From *The Trade Cycle* to the “Essay in Dynamic Theory”: The Harrod-Keynes Correspondence, 1937–1938

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It appears to me that your new scheme resembles Maynard’s *Treatise* one ... in that, setting out to be an analysis of *causes*, it ends by being a recognition of *results*.—Robertson to Harrod, 1934

1. Introduction

Both in the foreword of his 1936 book *The Trade Cycle* and in the first lines of his well-known 1939 article on economic dynamics, Roy Harrod claims that his theory is based on two principles: “It ... consists in a marriage of the ‘acceleration principle’ and the ‘multiplier’ theory” (Harrod 1936, vii). The 1939 “Essay in Dynamic Theory,” according to Harrod, is meant to provide “a development and extension of certain arguments advanced in [his] *Essay on the Trade Cycle*” (1939, 14). This claim of a theoretical continuity between the two works must not be taken too literally. In fact, after publishing *The Trade Cycle*, Harrod discussed...
his work with many of his fellow economists, and his "Essay" resulted from the exchange of views that occurred in those "years of high theory."

I focus here on one of the possible sources that stimulated the evolution of Harrod's thought, namely, his correspondence with John Maynard Keynes in 1937–38. I do not want to suggest that this was the only external source influencing Harrod; I merely think that his exchange of opinions with Keynes might provide some singular and useful hints toward understanding certain features of his theory. Before entering into the details of this correspondence, let me remind the reader of the contingencies that stimulated the exchange of letters between Harrod and Keynes.

Keynes read The Trade Cycle only after its publication in September 1936. At the end of March 1937, Keynes was planning to lecture on Harrod's book. He therefore made some preparatory notes and observations and sent them to Harrod. Keynes's argument was based on a misinterpretation of Harrod's thesis; Harrod's reply and Keynes's rejoinder led to a long exchange of letters, which are now collected in Keynes 1973 (150–79).

At the beginning of August 1938, Harrod announced to Keynes that he had "just finished writing [his] restatement of the "dynamic theory," which is . . . a great improvement on [his] book" (3 August 1938, in CW 14:301). The discussion on the first draft of Harrod's paper extended from August through December 1938 (CW 14:321–50). In this exchange of views, Harrod and Keynes addressed the focal points of Harrod's analytical setup. The misunderstandings, the concessions and resistances, the fallacies and the omissions can thus reveal some of Harrod's theoretical concerns, some partially or totally concealed in his published papers and books. In particular, Harrod's reaction to Keynes's interpretation of The Trade Cycle shows how Harrod minimized the extent of the changes he made when writing the "Essay," which amount to more than a mere development or extension. Although it is true that Harrod's dynamics in 1939 was still based on the multiplier and acceleration principles, it nevertheless differed from The Trade Cycle with regard to both its logical structure and the analytical role of the acceleration principle.

I will explore here the difficulties Harrod and Keynes experienced in trying to understand each other, focusing on the conceptual distance underlying their thoughts and on the mistakes it fostered on both parts. On the one hand, their exchange illustrates the difficulty of integrating within a conceptual scheme a notion not belonging to it, for example, Harrod's notion of justified growth, which Keynes could not figure out at first.

On the other hand, sometimes wrestling with alien concepts stimulates new interpretations of the most troublesome old notions, enabling creative theorists to radically modify systems of thought. Such was the case with the algebraic formulation Keynes devised to summarize The Trade Cycle, a condensation that greatly impressed and probably inspired Harrod. Moving from Harrod's interpretation of the formula and the following comments from Keynes, I will attempt to show how Harrod's thought evolved as regards the causal structure of relationships among variables. He also reconsidered the role of time in making decisions, specifically concentrating on the determination of investment and on the accelerator.

2. The Trade Cycle and the "Essay"

The Trade Cycle is an inquiry into the determinants and consequences of saving and investment decisions in an economy. Producers acquire increments of capital equipment according to their expectations about the increase in demand for consumption goods. This is the logic underlying the accelerator principle (Harrod called it "the Relation"). The strength of the Relation depends on the state of technology, the interest rate, and any other considerations that might affect investment decisions, collectively called the third dynamic determinant. The complex of the saving decisions depends on the total income of the community, which in turn depends on investment through the multiplier. The size of the multiplier effect is affected by the first two dynamic determinants: the average propensity to save and the distribution of income.

One of the possible outcomes of the interaction of the multiplier and the Relation is a state of moving equilibrium. The expectation of an increase in the demand of consumption goods might indeed stimulate investment and, through the multiplier, income and consumption in such a way that

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1. Among those who corresponded intensively with Harrod during that period, Robertson, Marschak, and Meade are particularly significant. For an overview of the cross-influence between them and Harrod, see Young 1989, in which many unpublished letters and manuscripts are transcribed.

2. Hereinafter this volume will be cited as CW 14.

3. Harrod sent the first draft of the "Essay in Dynamic Theory" on 6 August 1938: "I am at the moment more interested in it than in my wretched Ad dressed Presidential" (Harrod to Keynes, Keynes Papers, file EJ 1/5).
the expectations of growth are satisfied. In that case, the economic system would experience a period of steady growth, even a continuously increasing rate of growth, which would reproduce the expansive expectations and conditions leading to a regular enlargement of the scale of production. The maintenance of such a favorable situation requires that any change occurring in the determinants is exactly balanced by an equal and opposite change in the resultant of the other determinants. Yet nothing ensures that changes in the distribution of income, in the propensity to consume, or in the technology, triggered by the very increase in income that they themselves determine, precisely compensate each other. If some of these changes should cause a decrease in the flow of investments, the interaction of multiplier and accelerator would lead to a cumulative depressive process. The trade cycle consists of a succession of phases of progressive and regressive cumulating processes, ruled by the alternation of the value of the determinants.

As regards the “Essay in Dynamic Theory,” it might be useful to remember how the equation describing warranted growth is defined. Let

\[ s = \frac{S}{Y} \quad (1) \]

and

\[ C_p = \frac{I}{\Delta Y}. \quad (2) \]

These equalities define \( s \) and \( C_p \) by income \( Y \), income changes \( \Delta Y \), savings \( S \) and investment \( I \). Solving (1) and (2) with respect to \( I \) and \( S \) and equating the right-hand sides, Harrod obtained

\[ G = \frac{\Delta Y}{Y} = \frac{s}{C_p}. \quad (3) \]

This expression defines the growth rate of income at a given point in time. All these variables are ex post records of what happens in the system, and Harrod refers to them as “actual” values of the variables. In particular it has to be noted that \( C_p \), that is, the actual value of the ratio of investment and increase of income, reflects unplanned variations in the stocks (that are by definition considered part of ex post investment).

Sometimes \( C_p \) assumes a particular value \( C \), to which there corresponds no undesired accumulation of stock. In that case, the growth rate appears entirely “justified by the circumstances” and is considered an equilibrium quantity. Accordingly, Harrod expressed “warranted growth” as follows:

\[ G_w = \frac{s}{C}. \quad (4) \]

The equilibrium rate thus defined is highly unstable. Any deviation from it would tend to be amplified, rather than to be automatically corrected. For example, insufficient expenditure on investment would cause, through the multiplier process, unforeseen accumulation of stock that would be interpreted by entrepreneurs as a signal of excess investment. They would further reduce investment, thus reproducing and enlarging the initial difficulties.

Up to this point, Harrod restricted his analysis to the very short term. In the long run, Harrod considered that growth itself triggers changes in \( s \) and \( C \) and, consequently, in the equilibrium rate of growth. When income increases (because \( G > G_w \)), one has to expect the propensity to save to increase as well. This in turn determines that \( G_w \) chases \( G \). Resources being limited, depending on population increases and technological changes, there exists a limit to actual growth \( G \). When this limit is reached, \( G_w \) grows larger than \( G \), starting the cumulative process in the reverse direction, symmetrically repeating itself.

3. Steady Growth

On 31 March 1937, a few months after the publication of *The Trade Cycle*, Keynes sent to Harrod some notes on it, in which he raised the point that Harrod’s “theory in the form in which [he] finally enunciate[d] it is not correct, being fatally affected by a logical slip in the argument” (CW 14:150–51). Keynes was led to this conclusion by a misinterpretation of Harrod’s analytical method. This error on Keynes’s part, along with his surprise when Harrod explained to him why he was wrong, reveals the difficulty even distinguished thinkers experience in grasping concepts not in use in current debate.

On his first reading, Keynes did not understand that Harrod’s *steady growth* was an equilibrium *status* of the system, a condition capable of self-reproduction without the help of exogenous forces to sustain it. After all, the idea that the level of activity of the economic system, its consumption, income, and investments might possibly increase steadily

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4. According to Harrod, the action of the static and dynamic forces was not responsible for the determination of the equilibrium level or rate of growth, respectively, but rather for their *variations*. The maintenance of equilibrium only occurred when the resultant of the forces is zero, meaning that the opposite tendencies exactly balance each other. Here the analogy to Newtonian physics is precise: forces are the cause of change, while persistence of a state is characterized by the reciprocal canceling out of all the forces involved.
(that is, continuously and regularly) without growth being caused by some exogenous force, was as alien to economic thought of the 1930s as Galileo’s idea of uniform motion in a straight line was alien to Aristotelian physics, according to which motion had to be sustained by air. It is therefore not surprising that this aspect of Harrod’s analysis was not understood by many of his contemporaries. In fact, for the next fifteen years, economic dynamics ignored Harrod’s analysis in terms of the rate of growth.

Not having initially grasped this subtle innovation, Keynes could obviously not appraise its methodological counterpart. Keynes’s criticism, as perceived by Harrod himself, supposed that changes within the economic system occurred per saltum, while Harrod’s dynamics actually referred to steady (that is, continuous) changes. Harrod used various expressions such as these to represent two different orders of questions: “the difference between the two sets of problems is analogous to the difference of getting a train to move and the dynamics of a train in motion at a constant velocity” (Harrod 1934, 478). Harrod’s analysis pertained to the second issue: it presupposed that the economic system experiences growth at a constant rate and aimed to determine the conditions at which such a status might be maintained. Keynes’s calculations concerned the first problem instead, since he was considering sudden variations in investment. Keynes thus interpreted Harrod’s procedure as “a logical slip in the argument” and “a straightforward slip in arithmetic” (Keynes to Harrod, 31 March 1937; in CW 14:151, 155–57).

After reading Harrod’s reply, Keynes promptly admitted his mistake:

I have undoubtedly misunderstood you and there is no question of an arithmetical slip. But the odd thing is that, having invented so interesting a theory, you should not have mentioned it in the book! For I do not see how I could possibly have understood what you were driving at in the passage where I accuse you of an arithmetical slip. Indeed, I should doubt whether any reader who has not talked or corresponded with you could be aware that the whole of the last half of the book was intended to be in relation to a moving base of steady progress (CW 14:170).

4. The Fundamental Equation

After having correctly interpreted Harrod’s thesis, Keynes proposed a formulation worth considering carefully, for it provides some suggestions that Harrod took into account while writing the “Essay.” Keynes’s formulation, in fact, matches the “fundamental equation” of Harrod’s dynamics in many respects, which is essentially based on the same elements connected by an equivalent functional relationship. In both cases, there is an inverse relationship between a rate of growth and the product of the multiplier and the accelerator. On the other hand, Keynes’s formulation differs from the analytical framework of The Trade Cycle.

7. Interestingly enough, Robertson had pointed out to Keynes that Harrod was trying to lay down the conditions for a steady advance and that he was chiefly interested in a growing economy (Robertson to Keynes, two undated letters between 28 January and 27 March 1937, Keynes’s papers, file C03/47). Only after the latter of Robertson’s remarks did Keynes respond, but without mentioning growth: “It is only in the last week or two that I have been able to read Roy’s book carefully” (27 March 1937, file C03/47). The lecture notes were sent to Harrod four days after this letter to Robertson.

8. The parallels are quite easy to spot. In spite of that, most recent studies of the evolution of Harrod’s dynamics or of the Harrod–Keynes correspondence tend to ignore it, for some reason I frankly cannot understand. I refer in particular to Kregel 1980, Varri 1990, and especially Young 1989. The last presented valuable aspects, among which I particularly appreciate the illustration of the network of dense interchanges between Harrod and his contemporaries. On the other hand, Young prefers to look for unlikely similarities between Harrod’s formula and Cassel’s equations (from which “Harrod’s equation can be derived”), or Kalecki’s or Lange’s formulation, that “may even have catalyzed the discovery of Harrod’s ‘fundamental equation’” (Young 1989, 150–58; emphasis added). I do not want to deny that Harrod might have incorporated influences other than Keynes’s. I only affirm that Harrod was familiar with Keynes’s suggestions; that before Keynes’s formulation, Harrod had not thought of such a representation of The Trade Cycle; and that Harrod spent some time thinking about and experimenting with it. “Your algebraic formulation is extremely helpful,” wrote Harrod to Keynes, 15 April 1937 (CW 14:175). As far as I know, M. Pugno is the only author who did not ignore the similarities between Keynes’s and Harrod’s formulas. Unfortunately, after correctly identifying the analogies between the two equations, Pugno did not inquire into the different analytical and methodological role of the terms in the two equations and therefore drew quite a different conclusion from the one presented here.
both in its logical structure and in a few details. Moreover, both Keynes's equation and The Trade Cycle differ from the "Essay" with respect to the same features. Comparing these three different interpretations of the interaction of the multiplier and the Relation can therefore provide some clues for understanding the evolution of Harrod's dynamic theory.

Keynes suggested that the fundamental equation expressing the rate of growth of capital should be

\[ y = 1/(M \cdot R - 1), \]

where \( M \) stands for the average value of the multiplier while \( R \) is the acceleration coefficient (CW 14:171). Keynes did not specify how he obtained this equation. The following is one possible procedure.\(^9\)

Let \( M = Y_i/I_i \) and \( R = K_i/Y_i \).

If during a unit period of time, capital grows at a constant rate \( y \), we have \( K_i = (1 + y)K_{i-1} \). The value of \( y \) can be calculated easily by substituting from the definitions given:

\[ y = \frac{\Delta K_i}{K_{i-1}} = \frac{I_i}{K_{i-1}} = \frac{Y_i}{M} \cdot \frac{R \cdot Y_i}{1 + y} = \frac{1 + y}{M \cdot R}, \]

from which \( y \cdot M \cdot R = 1 + y \) and, finally, Keynes's equation (5) are derived.\(^10\)

Moving from equation (5), Keynes deduced some clues for interpreting Harrod's theory. First, he noticed that a necessary condition for a constant growth rate is that the product \( M \cdot R \) is constant. That is to say, steady growth requires that changes in some of the dynamic determinants exactly match variations in the others, according to an inversely proportional relationship (CW 14:171).

Incidentally, it must be observed that when adopting a formulation analogous to the one suggested by Keynes, the multiplier and accelerator acquire a different epistemological status. On the one hand, in the "Essay" the three dynamic determinants are substituted by the variables \( s \) and \( C \) (or \( M \) and \( R \), in Keynes's formulation); the qualitative discussion of the resultant of the dynamic forces in The Trade Cycle thus gives way to a quantitative link.\(^11\) On the other hand, the Relation and multiplier are no longer considered the mechanisms of two interacting processes, but independent coefficients codetermining the growth rate. The coefficients \( s \) and \( C \) and the growth rate \( G \) were entirely new concepts, needing interpretation.

Second, Keynes pointed out that, in general, \( M \) tends to fall as income increases; a constant growth rate thus requires steady growth of \( R \) and therefore a continuous decrease in the rate of interest (CW 14:171–72). Only in a subsequent letter did Keynes make explicit the relationship between \( R \) and the interest rate (see section 5 below).

Third, Keynes wrote, "it is only by a miracle or a careful design that the values of \( M \) and \( R \) will be such as to be consistent at the same time with steady growth and full employment" (CW 14:171). Thus, "steady growth and full employment are different criteria of policy. Full employment may require unsteady growth and steady growth may involve unsteady employment" (173). Harrod obviously agreed that "steady growth and full employment do not provide the same criterion. As between them I do not favour steady growth. On the contrary I say several times that, starting with the slump, to damp growth down to what could be steadily maintained would involve perpetuating existing unemployment, which would be intolerable" (CW 14:176). In the "Essay," Harrod faces this point more directly, acknowledging that the warranted growth rate is not uniquely determined once and for all but depends, among other things, on the level of activity.

The fourth of Keynes's remarks was the most painful for Harrod, and it seriously influenced the further development of both Harrod's theory and his debate with Keynes. According to Keynes, The Trade Cycle was not an appropriate title for Harrod's book: "I do not see that the theory has an application worth mentioning to the trade cycle." Without giving further details at the time, Keynes maintained that growth is a problem pertaining to long period analysis (CW 14:173). Keynes attributed to Harrod the assumption that the boom is a phase of steady growth (171). Keynes then rejected this hypothesis, asserting that "the maintenance of steady growth is at all times an inherent improbability in conditions of laissez-faire.... Both the boom and the slump, that is to say the whole of the cycle, are characterised, I should have thought, by none of

9. Both \( M \) and \( R \) can be expressed as the ratio of investment and capital to consumption, rather than to income, which would correspond more directly to Harrod's formulation in The Trade Cycle. Nonetheless, I refer to income in order to underline the formal similarity between Keynes's approach and that in the "Essay."

10. To be precise, Keynes expressed his formula in percentage, rather than in absolute terms. It thus read \( y = 100/(M \cdot R - 1) \).

11. De facto Harrod had ignored Keynes's warnings about the heterogeneity of the magnitudes involved. This was to have further consequences (see section 6 below).
the conditions of steady growth which you are assuming to be present” (173).

Harrod reacted vigorously: “In spite of what you say, I still think that my book concerns the trade cycle; nay more, subject to further criticism, that it contains the essence or germ of the theory of the trade cycle” (in CW 14:174). In his reply, Harrod emphasized the lack of symmetry between growth at a constant or increasing rate, justifying entrepreneurs’ decisions, and any fall in the pace of growth, causing a ruinous depression. “It is always possible to accelerate, but it is not possible to decelerate without starting again from the zero line” (in CW 14:174–75). In other words, Harrod was thinking of the upward phase of the cycle in terms of a growth pace temporarily feasible for the economy. This notion was not strictly coincident with that of equilibrium, since it also included more than justified investment. Keynes’s criticism indicates he did not understand that Harrod was less concerned with equilibrium per se than with the instability of the equilibrium state.12

Harrod’s reply to the crucial question whether growth is a matter of long- or short-run consideration eventually channeled the following debate and constituted the main point of disagreement between Harrod and Keynes. Still, as I shall try to show, this was not at all obvious, even to them.

5. The Accelerator: Actual, Normal, and Equilibrium Values

For Harrod, the problem was that of divorcing the component of global investment dependent on long-run considerations from the part depending on short-period perspectives:

You think I am wrong in making investment a function of current growth only. Granted. Suppose only half were governed by current growth, the rest by long-period planning. My theory is substantially intact. It remains true that the growth of consumption cannot slow down without producing a great recession; but in this case the recession would only have to be such as to reduce savings to half their usual level. Personally I believe by far the greater part of investment rests on an immediate prospect of an increase of demand. People do not build factories for use some years hence nor houses that will remain unwanted. Why should they? They increase equipment at the last feasible moment to save interest. Moreover if you try looking more than a year or so ahead everything becomes so violently uncertain. (Harrod to Keynes, in CW 14:175–76)

To understand Keynes’s viewpoint, it is necessary to examine carefully his interpretation of the fundamental equation \( y = 1/(M \cdot R - 1) \).

First, Keynes discussed which variables have to be considered as dependent and which as independent: “I should hold that \( y \) is determined by the rate of interest and the state of expectations, that \( M \) is determined by individual psychology and institutions, and that, in the short period, \( R \) is dragged at the chariot wheels of \( y \) and \( M \).” Thus, in the short run, \( R \) must be considered a dependent variable, for if, in consequence of better expectations, the rate of growth of capital should increase, there would follow a change in the relative prices of capital and consumption goods such as to reduce \( R \) to a level compatible with the new \( y \), given the value of the multiplier (see CW 14:178).

Keynes then raised a second point: “On the other hand, the strength of your approach lies in the idea that \( R \) has, so to speak, a normal long-period value which is a function of the rate of interest” (CW 14:177). This is the source of the first of his criticisms relative to Harrod’s long-run interpretation of the trade cycle. “The factors determining the normal value of the relation have little or no bearing on the trade cycle; and it was those I thought you were discussing in much of your book” (CW 14:178).

The distinction introduced by Keynes had far-reaching consequences. To appreciate them, it is worth considering his innovative description of the cycle, applying the new concept of “normal value of the Relation” (denoted by \( R_n \)). Consider a state of the economic system in which \( R = R_n \), and suppose that there occurs a change in the expectations or in the value of the multiplier. Being a dependent variable, \( R \) should then adapt to the new condition, thus diverging from \( R_n \), which depends instead on long-run considerations. There would follow, for instance, \( R < R_n \); this would stimulate economic activity, \( y \) would increase causing a further fall in \( R \) and thus widen the gap between \( R \) and \( R_n \) (CW 14:177–78).

Harrod could not remain indifferent to this formulation of the instability principle, and indeed we find traces of it in the “Essay.” Rather than

12. In the “Essay” Harrod explicitly shifts the emphasis to stability. But for favoring the symmetry of the cumulative, widening gap between actual and warranted growth over the asymmetry of boom and crisis, Harrod paid a price. He had to renounce his explanation of the difference between “the gentle nature of the revival” and the destructive character of the crisis.
pointing out the similarities between the two approaches, I shall concentrate instead on the differences, which seem to reveal some peculiarities of the evolution of the ideas of the accelerator and investment from *The Trade Cycle* to the “Essay.”

Let me thus compare Keynes’s equation

\[ y = 1/(M \cdot R - 1) \]  \hspace{1cm} (5)

with Harrod’s equations describing the actual and warranted rates of growth:

\[ G = s/C_p \]  \hspace{1cm} (3)

and

\[ G_w = s/C. \]  \hspace{1cm} (4)

As Keynes himself pointed out, if the Relation is assumed to be constant, then the rates of growth of capital \((y = 1/K)\) and of income \((G = \Delta Y/Y)\) are equal (Keynes 1973, 171). On the other hand, both Keynes and Harrod agreed that trade cycle analysis cannot overlook the possibility, and indeed the actual occurrence, of changes in \(R\). Therefore, working with one variable inherently presents differences from calculations with the other, in spite of the similarity of the respective equations. Entrepreneurs and consumers only have the power to decide how much to invest and save, not what the level of income will be. Therefore, \(y\) is an expression of the process of decision and must be interpreted as an independent variable. \(G\), on the contrary, represents the global effect of saving and investment decisions and is a dependent variable. More accurately, \(G\) is the result, registered ex post, of decisions actually made, while \(G_w\) expresses the result of decisions that the entrepreneurs ought to have made in order to be exactly satisfied.\(^{13}\)

Regarding the multiplier, the two economists fundamentally agreed. Keynes maintained that for trade cycle analysis, \(M\) must be considered an exogenous variable, though its value probably falls as \(y\) increases. In his “Essay,” Harrod treated \(s\) similarly: he considered it given in the short run but, in the course of the cycle, subject to fluctuations due to variations in \(Y\); the changes in this variable were themselves part of the cycle mechanism.

Regarding \(R\) and \(R_n\), and \(C\) and \(C_p\), their opinions radically diverged. First, Keynes distinguished \(R\) from \(R_n\) for the length of the reference period. In the short run, \(R\) is dependent on \(y\) and \(M\), whereas \(R_n\) is independent of the present state of business and only refers to the long-run rate of interest. On the contrary, Harrod’s \(C\) and \(C_p\) both concern a single instant, a period of infinitesimal length, during which they must be considered as data (this point is of fundamental importance and will be discussed below in section 7). The second notable difference is that \(R\) stands for the ratio between capital and income, while \(C\) represents its marginal value \(\Delta K/\Delta Y\). Comparison of \(R\), \(C\), and the Relation lucidly shows how Harrod’s notion of investment developed in those few years.

The Relation in *The Trade Cycle* expressed a determination nexus, converting entrepreneurs’ expectations and the technical conditions of production into investment decisions. Given the technique, the rate of interest, the intensity level of capital utilization (though Harrod did not mention this factor), and any other relevant considerations, to increase the production of consumption goods it is necessary to employ a corresponding quantity of additional machinery, labor, and so on. In Harrod’s view, investment thus depended on the prospective increase in the demand of consumption goods, in the ratio defined by the third dynamic determinant. *The Trade Cycle* thus provided a theory of the determinants and the consequences of the interrelations of two distinct but mutually interdependent orders of decisions, those regarding investment and those regarding saving (see Harrod 1936, especially 160–63).

For Keynes, investment essentially depended on the state of expectations. Given the multiplier, the rate of increase of capital \(y\) determines how the ratio of \(K\) to \(Y\) changes. Keynes’s idea seems to run as follows. Investment is an addition to capital stock, but, on the other hand, through the multiplier it also generates income. Given an initial level of capital \(K_0\), investment will increase capital to \(K_1 = K_0 + I_1\). Income, at the end of the period, will be \(Y_1 = M \cdot I_1\) (the reader is reminded that \(M\) represents the average and not the marginal multiplier). \(R\) is residually determined by the new values of capital and income, \(K_1\) and \(Y_1\). Since investment alters capital and income independently of each other, there is no guarantee that they are mutually consistent with a constant \(R\). Investment decisions are not represented by \(R\), but are the result of a multitude of other factors: the acceleration principle, in Keynes’s view, was not a determinant of investment. To clarify, let me calculate the new value of

\(^{13}\) The exact meaning of the latter term will be discussed below in section 8.
the Relation, corresponding to $K_1$ and $Y_1$:

$$R_1 = \frac{K_1}{Y_1} = \frac{K_0 + I_1}{M \cdot I_1} = \frac{1}{M} \left(1 + \frac{K_0}{I_1}\right) = \frac{1}{M \left(1 + \frac{1}{Y_1}\right)}.$$  

This result obviously matches the one that can be obtained by manipulating Keynes’s fundamental equation (5).

In his “Essay,” Harrod stated that an increase of a community’s income is an important determinant of its investment decisions and translated this axiom in the equations $I_s = C \cdot \Delta Y_e$ and $I = C_p \cdot \Delta Y$ (I have introduced the subscript $s$ to represent equilibrium values). Harrod, like Keynes, treated $C_p$ as a residual magnitude, determined by actual increases of income and investment (the latter including increments of capital planned by entrepreneurs and undesired variations of stocks). In other words, $C_p$ summarizes (given the ratio of investment to its actual effect on income and stocks. $C$ is merely a particular value $C_p$ might take, occurring in the special case when the combined effects on increased income balance, such that the expenditure on capital goods and the savings decisions do not give rise to unplanned changes in stocks.

Hence, not even in the “Essay” was the accelerator a determinant of investment. In fact, $C$ was not a ratio of investment to increase in income that entrepreneurs would find particularly satisfying or desirable. It is true that the suggested $C$ would be preferred to any actual $C_p > C$, but it is also true that entrepreneurs would prefer to it any $C_p < C$, which would make ex post investment more than justified.

Equations $I_s = C \cdot \Delta Y_e$ and $I = C_p \cdot \Delta Y$ therefore did not really represent the causal link the axioms claimed for them. What determined the investment in the “Essay” then? In Harrod’s growth equations no term represented the quantity of capital goods that entrepreneurs actually employed; after all, this was a matter that did not bother Harrod. While Keynes treated $y$ as dependent on the rate of interest and the state of expectations, for Harrod’s instability argument, only the ex post registration of undesired accumulation or shortage of stocks mattered, which the entrepreneurs would consider as an indicator of excess or deficit of investment and production. Obviously, the difference between Keynes’s $R$ and $R_n$ also entailed undesired variations of stocks or of the degree of capital utilization. Yet in Keynes’s view, these variations would influence expectations, whereas Harrod’s entrepreneurs interpreted $C \neq C_p$ as a signal of inadequate production and as an instruction to increase (or decrease) investment.

Harrod’s initial interpretation of the fundamental equation thus excluded expectations, which were at the core of both Keynes’s analysis and The Trade Cycle. While in his book Harrod compared the actual result of the entrepreneurs’ actions with the result they expected at the moment of deciding the amount of investment, in the 1939 article Harrod compared actual and warranted effects of some unexplained decisions independent of expectations. The difference in the interpretation of the acceleration principle in Harrod’s book and in his article was mirrored in the new nomenclature adopted in the “Essay.” Harrod no longer used “equilibrium” but preferred “warranted” instead, precisely defined as “the unknown variable, rate of growth, the value of which is found for solving the equation” (Harrod to Keynes, in CW 14:337).

The very concept of dynamics had changed, together with the notion of investment. In The Trade Cycle, dynamics was defined as the study of the set of forces (interpreted as causes; see above, note 4) determining the pattern of economic growth. In the “Essay,” Harrod reduced dynamics to the formulation and resolution of a system of equations “in which a rate of growth appears as an unknown variable” (1939, 17) and from which all causal nexuses were eliminated. Harrod’s dynamics thus turned from a science of causes into a science of effects that ignored what determined them.

14. Harrod had thus completed the shift of his concern from the growth of consumption to the growth of income. This generated some confusion for Keynes, since in his very first letter to Harrod on the “Essay” he felt the need to introduce into the discussion a new coefficient $t$ (as distinct from $C$) to represent the ratio of current output of investment industries to current output of consumption industries (CW 14:323). Twelve days later, Keynes abandoned the reference to the increase of consumption but maintained the symbol $t$ for the investment per unit increase of the output coefficient (333). In the course of the following discussion, both Harrod and Keynes adopted the symbol $t$ for $C$. (To avoid confusion I use the notation in the original Economic Journal article.) In reply to Keynes’s remark, Harrod pointed out that “the distinction between capital goods and consumption goods is clearly of great importance to the theory; but I have not made it. I feel that if I did make it I would have to embark on a rather elaborate development.” Harrod considered investment $I$ as the simple difference between production and consumption, so that “all output is lumped together and no distinction is made between fixed and liquid capital.” Investment thus included unplanned variations of stocks (CW 14:328).

15. Regarding expectations in The Trade Cycle, the following remark by Keynes is very appropriate: “it has dropped out in your treatment, I feel that, given the rate of interest, $y$ is a function of the widely fluctuating state of expectation (for which you substitute ‘the expectation of a steady growth of consumption,’ which does not hold in the short period)” (CW 14:178).

16. Harrod had, in fact, already changed terminology in his 1938 methodological essay, submitted as the presidential address to the Economic Section of the British Association; thus, the change occurred between the two letter exchanges with Keynes regarding The Trade Cycle and the “Essay,” respectively (see Harrod 1938, 403).
This change in perspective led to a paradoxical result. In spite of being the unknown variable in the fundamental equation, \( G_w \) was absolutely redundant in the argument of the "Essay." This variable functioned only to provide a term of comparison for \( G \), but the difference \( G - G_w \) was logically and formally equivalent to the difference \( C_p - C \). In discussing his fundamental equation, Keynes did not feel the need to introduce a term equivalent to \( G_w \); \( R_n \) had rather a different methodological status than \( C \), not only because \( R_n \) referred to the long run while \( C \) was instantaneous, but also because \( R_n \) was positively defined as a "normal" value of the variable \( R \) while \( C \) was negatively defined as that particular value of \( C_p \) that does not register an unwanted accumulation of stocks. The exchange of letters on the first draft of the "Essay" reveals more details about these aspects.

6. The Topic Harrod and Keynes Actually Discussed:

The Validity Clause for the Fundamental Equation

The first draft of the "Essay" was sent to Keynes on 6 August 1938. Keynes's first detailed comments on the paper were dated 17 August. Keynes asked for some restatements and for the explanation of a few terms,\(^{17}\) congratulated Harrod on his interpretation of the *Treatise on Money*, and then formulated a criticism that would absorb most of his and Harrod's attention in the course of their subsequent correspondence.

The whole affair appears quite curious, since many misunderstandings on both parts confused the terms of the debate and, in the end, obscured to Keynes and Harrod themselves the true problem behind Keynes's criticism. Keynes argued that the solution to Harrod's model is not independent of the values of its parameters, but that its instability is subject to the condition that \( C > s \) (CW 14:324). Harrod was never able to understand this comment of Keynes,\(^{18}\) while its implication first escaped the attention of Keynes himself, who only in his last letter explicitly realized that the above-mentioned condition does not regard the stability, but rather the very existence, of an equilibrium rate of growth.

Because of the reciprocal misinterpretations, the discussion often looks confused. Moreover, the modern reader of the correspondence is even more puzzled by mistakes in the transcription of some of the formulas\(^{19}\) and by the fact that the symbolic notation to which Harrod and Keynes refer in the correspondence is different from that in the "Essay." And in Keynes's opinion, to which I subscribe, it is a "very muddling symbolism" (CW 14:340), "so contrived as to lose sight of the dimensions of your quantities, which makes it very difficult to handle" (339). "I have found it practically impossible to work things out for myself in terms of your symbolism" (339). Yet it is not difficult to identify the cause of their inability to communicate if one heeds Keynes's suggestion to pay due attention to the dimensions of the magnitudes.\(^{20}\) Neither \( C \) nor \( s \) is a homogeneous quantity, \( s \) being a pure number and \( C \) depending on the length of the period, \( \Delta t \). Keynes thus correctly points out that "it is dangerous to drop [the length of the period] out in the first instance, because it confuses the dimension to drop the time dimension out of sight by defining it as a unity. (This is a point about which a mathematician is very particular)" (CW 14:347–48).\(^{21}\) In what follows, I shall thus insert a reference to the length of the period. The reader should take notice that \( s \), being a number, is not associated with a unit of measure; \( C \) is measured in time units. Income, \( Y \), savings, \( S \), and investment, \( I \), are flows and are therefore to be measured in some money unit divided by a time unit (obviously the same time unit used for \( C \)). The rates of growth \( G, G_w \), and \( y \) are measured in (time unit)\(^{-1}\).

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19. See especially Harrod's letters to Keynes of 6 and 22 September 1938. The correspondence relating to the first draft of the "Essay" is collected in file E3/1/5 of Keynes's Papers.

20. Unfortunately, too often economists, whether or not they are mathematicians, are not cautious enough about the dimensional nature of their variables. Consider, for instance, Kregel's essay on this same Harrod-Keynes correspondence (Kregel 1980). Having ignored Keynes's suggestion, he did not notice that Harrod and Keynes unknowingly referred to different formulas (as will be explained here). Kregel thus somewhat obscured the accurateness of other parts of his insights by confusing, as Keynes and Harrod did, the problems related to instantaneous analysis and stability of equilibrium.

21. Strangely enough, only at the end of the exchange of letters did Keynes pay due attention to his own advice. The instability condition \( C > s \), which he identified from the first letter on, compares the magnitude of two nonhomogeneous quantities. Only in his last letter, when he was already dealing with the existence of equilibrium rather than its stability, did Keynes introduce \( \Delta t \) to his calculations, that is, "the interval elapsing between new sets of entrepreneurs' decisions" (CW 14:346).
The origin of the misunderstandings seems to me to reside in the fact that Harrod and Keynes attributed different degrees of importance to the flow of time in economic behavior. However, in the course of their discussion, this difference was concealed by their practice of omitting the time indices. I shall now reintroduce these indices, to show that their reasonings were based on different assumptions. This will provide the instruments for discussing Harrod's notion of instantaneous analysis, which is essential to understanding the "Essay" and the debate with Keynes.

Harrod repeatedly stressed that his analysis only concerned a single instant. He obviously did not feel the need to specify to which time his variables referred, since they all belonged to the same instant. Moreover, if $\Delta t$ was sufficiently small, the difference, for example, between $Y_t$ and $Y_{t-\Delta t}$ would be irrelevant. Harrod could thus explicitly declare that in defining the growth rate of income, it did not matter whether one referred to the income of the preceding or the current period:

$$ G = \frac{\Delta Y_t}{Y_{t-\Delta t}} \cong \frac{\Delta Y_t}{Y_t} $$

(Harrod 1939, 16 and 20).

Having established this premise, the axiom that "the rate of increase of a community's income is an important determinant of its supply of savings" can be formulated in two ways, either

$$ S_t = s \cdot Y_t $$  \hspace{1cm} (1a)

or

$$ S_t = s \cdot Y_{t-\Delta t} $$  \hspace{1cm} (1b)

Assumption (1a) presupposes that people save a fraction of their incomes from the preceding period. This is Robertson's hypothesis, which Harrod had previously employed in The Trade Cycle to account for some peculiarities of the business cycle's turning points (see Harrod 1936, 128–38). Using (1a), the calculation of the growth rate of $Y$ is enormously simplified; but at time $t$, consumption and saving do not add up to total income—a serious drawback. This happened in the case Harrod considered in his letter of 21 August (CW 14:328). The alternative formulation (1b), although introducing some complications in the calculation of the growth rate, is free of this difficulty; it therefore implies the strict equivalence of investment, regarded as the constituent of income other than consumption, and saving, defined as the amount by which income exceeds consumption.

The origin of the misunderstandings seems to lie in the fact that Harrod calculated using (1a), but argued with Keynes supposing (1b). Harrod obviously felt justified doing so because of his assumption that $\Delta t$ was sufficiently small to be able to disregard it. Ignoring the time indices, Harrod thus wrote (1), thought (1b), and calculated with (1a). Keynes consistently used only (1b), since he could not believe $\Delta t$ to be irrelevant.22

Before examining the consequences of this, let me calculate the growth rates corresponding to the two assumptions. The equations representing investment and the equality between saving and investment must be taken into account:

$$ I_t = C_{\Delta t} \cdot \frac{\Delta Y_t}{\Delta t} $$  \hspace{1cm} (2)

$$ I_t = S_t $$  \hspace{1cm} (6)

Combining (1a), (2), and (6) and (1b), (2), and (6), it is easy to obtain, respectively, the growth rate of the "Essay" and Keynes's growth rate:

$$ \hat{G}_{\Delta t} = s/C_{\Delta t} $$  \hspace{1cm} (7a)

$$ \hat{G}_{\Delta t} = \frac{s}{(C_{\Delta t} - s \cdot \Delta t)} $$  \hspace{1cm} (7b)

These two equations mean that, if at time $t - \Delta t$ the system was in equilibrium and the parameters $s$ and $C$ remained constant, then corresponding to alternative assumptions (1a) and (1b), the only increase of income in period $\Delta t$ that would not entail unexpected accumulation of stocks would be, respectively,

$$ \Delta Y_t = \hat{G}_{\Delta t} \cdot Y_{t-\Delta t} \cdot \Delta t $$

$$ \Delta Y_t = \hat{G}_{\Delta t} \cdot Y_{t-\Delta t} \cdot \Delta t $$

It is now possible to verify Harrod's premise that if $\Delta t$ were small the two solutions would not much differ:

$$ \lim_{\Delta t \to 0} \hat{G}_{\Delta t} = \lim_{\Delta t \to 0} \frac{s}{C_{\Delta t} - s \cdot \Delta t} = \frac{s}{C_{\Delta t}} = \hat{G}_{\Delta t} $$

22. Keynes had considered assumption (1b) already when proposing his fundamental equation. His formula (5) in fact presupposes that $\Delta K_t/\Delta K_{t-1} \neq \Delta K_t/K_t$; in his calculation he uses the first of these rates of growth (see section 4); the second rate would have led to $y = 1/(M \cdot R)$. 

Yet Keynes too was right in claiming that the terms on the right-hand side of (7b) must not be overlooked. If \( s \cdot \Delta t > C_{\Delta t} \), the denominator would be negative and \( \ddot{G} \) would not describe growth but a cumulative downfall, or even an alternation of positive and negative values of \( Y \) if the values of \( s, C, \) and \( \Delta t \) were such that \( \ddot{G} < -1 \). Harrod, who probably never bothered solving the second set of equations, could not, therefore, imagine that such a difficulty would emerge, nor understand Keynes's remark. As a matter of fact, equation (7a) is free of this problem, because both terms on its right side are positive. 23 Keynes's suggestion that in practice \( C_{\Delta t} > s \cdot \Delta t \) is not a sufficient condition also seems legitimate, since for the rate of growth to be feasible, \( C \) must be much larger than \( s \cdot \Delta t \) (about forty times larger, in order to have \( \ddot{G} \approx 2.5\% \) (CW 14:349).

Unfortunately, Keynes's and Harrod's attempts to settle this matter ultimately diverted their attention from the peculiarity of Harrod's method, which was the root of their problems. Their ensuing debate on questions of stability and the existence of an equilibrium relegated the problem of instantaneous analysis, which was the real core of dissent, to low priority.

In other words, it seems to me that Keynes was not able to force Harrod 23. Keynes repeated his criticism many times and the following seems to me one of his clearest formulations. From equations \( S = s \cdot Y, I = C \cdot \Delta Y, \) and \( I = S \), Keynes formulates the equilibrium equality as \( C \cdot \Delta Y_w = s \cdot Y \). 'Suppose . . . that the ex-ante growth of output exceeds the warranted growth in the assumed unit of time by an amount X, then (I say) you are assuming that

\[
C \cdot (\Delta Y_w + X) > s \cdot (Y + X),
\]

where \( C \) and \( s \) have the same values as before, that is, \( C > s \) (for that unit of time). The longer the unit of time, the less likely is this to be true" (CW 14:333). Here Keynes repeated Harrod's mistake, that is to say, he thought in terms of (1b) but used Harrod's result based on (1a). Again, the use of time indices would have avoided this error. Equation (i) should be restated as:

\[
(1/\Delta t) \cdot C_{\Delta t} \cdot (\Delta Y''_w + X_t) > s \cdot (Y_{t-\Delta t} + \Delta Y_t + X_t),
\]

(ii)

Now the role of the period length \( \Delta t \) is really visible. Harrod realized that in Keynes's formulation (i) the term \( \Delta Y_t \) is missing on the right-hand side, and thus replied: "In any warranted position \( C > s \), since

\[
C \cdot \Delta Y = s(Y + \Delta Y).
\]

(iii)

\( C \) here stands for the required increment of capital" (CW 14:335). It was now Harrod's turn to be on the right track: from (iii)—which presupposes assumption (1b)—he could easily have obtained equation (7b), but he soon got lost, reviving instead the result (7a). Rather than proclaiming that in equilibrium \( C > s \) is always satisfied, the correct procedure would have been to complete equation (iii) with the appropriate time indices

\[
(1/\Delta t) \cdot C_{\Delta t} \cdot \Delta Y_t = s(Y_{t-\Delta t} + \Delta Y_t),
\]

(iii)

and then to transform it into

\[
\Delta Y_t \cdot (C_{\Delta t} - s \cdot \Delta t) = s \cdot Y_{t-\Delta t} \cdot \Delta t.
\]

(iv)

The right-hand side of (iv) being positive, the two factors on the left must have the same sign, that is, if \( \Delta Y_t > 0 \) then \( C_{\Delta t} > s \cdot \Delta t \), and if \( \Delta Y_t < 0 \), then \( C_{\Delta t} < s \cdot \Delta t \). The debate rested on very confusing terms, and neither party could be convinced. In the following letter, Keynes saw the way out of the dilemma (but unfortunately it was too late): "I quarrel with the first sentence in your letter . . . where you write that in any warranted position \( C \cdot \Delta Y = s(Y + \Delta Y) \). This seems to me wrong in its dimensions. I suggest that the equation should run \( C \cdot \Delta Y/dJ = s(Y + \Delta Y) \), or if you like \( C \cdot \Delta Y = s \cdot \Delta J \cdot (Y + \Delta Y) \), where \( dJ/dJ \) is the rate at which income has increased. \( J \) stands for time" (CW 14:337-38).

24. Stated correctly, rather than \( y = 1/(M \cdot R - 1) \), equation (5) should read

\[
y_{\Delta t} = 1/(M \cdot R - \Delta t).
\]

(5a)

Let me remind the reader of Keynes's claim that if the Relation is constant, the growth rate of capital is equal to the growth rate of income (CW 14:171; see above, section 3). Supposing then that \( R \) is constant and substituting in (5a), \( y \) by \( \ddot{G}, M \) by \( 1/\Delta t \) and \( R \) by \( C \), we obtain equation (7b), which is identical to the revised version of the fundamental equation suggested by Keynes in April 1937.
and mutual relations of the factors concerned are self-consistent and consistent with normal economic motives" (1934, 478).

In his 1938 methodological essay (written after the publication of The Trade Cycle, while he prepared the “Essay”), Harrod restates this rule by analogy with static analysis, in which he distinguishes two stages. The first consists of tracing a map of relationships among relevant magnitudes, that is, in drawing a “simultaneous chart or survey of the economic field” (Harrod 1938, 387; emphasis added); the next step entails the analysis of the “general laws concerning the succession of events” (Harrod 1938, 386; emphasis added). In his 1936 book and the 1939 article, Harrod was consistent with this method, although the two approaches should be differentiated.25

In The Trade Cycle “simultaneity” was intended as a logical relation, while in the “Essay,” Harrod literally confined his analysis to a single instant. In his 1936 book, in fact, Harrod was concerned with processes whose causes (the determinants) had to be distinguished from their effects. He examined causes and consequences not only in their logical linking but also in their time structure: entrepreneurs make their investment decisions with the prospective, future uses of capital goods in mind (see, for instance, Harrod 1936, 88, 160). The Trade Cycle described a world where past and future are not symmetric, where expectations are, therefore, all important. In 1939, however, “the analysis relates to a single point in time” (Harrod 1939, 24), and if the temporal horizon is reduced to a single instant, there can be no room for the distinction between past and future. This reinterpretation of “simultaneity” reflected both the change of role of the acceleration principle and the shift of emphasis from the causes to the effect of investment.

Furthermore, there were other reasons favoring the adoption of a new view of simultaneity, because the formal treatment of the “Essay” imposed some additional analytical constraints. In particular, to determine the warranted growth rate and to explore the consequences of any deviation from the equilibrium state, it was necessary that the coefficients C and s remained constant. Given his decision to define dynamics as the formulation and resolution of a fundamental equation whose unknown variable was a rate of growth, Harrod was then forced to choose between two alternatives only: either to regard s and C as constants, or to restrict his analysis to a domain where they could be treated as if they were constant.26 Harrod was conscious of this requirement, but he refused to assume that multiplier and accelerator remain unchanged during the cycle. How to account then for their variations? Here it becomes evident why and how Harrod applied the methodological distinction worked out in his 1938 presidential address. The cartographic stage, which really defined economic dynamics, consisted in the determination of the equilibrium growth rate in one single instant and the study of its stability in that neighborhood. Analysis of the cycle followed: only by considering the sequence of events far from equilibrium was it possible to account for the changes in s and C, the parameters that defined equilibrium itself.27

Thus runs the rationale for Harrod’s decision to confine his analysis to a period Δt → 0. Strictly speaking, only an interval of infinitesimal length assures the constancy of the coefficients. On the other hand, one has to ask oneself what is the meaning of Δt, and in the end Harrod agreed with Keynes that “the relevant period is as you say that within which orders are readjusted” (in CW 14:338). In Keynes’s view, time truly flowed, determining a symmetry break between the past, given and known, and the future, uncertain and indeterminate. Keynes would not even dream of considering a vanishing interval: Δt cannot tend to zero, but is of finite and not negligible length. During this interval something relevant might happen to s or C. For instance, if Δt were long enough, any positive G would entail perceptible increases of the income level that might, in turn, cause an increase in the propensity to save; it might thus happen that a casual deviation from equilibrium (say G > Gw) generates, through the combined effect of ΔY and Δs, an increment of total savings capable of financing the extra investment. In that case, Harrod’s principle of instability would not hold: the new rate of growth might be as “warranted” as the old, and the corresponding equilibrium would be neutral or even stable.

25. In the first (1938) draft of the “Essay” (now in Harrod’s Papers, file IV-113), on two occasions Harrod marked out the discontinuity between the two parts of his treatment, the first—“the strictest part of dynamic theory”—concerned with instantaneous analysis and the second—trade cycle analysis—concerned with the succession of phenomena: “the strictest part of the theory ceases here.” Both remarks were later deleted in the course of revisions to the text.

26. Nowadays, a third alternative is available that is actually considered by recent writers on business cycles and dynamics, but then it was far beyond Harrod’s and his contemporaries’ mathematical capacities. One can consider s and C as functions of Y or its differentials and formulate a nonlinear functional equation.

27. Kregel (1980) points out that most interpreters of Harrod’s theory have ignored the distinction between these two analytical domains. This has given rise to the misinterpretation of Harrod’s theory as the long-run counterpart of Keynes’s.
The discussion of this case was long and painful, for although Harrod agreed that the scenario was possible, he doubted its relevance: “If you think the point sufficiently important in practice or theory I will insert a footnote” (in CW 14:338). At first, Keynes insisted that this was not a footnote matter, but Harrod finally proved that even an unrealistically high marginal propensity to save could not result in any but a minimum effect on the average propensity. Harrod also managed to persuade Keynes that in his model only the average $s$ was important, since it was the total saving that must equal aggregate investment.

Harrod, however, was not convinced that this proof was essential to his argument, supplying it only to convince (or to please) Keynes. In the course of the entire correspondence, Harrod contended that it was necessary to keep distinct the three problems of equilibrium, its stability, and the cycle and that the proper procedure was to analyze the situation instant after instant. Before showing that his “proposition regarding instability, rigidly proved for $s$ constant, also holds with $s$ variable save in unlikely circumstances.” Harrod confirmed “the formal point that it is not to be supposed that $s$ changes in response to changes in the rate of growth, but only to changes in the level of income. A changed rate of growth will have to endure some time before an appreciable consequential change in the level of income occurs” (in CW 14:342).

28. Even though the length of the period is the matter in dispute, and notwithstanding Keynes’s explicit insistence (CW 14:337–38, 346–47), in his proof Harrod consistently neglected the time subscripts in his variables. Harrod argued by *reductio ad absurdum*, considering first a positive $\Delta t$ and its consequences on variations of $s$, concluding that with regard to the determination of the total amount of savings, the marginal propensity to save was a trifling matter with respect to the average propensity. Anyway, he did not explain how long $\Delta t$ was not what determined its duration—although in the “Essay,” when taken to task by Keynes, he considered it to be about six months (Harrod 1939, 28). Harrod probably thought he was not concerned with that problem, since equation (7a) $\bar{G}_{\Delta t} = s/C_{\Delta t}$ is valid independently of $\Delta t$ (a different choice of $\Delta t$ would involve changes only in $C_{\Delta t}$ and $\bar{G}_{\Delta t}$, but not in their product; see Harrod to Keynes, in CW 14:332; Harrod 1939, 17); he could not, therefore, imagine any obstacle in reducing it to an infinitesimal. In Keynes result (7b), on the contrary, $\Delta t$ explicitly determined $\bar{G}$, and he therefore had to ask himself on what it depended: “I am not quite clear what determines the relevant value of $[\Delta t]$ in practice. But it seems to me to depend on the period of production” (CW 14:347).

29. If Harrod’s proof is considered separately from the lewers showing he was reluctantly modifying his text as a concession to the editor of the *Economic Journal*, it might indeed appear that Harrod was not consistent. G. L. S. Shackle’s interpretation (written before these letters were published) is that it is “when he seeks to analyze the nature of consequences of a departure from warranted growth that Harrod finds the absence of an explicit apparatus of time-lags most inconvenient” (1967, 261).

30. Not only a changed rate of growth but any non-zero rate of growth (the warranted one included) adjusts the absolute amount of income and thus the propensity to save. The dynamic equilibrium itself generates disequilibrium.

8. Ex Ante and Ex Post: More about the Accelerator

Harrod did not assume that in the long run $C$ was constant; on the contrary, his explanation of the cycle presupposed changes in $C$ as well as $s$. Discussing stability, Harrod referred again to the methodological argument of instantaneous analysis: “$C$ may also be expected to vary with the size of income, e.g., owing to the occurrence of surplus capital capacity from time to time, but the same argument for regarding it as independent of the rate of growth at a particular point of time applies” (1939, 25). From this point of view, the debate on the nature and variability of $C$ did not differ much from the discussion of $s$, here just examined. Nonetheless, it reveals new insights into Harrod’s notion of investment, as compared to Keynes’s emphasis on uncertainty and expectations in the analysis of investment decisions.

In section 5, I argued that Harrod’s definition of $C$ was *negative*. In fact, $C$ was thought to be the coefficient describing the level of investment “which entrepreneurs regard as ideally suited to the output which they are undertaking in that period” (Harrod 1939, 19), where “ideally suited” did not refer to the entrepreneurs’ desires but only to the absence of ex post unplanned variations of stocks. $C = C_p$ was therefore not considered for what it was, but for what it was not. It was only that particular ratio $I/\Delta Y$ accumulating no residuals in the form of stocks. $C$ was not defined on its own but only as a particular value of $C_p$; it was nothing else than a watershed between two domains where instability prevailed.

Keynes, with his “animal spirits” in mind, treated investment as the result of decisions in conditions of uncertainty. All his comments on $C$ reflect this viewpoint and remind one of the discussion of $R$ that occurred...
Harrod took no notice of the thrust of Keynes's point and casually replied: "I have inserted a paragraph about confidence" (in CW 14:337); in the published version of the essay we thus read that C is "the amount of capital per unit increment of output required by technological and other conditions (including the state of confidence, the rate of interest, etc.)" (1939, 18). It is scarcely surprising that Harrod did not accept Keynes's suggestion. According to him, C was not a coefficient accounting for the factors influencing investment decisions, but rather an a posteriori summing-up of the conduct the entrepreneurs ought to have followed to be "satisfied" with their action.

Moreover, in the "Essay" decisions were never discussed, since Harrod only considered the consequences in his analysis. Only two parameters referred to investment and neither was concerned with economic actions, C_p representing the actual increment of capital per unit of output, and C, a particular case of C_p.

To distinguish between them, Harrod used the term ex ante for C and ex post for C_p. The potential deceptiveness of this choice did not escape Keynes's attention. His remark promptly summarized the peculiarity of Harrod's new terminology: "C is most certainly not ex ante strictly speaking. Ex ante C is the investment entrepreneurs actually plan to make. Your C is the addition they ought to plan to make. C is the planned investment which would equate ex ante and ex post investment" (CW 14:322). Harrod's reply again dismissed Keynes's point, since in the final version he merely added a note specifying that his notion was different from the one now prevailing: "with regard to ex-ante I do not think it matters using this in a somewhat different sense of Lindahl, since the definition is given" (in CW 14:337). While Keynes pointed out that Harrod's notion of investment did not account for the actual plans, Harrod did not feel bound to modify it. The entrepreneurs' plans and desires did not pertain to his dynamics, which only concerned equilibrium. When he wrote that C is the increment of capital per unit of income "which is desired" (1939, 22), Harrod did not mean a level that is desired by capitalists (who would prefer a lower ratio that would more than justify their investments) but only the marginal increment that it is necessary for growth to be in equilibrium.32

Keynes raised a point that has been overlooked by Harrod's interpreters, who have transformed the "Essay" into a theory of economic growth. C must not be interpreted as the capital-output ratio; it is rather "the marginal rate of investment corresponding to additional income, which is significant" (CW 14:342): "I stress that it is ΔK/ΔY which is relevant, not K/Y" (346).

If growth represents a growth of population and not of standard of life, there is, indeed, no reason for much difference between K/Y and ΔK/ΔY. . . . But let us suppose a stationary population in an old country, the rate of growth relating solely to the standard of life; or, even worse, a declining population. Let us suppose that the assumed

31. Further on in his fundamental equation, while introducing long-range capital outlays—increments of capital not directly dependent on increases of income—Harrod referred to the elements suggested by Keynes: "There are doubtless numerous factors, including the state of confidence and the rate of interest, affecting the volume of such outlay" (1939, 26–27). Whereas Keynes's remark was applicable to the short run, the distinction Harrod introduced was only meant to reduce the impact of the acceleration principle, which, untempered, led to distortions due to long-run plans.

32. This point was also raised by Asimakopulos (1985, 623), who compared Harrod's 1939 "Essay" with subsequent versions of his dynamics.
rate of interest has been in force for a long time, so that all the capital which can be advantageously employed at the rate of interest with the existing level of income is already employed. Let us suppose that the transport system and public utilities and housing are all on a satisfactory standard, and no foreign investment. Let us suppose disarmament, sinking funds rampant, distribution of profits conservative and belated, incomes unequally divided, fairly good employment so that there is large dissaving on this head. (CW 14:348)

Keynes argued that if such a state of affairs prevailed, one would have to expect that marginal investment would be much lower than the capital-output ratio, and that correspondingly there might arise some further difficulties in proving the instability principle (see CW 14:347–49).

There is a further consideration that might accentuate the difference between marginal and average values of C, that is the fact “that there are large fluctuations in unused capacity (or in capacity employed below its maximum) or in inventories in both” (334–35). If there is all-around surplus capacity, production might be increased without employing additional capital equipment. “Thus you are assuming that there is no surplus capacity” (CW 14:335; see also 349), or that, in effect, the degree of utilization of capital remains constant. Challenged again, Harrod did not change his mind. In his view, “the quantity of capital required or proportion of income saved are data, analogous to the demand schedules of static theory. (Data, the values of which, like those of demand schedules, change from time to time)” (in CW 14:337).

9. Concluding Remarks

In spite of the fame of his “Essay in Dynamic Theory,” Harrod failed to convert his fellow economists to his dynamic viewpoint. By “Economic Dynamics” Harrod meant a theory of the determination of the instantaneous rate of growth of an economic system, for which “a new method of approach—indeed, a mental revolution—is needed” (Harrod 1939, 15; see also Harrod 1938, 402–5). Both the features Harrod advocated for dynamics have been neglected or misinterpreted by scholars in the field, who almost unanimously accepted Ragnar Frisch’s conception (1936).

This is not the appropriate place for comparing premises and developments of these two conflicting views of economic movement. Rather, what I would like to point out is that the Harrod-Keynes correspondence reveals seeds of some of the difficulties that later prevented the revolution in thought advocated by Harrod.

The first aspect that initially escaped the attention of Keynes and his contemporaries is the fact that the dynamic analysis concerned the determination of the equilibrium rate of growth. This had not been noticed at all when The Trade Cycle and the “Essay” were published; only after World War II, when Harrod restated his fundamental equation in a book was it paid due attention.33 Hicks’s remark represents the situation well: “I could kick myself for not having seen it before. After all, the essential ideas which I am taking from Mr. Harrod are not new ideas, put forward by him the first time in 1948; if one had eyes to see, one could have seen them nearly a decade ago. It is, however, quite clear that neither I myself, nor (as far as I know) anyone else, seems to have seen them” (Hicks 1950, 7). In the second place, students of economic dynamics did not receive the method of instantaneous analysis favorably: according to Ragnar Frisch’s (and the generally accepted) view, a system is dynamic only if at least one of its equations refers to different points in time (Frisch 1936).34

Harrod’s idea of equilibrium growth met with a strange fate. In The Trade Cycle, it was meant to express the possibility of economic growth without postulating the existence of exogenous forces causing it. For traditional theory this was an outrageous proposition: if motion becomes theoretically conceivable on its own, so does crisis, a behavior of the system that the adherents of static equilibrium theory wanted to deny, unless the deviation from stability was determined by causes alien to the normal functioning of economic activity. In the “Essay,” the fundamental equation represented a particular state of the system characterized by no undesired accumulation of stocks, variations in the degree of utilization of machinery, or other incentives to adjust the level of activity.35 Such a state was not given once and for all but referred to a single point in time; the growth rate describing it could (and indeed, does) fluctuate in the course of the cycle.

33. The title itself of this book, Towards a Dynamic Economics, signals both the necessity of a new approach to the subject and the general and provisional nature of Harrod’s theory.
34. This definition, if broadly interpreted, comprehends not only difference and integral equations but also differential equations. Like Harrod, Hicks followed a path alternate to that traced by Frisch, but his definition differs as well from that proposed by Harrod. According to him, in economic dynamics all variables have to be dated (Hicks 1950, 10).
Modern readers of Harrod, ignoring this specificity and interpreting his result in light of Frisch’s notion of dynamics, have understood the fundamental equation as a definition of a growth path. Some of Harrod’s own expressions have indeed contributed to misguide scholars away from the methodological aspect of his considerations on instantaneous analysis. Yet, taken in context, such expressions as “line of output,” “line of advance,” “path of growth,” and “line of growth” should help clarify that warranted growth is a moving equilibrium and must be distinguished from static equilibrium (Harrod 1939, 22–23). Further on, Harrod fully specified that “there is no unique warranted rate; the value of warranted rate depends upon the phase of the trade cycle and the level of activity” (1939, 30). Interpretations of Harrod’s theory as dynamics of trajectories (as in the manner of Frisch’s dynamics) neglect to distinguish between the two stages of analysis to which Harrod dedicated his methodological essay in 1938, that is, the cartographic study of the system instant after instant and the subsequent integration of these points in time as a sequence of events.

Harrod proposed to keep these stages distinct in order to found dynamics on the same methodological principles as statics. Transposing this procedure to dynamics, Harrod treated the coefficients s and C as the data from which the fundamental proposition about the growth of the economic system is derived; only afterwards, in studying the cycle, were changes in the data considered.

Determining the rate of growth and its stability, Harrod thus transposed to dynamics the ceteris paribus clause, in reference to his fundamental conditions s and C. Harrod was certainly conscious that, strictly speaking, they were not parameters but endogenous variables; but since his unknown term (the rate of growth) monopolized his only equation, Harrod had to treat these variables as data to be able to solve the equation. His only way out, given his limited mathematical toolbox, was to confine the domain of his analysis to a period of time so short that nothing had time to change. In other words, Harrod knew that neither G, s, C, nor s were independent variables, but he was forced to ignore the fact. This seems to be the gist of Keynes’s protest that it is not possible to “assume absolute rigidity of s and C and a departure from warranted growth. You have to make some assumptions as to the changes in s and C in unwarranted conditions” (CW 14:334).

Keynes thus noticed the weakness of the analytical structure of the “Essay,” but, instead of locating its cause in the peculiarity of Harrod’s method, he thought Harrod was implicitly assuming constancy for his coefficients. But Harrod explicitly refuted this hypothesis, and while he desperately tried to prove to Keynes that his analysis, in the domain he chose for it, had no need or desire to adopt it, the sense of Keynes’s criticism escaped his attention. During the entire correspondence, Harrod firmly believed it was not his concern.36

Harrod was aware that Keynes’s criticism brought to the fore some sort of difficulty, but he thought he could respond in the second part of his analysis, dedicated to the study of long-run changes. Variations in the coefficient were not an analytical problem for The Trade Cycle but only for the later developments of Harrod’s theory. The quest for an equilibrium state in the 1936 book aimed to show how the combined effect of the two sets of independent decisions regarding saving and investment could justify the expectations on which these decisions depended. Harrod was, therefore, merely interested in showing the existence of some range of dynamic forces capable of causing such a result in the course of the period of finite length during which the multiplier and the accelerator unfolded their effects.37

36. In the course of the correspondence of 1935 on The General Theory, Harrod similarly reacted to Keynes’s dislike for the assumption of independence between economic magnitudes. In fact he interpreted Keynes’s criticism of the traditional theory of interest as regarding the validity of the ceteris paribus assumption rather than the logical and methodological issues. On that occasion as well, Harrod tried to define a domain in which the classical assumptions were valid and Keynes’s criticism would not apply; there, according to Harrod, the Marshallian toolbox provided the appropriate and logically consistent analytical instrument. Harrod also applied the same strategy in responding to Sraffa’s 1926 criticism to the nonindependence between supply and demand curves and to the logic underlying the construction of the supply curve. There as well Harrod interpreted Sraffa’s attack to be regarding the validity of the assumption of increasing costs rather than regarding Marshall’s analytical method. He then neutralized it by inventing the concept of marginal revenue (as later renamed), which enabled him to consider neoclassical theory as a special case of a continuum whose extremes are Cournot’s monopoly and Marshall’s free competition. I discuss Harrod’s attitude toward traditional theory and toward Keynes’s and Sraffa’s criticism of it in Besomi 1992.

37. In The Trade Cycle, Harrod often revealed his dislike for time-lag theories of the cycle, which he considered to be “premature when the fundamental propositions relating to velocity and acceleration remain unformulated” (Harrod 1936, viii): “all reference to time-intervals in this topic are highly dangerous; it is so easy to give plausible explanations on the basis of a time-lag hypothesis, the hypotheses that may be introduced are so many and various that with their aid the facts can be made to fit any theory” (viii). On the other hand, Harrod recognized that “the fact that net investment is undertaken with a view to facilitating production in the future is clearly a central one; and the interval that elapses between placing an order for, or beginning to undertake the construction of, capital goods and their use in the productive process can hardly be neglected. Who places such an order gives a hostage to fortune; his judgment can be vindicated after the interval has elapsed” (88).
The main result of the “Essay,” the instability proposition, was obtained by comparing the situation of two adjacent instants. If at any point in time entrepreneurs ascertained undesired changes in the utilization of machinery or in the volume of their stocks, they would slow down the pace of investment, which in turn entailed further accumulation of stocks (or conversely, if stocks at first were running down). Yet in Harrod’s paper there was no clue suggesting how the two instantaneous states were related, nor any precise suggestion regarding changes in the coefficients $s$ and $C$ due to the variations in the level of activity. In Harrod’s theoretical setup, it is therefore impossible to integrate the entire course of events in a unique picture; one can only recalculate everything instant after instant, or at best try to identify some domains in which changes are qualitatively homogeneous. The instability principle as stated by Harrod does not permit rigorous proof, but must be intuitively grasped.

It thus seems to me that the misfortune met by Harrod’s dynamic conception mainly depended on the fact that the interrelations and retroactions between Harrod’s chosen variables are much more entangled than his equation can possibly represent (and Harrod himself seemed from time to time to have been aware of that), even after expectations disappeared from the scene. This seems to explain why both Keynes and the “econometricians” could praise Harrod for some of his ideas, but at the same time be dissatisfied by his procedure as a whole.

The econometricians, who intended to formulate complete sets of functional equations capable of representing the time path of the unknown magnitudes were, for many years after the publication of the “Essay,” incapable of dealing with the intrinsic nonlinearity resulting from the feedback between variables. Only since advances in mathematical techniques in nonlinear analysis became available in economics (as from Goodwin’s 1951 paper) were Harrod’s ideas integrated within the notion of dynamics developed by Frisch; and the principle of instability was formalized in the metaphor “knife-edge.” This approach definitely dismissed Harrod’s dynamic conception, since in the new context the rate of growth of the variables was a by-product of the solution of the functional equation: it was deprived of any analytical value and was reduced to a component of a trajectory that Harrod would not and could not calculate.

On the other hand, Keynes became interested in the problem, deeply rooted in the classical and Marxian tradition and revived by Harrod after many decades of neglect, of whether and under which conditions there exists some form of self-sustaining economic growth. The economy that was the object of Keynes’s concern was one in which making decisions and recording their results took time: in his view, new decisions actually reflected the newly formed expectations and the disappointment of the old ones. Hence his predilection for the assumption (1b) above on the determination of savings and his concern for the question of the length of the relevant period $\Delta t$.

While in the successive versions of his dynamic theory Harrod remained consistent with the approach outlined in the “Essay,” in his exchanges with Harrod, Keynes preferred to reapproach the problem (and the approach) of The Trade Cycle. Faced with the contrasting notions of a dynamics outside time—because time, when reduced to an instant, disappears—and the dynamics of the econometricians describing a “changeless change . . . in timeless time” (Koyré 1965, 11), Keynes could not choose. His discussion with Harrod shows that he was inclined toward a dynamics “full of the natural give and take of a world where people must find out, compare, decide, before they act; then register results and make fresh plans and decisions.”

What Keynes held against Harrod in the course of their debate, and what Harrod did not understand, is that in dynamics time must be ordinary time, full of hopes and disenchantment, enabling one to recognize the importance of uncertainty regarding the future course of events, as distinct from the certitudes of the past.

38. See, for example, Medio 1979, section 1.3. This criticism was already raised by Marschak in a note entitled “Remarks on R. F. H.’s “Essay in Dynamic Theory,”” drafted on the first version of Harrod’s article (in the Harrod Papers, file V-113a). It is not possible to guess why Harrod did not take into account this comment (he certainly saw Marschak’s remarks before revising his text, as it emerges from some of the corrections that were introduced at Marschak’s suggestion), since Harrod does not seem to have taken up the matter in his reply to Marschak (some passages from two letters to Marschak are transcribed in Young and Lee 1993, 103–7).

39. It is well known that the possibility of integrating functional equations is restricted to a very limited and unrepresentative set of them, namely linear equations and a few others; in general, solutions are indeed calculated point by point with the use of computers. This remark is therefore not meant to criticize Harrod’s principle in itself, but only his claim that his result is “rigorously proved.”

40. These words were originally used by Shackle (1967, 270) to characterize The Trade Cycle, but they seem more appropriate to describe Keynes’s conception. For Keynes’s remarks about the limits of Harrod’s notion of “expectation,” see note 15.
References


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