Debt-Overhang Banking Crises: Detecting and Preventing Systemic Risk

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Abstract

This paper shows how the debt overhang distortion on bank lending can generate a self-fulfilling expectations banking solvency crisis accompanied by a plunge of bank asset values and a contraction of bank lending and economic activity. Some signals of the existence of this type of systemic risk include: a high volatility and the presence of two modes in the probability distribution functions of the returns of bank-issued bonds and of portfolios of bank-issued bonds and equities; and a high correlation between bank-issued bond returns. To prevent this risk, macroprudential regulation should limit the common default risk exposure of banks to the economic and financial cycle, using capital requirements with higher risk weights for more cyclical assets.

Keywords: Multiple equilibria; self-fulfilling expectations; financial contagion; financial fragility; macroprudential regulation.

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1 Introduction

Some of the most severe contractions of economic activity are accompanied by banking crises.\textsuperscript{1} In a typical economic and banking crisis, bank asset values drop, a large number of banks default or become insolvent, bank lending and economic activity contract. The period of economic weakness and banking stress tends to be prolonged. The change in economic fundamentals appears small relative to the severity of the financial and economic effects.

To study this type of economic and banking crisis, we introduce a mechanism based on the debt overhang distortion on bank lending.\textsuperscript{2} Two features of the banking system play an important role: the liabilities of banks distort their lending choices, inducing them to lend less than the optimal amount of funds; and the value of bank assets is sensitive to economic prospects. These features make the economy financially fragile—a pessimistic view of the economy can be self-fulfilling and lead to a crisis: if the economy is expected to perform poorly, then the value of bank assets declines, the bank risk of default rises, and the associated debt overhang distortion worsens; this leads to a contraction in bank lending and to a de-

\textsuperscript{1}In the U.S., the recessions that were accompanied by banking crises—the ones that began in 1873, 1893, 1907, 1929, and 2007—were all especially severe (Bordo and Haubrich 2010, Table 1 and Figure 1). During the Great Depression, in particular, the rise of bank failures and the fall of bank loans in the years 1930-1933 were decisive in making the recession so deep and so long (Bernanke 1983).

\textsuperscript{2}Myers (1977) is the seminal article that describes how the existing debt of firms discourages their investment. The marginal cost of a firm’s new investment is borne by the equity holders (or by junior creditors). The marginal return, however, is seized by the senior creditors in the event of default. The higher the firm’s probability of default, the lower the equity-holders’ expected marginal return, the smaller their incentive to invest, the lower the firm’s investment level. In the case of banks, their existing liabilities discourage their lending.
cline in economic activity, which confirms the initial pessimistic view (see Figure 1).³

In this mechanism, the fragility of the banking system results from the interaction between the loan-granting activity of banks and the sensitivity of their assets to aggregate economic conditions. No role is played by the deposit-receiving activity of banks and by the liquidity mismatch between their assets and liabilities, which instead are crucial in standard models of financial fragility. This debt-overhang explanation seems, then, more promising than the traditional liquidity-based ones for modern banking crises, which occur in the presence of institutions—deposit insurance and a lender of last resort—that are designed to prevent and solve liquidity crises.

We then study how systemic risk can be detected and prevented. We show that some signals of systemic risk include: high volatility and the presence of two modes in the probability distribution functions of the returns of bank-issued bonds and of portfolios of bank-issued bonds and equities; and high correlation between bank-issued bond returns. To prevent systemic risk, macroprudential regulation should limit the common default risk exposure of banks to the economic and financial cycle, using capital requirements with higher risk weights for more cyclical assets.

This paper is most closely related to the growing literature that studies the debt overhang distortion in the banking sector. Wilson and Wu (2010) and Wilson (2012) study how to efficiently recapitalize banks when bank lending is distorted by debt overhang, and show that purchases of preferred

³This mechanism is similar to the one studied by Lamont (1995), who shows that multiple equilibria can arise when firms’ investments are distorted by debt overhang and have positive spillovers, i.e. the value of a firm depends positively on other firms’ investment. In our model, banks play the role that firms play in Lamont’s model, banks’ loans play the role of firms’ investments, and, differently from Lamont’s model, the positive spillovers arise from the dependence of the bank asset value on economic prospects.
stock are less efficient than purchases of common stock or bank assets. Philippon and Schnabl (2013) introduce a financial contagion mechanism that is similar to the one at work in this paper. When a bank’s risk of default rises, the debt overhang distortion rises, and this induces the bank to contract its loans; at the aggregate level, this reduces payments to households, increases households defaults and raises the risk of default of other banks. They emphasize that this mechanism creates a negative externality, which renders the resulting equilibrium inefficient, and study how a government should optimally intervene with a recapitalization program. Bhattacharya and Nyborg (2013) also study optimal government recapitalization of banks that suffer from debt overhang problems. Banks have private information about the quality of their assets-in-place and new investment opportunities. Menus of bailout plans, made of equity injections and asset buyouts, are used as screening devices. Although they include the possibility of public benefits to bailouts in their analysis, they do not explicitly model cross-spillover effects. Finally, in their analysis of the objectives and tools of macroprudential regulation, Hanson, Kashyap and Stein (2013) point out that the debt overhang problem prevents banks from raising the socially-optimal amount of capital during a crisis, and leads them to shrink their assets and balance sheets excessively, which creates the need for policy intervention.\footnote{In addition, there is a growing literature that explores the aggregate implications of debt overhang on business investment, and includes Lamont (1995), Philippon (2009), Arellano, Bai and Kehoe (2012), Gomes, Jermann and Schmid (2013), Kobayashi and Nakajima (2014), and Occhino and Pescatori (2014, 2015).}

In the rest of the paper, Section 2 describes the model and the debt-overhang mechanism; Section 3 shows that this mechanism can give rise to a financial crisis; Section 4 investigates how systemic risk can be detected and measured; Section 5 discusses some regulatory tools that can prevent
the emergence of systemic risk; and Section 6 concludes.

2 Model

To describe the debt-overhang mechanism, we use a two-period model with a continuum of representative households, and a continuum of representative banks, both of unit measure. There is no aggregate fundamental shock and no aggregate uncertainty, i.e., in each equilibrium aggregate variables are non-stochastic—however, there can be multiple equilibria, and aggregate variables may differ across different equilibria.

Before describing the economy in detail, it is helpful to briefly outline the initial financial arrangements between households and firms. Each household owns one share of each bank. In addition, households are creditors of banks—the face value of this bank debt is $b$, payable at the end of the second period, but banks may default on their debt, so the actual payoff of this bank debt may be lower. Banks hold financial assets promising a payoff $\pi(Y)$ at the end of the second period, where $Y$ is the aggregate output produced with loans, with households as their counterpart for this financial position.

Banks

The two key features for the debt-overhang mechanism are that banks’ loans are distorted by the overhang of the existing bank liabilities and that the value of banks’ assets is sensitive to the aggregate economic activity. The latter feature, which is absent in Lamont’s model of firms’ debt overhang, is the one that creates the positive spillovers in our model of banks’ debt overhang. To model these two features, we assume that each bank, initially, has financial liabilities (e.g., deposits, interbank loans, long-term bonds) with face value $b$ due at the end of the second period, and owns
financial assets (e.g. equity, corporate and government bonds, asset-backed securities, previously granted loans) promising a payoff $\pi(Y)$ at the end of the second period that is strictly increasing in $Y$, the aggregate output produced with loans. Each bank also owns an amount of real funds $m$.

In the first period, each bank distributes dividends $d_1$ to households and grants new loans $l$, subject to the constraint

$$d_1 + l = m$$

Banks do not take any other decision.

In the second period, loans are used for production. The output produced with each individual bank’s loans $l$ is

$$y = \omega f(l)$$

where $\omega$ is a log-normally distributed idiosyncratic shock, and $f(l) \equiv Al^\alpha$ is a production function, with $A > 0$, and $\alpha \in (0, 1)$.

Aggregate output is

$$Y = E\{y\} = E\{\omega\} f(l)$$

where $E$ is the expectation over the idiosyncratic shock $\omega$.

Each bank receives all the output, $y$, in return of its loans. It also receives the return $\pi(Y)$ on its assets. If the sum of the two is less than the face value of its liabilities, $y + \pi(Y) < b$, then the bank defaults, repays

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5 As in most of the debt-overhang literature, including the two related papers of Lamont (1995) and Philippon and Schnabl (2013), we examine the economic implications of a given capital structure, without explaining it.

6 To focus on the main mechanism, we lump the financial and production sectors together, assuming that banks receive all the output produced with loans. The mechanism, however, does not depend on this assumption and would be at work even if firms were modeled separately, banks received only part of the output produced, and firms distributed the rest to households as dividends.
\( y + \pi(Y) \) to the creditors, and does not distribute any dividend. Otherwise, the bank repays the entire face value \( b \) to the creditors and distributes the rest, \( y + \pi(Y) - b \), as dividends. The debt payoff to the creditors is, then,

\[
\min(y + \pi(Y), b)
\]

and dividends are

\[
d_2 = y + \pi(Y) - \min(y + \pi(Y), b) = \max((y + \pi(Y) - b, 0)
\]

(4)

Notice that all decisions are taken before the realization of the idiosyncratic shock \( \omega \), and banks are ex-ante identical, so all banks make the same decision. Ex-post, however, banks are heterogeneous, and this implies that some will default and some will not.

Because of the crucial role played by the sensitivity of the asset payoff \( \pi(Y) \) to aggregate output, we add some more specific assumptions about the assets held by banks. Each bank holds a portfolio of three types of financial assets: \( \bar{a} \geq 0 \) units of a risk-free asset with unit payoff; \( a_Y \geq 0 \) units of an asset promising a payoff equal to \( \bar{Y} + \lambda(Y - \bar{Y}) \), where \( \bar{Y} > 0 \) is a constant level of output and \( \lambda > 0 \); and \( a_V \in [0,1) \) units of an asset promising a payoff equal to the value \( V(Y) = Y + \pi(Y) \) of the representative bank in the second period. The latter can be thought of as a portfolio of equity and liabilities of the representative bank, and represents cross-participations and interbank lending. The bank’s asset payoff is then

\[
\pi(Y) = \bar{a} + a_Y [\bar{Y} + \lambda(Y - \bar{Y})] + a_V Y
\]

(5)

where \( \bar{\kappa} \equiv \frac{\bar{a} + a_Y (1-\lambda)Y}{1-a_V} \) and \( \kappa_Y \equiv \frac{a_Y \lambda + a_V}{1-a_V} \).
Households

The households’ objective function is

\[ u(c_1) + \beta u(c_2) \]

where \( \beta \in (0,1) \), the utility function satisfies \( u'(c) \equiv e^{-\gamma} \), with \( \gamma > 0 \), and \( c_1 \) and \( c_2 \) are the non-stochastic consumption levels in the two periods.

Households don’t take any decision. They enter the first period holding the opposite financial position of banks: a short position in the financial assets held by banks, and claims to the banks’ liabilities.

In the first period, they receive an endowment \( e_1 \) and banks’ dividends \( d_1 \), so their first-period consumption is

\[ c_1 = e_1 + d_1 \]  \hfill (6)

In the second period, they receive an endowment \( e_2 \), they pay the financial assets’ payoff \( \pi(Y) \) to banks, and they receive dividends \( E\{d_2\} \) and debt payoff \( E\{\min(y + \pi(Y), b)\} \) from banks. Their consumption in the second period is

\[ c_2 = e_2 - \pi(Y) + E\{d_2\} + E\{\min(y + \pi(Y), b)\} \]  \hfill (7)

Bank’s problem

Each bank is owned by the representative household, so it makes its choices to maximize the representative household’s objective function, discounting the future using the non-stochastic discount factor

\[ \Lambda = \frac{\beta u'(c_2)}{u'(c_1)} \]  \hfill (8)

The following is the bank’s problem:

\[ \max_{d_1, d_2, l, y} \{d_1 + E\{\Lambda d_2\}\} \text{ subject to (1), (2) and (4)} \]  \hfill (9)
given Λ and Y.

Using the fact that the discount factor Λ is non-stochastic, the first-order condition is

\[ \Lambda \frac{\partial E\{\max(\omega f(l) + \pi(Y) - b, 0)\}}{\partial l} = 1 \]

Restricting attention to equilibria where \( b > \pi(Y) \), so that there is a positive probability that banks default, the first-order condition becomes

\[ \Lambda E\{\omega\} f'(l) \Phi(\zeta(l, Y)) = 1 \]

where

\[ \zeta(l, Y) = \frac{\ln(E\{\omega\} f(l)/(b - \pi(Y))}{\sigma} + \sigma/2 \]

Φ(·) is the cumulative distribution function of a standard normal, and \( \sigma \) is the standard deviation of \( \ln(\omega) \). For intuition, it is helpful to interpret \( \zeta \) as the normalized distance between \( E\{\omega\} f(l) \) and \( b - \pi(Y) \), i.e., the distance to default; \( \Phi(\zeta) \) as the adjusted probability of full debt repayment, i.e., of \( \omega f(l) + \pi(Y) - b \geq 0 \); and \( 1 - \Phi(\zeta) \) as the probability that the bank defaults on its liabilities.

In equilibrium, \( E\{\omega\} f(l) = E\{y\} = Y \), so

\[ \zeta(l, Y) = \delta(Y) = \frac{\ln(Y/(b - \pi(Y))}{\sigma} + \sigma/2 \quad (10) \]

and the first-order condition becomes

\[ \Lambda E\{\omega\} f'(l) \Phi(\delta(Y)) = 1 \quad (11) \]

This first-order condition implies that, since \( \Phi(\delta) \) is less than one, bank loans \( l \) are lower than they would be without risk of default and debt overhang. What distorts the bank’s lending decision is the anticipation that, in the event of default, the marginal benefit of lending will accrue to the bank’s creditors, not to the equity holders. Consider the bank’s
marginal decision to lend one extra-unit of resources. This unit increases the expected revenue by the marginal expected product

$$\partial E\{\omega f(l)\}/\partial l = E\{\omega \} f'(l)$$

However, this unit also increases the expected debt repayments to the bank’s creditors by

$$\frac{\partial E\{\min(\omega f(l) + \pi(Y), b)\}}{\partial l} = (1 - \Phi(\delta(Y)))E\{\omega \} f'(l)$$

since the marginal benefit of lending will be reaped by the creditors in the event of default, and this discourages the bank’s lending. The default probability, $1 - \Phi(\delta)$, acts like a tax discouraging lending, and is the correct indicator for the size of the debt overhang distortion.

Bank loans have positive spillovers—the decision of other banks to increase aggregate lending, raises aggregate output $Y$, and raises the value of the assets held by a bank. This positive spillover generates a contagion mechanism: when a bank’s risk of default, $1 - \Phi(\delta)$, rises, the debt overhang distortion rises, and the bank contracts its loans $l$; the contraction of bank lending worsens aggregate economic conditions $Y$, decreases the value of other banks’ financial assets $\pi(Y)$ and raises their risk of default.\(^7\)

\(^7\)The literature has described other contagion mechanisms that can transmit solvency risk from bank to bank. The contagion mechanism may be direct, as in Rochet and Tirole (1996): if banks lend to each other or invest in each other’s equity, a rise in the risk of default of a bank lowers the value of the other banks’ claims to that bank, and raises directly their risk of default. More often, the contagion mechanism has two parts, as in our model: first, a rise in the risk of default of a bank induces the bank to reduce its asset holdings, i.e. to sell its securities or to reduce its loans; second, the decision to disinvest by the bank reduces the return on the other banks’ investment, and raises their risk of default. An example of a two-part mechanism is the following. If banks target a constant leverage ratio for risk-management or regulatory purposes, or a pro-cyclical one as in Adrian and Shin (2013), an initial loss at a bank induces that bank to de-leverage and sell its assets; if those assets are not perfectly liquid, this depresses their price and
Bank loans are strategic complementary—a bank’s expected marginal return of loans rises as other banks’ loans rise. The decision of other banks to increase their lending lowers a bank’s risk of default and debt overhang distortion, and raises its expected marginal return of lending, $E\{\omega\} f'(l) \Phi(\zeta(l, Y))$, for any given level of bank loans $l$, encouraging its lending. Let

$$\Phi_Y \equiv \frac{\partial \ln(\Phi(\zeta(l, Y)))}{\partial \ln(Y)} = \frac{\Phi'(\delta(Y))}{\Phi(\delta(Y))} \frac{1}{\sigma b - \pi(Y)} \frac{Y}{\pi(Y)} > 0 \quad (12)$$

be the elasticity of the probability of debt repayment to aggregate output, evaluated in equilibrium. As in Lamont (1995), debt overhang generates a strategic complementarity in an economy that exhibits positive spillovers.

This strategic complementarity can amplify small shocks and could explain economic booms and busts accompanied by corresponding booms and busts in bank asset values and lending activity. More importantly, it has the potential to generate multiple equilibria and can give rise to a self-fulfilling expectations financial crisis. If there are pessimistic views of the financial sector, bank asset values drop, banks’ risk of default and debt overhang distortion rise, and this leads to under-lending and self-fulfilling poor performance of the financial sector.

**Equilibria**

The variables $\{m, b, e_1, e_2\}$ are given and are treated as parameters.

An equilibrium is a set of values $\{d_1, d_2, l, y, Y, c_1, c_2, \Lambda\}$ that satisfy equations (3), (6), (7) and (8), and that solve the generates losses at other banks holding the same assets. Another example, based on the one described by Lagunoff and Schreft (2001), works as follows: an initial portfolio loss induces a bank to disinvest; this lowers the return of other banks investing in the same portfolio because the return on a bank loan depends positively on whether other banks continue to finance that project, or because it depends on aggregate economic activity which in turn depends on other banks’s lending decisions.
bank’s problem (9), where the function $\pi(Y)$ is given by equation (5).

If $\pi(Y)$ did not depend on $Y$, the equilibrium would be unique, as we show in Appendix A. However, the dependence of $\pi(Y)$ on $Y$ and the associated spillovers raise the possibility of multiple equilibria. Multiple equilibria arise when the sensitivity of the banks’ financial assets to aggregate economic activity, $\kappa_Y$, is high.

To compute an equilibrium, first, use the equilibrium equations to obtain the following equations:

$$
\begin{align*}
c_1 &= e_1 + m - l \\
c_2 &= e_2 + Y \\
\Lambda &= \frac{\beta u'(e_2 + Y)}{u'(e_1 + m - l)}
\end{align*}
$$

Then, substitute the previous expression for $\Lambda$ into the equilibrium first-order condition (11), and obtain the following system of two equations in the two unknowns $l$ and $Y$:

$$
\frac{\beta u'(e_2 + Y)}{u'(e_1 + m - l)} E\{\omega\} f'(l) \Phi(\delta(Y)) = 1
$$

$$
Y = E\{\omega\} f(l)
$$

where the functions $\pi(Y)$ and $\delta(Y)$ are given by equations (5) and (10).

After finding a solution $\{l, Y\}$ to the system, compute the values for the

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8The expression for $c_2$ follows from:

$$
\begin{align*}
c_2 &= e_2 - \pi(Y) + E\{d_2\} + E\{\min(y + \pi(Y), b)\} \\
c_2 &= e_2 - \pi(Y) + E\{\max((y + \pi(Y) - b, 0)) + E\{\min(y + \pi(Y), b)\} \\
c_2 &= e_2 - \pi(Y) + E\{y + \pi(Y)\} \\
c_2 &= e_2 + Y
\end{align*}
$$
other equilibrium variables \(\{d_1, d_2, y, c_1, c_2, \Lambda\}\) using equations (1), (2), (4), (6), (7) and (8).

Finally, check that this set of values is an equilibrium by checking that, given \(\Lambda\) and \(Y\), \(\{d_1, d_2, l, y\}\) solve the bank’s problem (9).

For each equilibrium, we then compute the asset values using the discount factor \(\Lambda\). The banks’ bond value is equal to \(\Lambda E\{\min(y + \pi(Y), b)\}\), while the banks’ equity value is equal to \(d_1 + \Lambda E\{d_2\}\). The value of banks’ assets is equal to the sum of the bond and equity value, i.e. \(d_1 + \Lambda (E\{y\} + \pi(Y))\). The capital ratio is defined as the ratio of the equity value to the asset value.

The risk-free rate is equal to \(1/\Lambda - 1\). Notice that, since in each equilibrium there is no aggregate uncertainty, the expected rate of return of any asset is equal to the risk-free rate. The bond yield (which is not an expected rate of return), is equal to the ratio of the bond face value \(b\), to the bond value, as defined above, minus one. The bond spread is the difference between the bond yield and the risk-free rate.

## 3 Financial fragility

To illustrate the mechanics of a debt-overhang banking crisis, we set the parameter values so that the model has two equilibria, a normal equilibrium and a financial crisis equilibrium, and the normal equilibrium describes an economy in a normal (non-crisis) state.

### Parameter values

The parameter values are listed in Table 1. One period is one year. Both \(A\) and \(E\{\omega\}\) are normalized to 1. The parameter values \(\alpha = 0.3, \beta = 0.99,\) and \(\gamma = 1\) are standard. The annual volatility of the idiosyncratic productivity shock is \(\sigma = 0.8\), which implies a quarterly volatility of 40%. As we
will see, with this high value for the volatility, the probability distributions of the returns of an individual bank’s bonds and assets has two modes. If we chose a lower, more standard, value, for instance a quarterly volatility of 20%, all the qualitative results would continue to hold, except that there would be only one mode in those probability distributions.

The first-period household endowment $e_1$ is set so that, in the normal equilibrium, the first-period aggregate consumption $c_1$ is 5 times bank loans $l$, to match, approximately, the 2013:Q4 ratio of Personal Consumption Expenditures to new loans—new loans are computed by dividing total loans by their average maturity, that is 3.23 years (Commercial banks, Federal Reserve Call Report). The second-period household endowment $e_2$ is set so that, in the normal equilibrium, the consumption growth rate $c_2/c_1 - 1$ is 2.5%.

The bank balance sheet parameters, $m$, $\bar{a}$ and $b$, are set so that, in the normal equilibrium: the default probability is equal to 2.24%, which matches the average default rate (U.S. All Corporates, 1984:I—2011:IV, Moody’s); the bank equity-asset ratio is equal to 11 percent, which matches, approximately, the 2013:Q4 ratio of equity to assets (Commercial banks, Federal Reserve Call Report); and the first-period dividends $d_1$ are equal to 4 percent of equity, which implies an equity-dividend ratio of 25.

We set $a_V = 0.0075$, so each bank holds claims to the assets of other banks for 0.75% of the value of the representative bank, to match, approximately the 2013:Q4 ratio of interbank loans to total assets (Commercial banks, Federal Reserve Statistical Release, H.8).

We set $\bar{Y}$ equal to the level of output $Y$ in the normal equilibrium, and $a_Y = 3$, so that the value of assets that depend directly on aggregate output $Y$ is equal to 3 times the new loans $l$. This approximately matches the 2013:Q4 ratio of risky assets to new loans—new loans are computed as described above, while risky assets are computed as the difference between
total assets and the sum of Treasury and agency securities, new loans, inter-
bank loans, and cash assets (Commercial banks, Federal Reserve Statistical
Release, H.8).

The sensitivity $\lambda = 2$ is set high enough so that the model has two
equilibria. These parameter values imply that $\kappa_Y = 6.0529$. The next
section is devoted to compare this economy to other economies with lower
$\lambda$ and $\kappa_Y$.

The normal equilibrium and the financial crisis equilibrium

The values of the key variables in the two equilibria are summarized in Ta-
ble 2. Relative to the normal equilibrium, economic activity is dramatically
lower in the financial crisis equilibrium. The default probability jumps from
2.25% to 71.47%, raising the debt overhang. Loans are 39.17% smaller, and
output produced with loans drops by 13.86%. The risk-free rate drops to
a large negative value, while the bond spread jumps to 4.46 percentage
points. As a result, the bond yield decreases, and the bond value increases.
This latter result depends on parameter values—in general, the change in
the bond yield and in the bond value can be positive or negative, depending
on the opposite forces of the lower risk-free rate and of the higher bond
spread. The banks’ equity value plunges by 52.22%. The value of bank
assets rises by 5.96%, although this result is also dependent on parameter
values. The capital ratio drops from 11% to 4.96%.\(^9\)

\(^9\)The first-period dividends $d_1$ increase during the crisis. This is the consequence of
the limited set of options available to banks. The banks’ initial funds $m$ are constant,
and banks can only lend them or distribute them as dividends, so a drop in $l$ during the
crisis necessarily entails a rise in $d_1$. This model prediction is not robust to modifications
of the model where banks have additional options to raise or use funds. For instance,
if banks could borrow short-term from households and if short-term borrowing dropped
during a crisis, as it happens in reality, dividend distributions would drop as well. For
this reason, it is better to interpret the rise of $d_1$ in the model more broadly as a rise
4 Systemic risk

In this section, we investigate how systemic risk can be detected and measured before a financial crisis occurs. First, we study what distinguishes robust economies, the ones where the equilibrium is unique, from fragile economies, the ones where a financial crisis can occur. Then, we study how to detect the emergence of systemic risk in a specific economy over time. Finally, we study how to measure changes in systemic risk in a fragile economy over time.

To tackle these issues, we add an assumption about the likelihood of a financial crisis in the fragile economy. We assume that, if the economy has multiple equilibria, then a sunspot variable is realized right before banks make their decisions. The sunspot variable leads the economy in the financial crisis equilibrium, with probability \( q \in (0, 1) \), and in the normal equilibrium with probability \( 1 - q \). For illustrative purposes, we set \( q \) equal to 10%, so that on average there is a financial crisis every 10 years.

Detecting the existence of systemic risk

Some indicators of the existence of systemic risk are not reliable. The most obvious signal would be a high value for the sensitivity of banks’ financial assets to aggregate economic activity, or a high value of \( \kappa_Y \) in our model. However, the precise threshold that leads to multiple equilibria depends on the specifics of the economy, including the other parameters’ values. Hence, a high value for this sensitivity does not necessarily signal the existence of systemic risk.

Considering Table 2, one would be led to consider signals such as high default probability and leverage, low expected values of real activity, low in the cash outflow from banks to households, resulting from several factors including a drop of short-term borrowing. In the rest of the paper, we will disregard this model prediction about \( d_1 \).
equity values, low risk-free rates and high bond spreads. However, none of these signals is reliable, because all of these signals can be present in a robust economy.

To see this point, we compare our fragile economy with a robust economy with the same default probability and equity-asset ratio—In the latter economy, $\lambda = 1$, $a/Y = 3.4858$, and $b/Y = 6.7929$. As shown in Table 3, the expected values are very similar in the two economies, even though one is fragile and the other is robust. The only small differences are that, in the fragile economy, the risk-free rate (as well as the expected rate of return on a generic asset) tends to be slightly lower and the bond spread tends to be a little larger. In sum, this comparison shows that it is hard to detect the existence of systemic risk looking only at default probability and leverage, expected values and asset values, interest rates and spreads.

A clearer signal of financial fragility comes from the probability distributions of variables. These distributions can be obtained directly from the public’s expectations or indirectly from the prices of derivative securities. In the fragile economy, differently from the robust economy, the support of the probability distributions for aggregate real variables, such as loans and output, and financial variables, such as future interest rates and asset values, includes two values, the ones listed in Table 2. Standard deviations are non-zero in the fragile economy only. A measure of the probability of a financial crisis can be obtained from the probability of the worse mode. More generally, one would look for the presence of two modes and a larger standard deviation in the probability distributions.

The clearest signals of financial fragility come from the probability distributions of asset returns. We look at the returns of a risk-free bond promising a constant payoff at the end of the second period; portfolios of banks’ assets, debt and equity; the individual assets, debt and equity of each bank. As mentioned earlier, asset values after the realization of
the sunspot variable are computed using the discount factor \( \Lambda \). Asset values before the realization of the sunspot variable are computed simply as a weighted average (using the probabilities of the sunspot variable as a weight) of the post-realization asset values. From these asset values, we can compute two sets of rates of return: the returns from before the realization of the sunspot variable to immediately after the realization—the before-after-sunspot returns; and the returns from before the realization of the sunspot variable to the end of the second period—the total returns.

The rates of return from immediately after the realization of the sunspot variable to the end of the second period are not that interesting: the rates of return of the portfolios of banks’ assets, bonds and equity are the same as the risk-free rate, since there is no residual aggregate uncertainty; the ones of the individual banks’ assets, bonds and equity only reflect the realization of the idiosyncratic shock \( \omega \).

Table 4 shows the probability distributions of the total returns of the risk-free bond and of the portfolios. The probability distributions of both the equity portfolio and the bond portfolio include two values, differently from the robust economy. A measure of the probability of a financial crisis can be obtained from the probability of the worse return of the equity portfolio. The total return of the risk-free bond is, obviously, risk free.

Table 5 shows the probability distributions of the before-after-sunspot returns of the risk-free bond and of the portfolios. The probability distributions of the before-after-sunspot returns of individual banks’ assets, bonds and equity are the same as the ones of the corresponding portfolios. The probability distributions of the risk-free bond and of the equity portfolio include two values, differently from the robust economy. A measure of the probability of a financial crisis can be obtained from the probability of the worse return of the equity portfolio. The before-after-sunspot return of the risk-free bond is higher in a financial crisis. This is because the risk-free
rate drops to a negative value in the financial crisis (see Table 2), and this boosts the price of the risk-free bond. This effect is there for the prices of other assets as well, but, for other assets, it is offset by the drop of their expected payoffs. Risk-free bonds, then, are excellent hedges against the risk of a financial crisis.

Figure 2 plots the probability distributions of the total returns of an individual bank’s assets, bonds and equity. For bonds and assets, they tend to be bimodal as well. Equity returns, however, don’t show any bi-modality because of the long-call option feature of equity (as opposed to the short-call option feature of bonds). The changes in the distribution of returns of individual equities are hardly visible because of the large idiosyncratic risk of equities. A measure of the probability of a financial crisis can be obtained from the ratio of the second mode to the sum of the two modes of the probability distribution for assets. This measure, 0.1104, is close to the true probability of a financial crisis, 10%, and is increasing in it.

Table 6 compares the mean, standard deviation and correlation coefficient of the total returns of individual banks’ assets, bonds and equity. The volatility of returns of individual banks’ bonds is much larger in the fragile economy, with smaller recovery rates, which explains why bond spreads are higher. Correlations between different banks’ returns of assets, bonds and equity also increase, especially for bonds. The correlation coefficient in the robust economy is zero, since there is no aggregate uncertainty and the idiosyncratic risk is independent across banks. Correlation coefficients become positive in the fragile economy because of the possibility of a financial crisis that affects all returns. The correlation of bond returns is especially affected. The correlation between different asset classes (assets, bonds and equity) increases as well, as shown in Table 7.

To summarize, the following are some signs of financial fragility. The probability distribution of real variables and of returns of portfolios of eq-
uities and bonds becomes bi-modal, with a higher standard deviation. The distribution of returns of individual bonds becomes bi-modal, with a higher standard deviation. Correlations generally rise, especially across bonds of different banks.

**Detecting the emergence of systemic risk**

We next turn to detecting the emergence of systemic risk in a specific economy over time. Suppose that the economy is initially robust, but then an increase in $\lambda$ (due to an increase in $a_Y$) makes it fragile, i.e., a financial crisis equilibrium emerges besides the normal equilibrium. The increase in $\lambda$ leaves the normal equilibrium unchanged (the default probability and all expected values remain the same), and simply adds the possibility of a financial crisis. Before the transition, the economy has a unique normal equilibrium, the same as the one in Table 2; after the transition, it has the two equilibria in Table 2.

The emergence of systemic risk, the transition from a robust economy to a fragile economy has a sudden effect on expected values. Table 8 shows that the following changes in expected values would be associated with the emergence of a financial crisis equilibrium: a surge of the default probability, a drop of the expected values of loans, output, equity values (and capital ratios) a drop of the risk-free rate (and more generally of expected returns) and a surge of the bond spread. Bond values and bond yields are less informative because of the countervailing forces of the drop of the risk-free rate and the increase in the bond spread.

However, the above changes can be due to several other causes. For instance, a drop of aggregate productivity $A$ has similar effects (see Table 9). Hence, it is, again, better to rely on probability distributions of variables and of returns. The distribution of aggregate variables and of portfolio
returns, will tend to have two modes, and a higher standard deviation.\textsuperscript{10} The same is true for individual banks’ bond returns; returns of individual banks’ equities and bonds will tend to have higher correlations among each other (see Table 10). A second mode and a long tail of negative returns appear in the probability distribution of each bank’s asset and bond total returns (see Figure 3).

**Measuring an increase in systemic risk**

We finally study how to measure an increase in systemic risk in a fragile economy over time. Suppose that the economy continues to have the same two equilibria, but the probability $q$ of the financial crisis equilibrium rises.

The changes that would signal the emergence of systemic risk would also signal a rise of systemic risk. For instance, a surge of the default probability, a drop of the expected values of loans, output, equity values (and capital ratios), a drop of the risk-free rate (and more generally of expected returns) and a surge of the bond spread\textsuperscript{11} (see Figure 4). Again, it is better to rely on probability distributions of portfolios and individual banks’ securities. Two modes tend to appear, and standard deviations and correlations tend to increase. Notice, however, that this is only true in the region where the probability of a financial crisis is small. (see Figures 5, 6 and 7).

If the probability $q$ is small, a measure of the probability of the financial crisis can be obtained from the probability of the second mode in the case of portfolio returns, or the ratio of the second mode to the sum of the two modes of the probability density function of returns of individual banks’ assets (see Figure 8).

\textsuperscript{10}Most tables and figures of comparison are omitted because they are similar to the ones in the previous section.
\textsuperscript{11}The expected values can be easily computed by averaging the values of Table 2.
5 Some regulatory tools to prevent the emergence of systemic risk

The potential for multiple equilibria arises from the positive spillovers of bank loans generated by the dependence of the expected marginal return of loans on aggregate lending. This dependence, in turn, follows from the dependence of bank asset values on aggregate output and, more generally, from the common default risk exposure of banks to the economic and financial cycle. This suggests that, in order to prevent the emergence of the type of systemic risk described in this paper, macroprudential regulation should aim at limiting the elasticity \( \Phi_Y \) of the repayment probability to aggregate output, defined in (12). Notice that this is different from limiting the individual risk of default of a financial institution, or the overall risk of default of the financial system. In fact, limiting the latter does not necessarily eliminate the financial fragility—the risk of default may be tiny even in a financially fragile economy, provided that the probability \( q \) of a financial crisis is tiny.

There are a few regulatory tools that can be used to limit the elasticity \( \Phi_Y \) of the repayment probability to aggregate output. A first tool is restricting banks from investing in a subset of assets that are sensitive to the business cycle or to the financial sector—whenever this restriction is not in contrast with bank business model. As is clear from equation (5), limiting interbank lending and investment, and more generally bank interconnectedness, has an especially large impact on limiting the sensitivity. The rationale behind this restriction is not the traditional one. Legislation that restricts banks from trading and investing mainly aims at preventing banks from using deposits for speculative activities.\(^{12}\) There may be,

\(^{12}\)In the U.S., the Glass-Steagall legislation (Banking Acts of 1933 and 1935) separated commercial banks from investment banks and securities firms; prohibited the former
also, concerns about potential conflicts of interest. This paper, however, suggests that restricting banks’ speculative investment in securities helps reduce the systemic risk associated with the correlation between bank assets and economic activity.

In the case of bank investment that cannot be restricted, macroprudential regulation could impose capital requirements aimed at limiting the bank default risk exposure to the economic and financial cycle. This capital requirement, aimed at eliminating the financial fragility, is in addition to any capital requirement aimed at ensuring the safety of the individual financial institutions. As noted by Hanson, Kashyap and Stein (2013), the capital should be in the form of common equity, assuming that managers act in the interest of the equity holders, not in the form of preferred equity or of other senior types of capital, which still create a debt overhang distortion on lending.

To set the capital requirements, a tool that is well suited is a stress test that evaluates the capital strength and the bank risk in the event of a financial crisis. Alternatively, a capital ratio could be used with risk weights set to limit the elasticity $\Phi_Y$ of the repayment probability to aggregate output. From equation (12), it follows that

$$\Phi_Y = \frac{\Phi'(\delta)}{\Phi(\delta)} e^{\delta^2/2} \sigma' \pi'(Y)$$

(13)

For standard parameter values, for instance for $\sigma \leq 1$ and for $\delta \geq 1$, the right hand side is a decreasing function of the distance to default $\delta$. Hence, in order to limit $\Phi_Y$, macroprudential regulation could require a larger distance to default $\delta$ whenever the sensitivity $\pi'(Y)$ of assets to aggregate output from investing in non-investment grade securities for their own portfolio, and from dealing, underwriting and distributing non-government securities; and prohibited the latter ones from taking deposits. the Volcker rule (2010 Dodd-Frank Act) restricts banks and affiliates from proprietary trading, while allowing hedging as well as trading, market-making and dealing as services for customers.
output is higher. That is, it could require a higher capital ratio, which entails a larger distance to default $\delta$ and a lower risk of default $1 - \Phi(\delta)$, for banks with assets that are more sensitive to the cycle—banks with higher-beta assets. Risk weights, then, should be set higher for higher-beta assets that are more sensitive to the economic and financial cycle, and could be set negative for assets that co-vary negatively with the cycle.

6 Conclusions

In this paper, we have studied the risk of a debt-overhang banking crisis—a self-fulfilling expectations crisis that can arise because banks’ assets are sensitive to aggregate conditions, and banks’ liabilities distort their lending. We hope that the debt-overhang mechanism that we have described will be useful to explain a variety of economic and banking crises where a rise of the risk of default in the banking sector is accompanied by the plunge of assets sensitive to economic activity, such as financial securities, sovereign debt and the exchange rate. One example is a banking crisis associated with a crash of markets for financial securities, including equities, bonds and asset-backed securities. Another example is a banking crisis associated with a sovereign default. If banks invest in government bonds, then the economy is vulnerable to a debt-overhang financial crisis where economic activity declines, the tax revenue drops, the government defaults, bank balance sheets weaken, and the debt overhang distortion rises and discourages lending. A final example is an economic and banking crisis associated with a currency crisis. If bank assets are denominated in the domestic currency, while bank liabilities are denominated in a foreign currency, and if the exchange value of the local currency depends on domestic economic activity, then the economy is vulnerable to a debt-overhang financial crisis where domestic economic activity declines, the local currency depreciates,
bank balance sheets weaken, and the debt overhang distortion rises and
discourages lending.

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Federal Reserve Bank of Cleveland or of the Board of Governors of the
Federal Reserve System.

A Uniqueness of equilibrium when bank assets are not sensitive to aggregate output

In this appendix, we show that, if \( \pi(Y) \) does not depend on \( Y \), i.e. \( \pi(Y) = \bar{\pi} \)
constant, then the equilibrium is unique.

First, we show that the solution for \( l \) in the bank’s problem (9) does
not depend on \( Y \) and is increasing in \( \Lambda \). The bank’s problem is equivalent
to

\[
\max_l \{(m - l) + \Lambda g(l)\}
\]

given \( \Lambda \), where \( g(l) \equiv E\{\max((\omega f(l) + \bar{\pi} - b, 0)) \} \) is increasing. Since the
problem does not depend on \( Y \), the solution does not depend on \( Y \) either.
To show that the solution for loans \( l \) is increasing in \( \Lambda \), consider \( \Lambda_1 < \Lambda_2 \),
and let \( l_1 \) and \( l_2 \) be the arg max of the previous problem with, respectively,
Λ = Λ₁ and Λ = Λ₂. By definition,

\[(m - l_2) + \Lambda_2 g(l_2) \geq (m - l_1) + \Lambda_2 g(l_1)\]
\[(m - l_1) + \Lambda_1 g(l_1) \geq (m - l_2) + \Lambda_1 g(l_2)\]

Summing side by side,

\[(m - l_2) + \Lambda_2 g(l_2) + (m - l_1) + \Lambda_1 g(l_1) \geq (m - l_1) + \Lambda_2 g(l_1) + (m - l_2) + \Lambda_1 g(l_2)\]
\[\Lambda_2 g(l_2) + \Lambda_1 g(l_1) \geq \Lambda_2 g(l_1) + \Lambda_1 g(l_2)\]
\[(\Lambda_2 - \Lambda_1)(g(l_2) - g(l_1)) \geq 0\]
\[g(l_2) - g(l_1) \geq 0\]
\[l_2 - l_1 \geq 0\]

which completes the proof.

Next, we use the equilibrium equations to obtain that the discount factor is given by

\[\Lambda = \frac{\beta u'(e_2 + E\{\omega\} f(l))}{u'(e_1 + m - l)}\]  

(14)

so it is a strictly decreasing function of the equilibrium aggregate loans \(l\).

Recall that the solution for loans \(l\) in the bank’s problem does not depend on \(Y\) and is increasing in \(\Lambda\). It follows that there is a unique set of values \(\{l, \Lambda\}\) that satisfy equation (14) and such that \(l\) solves the bank’s problem given \(\Lambda\).

Finally, using the equilibrium equations, it is easy to show that there is a unique set of values \(\{d_1, d_2, l, y, Y, e_1, e_2, \Lambda\}\) that satisfy the equilibrium conditions, i.e., the equilibrium is unique.

**References**


Cycle Model. European Economic Review 73, 58-84.


**Parameter values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$E{\omega}$</td>
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<tr>
<td>$\alpha$</td>
<td>0.3000</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9900</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.8000</td>
</tr>
<tr>
<td>Consumption-loans ratio</td>
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</tr>
<tr>
<td>Consumption growth rate</td>
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<tr>
<td>$\bar{a}/Y$</td>
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<tr>
<td>$a_V$</td>
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</tr>
<tr>
<td>$a_Y$</td>
<td>3.0000</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2.0000</td>
</tr>
<tr>
<td>$\bar{\kappa}/Y$</td>
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<tr>
<td>$\kappa_Y$</td>
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</tr>
<tr>
<td>$b/Y$</td>
<td>7.2043</td>
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<tr>
<td>Dividends-equity ratio</td>
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<tr>
<td>Equity-asset ratio</td>
<td>0.1100</td>
</tr>
</tbody>
</table>

Table 1: Values of parameters and of selected variables in the normal equilibrium.
Equilibrium values in the fragile economy

<table>
<thead>
<tr>
<th></th>
<th>Normal equilibrium</th>
<th>Crisis equilibrium</th>
<th>Percent difference</th>
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</thead>
<tbody>
<tr>
<td>Default probability</td>
<td>0.0225</td>
<td>0.7147</td>
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<td>Loans</td>
<td>0.1667</td>
<td>0.1014</td>
<td>-39.1730</td>
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<tr>
<td>Output</td>
<td>0.5766</td>
<td>0.4968</td>
<td>-13.8552</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>0.0354</td>
<td>-0.1297</td>
<td></td>
</tr>
<tr>
<td>Bond spread</td>
<td>0.0001</td>
<td>0.0448</td>
<td></td>
</tr>
<tr>
<td>Bond yield</td>
<td>0.0355</td>
<td>-0.0849</td>
<td></td>
</tr>
<tr>
<td>Asset value</td>
<td>4.5079</td>
<td>4.7768</td>
<td>5.9640</td>
</tr>
<tr>
<td>Bond value</td>
<td>4.0120</td>
<td>4.5399</td>
<td>13.1557</td>
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<tr>
<td>Equity value</td>
<td>0.4959</td>
<td>0.2369</td>
<td>-52.2234</td>
</tr>
<tr>
<td>Equity-asset ratio</td>
<td>0.1100</td>
<td>0.0496</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Equilibrium values in the normal equilibrium and in the financial crisis equilibrium.
Expected values in the robust and fragile economies

<table>
<thead>
<tr>
<th></th>
<th>Robust economy</th>
<th>Fragile economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default probability</td>
<td>0.0917</td>
<td>0.0917</td>
</tr>
<tr>
<td>Loans</td>
<td>0.1667</td>
<td>0.1601</td>
</tr>
<tr>
<td>Output</td>
<td>0.5849</td>
<td>0.5687</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>0.0354</td>
<td>0.0188</td>
</tr>
<tr>
<td>Bond spread</td>
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<td>0.0046</td>
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<td>Bond yield</td>
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<tr>
<td>Asset value</td>
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<td>4.5348</td>
</tr>
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<td>Bond value</td>
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</tr>
<tr>
<td>Equity value</td>
<td>0.4448</td>
<td>0.4700</td>
</tr>
<tr>
<td>Equity-asset ratio</td>
<td>0.1040</td>
<td>0.1040</td>
</tr>
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</table>

Table 3: Expected values in the robust and fragile economies. In the robust economy $\lambda = 1$, while in the fragile economy $\lambda = 2$. The parameters $\bar{a}$ and $b$ are changed so that the default probability and the equity-asset ratio are the same in the two economies.

Total returns of portfolios

<table>
<thead>
<tr>
<th></th>
<th>Normal equilibrium</th>
<th>Crisis equilibrium</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free bond</td>
<td>0.0161</td>
<td>0.0161</td>
<td>0.0161</td>
<td>0.0000</td>
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<tr>
<td>Asset portfolio</td>
<td>0.0292</td>
<td>-0.0832</td>
<td>0.0180</td>
<td>0.0337</td>
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<tr>
<td>Bond portfolio</td>
<td>0.0219</td>
<td>-0.0280</td>
<td>0.0169</td>
<td>0.0150</td>
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<tr>
<td>Equity portfolio</td>
<td>0.0924</td>
<td>-0.5613</td>
<td>0.0270</td>
<td>0.1961</td>
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</table>

Table 4: Total returns of the risk-free bond and of portfolios of bank securities in the fragile economy.
### Before-after-sunspot returns of portfolios

<table>
<thead>
<tr>
<th></th>
<th>Normal equilibrium</th>
<th>Crisis equilibrium</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free bond</td>
<td>-0.0186</td>
<td>0.1675</td>
<td>-0.0000</td>
<td>0.0558</td>
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<td>Asset portfolio</td>
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<td>0.0000</td>
<td>0.0178</td>
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<tr>
<td>Bond portfolio</td>
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<td>0.1169</td>
<td>-0.0000</td>
<td>0.0390</td>
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<tr>
<td>Equity portfolio</td>
<td>0.0551</td>
<td>-0.4959</td>
<td>0.0000</td>
<td>0.1653</td>
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</table>

Table 5: Before-after-sunspot returns of the risk-free bond and of portfolios of bank securities in the fragile economy.

### Total returns of bank securities

<table>
<thead>
<tr>
<th></th>
<th>Robust economy</th>
<th></th>
<th>Fragile economy</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>0.0354</td>
<td>0.1294</td>
<td>0.0000</td>
<td>0.0180</td>
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<tr>
<td>Bonds</td>
<td>0.0354</td>
<td>0.0038</td>
<td>0.0000</td>
<td>0.0169</td>
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<tr>
<td>Equity</td>
<td>0.0354</td>
<td>1.2378</td>
<td>0.0000</td>
<td>0.0270</td>
</tr>
</tbody>
</table>

Table 6: Moments of total returns of individual bank securities in the robust and fragile economies. In the robust economy $\lambda = 1$, while in the fragile economy $\lambda = 2$. The parameters $\tilde{a}$ and $b$ are changed so that the default probability and the equity-asset ratio are the same in the two economies.
Correlation of total returns of bank securities in the fragile economy

<table>
<thead>
<tr>
<th></th>
<th>Assets</th>
<th>Bonds</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
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<td>0.2034</td>
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<tr>
<td>Bonds</td>
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<tr>
<td>Equity</td>
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<td>0.1274</td>
<td>0.0293</td>
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Table 7: Correlation of total returns of bank securities in the fragile economy.

Expected values after the emergence of systemic risk

<table>
<thead>
<tr>
<th></th>
<th>Robust economy</th>
<th>Fragile economy</th>
<th>Percent difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default probability</td>
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<td>0.0917</td>
<td></td>
</tr>
<tr>
<td>Loans</td>
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<td>0.1601</td>
<td>-3.9173</td>
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<td>Bond spread</td>
<td>0.0001</td>
<td>0.0046</td>
<td></td>
</tr>
<tr>
<td>Bond yield</td>
<td>0.0355</td>
<td>0.0234</td>
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<tr>
<td>Asset value</td>
<td>4.5079</td>
<td>4.5348</td>
<td>0.5964</td>
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<td>0.4959</td>
<td>0.4700</td>
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<tr>
<td>Equity-Asset ratio</td>
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<td>0.1040</td>
<td></td>
</tr>
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</table>

Table 8: Expected values in the robust and fragile economies. In the robust economy $\lambda = 1$, while in the fragile economy $\lambda = 2$. The other parameters are the same.
Expected values after a productivity shock

<table>
<thead>
<tr>
<th></th>
<th>Robust economy</th>
<th>Productivity shock</th>
<th>Percent or difference</th>
</tr>
</thead>
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<td>-1.1807</td>
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<td>Risk-free rate</td>
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<td>0.0259</td>
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<tr>
<td>Bond spread</td>
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<td>0.0003</td>
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</tr>
<tr>
<td>Bond yield</td>
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<tr>
<td>Bond value</td>
<td>4.0120</td>
<td>4.0485</td>
<td>0.9091</td>
</tr>
<tr>
<td>Equity value</td>
<td>0.4959</td>
<td>0.4752</td>
<td>-4.1651</td>
</tr>
<tr>
<td>Equity-Asset ratio</td>
<td>0.1100</td>
<td>0.1050</td>
<td></td>
</tr>
</tbody>
</table>

Table 9: The effect of a contractionary productivity shock in a robust economy. In both economies, $\lambda = 1$. In the second economy, aggregate productivity $A$ is lower by 1%. The other parameters are the same.

Total returns of bank securities after the emergence of systemic risk

<table>
<thead>
<tr>
<th></th>
<th>Robust economy</th>
<th>Fragile economy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Std. Dev.</td>
<td>Mean Std. Dev.</td>
</tr>
<tr>
<td>Assets</td>
<td>0.0354 0.1211</td>
<td>0.0180 0.1235</td>
</tr>
<tr>
<td>Bonds</td>
<td>0.0354 0.0009</td>
<td>0.0169 0.0201</td>
</tr>
<tr>
<td>Equity</td>
<td>0.0354 1.1002</td>
<td>0.0270 1.1462</td>
</tr>
</tbody>
</table>

Table 10: Moments of total returns of individual bank securities in the robust and fragile economies. In the robust economy $\lambda = 1$, while in the fragile economy $\lambda = 2$. The other parameters are the same.
Self-fulfilling expectations crisis

Poor economic prospects $\rightarrow$ Bank asset value declines

Bank lending contracts $\leftrightarrow$ Debt overhang worsens

Figure 1: Main debt-overhang mechanism leading to a self-fulfilling expectations banking crisis.
Probability distribution function of total returns of bank securities

Figure 2: Probability distribution function of total returns of bank securities. The solid lines and the diamond signs refer to a fragile economy where $\lambda = 2$, the dashed lines and the square signs refer to a robust economy where $\lambda = 1$. The parameters $\tilde{a}$ and $b$ are changed so that the default probability and the equity-asset ratio are the same in the two economies.
Figure 3: Probability distribution function of total returns of bank securities. The solid lines and the diamond signs refer to a fragile economy where \( \lambda = 2 \), the dashed lines and the square signs refer to a robust economy where \( \lambda = 1 \). The other parameters are the same.
Figure 4: Expected values as functions of the probability $q$ of a financial crisis.
Standard deviations of before-after-sunspot returns

Figure 5: Standard deviations of before-after-sunspot returns as functions of the probability $q$ of a financial crisis.
Figure 6: Standard deviations of total returns as functions of the probability $q$ of a financial crisis.
Correlations of total returns

Figure 7: Correlations of total returns as functions of the probability $q$ of a financial crisis.
Figure 8: Probability distribution function of total returns of bank securities. The solid lines and the diamond signs refer to a fragile economy where the probability of a financial crisis is equal to $q = 10\%$, the dashed lines and the square signs refer to a fragile economy where the probability of a financial crisis is equal to $q = 20\%$. The other parameters are the same.