Problem Set 3: Economic Growth

TO BE HANDED IN MY PIGEON HOLE OR SENT VIA E-MAIL BY
TUESDAY, 23RD NOVEMBER, NOON (12,00 A.M.)

Exercise 1: Applying the Solow growth model

1. Imagine that there are two countries: America, called from here onwards country A, and Belgium, called B. Both countries have the following production function:

\[ Y = F(K, N) = K^{1/2} N^{1/2} \]

where Y stands for output, K for physical capital stock and N for labour force.

a. Does this production function have constant returns to scale? Explain.

b. Calculate the production function per worker: \( y = f(k) \).

c. Assume that neither country has population growth or technological progress and that 5% of capital depreciates every year. Assume further that country A saves 10% of output each year and country B saves 20% of output each year.

c.1) Find the steady state level of capital per worker for each country.

c.2) Find the steady state levels of output per worker and consumption per worker.

d. Suppose that both countries have an initial capital stock equal to 2.

d.1) What are the levels of income per worker and consumption per worker?

d.2) Show how the capital stock per worker will evolve over time in both countries. For each year, calculate income per worker and consumption per worker. How many years will it be before consumption in country B is higher than consumption in country A (Suggestion: you may want to use an Excel file and report Table’s results)?

d.3) Explain why consumption in country B ends up to be higher than in country A.
Exercise 2: Growth and Development Accounting. Applied stuff!

Consider the following production function, which holds for every Italian region and Macro-region:

\[ Y_{it} = A_{it} F(K_{it}, N_{it}) = A_{it} (K_{it}^{1-\alpha} N_{it}^{\alpha}) \]

where \( Y_{it} \) is value added in region (Macro-region) \( i \) at time \( t \), \( K \) is the net capital stock; \( N \) is the number of workers and \( A \) is the so-called Solow residual, also named Total Factor Productivity (i.e. TFP), which proxies for the level of technology.

Assume that the production function exhibits constant returns to scale.


Hints:
--The Italian Macro-regional divide is the following:
- North-West: Lombardy, Piedimont, Valle d'Aosta, Liguria;  
- North-East: Trentino-Alto Adige, Veneto, Friuli-Venezia Giulia, Emilia Romagna;  
- Center: Tuscany, Umbria, Marche and Latium 
- South: Abruzzo, Molise, Campania, Apulia, Basilicata, Calabria, Sicily and Sardinia.

--In calculating the labour share, consider both employees and self employed, and assume that a self-employed gains as much as an employee on average, each year. Put in other terms, the total number of workers is given by the sum of employees and self-employed and each guy gains, on average, the same amount of money each year.

--As for empirical analysis, you should pass from continuous to discrete time. The time span you must consider is 5 years for the first four sub-periods and 4 years for the last one. Please refer to Robert J. Barro and Xavier Sala-i-Martin (1995): Economic Growth, The MIT Press. Chapter 10.4: Growth Accounting, pp.346-348.

b) Which is the region having the highest rate of technological progress? And the lowest?

c) Is there any relation between TFP and GDP growth? 
Hint: calculate the correlation coefficient or map the two series.

d) According to your calculations, what drives GDP growth in Italian regions? Is that factor accumulation or technological progress? Briefly discuss your results and highlight the possible problems your analysis faces.

Data sources:
GDP, Number of workers and Wage Bill from ISTAT, Conti Economici Regionali, 2008.