\kappa_w}{1-\lambda})(1-\lambda)} (\delta^{sw})^{-1} \lambda E_t \Delta a_{t+1} \end{aligned} \quad (74)$$

where

$$\delta^{sw} = \left(1 - \frac{\lambda}{(1-\lambda)} \frac{1 + \mu + \phi}{1 + \mu} \frac{\kappa_w}{(1 + \beta + \frac{\kappa_w}{1-\lambda})} \right)$$

setting $\mu = 0$ (as in our baseline model) we get:

$$\delta_{EB}^{sw} = 1 - \frac{\lambda}{(1-\lambda)} \frac{(1 + \phi) \kappa_w}{(1 + \beta + \frac{\kappa_w}{1-\lambda})} \quad (75)$$

where the subscript EB stands for *Encompassing Bilbie's model*. The coefficient in the *baseline model* (BM) is instead

$$\delta_{BM}^{sw} = 1 - \frac{\lambda}{(1-\lambda)} \frac{(1+\phi)\kappa_w}{(1+\beta+\kappa_w)} \quad (76)$$

Notice that, *ceteris paribus*, the threshold value of the share of non-Ricardian agents required to invert the slope of the IS curve is larger under the labor market specification proposed in this Appendix with respect to that necessary in the baseline model.

2.7 The complete log-linearized model

The following equations summarize log-linear equilibrium dynamics (as in Bilbie deviations are from the flexible price equilibrium):

$$\begin{aligned} (M1) \pi_t &= \beta E_t \pi_{t+1} + \kappa_p \tilde{\omega}_t && \text{NKPC} \\ (M2) \pi_t^w &= \beta E_t \pi_{t+1}^w + \frac{\kappa_w}{1-\lambda} [\chi x_t - \tilde{\omega}_t] && \text{Wage Inflation Curve} \\ (M3) \tilde{\omega}_t &= \tilde{\omega}_{t-1} + \pi_t^w - \pi_t - \Delta \omega_t^{Flex} && \text{Real Wage Gap} \\ (M4) x_t &= E_t x_{t+1} - (1-\lambda) E_t \left(i_t - \pi_{t+1} - r_t^{flex} \right) - \lambda E_t \Delta \tilde{\omega}_{t+1} && \text{IS curve} \end{aligned}$$

Equation (M1) is the NKPC obtained from the firms' price setting problem. The variable $\tilde{\omega}_t = \omega_t - a_t$ represents the *real wage gap*, which is defined as the gap between the current and the natural equilibrium real wage. Given the linear in labor production function it follows that $mc_t = \omega_t - y_t + l_t = \omega_t - a_t = \tilde{\omega}_t$. For this reason $\tilde{\omega}_t$ appears on the RHS of equation (M1). The *real wage gap* in the NKPC identifies a *labor demand gap* being equal to the difference between the current wage and the marginal productivity of labor. The parameter $\kappa_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$ is the slope of the NKPC. Equation (M2) is a wage inflation curve, similar to that in Erceg et al (2000) with slope $\kappa_w = \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w}$. Symmetrically to the NKPC, the term $[\chi x_t - \tilde{\omega}_t]$ in (M2) identifies a *labor supply gap* being equal to the difference between the marginal rate of substitution of the Ricardian consumer and the real wage.

2.8 Determinacy analysis

In this section, we prove numerically the conditions for the determinacy of the *REE* employing the same forward looking and contemporary rules considered in the baseline model. Thus,

$$i_t = \phi_\pi \pi_{t+j} \text{ with } j \in (0, 1). \quad (77)$$

As in Bilbie (2008), sticky prices lead to the inversion of the Taylor principle in the IADL region of the parameter space. However, consistently

with our baseline model, we show that with staggered wages the inversion of the Taylor principle is confined to implausible parameterization both under a forward looking and a contemporary rule.

Figure 1 and Figure 2 depict indeterminacy regions in the parameter space (ϕ_π, λ) , obtained by numerical simulations of the forward looking and the contemporary rule respectively. The two figures can be directly compared with Fig. 2 (panel B) and Fig. 4 (panel B) of the baseline model, where we use the same calibration. Notice that for both rules (contemporary and forward), the value of the share of non-Ricardian consumers λ which guarantees determinacy with a value of $\phi_\pi < 1$ is approximately equal to 0.9. In the baseline model instead we get a lower threshold value of λ closest to 0.8.

Summing up we show that, in the sticky wages case, the inversion of the slope of the IS curve requires a higher share of non-Ricardian agents with respect to our baseline model. Thus, the adoption of a labor market which, under flexible wages, nests that in Bilbiie's reinforces our baseline result, namely that nominal wage rigidity leads to the inversion of the slope of the IS curve just for values of the share of non-Ricardian agents which are empirically implausible. This means that, we can more than confirm that wage stickiness *generically* restores the standard determinacy proprieties of the NK model.

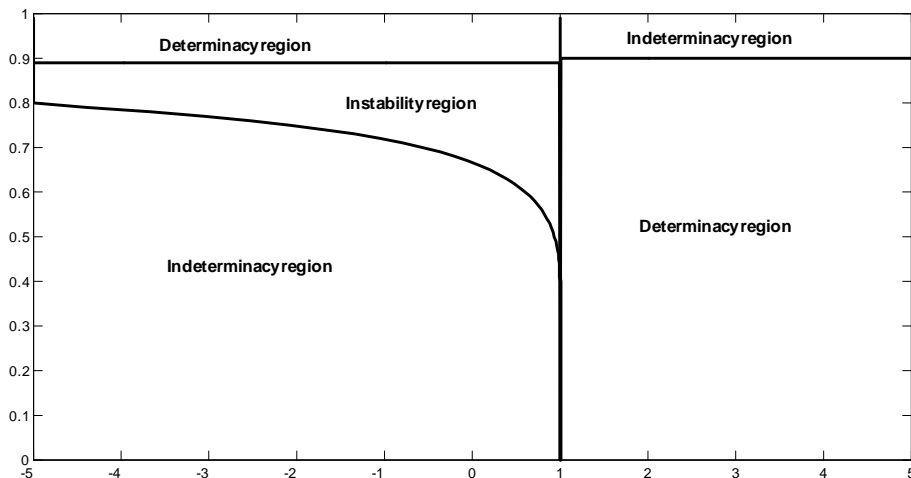


Figure 1. Determinacy and Indeterminacy regions under the rule:

$$i_t = \phi_\pi \pi_{t+1} \text{ with sticky wages}$$

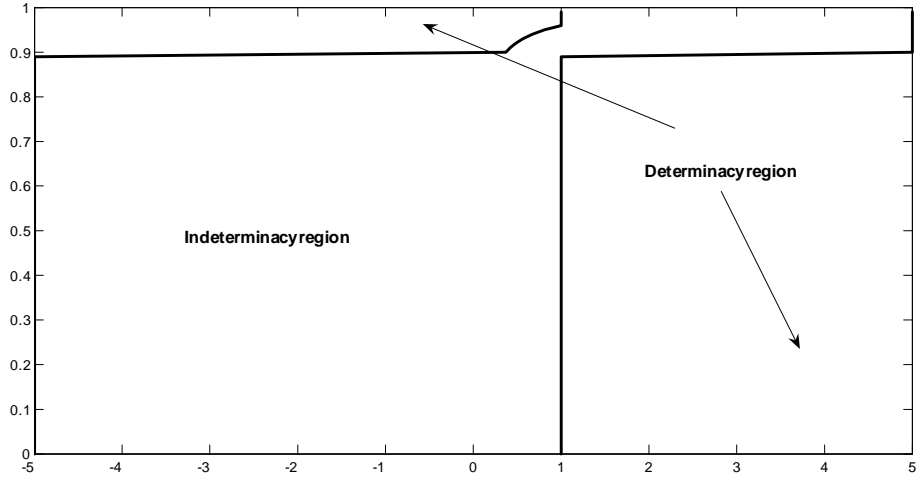


Figure 2. Determinacy and Indeterminacy regions under the rule:
 $i_t = \phi_\pi \pi_t$ with sticky wages