1. Consider the standard Diamond OLG model analyzed in the lectures. Assume that
the utility function is

\[ U(c_{1t}, c_{2t+1}) = \ln c_{1t} + \frac{1}{1 + \theta} \ln c_{2t+1} \]

and the production function is

\[ y_t = f(k_t) = A k_t^\alpha - \delta k_t. \]

Then:

(a) Solve for the saving function and the optimal consumption in period 1 and period 2.
Does the saving function (or the marginal propensity of consumption in the first
period) depend on the interest rate? Why?

(b) Find the competitive equilibrium. Then, characterizes the steady state equilibrium
and show that is globally stable.

(c) What is the effect on the equilibrium of:
   (i) an increase in \( \theta \);
   (ii) an increase in \( n \) (rate of population growth);
   (iii) an increase in \( A_t \). What if there is a positive rate of growth of \( A : A_{t+1} = (1 + g)A_t \) with \( g > 0 \), as in the Solow model of growth?

(d) Derive explicit conditions under which the steady-state equilibrium is dynamically
   efficient. Using plausible numbers discuss whether dynamic efficiency is "realistic".

2. Using the same model as above assume now that people work both when they are
   young and when they are old.

(a) Solve for the saving function and the optimal consumption in period 1 and period
   2. Does the saving function (or the marginal propensity of consumption in the
   first period) depend on the interest rate? Why?

(b) Derive explicit conditions under which the steady-state equilibrium is dynamically
   efficient and show that dynamic efficiency is more likely in this case with respect
   to the previous exercise.
3. **Fiscal Policy.** Use the model in exercise 1.

(a) **Effects of government debt.** Assume that the government issues a certain level of debt $B_t$ and gives the resources to the old generation in period $t$. Then, in all the future periods the government will tax the young (lump-sum taxation) such that the per-capita level of debt remain constant $d = \frac{B_t}{N_t} = \frac{B_{t+1}}{N_{t+1}} = \frac{B_{t+2}}{N_{t+2}} \cdots$. Show what happens to the steady state capital stock. Is this policy Pareto-efficient? Discuss. (HINT: find the value of taxes needed to keep the per-capita government debt constant, given the government budget constraint, and then see what happens to the saving decision of agents)

(b) **Effects of government spending.** Assume that the government spends $G_t$ financed by lump sum taxation on the young agents, such that: $G_t = \tau_{1t}$. Show what happen to the steady state capital stock. Is this policy Pareto-efficient? Discuss. What happens if there is a temporary increase in expenditure, let’s say an increase from $G_t$ to $\tilde{G}_t$ where $\tilde{G}_t > G_t$ that last only for $T$ periods?

(c) **Financing expenditure.** Assume now that in (b) the expenditure is partly financed by government debt and partly by the lump sum taxes on the young. Will a switch from tax to bond financing have any effect on the steady state of the model? Why?