Appendix 1. The Model

(A) The Model with variable capital

1) Household

Given the utility function

$$U = \left\{ \left[bC^{\frac{\eta-1}{\eta}} + (1-b)\left(\frac{M}{P}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} (1-L)^{e} \right\}^{1-\chi} / (1-\chi) \tag{1}$$

the first order condition for the representative households are the following:

$$\frac{W_t}{P_t} = \frac{eC_t\left[1+\bar{b}\left(\frac{m_t}{C_t}\right)^{\frac{\eta-1}{\eta}}\right]}{1-L_t} \tag{2}$$

$$\frac{U_m(t)}{U_C(t)} = \bar{b}\left(\frac{C_t}{m_t}\right)^{\frac{1}{\eta}} = \frac{i_t}{1+_{i_t}} \tag{3}$$

$$E_t\left(\frac{U_C(t)}{U_C(t+1)}\beta(1+r_t)\right) = E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\frac{-1}{\eta}}\left(\frac{cm_t}{cm_{t+1}}\right)^{\frac{1}{\eta}-\chi}\left(\frac{1-L_t}{1-L_{t+1}}\right)^{e(1-\chi)}\beta(1+r_t)\right] = 1 \tag{4}$$

where W_t = nominal wage; P_t = general price level; C_t = consumption; $m_t = \left(\frac{M_t}{P_t}\right)$ = real money balances; $\bar{b} = \frac{1-b}{b}$; $cm_t = \left[bC_t^{\frac{\eta-1}{\eta}} + (1-b)m_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$; L_t = labour supply; $U_X(t)$ = marginal utility with respect to the argument X (for $X = C, m, L$); i_t = nominal interest rate; r_t = real interest rate.

2) Pricing equations

Final good producers use the following technology

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{5}$$

Their demand for intermediate inputs is therefore equal to

$$Y_{i,t+j} = \left(\frac{P_{i,t}}{P_{t+j}}\right)^{-\theta} Y_{t+j} \tag{6}$$

The problem of the representative intermediate goods producer firms that reset the price is

$$\underset{\{p_{it}\}}{Max} \quad E_t\left(\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}F_{t+j}\right) = E_t\left(\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}\left[\left(\frac{P_{i,t}}{P_{t+j}}\right)Y_{i,t+j} - TC_{i,t+j}(Y_{i,t+j})\right]\right) \tag{7}$$

$$s.t. \qquad Y_{i,t+j} = \left(\frac{P_{i,t}}{P_{t+j}}\right)^{-\theta} Y_{t+j}$$

where $\Delta_{t,t+j}$ represents the real discount factor from t to $t+j$ applied by the firm to the stream of future real profits[1]; F = real profits, P_i = price set by the firm, TC_i = real total costs. The optimal price fixed by re-setting firms in period t is

$$P_{i,t}^{*}=\left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}MC_{i,t+j}P_{t+j}^{\theta}Y_{t+j}}{E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}P_{t+j}^{\theta-1}Y_{t+j}} \tag{8}$$

where MC_i = real marginal cost of producer i. Note that (8) can be written as $P_{i,t}^{*}=\left(\frac{\theta}{\theta-1}\right)\frac{\Psi(t)}{\Phi(t)}$, where

$$\Psi(t)=MC_{i,t}P_t^{\theta}Y_t+\alpha\beta E_t\left[\Psi(t+1)\right] \tag{9}$$

$$\Phi(t)=P_t^{\theta-1}Y_t+\alpha\beta E_t\left[\Phi(t+1)\right] \tag{10}$$

The price of the final good is given by

$$P_t=\left[\int_0^1 P_{i,t}^{1-\theta}di\right]^{\frac{1}{1-\theta}} \tag{11}$$

3) Technology

Denoting by q_t the real user cost of capital, the cost minimisation problem of a representative intermediate goods producer firm i is

$$\underset{\{K_{i,t-1},L_t\}}{MIN}\qquad q_tK_{i,t-1}+\overbrace{w_t}^{W_t/P_t}L_{i,t}$$

$$s.t.\qquad Y_{i,t}=A_t(K_{i,t-1})^{1-\sigma}(L_{i,t})^{\sigma}$$

which gives the following usual first order conditions

$$q_t=A_t(1-\sigma)\left(\frac{L_{i.t}}{K_{i,t-1}}\right)^{\sigma}MC_{i,t} \tag{12}$$

$$w_t=A_t\sigma\left(\frac{K_{i,t-1}}{L_{i.t}}\right)^{1-\sigma}MC_{i,t} \tag{13}$$

Combining these two equations with the production function yields the equations for the demand of labour and capital and for the marginal cost

$$L_{i,t}^{d}=\frac{Y_{i,t}}{A_t}\left[\frac{\sigma}{1-\sigma}\frac{q_t}{w_t}\right]^{1-\sigma} \tag{14}$$

$$K_{i,t-1}^{d}=\frac{Y_{i,t}}{A_t}\left[\frac{1-\sigma}{\sigma}\frac{w_t}{q_t}\right]^{\sigma} \tag{15}$$

$$MC_{i,t}=\frac{1}{A_t}\left[\frac{w_t}{\sigma}\right]^{\sigma}\left[\frac{q_t}{1-\sigma}\right]^{1-\sigma} \tag{16}$$

[1] For simplicity, we will set that equal to β, the real discount factor in the utility function.

4) Market clearing

The aggregate resource constraint is

$$Y_t = C_t + X_t \tag{17}$$

where $X_t = \left[\int_0^1 X_{z,t} dz\right]$ and X_t = aggregate investment, while $X_{i,t}$ = investment of the intermediate goods producer i. $X_{i,t}$ is given by the following capital accumulation equation for the single intermediate goods producer i

$$K_{i,t} = (1-\delta)K_{i,t-1} + X_{i,t} \tag{18}$$

where δ = depreciation rate. This linear equation can be aggregated over all the intermediate goods producers and then substituted into the aggregate resource constraint to get

$$Y_t = C_t + K_t - (1-\delta)K_{t-1} \tag{19}$$

Market clearing on the capital and labour markets requires

$$K_{t-1} = \left[\int_0^1 K_{i,t-1}^d di\right] \tag{20}$$

$$L_t^d = \left[\int_0^1 L_{i,t}^d di\right] = L_t^s \tag{21}$$

Following Yun (1996) the equation to link intermediate goods output and final good output is given by

$$IO_t = \left[\int_0^1 Y_{i,t} di\right] = \left[\frac{P_t}{\widetilde{P}_t}\right]^\theta Y_t \tag{22}$$

where $\widetilde{P}_t = \left[\int_0^1 P_{i,t}^{-\theta} di\right]^{\frac{-1}{\theta}}$ and IO_t = 'aggregator' of intermediate goods output.

Finally, exploiting the property that, given the Cobb-Douglas production function for intermediate goods producer, the ratio $\left[\frac{K_{i,t-1}}{L_{i,t}}\right]$ is the same across all firms i, it is possible to aggregate to obtain:

$$IO_t = A_t K_{t-1}^{1-\sigma} L_t^\sigma \tag{23}$$

$$L_t^d = \frac{IO_t}{A_t}\left[\frac{\sigma}{1-\sigma}\frac{q_t}{w_t}\right]^{1-\sigma} \tag{24}$$

$$K_{t-1}^d = \frac{IO_t}{A_t}\left[\frac{1-\sigma}{\sigma}\frac{w_t}{q_t}\right]^\sigma \tag{25}$$

$$MC_t = \frac{1}{A_t}\left[\frac{w_t}{\sigma}\right]^\sigma\left[\frac{q_t}{1-\sigma}\right]^{1-\sigma} \tag{26}$$

The model is closed by the equation $r = q - \delta$.

(B) The Model with fixed capital

Both the household problems and the pricing problem of the resetting firms do not change, and thus neither do the first order conditions. The difference is given by the technology of intermediate goods producers, now given by

$$Y_{i,t} = A_t L_{i,t}^{\sigma} \tag{27}$$

The labour demand and the real marginal cost of firm i is therefore

$$L_{i,t}^{d} = \left[\frac{Y_{i,t}}{A_t}\right]^{\frac{1}{\sigma}} \tag{28}$$

$$MC_{i,t} = \frac{1}{\sigma} A_t^{-\frac{1}{\sigma}} w_t Y_{i,t}^{\frac{1}{\sigma}-1} \tag{29}$$

The aggregate resource constraint is now simply given by

$$Y_t = C_t \tag{30}$$

and the link between aggregate labour demand and aggregate output is provided by

$$L_t^d = \left[\int_0^1 L_{i,t}^d di\right] = \left[\frac{Y_t}{A_t}\right]^{\frac{1}{\sigma}} \left[\frac{P_t}{\overline{P_t}}\right]^{\frac{\theta}{\sigma}}$$

where $\overline{P_t} = \left[\int_0^1 P_{i,t}^{-\frac{\theta}{\sigma}} di\right]^{-\frac{\sigma}{\theta}}$.

Note that now marginal costs depend upon the quantity produced by the single firm, given the decreasing returns to scale. In other words, different firms charging different prices would produce different levels of output and hence have different marginal costs. Consider the optimal reset price formula in a non-stochastic steady state. This is still described by

$$P_{i,t}^{*} = \left(\frac{\theta}{\theta-1}\right)\frac{\Psi(t)}{\Phi(t)} \tag{31}$$

$$\Phi(t) = P_t^{\theta-1} Y_t + \alpha\beta E_t\left[\Phi(t+1)\right]$$

$$\Psi(t) = MC_{i,t} P_t^{\theta} Y_t + \alpha\beta E_t\left[\Psi(t+1)\right]$$

The $MC_{i,t}$ in $\Psi(t)$ is now increasing over time, since

$$MC_{i,t+j} = \frac{1}{\sigma} A_{t+j}^{-\frac{1}{\sigma}} w_{t+j} \left(\frac{P_{i,t}^{*}}{P_{t+j}}\right)^{-\theta\left(\frac{1}{\sigma}-1\right)} Y_{t+j}^{\left(\frac{1}{\sigma}-1\right)}$$

and $P^*_{i,t}$ is fixed until the new resetting. The variable $\Psi(t)$ needs therefore to be deflated accordingly to make it stationary. In a non-stochastic environment,

$$\Phi(t) = \sum_{j=0}^{\infty} (\alpha\beta)^j P_{t+j}^{\theta-1} Y_{t+j} \tag{32}$$

$$\Psi(t) = \sum_{j=0}^{\infty} (\alpha\beta)^j MC_{i,t+j} P_{t+j}^{\theta} Y_{t+j} = \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{1}{\sigma} A_{t+j}^{-\frac{1}{\sigma}} w_{t+j} \left(\frac{P^*_{i,t}}{P_{t+j}}\right)^{-\theta(\frac{1}{\sigma}-1)} Y_{t+j}^{(\frac{1}{\sigma}-1)} P_{t+j}^{\theta} Y_{t+j} \tag{33}$$

Substituting (32) and (33) in (31) yields a dynamic equation that links $P^*_{i,t}$ to aggregate variables.

$$P_{i,t}^{*1+-\theta(\frac{1}{\sigma}-1)} = \left(\frac{\theta}{\theta-1}\right) \frac{\sum_{j=0}^{\infty} (\alpha\beta)^j \frac{1}{\sigma} A_{t+j}^{-\frac{1}{\sigma}} w_{t+j} Y_{t+j}^{\frac{1}{\sigma}} P_{t+j}^{\frac{\theta}{\sigma}}}{\sum_{j=0}^{\infty} (\alpha\beta)^j P_{t+j}^{\theta-1} Y_{t+j}} \tag{34}$$

In a non-stochastic steady state A_t, Y_t and w_t are constant over time, while $P_{t+1}/P_t = \gamma$, hence substituting it yields

$$\Phi(t) = P_t^{\theta-1} Y \sum_{j=0}^{\infty} (\alpha\beta\gamma^{\theta-1})^j \tag{35}$$

$$\Psi(t) = \frac{1}{\sigma} A^{-\frac{1}{\sigma}} w Y^{\frac{1}{\sigma}} P_t^{\theta/\sigma} P_{i,t}^{*-\theta(\frac{1}{\sigma}-1)} \sum_{j=0}^{\infty} (\alpha\beta\gamma^{\theta/\sigma})^j \tag{36}$$

Substituting the expression for $\Phi(t)$ and $\Psi(t)$ in (31) we can obtain a formula that links the reset price with the aggregate variables in the non-stochastic steady state and then solve for Y. It is clear, however, that the two summations in (35) and (36) need to converge. In particular, we need the following: $\alpha\beta\gamma^{\theta/\sigma} < 1$,i.e., $\gamma < (\alpha\beta)^{-\sigma/\theta}$. Putting $\alpha = 0.75, \beta = 0.99, \sigma = 0.67, \theta = 10$, we get $\gamma < 1.02$, which means an annual rate of growth of money lower than 8%.

Appendix 2. The Calvo-Fischer Case

Yun (1996) and Jeanne (1998) assume that the new price set in a generic period t is actually indexed to trend inflation. Hence, even if the firm is not allowed to revise its price, the latter grows at the same rate as trend inflation. Then the problem of the firm is

$$\underset{\{p_{it}\}}{Max} \qquad E_t\left(\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}F_{t+j}\right) = E_t\left(\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}\left[\left(\frac{P_{i,t}\Pi^j}{P_{t+j}}\right)^{1-\theta}Y_{t+j} - TC_{i,t+j}(Y_{i,t+j})\right]\right) \tag{37}$$

where $Y_{i,t+j} = \left(\frac{P_{i,t}\Pi^j}{P_{t+j}}\right)^{-\theta}Y_{t+j}$ and the optimal price is

$$P_{it}^* = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}MC_{t+j}\left(\frac{P_{t+j}}{\gamma^j}\right)^{\theta}Y_{t+j}}{E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}\left(\frac{P_{t+j}}{\gamma^j}\right)^{\theta-1}Y_{t+j}} \tag{38}$$

The steady state value is

$$\frac{P_{i,t}}{P_t} = \left(\frac{\theta}{\theta-1}\right)MC \tag{39}$$

which coincides with the flexible price steady state. Moreover, note that there is no upper value for the steady state rate of growth of money.

The log-linearised optimal price setting rule equation coincides with the log-linearisation of a typical Calvo framework around a zero money growth steady state

$$p_{it} - p_t = (1-\alpha\beta)E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left[\pi_{t,t+j} + mc_{t+j}\right] \tag{40}$$

which is also the case for the log-linearised general price level equation

$$p_{it} - p_t = \frac{\alpha}{1-\alpha}\pi_t \tag{41}$$

Putting them together one gets the usual New Keynesian Phillips Curve. Hence, a Calvo-Fischer structure delivers exactly the kind of equations used in most models in the literature.

References

Jeanne, O. (1998). Generating real persistent effects of monetary shocks: How much nominal rigidity do we really need? *European Economic Review 42*, 1009–1032.

Yun, T. (1996). Nominal price rigidity, money supply endogeneity and business cycle. *Journal of Monetary Economics 37*, 345–370.