Dynamic Models for Monetary Transmission

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Abstract

Monetary policies, either actual or perceived, cause changes in monetary interest rates. These changes impact the economy through financial institutions, which react to changes in the monetary rates with changes in their administered rates, on both deposits and lendings.

The dynamics of administered bank interest rates in response to changes in money market rates is essential to examine the impact of monetary policies on the economy. Chong et al. (2006) proposed an error correction model to study such impact, using data previous to the recent financial crisis. Parisi et al. (2015) analyzed the Chong error correction model, extended it and proposed an alternative, simpler to interpret, one-equation model: furthermore, they applied it to the recent time period, characterized by very low monetary rates.

In this paper we extend the Parisi et al. static model in a dynamic sense, by comparing it with more sophisticated equations such as dynamic linear models and stochastic processes.

The main contribution of this work consists in novel, more parsimonious and endogenous models: more precisely, dynamic linear models have been developed in order to capture the changing relationship between monetary rates and bank rates, while stochastic processes have been studied to provide endogenously determined and generalizable models. Secondly, this paper introduces a predictive performance assessment methodology, which allows to compare all the proposed mod-
1 Introduction

Monetary policies, such as variations in the official rate or liquidity injections, cause changes in monetary interest rates. These changes impact the economy mainly in an indirect way, through financial institutions, which react to changes in the monetary rates with changes in their administered rates, on both deposits and lendings.

The dynamics of administered bank interest rates in response to changes in money market rates is essential to examine the impact of monetary policies on the economy. This dynamics has been the subject of an extensive literature; the available studies differ, depending on the used models, the period under analysis and the geographical reference.

The relationship between market rates and administered rates is a complicated one and the evidence presented, thus far, is mixed and inconclusive. Hannan and Berger (1991), for example, examine the deposit rate setting behaviour of commercial banks in the United States and find that (a) banks in more concentrated markets exhibit greater rates rigidity; (b) larger banks exhibit less rates rigidity; and (c) deposit rates are more rigid upwards than downwards. Scholnick (1996), similarly, finds that deposit rates are more rigid when they are below their equilibrium level than when they are above; his finding on lending rate adjustment, however, is mixed. Heffernan (1997) examines how the lending and deposit rates of four banks and three building societies respond to changes in the base rate set by the Bank of England and finds that (a) there is very little evidence on the asymmetric nature of adjustments in both the deposit and lending rates, (b) there is no systematic difference in the administered rate pricing dynamics of banks and building societies, and (c) the adjustment speed for deposit rates is, on average, roughly the same as that for loan rates.

More recent papers on the same issue include: Ballester et al. (2009), Chong et al. (2006), Demirguc-Kunt and Huizinga (1999), Flannery et al. (1984), Maudos et al. (2004), Maudos et al. (2009).

The empirical evidence contained in all the previous papers can be sum-
marized in the following points: (a) bank rates react with a partial and delayed change to changes in the monetary rates; (b) the speed and the degree to which they follow these changes present substantial differences between the various categories of banking products and between different countries.

The previous conclusions have been obtained for a relatively stable time period, previous to the emergence of the recent financial crisis.

After 2008, however, they have witnessed substantial changes. From a macroeconomic viewpoint, monetary interest rates are now, in most developed economies, close to zero, or negative; from a microeconomic viewpoint, bank management has changed substantially, for the compression of interest margins and for the increase in regulatory capital requirements. The effects of the previous changes on the transmission of monetary policies have not been yet fully investigated. In particular, the current state of close-to-zero interest rates is of particular relevance, and, to our knowledge, Parisi et al. (2015) is the only paper that has concentrated on this topic. For this reason, among all the other cited works, Parisi et al. (2015), who extends Chong et al. (2006) error correction model, has become our reference paper, since (a) it analyzes the error correction model, which is the most commonly used in both the academic and the professional practice, and (b) it concentrates on the recent, post-crisis years.

When monetary rates are close to zero, the error correction model, albeit formally elegant, does not well capture the dynamic of administered rates, which appears strongly inertial.

The first aim of this paper is to broaden the model of Parisi et al. (2015) in a predictive performance comparison framework, by introducing a dynamic evolution through more sophisticated models able (a) to capture the changing relationship between bank rates and monetary rates and (b) to provide better results in terms of predictions.

This work first shows that the Parisi et al. model performs quite well in a predictive sense, but it also concludes that dynamic models are not inferior in terms of predictive performance, and, therefore, represent a valid alternative.

Secondly, the present work compares time-homogeneous linear models with dynamic linear models, able to capture the changing relationship between the level of interest rates and monetary rates, which is particularly significant in a context where and when the latter are very close to zero. This study confirms that the last years have been characterized by a regime-switching behavior and, therefore, can not be modeled with static equations.

Finally, this paper introduces stochastic processes as the continuous-time
version of time-homogeneous linear models: the main advantage of the first ones consists in the fact that they are endogenous, and, thus, that can be derived, estimated and predicted independently from any other variable. As underlined before, this feature can be particularly useful in the current situation of almost-zero monetary rates.

All the proposed methods are applied to data from the recent period (1999-2014), of a southern European country with a traditional banking sector: Italy.

The paper is structured as follows. Section 2 describes the proposed models and, in particular: Section 2.1 describes the Parisi et al. model; Section 2.2 motivates and introduces the new proposed models, with Subsection 2.2.1 concentrating on simple linear models and dynamic linear models, Subsection 2.2.2 describing stochastic processes and Subsection 2.2.3 comparing all the previous models. Section 2.3 provides the predictive performance environment used to compare the models. Section 3 shows the empirical evidence obtained from the application of the models and, in particular: Section 3.1 describes the available data; Section 3.2 presents the estimation results obtained when the models are applied to such data; Section 3.3 compares the models in a predictive sense. Finally, Section 4 concludes with some final remarks.

2 Methodology

2.1 Theoretical Framework

In line with the contribution of Chong et al. (2006), the relationship between monetary rates and administered bank rates can be analyzed with the use of the Error Correction Model (ECM), following the procedure proposed by Engle and Granger (1987). The model is based on two equations. A long-run relationship provides a measure of how a change in the monetary rate is reflected in the bank rate. A short-run equation, which includes an error correction term, analyzes variations of the administered interest rates as a function of variations in the monetary rates.

Parisi et al. (2015) analyzed and extended Chong et al. (2006), by computing their two equations separately and by proposing an alternative one-equation model. More precisely, they assumed that bank interest rates depend on their previous level when monetary rates are close to zero, in
order to allow for a slow and partial reaction of bank rates to monetary rates changes. Thus, they modeled bank administered interest rates as a function of monetary rates, their variations and the previous level of bank rates. Their complete model can be formalized as follows:

\[ BR_t = k + \beta \cdot MR_{t-1} + \gamma \cdot \Delta MR_t + \delta \cdot BR_{t-1} + \epsilon_t. \] (2.1)

In equation (2.1) \( BR_t \) and \( MR_t \) represent, respectively, the bank administered rates and the monetary rates at time \( t \); \( \beta \) is a regression coefficient that gives a measure of the extent of the monetary rate transmitted on bank rates in a long-term perspective; \( \gamma \) is the coefficient that explains the influence of the variations of monetary rates on the bank rates levels; \( \delta \) weights the auto-regressive component \( BR_{t-1} \); \( k \) is a constant that synthetizes all other factors that, in addition to the dynamics described by the regressors, may affect the transmission mechanism of the monetary policy on bank rates as, for example, the market power and the efficiency of a bank; finally, \( \epsilon_t \) is the error term.

The previous linear model can be equivalently written in terms of the variations of the administered rates:

\[ \Delta BR_t = k + \beta \cdot MR_{t-1} + \gamma \cdot \Delta MR_t + (\delta - 1) \cdot BR_{t-1} + \epsilon_t. \] (2.2)

The previous formulation is necessary in order to make all models comparable.

2.2 The proposed models

2.2.1 Linear Models and Dynamic Linear Models

The Parisi et al. (2015) model can be written in terms of either the level of bank interest rates (2.1) or their variations (2.2); for this reason the first objective of the analysis consists in understanding how they both depend on the levels or on the variations of monetary rates. The two equations can be formalized as two simple regression models, as follows:

\[ BR_t = k + \beta \cdot MR_t + \epsilon_t; \] (2.3)

\[ \Delta BR_t = k + \beta \cdot \Delta MR_t + \epsilon_t. \] (2.4)

5
While model (2.3) explains the levels of bank rates in terms of the level of monetary ones, equation (2.4) is a model for the variations of bank rates in terms of the variations of monetary rates. These models, albeit very simple, should be considered in practical applications, because (a) they give insights on the relationship between the two variables considered in this paper; (b) it is interesting to understand which one of the two equations is more significant during the recent time-period and (c) it is of interest to see how the significance of the two models changes over time.

We have anticipated that the relationship between bank rates and monetary rates has radically changed during the last years, reaching a situation of almost-zero monetary rates. In order to better analyze how bank rates react to changes in monetary rates, and how this reaction changes over time, the previous, simple models can be enriched with a dynamic structure.

More formally, dynamic linear models are a particular class of state-space models, in which the regression coefficients are allowed to vary over time. For this reason model (2.3) can be compared with the following:

\[
\begin{align*}
BR_t &= k_t + \beta_t MR_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2) \\
k_t &= k_{t-1} + \omega_{k,t}, \quad \omega_{k,t} \sim N(0, \sigma^2_{\omega,k}) \\
\beta_t &= \beta_{t-1} + \omega_{\beta,t}, \quad \omega_{\beta,t} \sim N(0, \sigma^2_{\omega,\beta})
\end{align*}
\] (2.5)

where the conditions on the quantities \((\epsilon_t, \omega_{k,t}, \omega_{\beta,t})\), for \(t = 1, ..., T\), have been fixed in order to make them independent and identically distributed.

Similarly, equation (2.4) can be compared with the dynamic linear model expressed in terms of variations of interest rates:

\[
\begin{align*}
\Delta BR_t &= k_t + \beta_t \cdot \Delta MR_t + u_t, \quad u_t \sim N(0, \sigma^2_u) \\
k_t &= k_{t-1} + \omega_{k,t}, \quad \omega_{k,t} \sim N(0, \sigma^2_{\omega,k}) \\
\beta_t &= \beta_{t-1} + \omega_{\beta,t}, \quad \omega_{\beta,t} \sim N(0, \sigma^2_{\omega,\beta})
\end{align*}
\] (2.6)

The dynamic linear models can be rewritten in a compact form, by using the following substitutions:

\[
K_t = \begin{bmatrix} k_t \\ \beta_t \end{bmatrix}, \quad F_t = \begin{bmatrix} 1 \\ MR_t \end{bmatrix}, \quad G_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_t = \begin{bmatrix} \omega_{k,t} \\ \omega_{\beta,t} \end{bmatrix},
\]

where the second member of the matrix \(F_t\) has to be substituted with \(\Delta MR_t\) for equation (2.6).

The dynamic model (2.5) can thus be re-written in the following way:
\[
\begin{cases}
BR_t = F^T_t K_t + \epsilon_t, \\
K_t = G_t K_{t-1} + W_t.
\end{cases}
\]

(2.7)

Similarly for model (2.6):

\[
\begin{cases}
\Delta BR_t = F^T_t K_t + \epsilon_t, \\
K_t = G_t K_{t-1} + W_t,
\end{cases}
\]

(2.8)

with \(F_t = \begin{bmatrix} 1 \\ \Delta MR_t \end{bmatrix}\).

We remark that models (2.3) and (2.4) are too simple to lead to a good predictive performance. However, the idea of extending the model proposed by Parisi et al. with a dynamic structure is tempting and, to do so, they are useful in order to be compared with dynamic models and, therefore, to study how the relationship between either the levels of interest rates or their variations changes over time.

2.2.2 Linear Models and Stochastic Processes

The model proposed by Parisi et al. (2015) and described in Section 2.1 can be simplified by slightly changing the initial assumptions: by considering that monetary rates are at the moment very close to zero, we can relax equation (2.2) and make it independent from the level of monetary rates. The result is the following:

\[
\Delta BR_t = k + \delta BR_{t-1} + \gamma \Delta MR_t + u_t.
\]

(2.9)

In order make it comparable with the other models we can write equation (2.9) in terms of the levels of administered rates:

\[
BR_t = k + (\delta + 1) BR_{t-1} + \gamma \Delta MR_t + \epsilon_t.
\]

(2.10)

According to the existing literature, we can consider the variations of monetary rates as a Wiener process: they can thus be used in order to represent the Brownian motion \(dW_t\). Consistently with this method we can write (2.9) in the alternative way:

\[
\Delta BR_t = k + \delta BR_{t-1} + \sigma \epsilon_t.
\]

(2.11)
The obtained result is particularly interesting: (2.11), in fact, corresponds to the discrete version of the Vasicek stochastic process, with \( k + \delta BR_{t-1} \) being the drift term, \( \sigma \) representing the volatility of the process and \( \epsilon_t \sim N(0, 1) \) corresponding to the geometric Brownian motion \( dW_t \) of the continuous-time equation.

Moreover, the linear regression model described in (2.9) can be extended in a Cox-Ingersoll-Ross (CIR) stochastic process, whose discrete-time version can be expressed by the following:

\[
\Delta BR_t = k + \delta BR_{t-1} + \sigma \sqrt{BR_{t-1}} \epsilon_t,
\]

(2.12)

where, again, \( \epsilon_t \sim N(0, 1) \) corresponds to the geometric Brownian motion \( dW_t \) of the continuous-time CIR equation; \( k + \delta BR_{t-1} \) is the drift term, in which \( \frac{k}{2} \) represents the mean long term level of the bank administered rates, \( \delta \) is the adjustment speed, while \( \sigma \) is the volatility.

In order to make equations (2.11) and (2.12) comparable, the latter can be written by substituting \( \epsilon_t \) with the monetary rates variations, thus obtaining the discrete formulation:

\[
\Delta BR_t = k + \delta BR_{t-1} + \sigma \sqrt{BR_{t-1}} \Delta MR_t + u_t.
\]

(2.13)

Also in this case the equation can be written in terms of the levels of bank administered interest rates, and the result is the following:

\[
BR_t = k + (\delta + 1) BR_{t-1} + \sigma \sqrt{BR_{t-1}} \Delta MR_t - 1 + \epsilon_t.
\]

(2.14)

In this way we have obtained two equations, (2.13) and (2.14), that can be compared with (2.9) and (2.10).

Finally, the two discrete formulations (2.11) and (2.12) can be interpreted as two specific solutions of the general family of non-parametric, time-homogeneous and continuous models:

\[
dBR_t = (k - \delta BR_{t-1}) dt + \sigma (BR_{t-1})^\beta dW_t,
\]

(2.15)

where \( \beta = 0.5 \) corresponds to the CIR process, while \( \beta = 0 \) represents the Vasicek model. Because of their large diffusion in many application, in Section 3 we will concentrate on both the Vasicek and the CIR specifications of the stochastic process (2.15), and we will consider their continuous versions with respect to their discrete formulations.
2.2.3 Models comparison

If we consider the second formulation of the Parisi et al. model (2.2), we can compare it with the proposed models (2.9) and (2.13). More formally, by using the notational index 1 for the coefficients of the Parisi et al. model, and the index 2 for the coefficients referred to the proposed model (2.9), the second one is a particular case of the first one with the following constraints on the parameters:

\[
\begin{align*}
    k_1 &= k_2, \\
    \gamma_1 &= \gamma_2, \\
    \delta_1 &= \delta_2 + 1, \\
    \beta_1 &= 0.
\end{align*}
\] (2.16)

The last equation in (2.16) is particularly interesting because it means that model (2.9) can be derived by (2.2) by eliminating the dependence on the level of monetary rates. Unfortunately, the CIR formulation (2.13) cannot be compared with the other models because of the presence of the volatility term, which is a function of \( \sqrt{BR_t - 1} \).

A full comparison of our proposed models with the Parisi et al. model can not be easily carried out in a statistical testing framework, as the models are, evidently, not nested; however, they can be compared in terms of predictive performance and, for this purpose, the next Subsection introduces an appropriate methodology.

A different comparison between the three models can be carried out by looking at their time dynamics. This is of particular interest in the context of interest rate risk modeling. For sake of simplicity we illustrate this comparison for the first three one-month rates and, then, for the general situation.

Thus, assume that:

\[
\begin{align*}
    BR(0)^{1,2,3} &= BR_0^{1,2,3}, \\
    MR(0)^{1,2,3} &= MR_0^{1,2,3}.
\end{align*}
\]

By using the same notation as before (index 1 for the Parisi et al. model (2.1), index 2 for the proposed model expressed by (2.10) which corresponds to the Vasicek discrete model, and index 3 for the CIR discrete model (2.14)), and by considering the equations that derive the levels, rather than the vari-
ations, of bank rates, for the first month ahead and for the Parisi et al. model we obtain:

$$BR_1^1 = MR_0^1 \beta^1 + \Delta MR_1^1 \gamma^1 + BR_0^1 \delta^1 + k^1,$$

whereas for the second and the third month ahead:

$$BR_1^2 = MR_0^1 \beta^1 (1 + \delta^1) + \Delta MR_1^1 [\beta^1 + \delta^1 \gamma^1] + \Delta MR_2^1 \gamma^1 + BR_0^1 (\delta^1)^2 + k^1 (1 + \delta^1);$$

$$BR_1^3 = MR_0^1 \beta^1 (1 + \delta^1 + (\delta^1)^2) + \Delta MR_1^1 [\beta^1 + \delta^1 (\beta^1 + \delta^1 \gamma^1)] +$$

$$\Delta MR_2^1 [\beta^1 + \delta^1 \gamma^1] + \Delta MR_3^1 \gamma^1 + BR_0^1 (\delta^1)^3 + k^1 \delta^1 (1 + \delta^1).$$

For the discrete Vasicek model (2.10), again assuming as initial values $BR_0^1$ and $MR_0^2$, we find the following equations for the first, the second and the third months ahead:

$$BR_1^2 = \Delta MR_1^2 \gamma^2 + BR_0^2 (1 + \delta^2) + k^2;$$

$$BR_2^2 = \Delta MR_1^2 [\gamma^2 (1 + \delta^2)] + \Delta MR_2^2 \gamma^2 + BR_0^2 (1 + \delta^2)^2 + k^2 (2 + \delta^2);$$

$$BR_3^2 = \Delta MR_1^2 [\gamma^2 (1 + \delta^2)^2] + \Delta MR_2^2 \gamma^2 (1 + \delta^2)] +$$

$$\Delta MR_3^2 \gamma^2 + BR_0^2 (1 + \delta^2)^3 + k^2 (1 + (2 + \delta^2)(1 + \delta^2)).$$

Finally, for the discrete CIR model (2.13) expressed in terms of the levels of bank interest rates the results are the following:

$$BR_1^3 = (\delta^3 + 1) BR_0^3 + \sigma^3 \sqrt{BR_0^3 \Delta MR_1^3},$$

$$BR_2^3 = \Delta MR_1^3 \sigma^3 (\delta^3 + 1) \sqrt{BR_0^3 + \Delta MR_2^3} \cdot \sigma^3 \sqrt{(\delta^3 + 1) BR_0^3 + \sigma^3 \sqrt{BR_0^3 \Delta MR_1^3} + k^3 +}$$

$$BR_0^3 (\delta^3 + 1)^2 + k^3 (\delta^3 + 2).$$
\[ BR_3^t = \Delta MR_1^3 \sigma^3 (\delta^3 + 1)^2 \sqrt{BR_3^0} + \]
\[ + \Delta MR_2^3 \cdot \sigma^3 (\delta^3 + 1) \sqrt{BR_3^0} + \sigma^3 \sqrt{BR_3^2 \Delta MR_3^1 + k^3} + \]
\[ + \Delta MR_3^3 \cdot \sigma^3 \sqrt{\Delta MR_1^3 \sigma^3 (\delta^3 + 1) \sqrt{BR_3^0} + MR_2^3 \cdot \sigma^3 \sqrt{\Delta MR_3^1 \sigma^3 (\delta^3 + 1) \sqrt{BR_3^0} + \sigma^3 \sqrt{BR_3^2 \Delta MR_3^1 + k^3} + k^3 (\delta^3 + 2)}} \]

From the above calculations we can derive a general iterative formula for the three models, in order to calculate bank interest rates at any time \( t \) \((BR_t^{1,2,3})\), as functions of the levels of bank rates at time \( t - 1 \) \((BR_{t-1}^{1,2,3})\).

For the Parisi et al. model (2.1) we obtain:
\[ BR_1^t = \delta BR_1^{t-1} + \beta^2 \left[ MR_1^0 + \sum_{s=1}^{t-1} \Delta MR_1^s \right] + \gamma^1 \Delta MR_1^t + k^1. \quad (2.17) \]

The Vasicek discrete model remains the same as expressed by equation (2.10) because it does not depend on the level of monetary rates:
\[ BR_2^t = (1 + \delta^2) BR_2^{t-1} + \gamma^2 \Delta MR_2^t + k^2. \quad (2.18) \]

Similarly, the discrete CIR version does not depend on the levels of monetary rates, thus it remains the same as expressed by equation (2.14):
\[ BR_3^t = (1 + \delta^3) BR_3^{t-1} + \sigma \sqrt{BR_{t-1}^t} \Delta MR_3^t + k^3. \quad (2.19) \]

Note that the second expression (2.18) is a particular case of (2.17) with the constraint \( \beta^1 = 0 \), which is consistent with (2.16). Finally, the CIR formulation (2.19) still remains different from the other two because of the presence of the term \( \sqrt{BR_{t-1}^t} \).

### 2.3 Predictive performance assessment

All the models proposed so far are quite heterogeneous, are based on different hypothesis and present various approaches, thus a general set-up is required in order to compare them on the same playing field. This can be provided, for example, by a predictive performance framework that we are going to illustrate in this Subsection. Doing so, we can enrich all the models with a validation procedure that has been firstly introduced by Parisi et al. (2015).
In order to predict bank rates, we need to estimate reasonable future values of monetary rates. Consistently with the literature, we assume that their variations follow a Wiener process.

More formally, assume that we want to predict the level of monetary rates for each of the next 12 months. Let $\Delta MR_i$ indicates the variation of the monetary rate in a given month. We then assume that all the $\Delta MR_i$ are independently and identically distributed Gaussian random variables, so that:

$$\begin{align*}
\Delta MR_i &\sim N(0, \sigma^2) \\
MR_i &= MR_{i-1} + \Delta MR_i \quad i = 1, ..., 12.
\end{align*}$$

Equation (2.20) describes a recursive procedure to obtain predictions of the monetary rates for a given year ahead, based on the Wiener process assumption. We can then insert the predicted monetary rates as regressor values in the models of the previous Subsection and, thus, obtain predictions for the administered bank rates.

### 2.3.1 Linear Models and Dynamic Linear Models

The two univariate linear models are quite easy to be predicted, and the corresponding equations are:

$$\begin{align*}
\hat{BR}_i &= k + \beta \cdot MR_i, \\
\hat{\Delta BR}_i &= \hat{BR}_i - \hat{BR}_{i-1}; \\
\hat{\Delta BR}_i &= k + \beta \cdot \Delta MR_i, \\
\hat{BR}_i &= \hat{BR}_{i-1} + \Delta BR_i.
\end{align*}$$

A different situation is the one regarding dynamic linear models. More precisely, considering the analysis of the relationship between the levels of interest rates, and concentrating on the compact form reported in (2.7), we can define the following:

$$\begin{align*}
f_t &= \mathbb{E}(BR_t | D_{t-1}), \\
Q_t &= \text{Var}(BR_t | D_{t-1}),
\end{align*}$$

where $D_{t-1}$ denotes the information provided by the first $t-1$ observations $BR_1, ..., BR_{t-1}$. In such a way the log-likelihood can be derived, as a function of the unknown parameters vector $K$: 

12
\[
\ell(K) = -\frac{1}{2} \sum_{t=1}^{T} \log |Q_t| - \frac{1}{2} \sum_{t=1}^{T} (BR_t - f_t)^T Q_t^{-1} (BR_t - f_t),
\] (2.22)

where \( f_t \) and \( Q_t \) both depend implicitly on the vector \( K \). The previous expression can be numerically maximized in order to obtain the maximum likelihood estimator of the unknown parameters’ vector \( \hat{K} \):

\[
\hat{K} = \arg \max_K \log \ell(K).
\] (2.23)

Moreover, denoting by \( H \) the Hessian matrix of \(-\ell(K)\), the inverse \( H^{-1} \) provides an estimate of the variance of the estimator. Once the parameters \( k_t \) and \( \beta_t \) have been estimated, together with their errors \( \omega_{k,t}, \omega_{\beta,t} \), future values for \( k \) and \( \beta \) can be calculated.

In order to achieve this objective we propose here two alternatives. The first method consists in extracting \( \omega_{k,t} \) and \( \omega_{\beta,t} \) from their distributions for the next twelve months of the year to be predicted \((\hat{\omega}_{k,i}, \hat{\omega}_{\beta,i}; i = 1, \ldots, 12)\), and thus estimating the parameters as follows:

\[
\begin{cases}
\hat{k}_i = \hat{k}_{i-1} + \hat{\omega}_{k,i}, \\
\hat{\beta}_i = \hat{\beta}_{i-1} + \hat{\omega}_{\beta,i}.
\end{cases}
\] (2.24)

The second alternative consists in considering the parameters \( \hat{k}_i \) and \( \hat{\beta}_i \) as fixed for the whole year, more precisely equal to the last values estimated through the maximization of the log-likelihood function:

\[
\begin{cases}
\hat{k}_i = k_T, \\
\hat{\beta}_i = \beta_T, \\
i = 1, \ldots, 12.
\end{cases}
\] (2.25)

In both cases monetary rates are predicted according to (2.20), and bank rates are calculated as:

\[
\hat{BR}_i = \hat{k}_i + \hat{\beta}_i \hat{MR}_i.
\] (2.26)

The dynamic linear model referred to the relationship between the variations of interest rates (2.8) presents similar results as regards both the estimation of the parameters through the maximization of the log-likelihood
function and the prediction of future values of the variations of bank administered rates.

2.3.2 Linear Models and Stochastic Processes

The proposed linear model, described in (2.10), can be used in order to predict future values for bank administered interest rates as follows:

$$\hat{BR}_i = k + \gamma \cdot \Delta M R_i + (\delta + 1) \cdot \hat{BR}_{i-1}.$$ 

In the previous Section we have described linear models as the discrete versions of a Vasicek and a CIR process: in the next Section, however, we will consider them in their continuous formulation, as described by equation (2.15).

If the parameters of the linear time-homogeneous regression models presented in this paper can be estimated by means of ordinary least squares, the two stochastic time-homogeneous continuous processes need a specific parameter estimation.

The three parameters $k$, $\delta$ and $\sigma$ of the Vasicek process can be calculated through the maximization of the log-likelihood function: according to the literature this procedure is standard practice, and it aims at finding the values of the parameters that maximize the probability of the observed outcome. In order to derive the likelihood function, two variables have to be defined:

$$\text{Var}_t = \frac{\sigma^2}{2\delta} (1 - e^{-2\delta \Delta t}), \quad \nu(BR_t, BR_{t+1}, \Delta t) = \frac{BR_{t+1} - \left[\frac{k}{\delta} + (BR_t - \frac{k}{\delta}) e^{-\delta \Delta t}\right]}{\sqrt{\text{Var}_t}}.$$ 

Thus the log-likelihood function can be derived:

$$\log \ell(K) = -\frac{N - 1}{2} \log 2\pi - \frac{N - 1}{2} \log \left(\frac{\sigma^2}{2\delta} (1 - e^{-2\delta \Delta t})\right) - \frac{1}{2} \sum_{t=1}^{T-1} \nu^2(BR_t, BR_{t+1}, \Delta t).$$

The parameters’ vector $\hat{K}$ can be easily found by maximizing the previous equation:

$$\hat{K} = (\hat{k}, \hat{\delta}, \hat{\sigma}) = \arg\max_K \log \ell(K).$$  

14
The parameters estimation of the CIR process is based on the same maximization procedure. Firstly, the following variables have to be defined:

\[ c = \frac{2\delta}{\sigma^2(1 - e^{-\delta t})}, \quad u = c BR_t e^{-\delta t}, \quad q = \frac{2k}{\sigma^2} - 1, \quad v = c BR_{t+1}. \]

Then, the log-likelihood function of the process can be derived:

\[
\log \ell(K) = (N - 1) \log c + \sum_{t=1}^{T-1} \left[ -u_t - v_t + \frac{q}{2} \log \left( \frac{v_t}{u_t} \right) + \log[I_q(2\sqrt{u_t v_t})] \right],
\]

where \( I_q(2\sqrt{uv}) \) is the modified Bessel function of order \( q \). The parameter vector \( \hat{K} \) is again found by maximizing the log-likelihood function:

\[
\hat{K} = (\hat{k}, \hat{\delta}, \hat{\sigma}) = \arg \max_K \log \ell(K). \tag{2.30}
\]

Once the parameters have been estimated, through the Monte Carlo estimation procedure described in the next paragraph and by considering the variations of monetary rates as a Wiener process, a number of scenarios is generated in order to predict future values for the bank administered interest rates, both for the Vasicek and the CIR equations.

2.3.3 Monte Carlo estimation

According to the standard cross-validation (backtesting) procedure, to evaluate the predictive performance of a model, we can compare, for a given time period, the predictions of monetary rates obtained with the previous equations with the actual values. To obtain a robust measurement we can indeed generate \( N \) scenarios of monetary rates, using (2.20), and obtain the corresponding bank rates, using either (2.1), (2.5), (2.10) or (2.14). On the basis of them we can calculate and approximate Monte Carlo expected values and variances of the predictions, as follows.

Let \( Y \) be a bank rate to be predicted at time \( i \), with unknown density function \( f_Y(y) \). The expected value of \( Y \) can then be approximated with

\[
\hat{\mathbb{E}}(Y) = \frac{1}{N} \sum_{k=1}^{N} y^{(k)}, \tag{2.31}
\]
and its variance with

\[ \text{var}(Y) = \frac{1}{N^2} \sum_{k=1}^{N} [y_k - \hat{E}(Y)]^2. \]  \hspace{1cm} (2.32)

A similar procedure can be obtained by considering \( Y \) as a bank rate variation, rather than a bank rate level.

In the next section we will use (2.31) and (2.32) to compare model predictive performances. Before proceeding, we would like to remark that the random number generation at the basis of the Monte Carlo algorithm is pseudo-random, and depends on an initial seed. Different seeds may lead to different results so that models can not be compared equally. We have thus decided to use the same random seed for all models, so that the differences in performances are not biased by the Monte Carlo random mechanism.

3 Data analysis and results

3.1 Descriptive analysis

The recent financial crisis has had a major impact on the banking sector and, in particular, has led to a change in the relationship between monetary and administered rates and, therefore, to the transmission mechanisms of monetary policies. In the Eurozone, characterized by one monetary authority (the European Central Bank), that regulates still fragmented national markets, this effect is particularly evident: southern European countries, differently from what expected, have benefited very little from the drop of monetary rates that has followed the financial crisis.

To investigate the above issues we focus on a southern European country, Italy, for which the transmission of monetary impulses is particularly problematic, given the importance of the banking sector and the difficult economic situation.

Accordingly, we have collected monthly time series data on monetary rates and on aggregate bank deposits administered rates from the statistical database provided by the Bank of Italy, for the period ranging from January 1999 to December 2014.

For the purposes of our analysis, the monetary rate used in this paper is the 1-month Euribor. This choice has been based on the fact that this rate has a greater correlation with the administered bank rate with respect
to the other monetary rates, such as the EONIA and the Euribor at 3 and 6 months, as can be seen in Table 3.1.

<table>
<thead>
<tr>
<th></th>
<th>EONIA</th>
<th>Euribor (1m)</th>
<th>Euribor (3m)</th>
<th>Euribor (6m)</th>
<th>Bank Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>EONIA</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euribor (1m)</td>
<td>0.9904</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euribor (3m)</td>
<td>0.9801</td>
<td>0.9951</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euribor (6m)</td>
<td>0.9701</td>
<td>0.9876</td>
<td>0.9972</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>Bank Rate</td>
<td>0.9488</td>
<td>0.9512</td>
<td>0.9453</td>
<td>0.9333</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 3.1: Correlation matrix between the EONIA rate, the Euribor rates and the Bank administered rate

Figure 3.1 represents the time series of the chosen monetary rates, along with that of the aggregate administered bank rates on deposits, for the considered time period.

Figure 3.1: The observed monetary and administered bank rates

From Figure 3.1 note that both the administered and the monetary rates rapidly decreased in 2008 and 2009, while in the last two years they have remained quite stable and close to zero. Moreover, the two curves seem to have the same shape between 1999 and 2008, while the relationship between
the two radically changes in the following years, leading to overlaps and different behaviors. In other words, the correlation pattern between the bank administered rate and the monetary rate shows a very heterogeneous behavior: before 2008 they seem to have a stable relationship; in 2008 they both dropped; after that time they look stable and close to zero, with a relationship that is indeed quite different from the one observed before the crisis.

To obtain further insights, in Figure 3.2 the histogram and the corresponding density estimate of the two rates are presented.

Figure 3.2: Distribution of the monetary and the administered bank rates

Figure 3.2 reveals that bank administered interest rates are more concentrated around their mean value, while monetary rates are quite spread.

It is also interesting to compare the distributions of the variations of the two rates, represented in Figure 3.3.

From Figure 3.3 note that the variations of the administered bank rates are more concentrated around zero, while monetary rates seem to have broader variations. Indeed, the behavior of $\Delta MR$ justifies the assumption of
considering the variations of monetary interest rates as a Wiener process, so that they can be modeled according to equation (2.20). For the same reason we can consider the linear model proposed in Section 2.2.2 as the discrete version of a stochastic process, thus justifying the use of Vasicek and CIR stochastic differential equations.

We have previously commented on the change in the relationship between the two rates, comparing the situation before and after 2009. This switching behavior can be easily seen by looking at the correlation matrix between the rates and their variations. Table 3.2 shows the correlations between the rates and between their variations in the two periods (1999-2008) and (2009-2014), before and after the financial crisis.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$BR, MR$</td>
<td>0.93</td>
<td>0.71</td>
<td>0.96</td>
</tr>
<tr>
<td>$\Delta BR, \Delta MR$</td>
<td>0.43</td>
<td>0.83</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 3.2: Correlation matrix between rates and their variations, in different periods

From Table 3.2 note that the correlation between the levels of bank and
monetary rates has decreased after 2009, while the correlation between the variations of the administered bank rates and those of the monetary rates has increased during the same period.

3.2 Model estimates

For the models proposed in Section 2.2, we now show the corresponding parameter estimates, considering the following four time series: (a) data from 1999 to 2007; (b) data from 1999 to 2008; (c) data from 2009 to 2013; (d) data from 1999 to 2013. This choice of data windows is consistent with the aim of investigating the switching behavior in the correlation structure of interest rates, which has occurred during the years 2008 and 2009. On the basis of this windows selection we intend to obtain predictions for the years 2008, 2009 and, finally, for the last available year, 2014. Predictions can be compared with the actual occurred value, thus giving a measure of model predictive performance.

Of course the choice of the four time-series is not necessary for the two dynamic linear models: those equations, in fact, are able to calculate time-varying coefficients and, thus, relationships between the involved variables without repeating the estimation procedure for 2008, 2009 and 2014.

We now show the parameter estimates for all the considered models, including the two simple univariate linear models, and the four periods we have chosen. For each linear model estimate we also report the corresponding \( t \)-value and the \( R^2 \) contribution.

3.2.1 Linear Models and Dynamic Linear Models

Table 3.3 shows the parameter estimates for the simple linear model expressed in terms of the levels of bank interest rates (2.3).

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>t</td>
<td>Coeff.</td>
<td>t</td>
<td>Coeff.</td>
</tr>
<tr>
<td>( k )</td>
<td>-0.133</td>
<td>-0.100</td>
<td>0.263</td>
<td>0.146</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.351</td>
<td>0.341</td>
<td>0.138</td>
<td>0.271</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.893</td>
<td>0.880</td>
<td>0.287</td>
<td>0.905</td>
</tr>
</tbody>
</table>

Table 3.3: Parameter estimates for the linear model in terms of the levels of bank interest rates
Consistently with the correlation matrix (Table 3.2), the parameter $\beta$ has decreased after 2008; similarly, the $R^2$ contribution has consistently dropped in the recent period.

Table 3.4 shows the parameter estimates for the simple linear model in terms of variations of bank interest rates (2.4).

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.149</td>
<td>5.444</td>
<td>0.131</td>
<td>6.344</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.219</td>
<td>0.254</td>
<td>0.683</td>
<td>0.341</td>
</tr>
</tbody>
</table>

Table 3.4: Parameter estimates for the linear model in terms of the variation of bank interest rates

From Table 3.4 it is clear that the univariate linear model for the variations of administered bank interest rates, calculated as a function of the variations of monetary rates, shows different results: first of all, the intercept term is not significant; secondly, $R^2$ values have an opposite trend with respect to those in Table 3.3, increasing during the last period; finally, also the coefficient $\beta$ shows an opposite trend, decreasing after 2008. This results are a further confirmation of the changing regime after 2009.

Figure 3.4 shows the estimated parameters for the dynamic linear model that explains how administered bank interest rates react to monetary rates (2.5). The black line stands for the mean values, while the red and the green lines represent the confidence intervals.

Figure 3.4: Parameters estimates for the dynamic linear model in terms of the levels of interest rates

Figure 3.4 shows a clear switching behaviour between bank rates and monetary rates during the period under analysis: more precisely, the graph
on the left represents the values estimated for the intercept $k_t$ from 1999 until 2013, while the graph on the right represents the values referred to the time-varying coefficient $\beta_t$ during the same years. From the first one it is clear that the intercept has remained substantially constant and quite stable until the crisis of 2009. Moreover, the second graph seems to underline the changing regime after the crisis, with a coefficient $\beta_t$ radically decreasing in the last few years, meaning a less and less significant relationship between administered rates and monetary rates.

Figure 3.5 shows the estimated parameters for the dynamic linear model that explains how the variations of administered bank interest rates change according to the variations of monetary rates (2.6).

Figure 3.5: Parameters estimates for the dynamic linear model in terms of the variations of interest rates

Differently from the previous situation, in this case the two graphs do not seem to be significant from an interpretative point of view. The graph on the left, in fact, shows that the intercept $k_t$ is always close to zero, while the coefficient $\beta_t$, which explains how the variations in bank rates react to changes in the variations of monetary rates, is very unstable and does not capture a clear pattern between the two variables.

### 3.2.2 Linear Models and Stochastic Processes

Table 3.5 shows the parameter estimates for the linear model (2.10) that has been proposed in Section 2.2.2. In order to be consistent with the other models and their estimated parameters, in Table 3.5 are reported the coefficients of the equation that explains bank administered interest rates as a function of their previous levels and of the variations of monetary rates. The notation is thus consistent with equation (2.10).
Table 3.5: Parameter estimates for the proposed model

Table (3.5) shows that our proposed linear model presents an interesting behavior. For the first years 1999-2007 and 1999-2008, and for the whole period 1999-2013, in fact, all variables (apart from the intercept) are significant to describe the levels of the bank administered interest rates. But the situation changes if we concentrate on the second period: within the years 2009-2013 the only significant variable is the autoregressive component. This is a clear evidence of the fact that, when rates are close to zero as in the current situation, administered interest rates are not affected by the variations of monetary rates but, rather, they depend only on their past values.

Table 3.6 shows the parameter estimates for the two stochastic processes introduced in Section 2.2.2 as the continuous-time versions of the previous linear model.

Table 3.6: Parameter estimates for the two stochastic processes: Vasicek and CIR

Table 3.6 presents in the first columns the estimated coefficients for the Vasicek model (equation (2.15) with \( \beta = 0 \)), for the four selected time-periods. Similarly, the last columns refer to the CIR stochastic process (equation (2.15) with \( \beta = 0.5 \)).

From a comparison between these results it is clear that the drift terms of the two models are quite similar to each others. The major differences regard the volatility term: this is much higher for the Vasicek model with respect to the CIR specification of equation (2.15): the reason for this behavior is due to the fact that the volatility term in the Vasicek model has to compensate the
absence of the multiplier $\sqrt{BR}$ in the second part of its equation. Finally, in both the models such a volatility is substantially lower during the last period (2009-2013), consistently with the actual situation of stable and close-to-zero interest rates.

### 3.3 Predictive performances

After having estimated the coefficients of the different models, we then predict monthly administered bank interest rates and their variations for 2008, 2009 and 2014, using a range of monetary rates scenarios, simulated from a Wiener process as previously described. In particular, for the 2014 prediction we performed the simulations by using the coefficients obtained both by considering the whole period (1999-2013) and the second part of the time range under examination (2009-2013). In the next Figures the estimated variations of bank administered interest rates are illustrated.

Firstly, in Figure 3.6 a comparison between the predictions for 2014 (data from 1999 until 2013) obtained with the simple, univariate linear model (2.4) and the dynamic linear model described in (2.6) is shown.

![Figure 3.6: The estimated variations of administered interest rates for 2014, obtained with the univariate linear model and with dynamic linear model by using coefficients calculated on the whole period 1999 - 2013](image)

In Figure 3.6 future values of the variations of bank administered interest rates obtained with the dynamic linear model have been calculated by estimating also the parameters $\hat{k}_i$ and $\hat{\beta}_i$ for 2014, according to equation (2.24).

Secondly, in Figure 3.7 a comparison between the predictions for 2014 (data from 1999 until 2013) obtained with the Parisi et al. model (2.2) and our proposed linear model described in (2.9) is shown.
Figure 3.7: The estimated variations of administered interest rates for 2014, obtained with the Parisi et al. model and our proposed linear model, by using coefficients calculated on the whole period 1999 - 2013.

In Figure 3.7 both the models have been considered in the formulation that derives bank interest rates variations as functions of the corresponding regressors.

Finally, in Figure 3.8 a comparison between the predictions for 2014 (data from 1999 until 2013) obtained with the two stochastic processes Vasicek and CIR is shown.

Figure 3.8: The estimated variations of administered interest rates for 2014, obtained with the Vasicek and the CIR stochastic processes, by using coefficients calculated on the whole period 1999 - 2013.

In order to better compare models, as a measure of predictive performance we have calculated the root mean square errors of the predictions from all the equations. Consistently with the previous Figures, here we present the prediction results in terms of variations of bank rates rather than on their levels. This because, in this case, all the predictions are more challenging, being the variations on a smaller scale.

In Table 3.7 the root mean square errors of the predicted variations
administered interest rates obtained with the simple linear models and the
dynamic linear models introduced in Section 2.2.1 are reported.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Univariate linear model</td>
<td>0.065</td>
<td>0.120</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>Dynamic linear model</td>
<td>0.067</td>
<td>0.119</td>
<td>0.015</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Table 3.7: A comparison between the root mean square errors of the predictions of ∆BR

The first column of Table 3.7 refers to the prediction errors for the year 2008, obtained with the two selected models, and using coefficients estimated on data from 1999 to 2007. Similarly, the second and the third columns report root mean square errors for 2009 and 2014. We have decided to compare predictions on these crucial years because they represent the breaking points before and after which the relationship between the rates radically changes. The objective is thus to verify whether the two models can adapt to such strong variations in the underlying economic system. Note that the last two columns both refer to estimations for 2014, but the first one uses coefficients estimated only by using the second period data, while the second one is based on estimations on the entire period 1999-2013.

Although Table 3.7 shows that the predictive performance looks quite the same for the two models, it is important to underline the substantial difference between the two models: the dynamic linear model, in fact, is able to capture the switching regime between two variables during a certain period of time, while the simple linear model is not. Moreover, the dynamic linear model adapts much better and much more to real data: if we consider the standard error of the dynamic linear model calculated on the whole period 1999-2013, in fact, we obtain \( SE = 1.3 \cdot 10^{-7} \), while the same quantity obtained with the simple linear model is \( SE = 0.0456 \). This is a further evidence of the fact that dynamic linear models are able to capture the dynamics of the relationship between two, or more variables, and are thus preferable at least for data description.

In Table 3.8 the root mean square errors of the predicted variations of administered interest rates obtained with the Parisi et al. model, our proposed linear model and the stochastic processes (Vasicek and CIR) are reported.

From the analysis of Table 3.8 some interesting conclusions emerge: (a) all the models predict quite well future variations of bank interest rates;
Table 3.8: A comparison between the root mean square errors of the predictions of $\Delta BR$

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Parisi et al. model</td>
<td>0.065</td>
<td>0.069</td>
<td>0.014</td>
<td>0.018</td>
</tr>
<tr>
<td>Proposed linear model</td>
<td>0.018</td>
<td>0.295</td>
<td>0.021</td>
<td>0.020</td>
</tr>
<tr>
<td>Vasicek</td>
<td>0.065</td>
<td>0.113</td>
<td>0.017</td>
<td>0.016</td>
</tr>
<tr>
<td>CIR</td>
<td>0.065</td>
<td>0.117</td>
<td>0.017</td>
<td>0.017</td>
</tr>
</tbody>
</table>

(b) the proposed linear model is the best challenging during the first time period, and, most interestingly, (c) the two stochastic processes are the best models in terms of predictive performance. This means that continuous, time-homogeneous models are preferable with respect to linear models, and this is an even more important result by considering that they are endogenous models, which means that data on monetary rates are not needed in order to predict future values of bank administered interest rates. Finally, their root mean square errors do not seem to increase even during a financial crisis (2008-2009), meaning that they more adaptive and able to change according to changes in the underlying economy.

4 Conclusions

The main contribution of this paper is in the improvement and extension of the model proposed by Parisi et al. (2015), able to explain variations of the administered bank rates as a function of monetary rates. We add to the model a dynamic structure, and we compare it with a series of more sophisticated models, such as dynamic linear models and stochastic processes.

We have shown the implications of our proposals on data for the aggregate Italian banking sector, that concerns the recent period, characterized by a substantial shift in the relationship between monetary and bank rates, with the former getting close to zero. In this context, we have shown that dynamic linear models can better capture the switching regime behavior observed in the recent years, as well as stochastic processes give the best performance results and are endogenously determined.

Future research in this topic may involve the use of multivariate dynamic equations and of time-inhomogeneous stochastic differential equations, in order to improve the model ability to adapt to dynamic changes.
Finally, a further extension should consider the microeconomic impact of the found relationships on the probability of default of both financial and non financial corporates, enriched with a systemic correlation perspective.

5 Acknowledgements

6 References


submitted.

