The due date for this assignment is \textit{Wednesday, November 10}.

1. Consider the model of investment analyzed in the lectures. Describe the effects of each of the following changes on the $K = 0$ and $q = 0$ loci, on $K(t)$ and $q(t)$ at the time of the change, and on their behavior over time. In each case, assume that $K$ and $q$ are initially at their long-run equilibrium values.

(a) A war destroys half of the capital stock

(b) The government taxes investment (firms pay the government $\gamma$ for each unit of capital they acquire, and receive a subsidy of $\gamma$ for each unit of \textbf{disinvestment} => HINT: add a term ($-\gamma I(t)$) to the $F(\bullet)$ function).

2. Let $H$ denote the stock of housing, $I$ the rate of investment, $p_H$ the real price of housing, and $R$ the rent. Assume that $I$ is increasing in $p_H$, so that $I = I(p_H)$ with $I'(\bullet) > 0$, and that $\dot{H} = I - \delta H$. Assume also that the rent is a decreasing function of $H : R = R(H), R'(\bullet) < 0$. Finally, assume that rental income plus capital gains must equal the exogenous required rate of return, $r : \frac{R+\dot{p}_H}{p_H} = r$.

(a) Sketch the set of points in $(H, p_H)$ space such that $\dot{H} = 0$ and $\dot{p}_H = 0$.

(b) What are the dynamics of $H$ and $p_H$ in each region of the resulting phase diagram? Sketch the saddle path.

(c) Suppose the market is initially in the long-run equilibrium, and that there is an unexpected permanent increase in $r$. What happens to $H$ and $p_H$ at the time of the change? How do $H$, $p_H$, $I$, and $R$ behave over time?

(d) Do the same as in (c), but assuming that the permanent increase in $r$ is known to happen in a future time period $T$.

3. Solve the model analyzed in the lectures, assuming that $p_K = 1$, $R(t, K(t), N(t)) = K(t)^\alpha N(t)^{1-\alpha}$ and, in turn:

(a) $G(I(t), K(t)) = \left(e^{I(t)/K(t)} - 1\right) K(t)$

(b) $G(I(t), K(t)) = \frac{3}{2} \left(\frac{I(t)}{K(t)}\right)^2 K(t)$

(c) $G(I(t), K(t)) = \tilde{G} \left(\frac{I(t)}{K(t)}\right) K(t)$