Productivity shocks, capital intensities, and bank interest rates

Enzo Dia, Lorenzo Menna

July 20, 2015

Abstract

This paper proposes a real business cycle model where banks face significant resource costs to provide financial services, aiming to study the impact of productivity shocks on the industrial and the banking sector. We focus in particular on the impact of these shocks on bank interest rates and margins, and on the interactions between the industrial and the banking systems. We find that resource costs explain a large share of bank interest margins, particularly when banks are inefficient. Idiosyncratic shocks hitting the productivity of the banking industry produce impacts that are always negligible on the rest of the economy. Positive productivity shocks hitting the industrial sector induce sizable increases of the rate on loans, and sharp increases in the total hours worked in the banking system. Following symmetric productivity shocks, the spread falls significantly at impact, then overshoots its long-run value. The pattern obtained from the model replicates very closely the empirical response of bank interest margins to the productivity shock obtained from a structural VAR.

† Ricercatore, Dipartimento di Economia, Metodi Quantitivi e Strategie di Impresa, Università degli Studi di Milano Bicocca, Piazza Ateneo Nuovo 1, Milano 20126, Italy.

‡ Dipartimento di Economia, Metodi Quantitivi e Strategie di Impresa, Università degli Studi di Milano Bicocca, Piazza Ateneo Nuovo 1, Milano 20126, Italy.

*Corresponding author. Email: enzo.dia@unimib.it
†Email: lorenzo.menna@unimib.it
1 Introduction

The industrial organization approach to the study of the banking industry beginning with Klein (1971) and Monti (1972) has given due consideration to the role of resource costs in modelling the banking firm. On the contrary, in most of the dynamic stochastic general equilibrium models where banks play a relevant role, as for example Gertler and Karadi (2011) or Gertler et al. (2012), banks have not been treated as firms with a significant resource cost base. Bank margins are thus shaped by the interest costs of deposits, by the cost of equity, or by the loan loss provisions on the loan portfolio. These models, as others analyzing the role of financial markets in the transmission of shocks, have helped improving our understanding of the links between financial and real variables, and have explored the possibility that innovations originating in the financial sector may generate important effects on macro variables such as output or unemployment. Other models that instead explicitly require resource costs in banking activity adopt a calibration strategy that take interest rates only as exogenous parameters from the data, while all other banking parameters are obtained endogenously. As a consequence, also in these models banks are not constrained by the availability and costs of resources. But the absence from the analysis of the resource cost structure of banks severs a potentially important transmission mechanism of the impulses from the real economy to the banking industry. And resource costs matter for banks.

Fig. 1 displays the relative share of total costs represented by interest costs, non-interest costs, and charge-offs for all the commercial banks insured by the FDIC. With the exclusion of the period of very high inflation, resource costs are by far the main determinant of the cost base. Even in periods of high inflation they are sizable, but in a low interest rates environment little else matters. The charge-offs are normally a small share of the cost base, to become large only during the heavy recessions of the 1930s and of 2009. Even during this periods of extreme stress, the average level of these costs for the banking system of the United States has remained significantly below that of
Figure 1: Relative shares of total costs represented by interest costs, non-interest costs, and charge-offs for all commercial banks insured by the FDIC of the U.S..

This paper proposes a real business cycle model where banks face significant resource costs to provide financial services, aiming to study the impact of productivity shocks on the industrial and the banking sector. We focus in particular on the impact of these shocks on bank interest rates and margins, and on the interactions between the industrial and the banking systems. We keep the model as simple as possible, by excluding from the analysis the monetary role of deposits. We thus focus on the traditional role of the banking industry to provide loans to finance the working capital of industrial firms.
The model is similar to that of Chari et al. (1995), although we focus on real variables only. The most important difference between the two approaches regards the way in which the banking industry is introduced in the general equilibrium framework. Chari et al. (1995) use data on bank interest rates as exogenous variables, and calibrate all other parameters defining the banking industry. We follow a different strategy. We examine bank balance sheet data for several countries and obtain the average capital intensities and measures of total factor productivity for the banking industry at the country level. Similarly, we use aggregate data on business lending to set the value of the cash in advance constraint for several countries. We obtain endogenously all other parameters defining the banking industry, including, crucially, interest rates. To evaluate the model, we check if the interest rates generated by the model are compatible with those observed in the data. Finally, we study the impact of productivity shocks on bank rates as well as on other banking and industrial sector variables for the different economies.

We find that the bank interest rates generated by the model match the data extremely well, and that resource costs are the main determinants of bank interest spreads. Differentials in bank productivity and in the productivity of the industrial sector explain most of the variability across countries of bank rates on loans and bank interest margins.

Idiosyncratic shocks hitting the productivity of the banking industry produce impacts that are always negligible on the rest of the economy. This result holds for any country, even those as Spain where banks are large relative to the rest of the economy, and quite inefficient. Interest costs are not large enough to induce significant reallocations of resources, even though banks productivity shocks produce sizeable effects on the interest rate on loans.

Shocks affecting the productivity of the industrial sector have a differentiated impact on banks of different countries, but the impact is always large. The total number of hours worked in banks is far more volatile than that of hours worked in other industrial sectors. Positive productivity shocks induce sizable increases of the rate on loans, driven in particular by the increase of the rate on deposits, but also by the increase of the interest rate spread. The latter grows because of the wage and capital rental rate hikes produced by the higher productivity of firms.

\[^1\] A similar strategy is also adopted by Goodfriend and McCallum (2007), who use the risk-free interest rate, the equilibrium spread and the share of labor employed in the banking industry as exogenous parameters.
When the productivity shock is symmetric for banks and firms, and banks are less capital intensive than firms (and provide evidence suggesting this to be the case for the US and for most European countries), the spread falls significantly at impact, than overshoots its long run value. Symmetric productivity shocks induce a strong increase in the cost of capital at impact and a more delayed and persistent increase in wages. Since banks have a lower capital intensity than the industrial sector, following the shock marginal costs rise less than productivity and interest margins decline. As the shock is felt through the system and wages gradually grow, however, banks respond by charging higher interest rates on loans and interest margins overshoot. To test the predictions of the model we estimate a structural VAR, analyzing the dynamics of bank interest spreads in response to technology shocks in the United States for the period 1956-2011. The pattern obtained from the model replicates very closely the empirical response of bank interest margins to the productivity shock obtained from the VAR.

Our analysis of technology shocks is closely related to that of Goodfriend and McCallum (2007), and our results, when comparable, are in line with their findings. In both models banks create liquidity, for industrial firms in one case, for households in the other, but since in both cases banks adopt a technology with constant returns to scale, the impact of the quantity of loans on the interest rate is relatively minor. Moreover, in both models labor costs are the main transmission mechanism from shocks originating in the economy to the banking industry and banks’ interest rates and spreads. The important difference between the models regards the role of other resources and productivity, as we impose restrictions on the parameters of the model from banking data on fixed assets and productivity, while in the model by Goodfriend and McCallum (2007) banks use physical capital as collateral, and the parameters for collateral and bank productivity are obtained endogenously in the calibration by matching data on spreads and interest rates.

In the next section we discuss the relevant literature, Section 3 lays out a real business cycle model with a banking industry that provides short-term loans to industrial firms facing a cash-in-advance constraint. Section 4 shows the results of the quantitative analysis of the model and discusses the implications of the results. Section 5 provides a summary of the paper’s conclusions.
2 Literature review

In this section we discuss the DSGE models that incorporate banks facing resource costs.

King and Plosser (1984) introduced money and banking into a real business cycle model, and explained the correlation between money and business cycles as the result of reverse causation. They modelled a financial industry composed of banks providing accounting services to households and firms that reduce the amount of time and other resources necessary to undertake market transactions. Households purchase consumption and investment goods using time and transaction services, and can use either currency or services provided by banks. The final goods industry needs financial services as an input in the production function. Financial firms use labor and capital to produce transaction services according to a standard production function with constant returns to scale. Banks issue deposits through simple bookkeeping entries and pass on to depositors the return to the portfolio of assets that they acquire less a fee for transaction services. On the assets side of the balance sheet, banks hold shares of firms producing final goods. This framework provides a theoretical background for the strong correlation between output and deposits that data display. In this model, banks potentially influence output, while they do not influence the price level, as the model follows the approach of Fama (1980) suggesting that direct costs for the provision of transaction services are not influenced by the composition of the portfolio of banks' liabilities.

In the model proposed by Christiano and Eichenbaum (1992) banks potentially influence the price system, and affect the transmission of monetary policy, to the extent that they allow the propagation of liquidity effects. They assume that households and firms face cash-in-advance constraints, and perfectly competitive financial intermediaries lend to final good producing firms the resources raised from households. The latter, however, can change the allocation of their savings only after shocks hitting the system are realized, otherwise they face significant costs in adjusting the sums lent to financial intermediaries. The analysis of the impulse-response functions suggest that under these assumptions nominal interest rates decline, while consumption and hours worked rise in response to a shock in the growth rate of money.

Chari et al. (1995) merges both previous approaches, as households, constrained in their ability
to manage liquidity, lend resources to banks that use labor and capital to provide deposit services for households, and short-term loans to finance the working capital needs of final good producing firms facing a cash-in-advance constraint. The liquidity effects of the model are produced by the assumption that households cannot adjust their currency holdings immediately in response to shocks. The model explains the positive correlation between broader monetary aggregates and output as the effect of productivity shocks on money, in line with King and Plosser (1984). In principle shocks to the productivity of banks influence output, as the productivity of the financial sector has a strong impact on the rate on loans, which is a significant cost factor for final goods producers. However, it turns out that the impact of shocks to banks’ productivity is irrelevant. It is important to note that the only data from the banking industry that are used as inputs in the model are the interest rates on loans and a parameter that depends on reserve requirements, while all other parameters are calibrated to match macroeconomic data, and respond endogenously to the other variables of the system.

In the model developed by Freeman and Kydland (2000) households can allocate their savings to three different assets: Capital, fiat money and bank deposits. Banks generate money by acting as intermediaries in the investment process, as they invest deposits raised from households in the capital stock of industrial firms, and reserves of currency, so that deposits finance investment, rather than working capital as in previous approaches. Banks do not make use of resources; they are rather a technology, actually a rule, transforming part of the capital stock in a liquid security that can be used for transaction purposes. The key feature of the model is the simple transaction technology developed, where households face both a cost of acquiring money balances proportional to the amount of goods consumed, and a fixed cost of using deposits, generating the endogenous responses of the velocity of money and the money multiplier over the business cycle. An important implication of the model is the capability to explain the persistence of money holdings as income and nominal interest rates fluctuate, without imposing any price rigidity.

The model by Freeman and Kydland (2000) has been widely used afterwards. Sustek (2010) introduced a two stage production function where the production of final goods requires a composite intermediate good on top of capital and labor. Households hold and rent both capital and the
intermediate good. Banks issue both checkable deposits, subject to reserve requirements, and
time deposits, investing the proceeds in capital and intermediate inputs. Banks thus finance both
physical capital and working capital, the last being represented by the intermediate good, as the
author assumes that this last depreciates entirely in one period. Households face a fixed cost to use
time deposits. Finally, although the production function of banks is not explicitly modelled, banks
face different, separate linear costs for servicing time and checkable deposits. The cost parameters
are then calibrated in order to obtain values for the interest rates on checkable and time deposits
in line with the available data. The main implication of the model is that when the central bank
adopts a Taylor rule, the deposits creation process implies that a monetary aggregate like MZM,
consisting predominantly of demand deposits, systematically leads output, in line with the available
empirical evidence.

In a similar fashion, making use of a limited participation model, Dressler and Li (2009) intro-
duce a role for credit intermediation independent from the assets of intermediaries financed with
deposits. In the model, a specific credit industry uses labor to provide credit services to households
and firms, while banks provide deposit services to households and loanable funds to the credit sec-
tor. Assets of banks different from cash reserves are liabilities of households and firms. Households
require credit to purchase durable goods, while they use currency or deposits to purchase non-
durable goods; firms finance investment with current cash flows and credit, in fixed proportions.
When a positive technology shock hits the system, the demand for loanable funds grows, pushing
up both household deposits and credit, because of the additional investment expenditure and the
increase of the nominal interest rates.

The models discussed so far focus on the monetary role of deposits, and analyze the impact of
technological shocks hitting industrial firms, which increase the issuance of loans and symmetrically
the amount of deposits funds demanded by banks. As the focus of the analysis are the quantities,
these models take bank interest rates as exogenous data and all other variables are calibrated to
match macroeconomic variables. The role of technological shocks hitting the banking industry is
seldom explored. The only exception in the traditional RBC literature is represented by Chari et al.
(1995), who find that they have an irrelevant impact on output, even if neither money nor banks
are neutral in the transmission of shocks.

More recently though Kollmann (2013) has proposed a two-country model in the context of the International Real Business Cycle literature, in which a global bank intermediates between savers and borrowers in the two countries. The bank of the model is constrained by its equity position, produces loans and deposits using a constant return to scale technology, and in the calibration of the model the parameters of the cost function are endogenous. An important feature of the model is that loan losses originating in a country generate a spillover to the other country as output declines in both, so that banks transmit shocks internationally. Kollmann has estimated the model using U.S. and euro area (EA) data, finding that during the 2007–09 recession banking shocks accounted for a significant share of the fall in U.S. and EA GDP. However, over the entire sample banking shocks explain a relatively small size of the unconditional variance of GDP, particularly in the United States.

Other studies have analyzed the interactions of technology, financial and monetary shocks by introducing banks in New Keynesian DSGE models with credit frictions. Goodfriend and McCallum (2007), in particular, develop a model where both the value of collateral and resource costs of the banking industry influence the transmission of shocks, generating either attenuating or accelerating effects, depending on the nature of the shocks. Banks are not neutral in the economy because household acquire bank deposits in the model to self-insure against liquidity risk, and borrow from banks in order to fund deposits. Loans are produced with a Cobb-Douglas technology requiring labor to manage and monitor loans, and collateral that enables a bank to enforce the repayment of loans with less monitoring and management effort. Households can use either bonds or the stock of capital that they own as collateral, but physical capital is less valuable than bonds. Goodfriend and McCallum use the model to analyze different interest rates and spreads, and the responses of rates and spreads to technology shocks, monetary policy shocks, and financial shocks hitting the productivity of banks’ labor or the value of the capital held as collateral. The exogenous parameters in the calibration of the banking industry are the risk-free interest rate, the equilibrium value of the spread, and the share of employment in the banking industry, while the relative share of the

\[2\] The details of the model are discussed in Goodfriend (2005).
different factors, the productivity of the bank, and the parameter measuring the weight of physical capital as collateral are all endogenous.

Curdia and Woodford (2010), following the approach of Goodfriend and McCallum (2007), analyze the costs and benefits of the introduction of a policy response to changes of bank interest spreads in the Taylor rule. The analysis is based on a DSGE model with financial frictions where banks use real resources to originate loans. Curdia and Woodford (2011) adopt the same approach to analyze the impact of the payment of interest on bank reserves as a tool of monetary policy. In both models the bank interest spread depends on the share of loans that will not be repaid and on the marginal resource cost, which is not constant as the cost function is assumed to be convex. In the calibration of the model the parameter of the resource cost function is obtained by assuming a steady-state credit spread equal to 2 percent per annum, together with zero steady-state default costs. They then define two types of credit shock, with the default cost shock affecting the intercept of the loan supply schedule, and an exogenous shock to the resource cost function affecting the slope parameter.

Gerali et al. (2010) have developed a banking model by adapting to the banking industry the Dixit-Stiglitz framework of monopolistic competition, and introduced the model in a DSGE with credit frictions and borrowing constraints. In the model households and entrepreneurs purchase baskets of slightly differentiated loans and deposit contracts from two classes of banks, wholesale and retail banks. Bank interest margins ultimately depend on on the banks’ capital-to-assets ratio and on the degree of interest rate stickiness, as both retail and wholesale banks are subject to adjustment costs when changing interest rates. The empirical estimates of the model using data for the euro area suggest that banks smooth monetary policy and technology shocks, but at the same time induce sizeable volatility, as shocks originating in the banking sector explain a large a share of the recession beginning in 2008.
3 The Model

3.1 Households

We consider an infinitely lived representative household whose utility depends positively on consumption \((c)\) and negatively on the number of hours worked \((h)\), and whose endowment of time in each period is normalized to one. The same good can be used either for consumption or for investment purposes, and household can allocate their savings to two alternative assets: Physical capital, \(k\) and deposits, \(d^s\). The representative household solves the following problem:

\[
\max_{c} \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \theta \frac{(1-h_t)^{1-\gamma} - 1}{1-\gamma} \right],
\]

s.t.

\[
c_t + k_t - (1-\delta) k_{t-1} + d^s_t = w_t h_t + r_d t_k_{t-1} + r_d t d^s_{t-1} + \Pi^f_t + \Pi^b_t,
\]

where \(w\) is the real wage, \(r_d\) is the gross interest rate on deposits, \(r\) is the rental rate for capital and \(\Pi^f\) and \(\Pi^b\) are dividends paid by firms and banks. \(\beta\) is the subjective discount factor, \(\sigma\) is the inverse elasticity of intertemporal substitution, \(\gamma\) governs the Frisch elasticity of labor supply and \(\theta\) is the utility weight on leisure. Notice that deposits are riskless, as the interest rate is set in advance. The first order conditions are:

\[
\frac{\partial}{\partial k_t} : \frac{c_t^{1-\sigma} - 1}{1-\sigma} = \beta E_t \left[ \frac{c_{t+1}^{1-\sigma} - 1}{1-\sigma} + \theta \frac{(1-h_{t+1})^{1-\gamma} - 1}{1-\gamma} \right],
\]

\[
\frac{\partial}{\partial h_t} : w_t h_t = \theta \frac{(1-h_t)^{-\gamma}}{c_t^{1-\sigma}},
\]

and

\[
\frac{\partial}{\partial d^s_t} : \frac{c_t^{1-\sigma}}{1-\sigma} = \beta E_t \left[ \frac{c_{t+1}^{1-\sigma}}{1-\sigma} \right] r_d^t.
\]

Equations (2) and (4) are the Euler conditions for capital and deposits, while equation (3) governs the labor supply.
3.2 Banks

We assume a continuum of symmetric banks in the interval $[0, 1]$. The banking sector is perfectly competitive and banks are owned by households. The liabilities of the representative bank are given by one period deposits ($d^d_t$) and its assets are one period loans ($l^s_t$) to firms. In order to supply loans, the bank has to employ labor and capital. The production function for loans is

$$l^s_t = \min \left[ d^d_t, z_t \left( h^b_t \right)^{1-\kappa} \left( k^b_t \right)^\kappa \right],$$

where $h^b_t$ are worked hours in banks and $k^b_t$ is the amount of capital rented by banks. $\kappa$ governs the contribution of capital to the production of loans. $z_t$ is the bank’s total factor productivity, which we assume to be stochastic. The objective of the bank is to maximise discounted dividend payments ($\Pi^b_t$) to households. In each period, funds available for dividend payment are given by

$$\Pi^b_t = d^d_t + r^l_t \cdot l^s_t - l^s_t - w_t h^b_t - r_t k^b_t,$$

where $r^l_t$ is the gross interest rate on loans. Optimality implies that

$$l^s_t = d^d_t = z_t \left( h^b_t \right)^{1-\kappa} \left( k^b_t \right)^\kappa.$$

Hence, the problem for the representative bank can be rewritten as

$$\max_{t=0}^{\infty} \beta^t c^{-\sigma}_t \left[ r^l_{t-1} \cdot z_{t-1} \left( h^b_{t-1} \right)^{1-\kappa} \left( k^b_{t-1} \right)^\kappa - r^d_{t-1} \cdot z_{t-1} \left( h^b_{t-1} \right)^{1-\kappa} \left( k^b_t \right)^\kappa - w_t h^b_t - r_t k^b_t \right],$$

where the discount factor $\beta^t c^{-\sigma}_t$ takes into account the marginal value to the household of one unit of profits. Cost minimization, together with the zero profit condition, yields the following equilibrium relations.

$$\beta E_t c^{-\sigma}_t z_t (1 - \kappa) \left( \frac{k^b_t}{h^b_t} \right)^\kappa \left( r^l_t - r^d_t \right) = c^{-\sigma}_t w_t,$$

Although economic profits are zero, dividends can differ from zero because banks pay labor and capital one period before earning the return on loans. In the steady state, discounting implies that dividends are positive.
and

$$\beta E_t c^{-\sigma}_{t+1} z_t \kappa \left( \frac{k^b_t}{h^f_t} \right)^{\kappa-1} (r^l_t - r^d_t) = c^{-\sigma}_t r_t. \tag{8}$$

Equation (7) is the banks’ demand for labor. The marginal cost in utility terms of hiring labor at time $t$, $c^{-\sigma}_t w_t$, is equal to the physical marginal product of labor, $z_t (1 - \kappa) \left( \frac{k^b_t}{h^f_t} \right)^{\kappa}$ times the discounted interest rate margin, $\beta E_t c^{-\sigma}_{t+1} (r^l_t - r^d_t)$. A similar interpretation holds for eq. (8), which governs the banks’ demand for physical capital.

### 3.3 Firms

We assume a continuum of symmetric firms in the interval $[0, 1]$. The goods market is perfectly competitive and firms are owned by households. The representative firm produces a good $y$ employing labor ($h^f$) and capital ($k^f$) according to the Cobb-Douglas production function

$$y_t = a_t \left( k^f_t \right)^{\alpha} \left( h^f_t \right)^{1-\alpha}, \tag{9}$$

where $a$ is total factor productivity, which we assume to be stochastic. The representative firm issues one period loans to pay a share $\mu$ of its labor and capital costs in advance, hence:

$$\left( w_t h^f_t + r_t k^f_t \right) \mu = l^d_t. \tag{10}$$

The objective of the firm is to maximize discounted dividend payments ($\Pi^f$) to households. In each period, funds available for dividend payments are

$$\Pi^f_t = l^d_t + y_t - r^l_{t-1} l^d_{t-1} - w_t h^f_t - r_t k^f_t. \tag{11}$$

---

The cash in advance constraint (10) works as follows. In each period, before production occurs, banks credit firms’ deposit accounts a value equal to $\left( w_t h^f_t + r_t k^f_t \right) \mu$ and firms write checks to labor and capital owners for the same amount. Banks transfer the funds on households’ accounts. At the end of the period, after all transactions not subject to the cash in advance constraint have been cleared, households recapitalize firms by writing checks for the amount necessary to pay back loans, and once checks are delivered, banks close their credit and debit positions. Interest margins thus contribute to the bank profits of the following period.
Equations (10) and (11) imply that dividends can be rewritten as

$$\Pi_t = y_t - r^l_{t-1} \left( w_{t-1} h^f_{t-1} + r_{t-1} k^f_{t-1} \right) \mu - \left( w_t h^f_t + r_t k^f_t \right) (1 - \mu).$$

(12)

Using equations (9) and (12), the problem of the representative firm can be written as

$$\max_{c_t} \sum_{t=0}^{\infty} \beta_t c_t^{-\sigma} \left[ a_t \left( k^f_t \right)^{\alpha} \left( h^f_t \right)^{1-\alpha} - r^l_{t-1} \left( w_{t-1} h^f_{t-1} + r_{t-1} k^f_{t-1} \right) \mu - \left( w_t h^f_t + r_t k^f_t \right) (1 - \mu) \right],$$

where the discount factor $\beta_t c_t^{-\sigma}$ takes into account the marginal value to the household of one unit of profits. Cost minimization, together with the zero profit condition, yields the following equilibrium relations:

$$c_t^{-\sigma} \left[ (1 - \alpha) a_t \left( k^f_t \right)^{\alpha} \left( h^f_t \right)^{-\alpha} - w_t (1 - \mu) \right] = \beta c_{t+1}^{-\sigma} r^l_t w_t \mu,$$

(13)

and

$$c_t^{-\sigma} \left[ \alpha a_t \left( k^f_t \right)^{\alpha-1} \left( h^f_t \right)^{1-\alpha} - r_t (1 - \mu) \right] = \beta c_{t+1}^{-\sigma} r^l_t r_t \mu.$$

(14)

Equations (13) and (14) collapse to standard labor demand and capital demand functions for $\mu$ equal to zero. When $\mu$ is bigger than zero, the firm takes into account the interest rate on loans, $r^l_t$.

### 3.4 Market Clearing Conditions

Equations (15)-(18) define the market clearing conditions for the capital market, the labor market, the market for deposits and the market for loans.

$$k_{t-1} = k^f_t + k^b_t,$$

(15)

$$h_t = h^f_t + h^b_t,$$

(16)

---

5 Also for firms, economic profits are zero. Dividends can differ from zero because firms incur a part $\mu$ of their labor and capital costs with a one period delay. In the steady state, discounting implies that dividends are negative.
\[ d_t^d = d_t^r, \quad (17) \]

\[ l_t^d = l_t^r. \quad (18) \]

The aggregate resource constraint

\[ c_t + k_t - (1 - \delta) k_{t-1} = y_t \]

can be obtained combining equations \(1\), \(6\), \(11\) and \(15\)-\(18\).

\section{Quantitative analysis}

\subsection{Calibration Strategy}

We adopt a two-sided calibration strategy. Parameters concerning households’ preferences and the industrial sector are calibrated taking into account the macroeconomic literature and aggregate data, while parameters concerning the banking sector are instead set to be consistent with our dataset on banks.

For all the countries under study, the time unit is a year and we set the subjective discount factor \(\beta\) to 0.96. This value is consistent with the RBC literature and implies a steady state interest rate on deposits equal to 4\% on an annual basis. Adopting an annual calibration, rather than the more common quarterly calibration, allows us to include debt with maturity longer than one quarter (and less than a year) in the liabilities of firms, without having to model multi-period loans. We set \(\theta\) to imply that agents spend 30\% of their time working in steady state. We assume logarithmic utility in both consumption and leisure (\(\sigma = \gamma = 1\)): these values are consistent with a balanced growth path and imply a Frisch elasticity around 2. The physical capital depreciation rate, \(\delta\), is set to 10\% on an annual basis. For all countries, the parameters \(\alpha\) and \(\mu\) are set equal to the capital income share over GDP and to our measure of the bank credit to GDP ratio, respectively. We normalize the total factor productivity of firms, \(a\), to one.

The most innovative part of our calibration strategy regards the way in which we set the pa-
parameters governing banks’ production function, $z$ and $\kappa$. We impose two restrictions on the model: in the steady state these parameters must fit both the ratio of the banking sector total factor productivity to the total factor productivity of the whole economy, and the ratio of the banking sector capital intensity to the capital intensity of the whole economy. Both these ratios are calculated from available data, whose sources and construction are described in detail in the next section. It is important to stress that all the restrictions on banks are obtained from banks’ balance sheet data aggregated at a country level. In particular, given the structure of the production function assumed, if the capital compensation share for banks is the same as that of the rest of the economy, the available data on banks’ loans per worker and capital intensities allow to obtain the total factor productivity of banks for each country. In a refinement of our calibration strategy, we do not constrain the the capital compensation share for banks to be identical to that of the rest of the economy, and we obtain it together with the total factor productivity of banks. However, this refinement does not change the outcome substantially, and the essential feature of our calibration strategy is that the productivity of banks is calculated exogenously from bank balance sheet data, rather than endogenously from the calibration of the model, and this is the crucial departure from the strategy followed by Chari et al. (1995).

In our extremely simple model, loans are the only bank asset, and deposits the only liability. In principle we should thus evaluate the ratio of loans to workers by considering only the workers allocated by banks to the core function of providing loans financed with deposit. However, it is not possible to allocate bank workers to different bank products. We thus analyze two extreme settings. In the first we assume that loans and other financial services are produced with the same production function and workers are thus allocated to different products in the same proportion. In this case the loan per worker measure becomes identical to the asset per worker measure, as the loan per worker measure has to be rescaled by dividing it by the loan to asset ratio. In the second, labor and capital resources are used to produce exclusively loans, while all other bank assets are implicitly acquired for a negligible resource cost.
In the first framework, the first restriction can be expressed as

\[ z = \frac{A/BW}{(BK/BW)^\kappa \cdot TFP}, \]

where \(A/BW\) and \(BK/BW\) are the asset to worker ratio and the capital intensity of the banking industry, and \(TFP\) is the total factor productivity of firms. The total factor productivity is obtained as

\[ TFP = \frac{GDP/W}{(K/W)^\alpha} \]

where \(GDP/W\) and \(K/W\) are GDP per worker and the capital intensity of firms. The second restriction is the following:

\[ \frac{k^b/h^b}{k/h} = \frac{BK/BW}{K/W}. \]

In the second framework, when all bank workers are assumed to be employed in the loans industry, the alternative first restriction becomes

\[ z = \frac{L/BW}{(BK/BW)^\kappa \cdot TFP}, \]

where \(L/BW\) is the loans to worker ratio.

We consider AR(1) shocks to the log-deviation of total factor productivity of firms and of banks from their respective steady states. We assume that for both shocks the standard deviation is 1.86\% and the autocorrelation 0.95 on a quarterly basis. This implies a standard deviation of 3.45\% and an autocorrelation of 0.8145 on an annual basis.\(^6\)

### 4.2 Data

Table 1 displays basic parameter values for the main variables of the model defining the banking system, for a sample of 13 countries.

Data on the capital stock, the number of workers, the capital compensation share in GDP, and

\(^6\)If \(\rho\) and \(\sigma_x\) are the autocorrelation and the standard deviation on a quarterly basis, the autocorrelation on an annual basis is computed as \(\rho^4\) and the standard deviation as \(\sigma_x \sqrt{1 + \rho^2 + \rho^4 + \rho^8}\).
### Table 1: Parameter values

<table>
<thead>
<tr>
<th>Ctr</th>
<th>$\bar{c}_k$</th>
<th>$\bar{GDP}_W$</th>
<th>$\alpha$</th>
<th>$TFP$</th>
<th>$\mu$</th>
<th>NIM</th>
<th>$\bar{BK}$</th>
<th>$\bar{BW}$</th>
<th>$\bar{L}$</th>
<th>$\bar{A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>154</td>
<td>84</td>
<td>0.38</td>
<td>12.5</td>
<td>0.67</td>
<td>1.74</td>
<td>80</td>
<td>4141</td>
<td>8006</td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>163</td>
<td>96</td>
<td>0.29</td>
<td>21.7</td>
<td>0.56</td>
<td>1.24</td>
<td>86</td>
<td>6695</td>
<td>17699</td>
<td></td>
</tr>
<tr>
<td>DE</td>
<td>141</td>
<td>83</td>
<td>0.34</td>
<td>15.4</td>
<td>0.53</td>
<td>1.04</td>
<td>135</td>
<td>11670</td>
<td>29193</td>
<td></td>
</tr>
<tr>
<td>DK</td>
<td>170</td>
<td>99</td>
<td>0.35</td>
<td>16.2</td>
<td>0.71</td>
<td>1.39</td>
<td>73</td>
<td>7523</td>
<td>16782</td>
<td></td>
</tr>
<tr>
<td>ES</td>
<td>114</td>
<td>64</td>
<td>0.35</td>
<td>12.1</td>
<td>0.69</td>
<td>2.40</td>
<td>149</td>
<td>4114</td>
<td>6936</td>
<td></td>
</tr>
<tr>
<td>FR</td>
<td>160</td>
<td>90</td>
<td>0.36</td>
<td>14.5</td>
<td>0.61</td>
<td>0.89</td>
<td>69</td>
<td>4821</td>
<td>29193</td>
<td></td>
</tr>
<tr>
<td>IE</td>
<td>112</td>
<td>110</td>
<td>0.47</td>
<td>11.7</td>
<td>0.67</td>
<td>0.60</td>
<td>79</td>
<td>16843</td>
<td>34915</td>
<td></td>
</tr>
<tr>
<td>IT</td>
<td>111</td>
<td>74</td>
<td>0.39</td>
<td>11.7</td>
<td>0.56</td>
<td>2.42</td>
<td>87</td>
<td>3758</td>
<td>6234</td>
<td></td>
</tr>
<tr>
<td>NL</td>
<td>137</td>
<td>79</td>
<td>0.28</td>
<td>20.4</td>
<td>0.72</td>
<td>1.73</td>
<td>111</td>
<td>6133</td>
<td>11421</td>
<td></td>
</tr>
<tr>
<td>PT</td>
<td>51</td>
<td>36</td>
<td>0.28</td>
<td>12.3</td>
<td>0.81</td>
<td>1.42</td>
<td>64</td>
<td>3417</td>
<td>5213</td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>138</td>
<td>89</td>
<td>0.30</td>
<td>20.2</td>
<td>0.66</td>
<td>1.07</td>
<td>28</td>
<td>8318</td>
<td>13081</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>98</td>
<td>79</td>
<td>0.30</td>
<td>19.8</td>
<td>0.68</td>
<td>1.91</td>
<td>103</td>
<td>6239</td>
<td>10549</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>112</td>
<td>84</td>
<td>0.31</td>
<td>19.7</td>
<td>0.23</td>
<td>3.51</td>
<td>47</td>
<td>2993</td>
<td>5733</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $\bar{c}_k$ is capital per worker in the industrial sector, $\bar{GDP}_W$ is GDP per worker, $\alpha$ is the capital compensation share in GDP, and real GDP, $TFP$ is the total factor productivity of the industrial sector, $\mu$ measures the relevance of the cash in advance constraint as a value of bank private credit to GDP, NIM is the net interest margin, $\bar{BK}$, $\bar{BW}$, and $\bar{L}$ are respectively, bank capital per worker, loans per worker, and assets per worker. The variables are expressed in thousands of US dollars.

Real GDP, of different countries are obtained from the GGDC Total Economy Growth Accounting Database. The value of the capital stock is calculated for the year 2004 by using the perpetual inventory method with a geometric depreciation rate, common for all countries, and by using investment series taken from the OECD National Accounts or national statistical sources. Residential capital is excluded from the analysis. To calculate the value of $\mu$ we use the Financial Structure Dataset of the World Bank developed by Thorsten Beck and Amin Mohseni-Cheraghlou and data provided from the BIS.

We use BIS data on the value of borrowing on the part of non-financial corporations (ANA), and total private non-financial sector (APA), and we calculate the ratio ANA/APA, measuring the share of borrowing from non-financial corporations with respect to that from private non-financial sector, measured in billions of the respective currency. Credit covers both loans and debt securities and is provided by domestic banks, all other sectors of the economy and non-residents. The private non-financial sector includes non-financial corporations, households and non-profit institutions serving households. The ratio we obtain measures the share of total private sector borrowing obtained by non-financial corporations.
We obtain from the Financial Structure Dataset the value of bank private credit to GDP, measuring the financial resources provided to the private sector by domestic banks as a share of GDP. Multiplying the value of the bank private credit to GDP ratio by the ratio ANA/APA measuring the share of borrowings of non-financial corporation, we obtain the ratio between bank credit to non-financial corporations and GDP. Coherently with our definition of investment and physical capital, we do not include mortgages and consumer credit in our lending measure. However, our figures do not include just Commercial and Industrial Loans, but also Commercial Real Estate Loans, which in the United States includes construction, land development, and other land loans, and loans secured by farmland, multifamily (5 or more) residential properties, and nonfarm non-residential properties. Business firms that operate in the real estate sector, in fact, use bank loans as those of other industrial sectors. Although the sector is often more highly leveraged than others because it can provide collateral that is relatively easy to price, it requires external finance to anticipate expenditure on labor and raw material, and to hire capital. The values of \( \mu \), displayed in Table 1, are not too far from the average of 0.62 for most European countries, while the value for the United States, 0.23, is significantly smaller.

Bank capital intensities have been calculated using data obtained from both Bankscope and SNL Financials. The dataset includes all banks that in any year between 2012 and 2005 had total assets of at least 10 bln dollars and loans of at least 5 bln dollars. Capital is measured by the fixed assets value, which measures the accounting value of property, plant and equipment. We have calculated country averages as a weighted average of the data of individual banks, with weights given by the share of deposits of the single bank on the country total. The average value across countries was in the reference year, of 85000 dollars per worker, and most countries were not too far from the average value. The most notable exceptions were Sweden, where bank capital per worker was just 28000 dollars, and Spain, where it was 149000 dollars. The capital intensity of banks in the United States, of 47000 dollars, is also significantly below the average. Data on bank loans per worker and of bank assets per worker have been obtained from the same dataset, and have been averaged across banks of each individual country with the same procedures used for capital intensities.
4.3 Steady state

Table 2: Steady state values

<table>
<thead>
<tr>
<th>Ctr</th>
<th>$\frac{BK}{BW}$</th>
<th>$\frac{K}{W}$</th>
<th>$\frac{Z}{A}$</th>
<th>$\frac{TFP}{MNIM}$</th>
<th>$\frac{MNIM}{NIM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>0.52</td>
<td>0.24</td>
<td>225</td>
<td>0.55</td>
<td>0.32</td>
</tr>
<tr>
<td>BE</td>
<td>0.53</td>
<td>0.18</td>
<td>369</td>
<td>0.31</td>
<td>0.25</td>
</tr>
<tr>
<td>DE</td>
<td>0.96</td>
<td>0.33</td>
<td>386</td>
<td>0.27</td>
<td>0.26</td>
</tr>
<tr>
<td>DK</td>
<td>0.43</td>
<td>0.18</td>
<td>483</td>
<td>0.26</td>
<td>0.18</td>
</tr>
<tr>
<td>ES</td>
<td>1.31</td>
<td>0.41</td>
<td>72</td>
<td>1.30</td>
<td>0.54</td>
</tr>
<tr>
<td>FR</td>
<td>0.43</td>
<td>0.22</td>
<td>799</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>IE</td>
<td>0.70</td>
<td>0.38</td>
<td>569</td>
<td>0.22</td>
<td>0.37</td>
</tr>
<tr>
<td>IT</td>
<td>0.78</td>
<td>0.33</td>
<td>122</td>
<td>0.93</td>
<td>0.38</td>
</tr>
<tr>
<td>NL</td>
<td>0.81</td>
<td>0.23</td>
<td>186</td>
<td>0.58</td>
<td>0.33</td>
</tr>
<tr>
<td>PT</td>
<td>1.26</td>
<td>0.32</td>
<td>110</td>
<td>0.90</td>
<td>0.63</td>
</tr>
<tr>
<td>SE</td>
<td>0.20</td>
<td>0.06</td>
<td>523</td>
<td>0.22</td>
<td>0.20</td>
</tr>
<tr>
<td>UK</td>
<td>1.06</td>
<td>0.31</td>
<td>126</td>
<td>0.81</td>
<td>0.42</td>
</tr>
<tr>
<td>USA</td>
<td>0.42</td>
<td>0.16</td>
<td>160</td>
<td>0.73</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Notes: $\frac{BK}{BW}$ is the ratio between the capital intensity of banks relative to that of the industrial sector, $\kappa$ is the steady state value of the share of capital remuneration for the banking industry, $\frac{Z}{A}$ is the steady state value of the ratio between the total factor productivity of banks and of the industrial sector, MNIM is the steady state value of the net interest margin predicted by the model, $\frac{MNIM}{NIM}$ is the ratio between the net interest margin predicted by the model and the actual value from the data.

The second column of Table 2 displays the ratio between the capital intensity of banks relative to that of the industrial sector ($\frac{BK}{BW}$), which we obtain from the data. Bank capital per worker is in most countries significantly lower than aggregate capital per worker, but the UK, Spain and Portugal are notable exceptions as capital intensities in these countries is larger than in the aggregate. The average value across countries is 0.72. In the United States the ratio is 0.42, while in Sweden, where it is lower, it is just above 0.2.

Table 2 displays the results of the model calibration under the assumption that loans are produced with the same proportion of workers as other assets.

The capital compensation shares obtained from the model display a far larger variation than what is observed for the industrial sector, and the values mirror the behavior of the capital intensities. With the exception of Spain, Portugal and UK, the value for banks is lower than that of the industrial sector, and in the United States the share of capital remuneration for banks is roughly one half of that of the industrial sector. The steady state solutions provide also a range of values for the total factor productivity of banks, and the interest margins of banks.
The ratio between bank total factor productivity and the total factor productivity of industrial sector varies substantially across countries, with values ranging from a minimum of 72 for Portugal to a maximum of 799 for France. It is interesting to note, however, that this ratio is close to the average of 318 for a large number of countries. Spain, Portugal, Italy and the UK emerge as the countries far below average, while France, Ireland, Sweden and Denmark are far above the average. The last two columns of the table show the bank interest margin generated by the model, and the share of the actual net interest margin that is explained by the theoretical interest margin produced by the model. The average spread predicted by the model is 0.55 percent, while the average net interest margin across the countries of the sample was in 2004 of 1.64 percent. The share of the actual interest margin predicted by the model ranges from 17 percent for France to 63 percent for Portugal, and it averages 33 percent across countries.

To put this result in perspective, it must be noted that the rate on deposits of the model is actually a market interest rate for savings, as deposits provide no monetary functions and these rates are not influence by the banks’ marginal costs or revenues. But deposits that provide monetary services pay a significantly lower rate; for example in the United States, over the period 1959:2012 the average rate on 3 months treasury bills was 5.01 %, while the average own rate of M2 was 3.31 %, and the average prime rate on loans was 7.54 %. These numbers imply that the mark-down on deposits rates with respect to 3 months treasury bill, explained 40 % of the interest margin of the period.

As importantly, actual interest margins reflect also market power, and loan loss provisions, which according to the empirical analysis conducted by [Dia and Giuliodori (2012)](Dia_and_Giuliodori_2012) on a similar sample of countries are both strongly significant determinant of bank interest margins. As both market power and risk are not modelled, it is plausible that these factors explain the remaining share of the interest margin.

In any case, however, even for this specification of the model, which is the more challenging of the two, the model suggests that resource constraints explain a sizable share of the spread for any country, a share that in many of them is larger than one half. The clear pattern emerging from these results is that particularly whenever banks are inefficient, resource costs are the main determinants
of bank interest margins. However resource costs explain a large share of bank interest margins even in the case of the most efficient banking systems.

Table 3: Steady state values

<table>
<thead>
<tr>
<th>Ctr</th>
<th>BK/BW</th>
<th>κ</th>
<th>TFP</th>
<th>MNIM</th>
<th>NIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>0.52</td>
<td>0.24</td>
<td>116</td>
<td>1.07</td>
<td>0.61</td>
</tr>
<tr>
<td>BE</td>
<td>0.53</td>
<td>0.18</td>
<td>140</td>
<td>0.80</td>
<td>0.65</td>
</tr>
<tr>
<td>DE</td>
<td>0.96</td>
<td>0.33</td>
<td>153</td>
<td>0.69</td>
<td>0.66</td>
</tr>
<tr>
<td>DK</td>
<td>0.43</td>
<td>0.17</td>
<td>221</td>
<td>0.56</td>
<td>0.40</td>
</tr>
<tr>
<td>ES</td>
<td>1.31</td>
<td>0.41</td>
<td>43</td>
<td>2.18</td>
<td>0.91</td>
</tr>
<tr>
<td>FR</td>
<td>0.43</td>
<td>0.19</td>
<td>147</td>
<td>0.84</td>
<td>0.95</td>
</tr>
<tr>
<td>IE</td>
<td>0.70</td>
<td>0.38</td>
<td>270</td>
<td>0.47</td>
<td>0.78</td>
</tr>
<tr>
<td>IT</td>
<td>0.78</td>
<td>0.33</td>
<td>73</td>
<td>1.55</td>
<td>0.64</td>
</tr>
<tr>
<td>NL</td>
<td>0.81</td>
<td>0.23</td>
<td>100</td>
<td>1.07</td>
<td>0.62</td>
</tr>
<tr>
<td>PT</td>
<td>1.26</td>
<td>0.33</td>
<td>72</td>
<td>1.37</td>
<td>0.56</td>
</tr>
<tr>
<td>SE</td>
<td>0.20</td>
<td>0.01</td>
<td>399</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>UK</td>
<td>1.06</td>
<td>0.31</td>
<td>74</td>
<td>1.38</td>
<td>0.72</td>
</tr>
<tr>
<td>USA</td>
<td>0.42</td>
<td>0.16</td>
<td>83</td>
<td>1.39</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Notes: $BK/BW$ is the ratio between the capital intensity of banks relative to that of the industrial sector, $κ$ is the steady state value of the share of capital remuneration for the banking industry, $\frac{TFP}{κ}$ is the steady state value of the ratio between the total factor productivity of banks and of the industrial sector, MNIM is the steady state value of the net interest margin predicted by the model, $\frac{MNIM}{NIM}$ is the ratio between the net interest margin predicted by the model and the actual value from the data.

Table 3 displays the results of the model calibration under the alternative assumption that all bank workers are employed in the production of loans. The average spread predicted by the model is now equal to 1.05 percent, and it explains on average a share of 66 percent of the observed net interest margin. Excluding from the sample just two countries, the United States and Sweden, the average share explained by the model rises to 72 percent. In the case of France, Spain and Portugal, the model explains more than 90 percent of the actual net interest margin. And even in the case of the United States the model explains 40 percent of the margin.

The assumptions behind the two extreme scenarios represent extreme, equally unrealistic cases, and each country may be closer to one or the other case depending on the relevance of labour intensive products different from loans that banks provide. For example, the second scenario is implausible for French banks as the system is organized around large universal banks that hold large insurance assets, which require significant amounts of labor; but the first is similarly implausible as investment banking activities swell the asset portfolio of bonds and derivatives that require very
little labor. On the contrary, the market of the United States, that is dominated by large commercial
banks is probably better approximated by the second scenario. The picture that emerges, though,
suggests that resource cost are by far the dominating factor in shaping bank interest margins across
all different countries.

4.4 The banking industry

Banks influence wages and the cost of capital both directly, as they employ resources to produce
loans, and indirectly, as bank loans are necessary to produce any good; however our results so far
indicate that even when \( \mu \) is large, the impact of banks on aggregate variables is negligible: banks
employ too little labor and capital to matter much in the markets for resources, and interest rates
are too small a share of industrial costs to generate a strong indirect effect. It is thus possible to
analyze the steady-state equilibrium of the market for bank loans in a partial equilibrium setting.
The loans supply and loans demand functions are derived in the appendix.

Given the weakness of the general equilibrium effects of the banking industry, the assumption of
constant returns to scale generates a flat loan supply schedule. And \( ? \) has provided strong empirical
evidence for the United States suggesting that financial intermediation has constant returns to scale.
Similarly, as loan rates are a small share of industrial firms’ total costs, the demand for loans is
an almost vertical line. As a consequence of the slope of the supply function, following an increase
in the total factor productivity of the industrial sector, the resulting higher demand for loans does
not generate higher rates on loans. However, as wages and the cost of capital rise with output, the
higher resource costs affects banks too, and the loan supply schedule shifts up, causing an increase
of the rate on loans.

Quite interestingly, since the demand for loans has little impact on the rate and on the spread,
this pattern remains similar for countries where banks’ weight is different, as the parameter \( \mu \) does
not influence significantly the rate on loans. For example in the case of banks in the United States,
where the value of \( \mu \) is one third of that of European countries, loan rates and hours and capital
employed respond as in their European counterparts. The only significant difference in the response
to changes in the total productivity across countries produced by the different values of \( \mu \) regards
the impact on the quantity of loans.

4.5 Impulse responses

We have analyzed the impact of a one standard deviation shock to productivity, for either the case of a symmetric shock hitting in the same way the productivity of industrial firms and banks, or asymmetric shocks. We have chosen three different countries that display quite different parameter combinations, the USA, Sweden and Spain. The impulse responses displayed are those obtained under the assumption that loans are produced with the same proportion of workers as other assets, but the ones obtained under the alternative assumption are virtually identical.

The main findings are the following: The basic impact of total factor productivity shocks hitting industrial firms is identical to that of the standard RBC model, suggesting that the model is quite robust. These same shocks, however, have very relevant effects on banks.

Fig. (2) and Fig. (3) display the impact of a shock on the productivity of the industrial sector on the three different countries. Shocks hitting the economy but not banks have a differentiated impact on banks of different countries, and the impact is always relevant. In particular, the impact on hours worked is always very large and quite independent of the parameter set. In all countries a 3 percent increase of total factor productivity generates a roughly 6 percent increase in hours worked. Hours in banks are thus far more volatile than hours in the rest of the economy. The impact on capital and on the spread depends instead on the capital intensity of the banking system, and its productivity, but it is always economically relevant. While in some countries the impact on the spread is limited, the impact on the rate on loans is always strong. The shock, in fact, generates a large impact on the rate on deposits, whose change is passed by the banking system to their borrowers. The impact on the spread is always positive because the increase of the rate on loans reflects also the higher wages and cost of capital that the bank suffers.

Fig. 4 and Fig. 5 compares the impact of a shock hitting the total factor productivity of banks on the economy of the three countries: Spain blue line, USA small green dashes, Sweden large red dashes.

Bank shocks have little relevance for the rest of the economy. The adjustments following the
shocks are in the expected direction, but the impacts are very small. This is true for any country, even those as Spain where banks are large relative to the rest of the economy and quite inefficient. Banks are simply too small to matter much in the market for labour and capital, and interest costs are not large enough to induce significant reallocations of resources in the rest of the economy, following bank productivity shocks. These results are so sharp that we seriously doubt that any more complicated model may produce substantially different results, nor that simply increasing the size of banks, for example by introducing housing investment, can make a substantial difference. For bank productivity shocks to generate large impacts on the business cycle important amplify-
Figure 3: Impulse response function of Hours worked in Banks, Loans Interest Rate, Hours worked in Firms, Capital of Banks, Capital of Firms, and Spread a one standard deviation shock to the total factor productivity of the industrial sector, for the following countries: Spain blue line, USA small green dashes, Sweden large red dashes.

The mechanisms at work, as for example may be the case when bank lending generates endogenous increases in the price of collateral influencing households’ wealth.

Bank productivity shocks generate a reduction of the rate on loans and of the spread that is inversely related to the productivity of banks. In Sweden, the country with the most productive banks, the impact of a one standard deviation shock is negligible, while in Spain, the country where banks are least productive, the same shock produces a relevant effect, as the loan rate declines by 4 basis points. Note that the reduction of the loan rate would be bigger in all countries if the deposit rate did not respond to the shock. As the increased productivity of banks pushes up the rental
rate of capital, a simple arbitrage condition requires the deposit rate to increase. The interest rate effect of bank productivity shocks is probably stronger than the model suggests, as deposit rates are sticky and adjust very slowly, while we have modelled them as perfectly flexible. We do not believe, however, that allowing for sticky deposit rates would substantially modify these results. Indeed, as discussed in the former section, and with more details in the appendix, the loan market quantities are almost completely determined by demand, while interest rates by the supply side of the market. The different behavior of the interest rate among different countries is simply a scale effect due to a zero lower bound on the interest rate spread. On the contrary, the impact of the shock on hours worked and capital employed in banks is almost perfectly symmetrical across countries, as banks use a small fraction of the resources of the economy, with a decline of both hours and capital employed proportional to the size of the increase of productivity, 3 percent.

Following the shock the spread declines, in line with the results of Goodfriend and McCallum (2007), but the impact is not very large, particularly in those countries where the banking industry is more productive.

5 Model predictions and actual data: An empirical analysis of symmetric shocks in the United States

In this section we compare the predictions of the model when symmetric productivity shocks hit the economy, with those obtained from a structural VAR. More in detail, we analyze the dynamics of bank interest spreads in response to technology shocks in the United States, as relatively long time series for both total factor productivity (TFP henceforth) and bank lending rates are available for this country.

We analyze a VAR including a variable measuring total factor productivity and one measuring bank interest spreads. Coherently with our theoretical model, these last are obtained by subtracting interest rates on three months T-bills from Bank Prime Loan Rates, as we neglect the monetary role of deposits and deposit rates of the model have to be interpreted as rates on riskless non-monetary assets. T-bill rates, Bank Prime Loan Rates and Total Factor Productivity at constant national
Figure 4: Impulse response function of Consumption, Wage, Deposit Rate, Rental Rate on Capital, Total Hours, and Capital following a one standard deviation shock to the total factor productivity of banks, for the following countries: Spain blue line, USA small green dashes, Sweden large red dashes.
Figure 5: Impulse response function of Hours worked in Banks, Loans Interest Rate, Hours worked in Firms, Capital of Banks, Capital of Firms, and Spread of a one standard deviation shock to the total factor productivity of banks, for the following countries: Spain blue line, USA small green dashes, Sweden large red dashes.
prices are obtained from FRED. Data cover the period 1956-2011 and are collected at an annual frequency. As we lack a sufficiently long time series for the total factor productivity of the banking sector, we are constrained to limit our empirical analysis to the estimate of the response of bank interest spreads to economy wide TFP shocks, assuming that productivity shocks hit the industrial and the banking sectors symmetrically.\footnote{Notice that the converse is not true, i.e. we can in principle allow for banking sector specific shocks.}

As we find evidence of non-stationarity in both the TFP and the spread, we log-difference and demean both series. The Akaike’s Information Criterion suggests that the best specification of our empirical model is a VAR(1), i.e.:

\[
\begin{bmatrix}
\Delta TFP_t \\
\Delta Spread_t
\end{bmatrix} = B \begin{bmatrix}
\Delta TFP_{t-1} \\
\Delta Spread_{t-1}
\end{bmatrix} + \begin{bmatrix}
u_{1,t} \\
\nu_{2,t}
\end{bmatrix},
\]

To identify the technology shock we follow Gali (1999), and we impose the restriction that only technology shocks have permanent effects on both TFP and bank spreads. Such an assumption is compatible with our theoretical model, since banking sector specific shocks do not affect permanently the industrial sector TFP. As a robustness check, we also identify the shock through short-run restrictions, assuming that shocks to the spread have no contemporaneous effect on TFP. The two approaches yield similar results.

The SVAR can be written as

\[
\begin{bmatrix}
\Delta TFP_t \\
\Delta Spread_t
\end{bmatrix} = C (L) \begin{bmatrix}
\varepsilon^A_t \\
\varepsilon^d_t
\end{bmatrix},
\]

where

\[
C (L) = [I - BL]^{-1} A_0^{-1}
\]

\(L\) is the lag operator and \(I\) is a two by two identity matrix. \(\varepsilon^A_t\) is the technology shock and \(\varepsilon^d_t\) represents other shocks that are assumed to have no permanent effects on TFP.\footnote{In the standard Blanchard and Quah (1989) approach, these shocks are considered demand shocks. In our model, our identifying assumption is that only technology shocks have permanent effects on both TFP and bank spreads. This assumption is compatible with our theoretical model, since banking sector specific shocks do not affect permanently the industrial sector TFP. As a robustness check, we also identify the shock through short-run restrictions, assuming that shocks to the spread have no contemporaneous effect on TFP. The two approaches yield similar results.}
tion requires that \( C(1) \) be lower triangular. In practice, we define \( Q = [I - B]^{-1} \Sigma \left\{ [I - B]^{-1} \right\}' \) and compute \( A_0 \) using the Choleski decomposition of \( Q \) as \( A_0 = [I - B]^{-1} \text{Chol}(Q) \).

Figures 7 and 6 display the impulse responses of first-differenced TFP and bank spread to a positive productivity shock obtained from the VAR and the model, respectively. The pattern obtained from the model replicates very closely the empirical response of bank interest margins to the productivity shock in the VAR. The spread falls significantly at impact, than overshoots its long run value. The behaviour of the spread can be understood by remembering that the banking sector has a lower capital intensity than the industrial sector. Following the shock, wages increase more gradually than the rental rate of capital, but remain higher for a longer period. As the banking sector uses labor more intensively than other industrial sectors, costs rise less than productivity in the short run and interest margins accordingly decline; over a longer horizon, however, as wages grow more than the cost of capital, banks’ resource costs increase more than those of the rest of the economy. As marginal costs increase, banks respond by charging the higher interest rates on loans generating the overshooting of interest margins. We verified that this behaviour disappears if we impose that banks have the same capital intensity of the industrial sector in the model.

Figure 8 displays the impulse responses of first-differenced TFP and bank spread to a non-technology shock obtained from the VAR. As expected, TFP does not respond significantly to the shock, in line with the predictions of the model, as banking sector shocks do not affect the TFP of the overall economy.

The forecasting error variance decomposition of TFP and Spread, displayed in Table 4 suggests that our model is correctly specified. The technology shock, in fact, explains almost all of the forecasting error variance for TFP at one year, five years and ten years horizons, and more than 50% of that of the spread at any horizon.

\( \epsilon_{tr} \) may be interpreted as banking sector specific shocks that have no long-run effect on the industrial sector TFP.
Table 4: Proportion of forecasting error variance for $\Delta TFP$ and $\Delta Spread$ explained by $\Delta TFP$ at horizons 1, 5, and 10 years.

<table>
<thead>
<tr>
<th>Var. Decomposition</th>
<th>1 Year</th>
<th>5 Years</th>
<th>10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta TFP$</td>
<td>98.84</td>
<td>97.41</td>
<td>97.41</td>
</tr>
<tr>
<td>$\Delta Spread$</td>
<td>50.29</td>
<td>55.25</td>
<td>55.25</td>
</tr>
</tbody>
</table>

6 Conclusion

In previous works that have introduced banks in DSGE models most of the parameters defining the banking industry are calibrated endogenously, with the only exception of bank interest rates that are set on the basis of actual data. Bank parameters only need to replicate the correct rates, while
the amount of resources they use, or their productivity are left unexplained and do not necessarily match observed data. To a large extent the limitations of this approach are relatively minor, as the amount of resources that bank use are a small proportion of those of the entire economy. Bank interest rates and the productivity of the banking industry, however, differ substantially across countries, and the relevance of the banking system as a source of finance for industrial firms is similarly different in different markets. This paper introduces banks in a real business cycle model largely along the lines of the previous literature, but the model is calibrated by imposing parameter values obtained from an original database on bank capital intensities for different countries, and by matching the data on the ratio between bank business lending and GDP for each country. Bank interest rates are instead obtained endogenously.

Bank interest rates generated by the model match the data extremely well, particularly when

Figure 7: Empirical Impulse Responses of first-differenced TFP and bank spread to a technology shock. Confidence intervals are computed using the Monte Carlo method with 1000 replications.
Taking into account that the model assumes perfect competition and the absence of risk. We thus conclude that resource costs are the main determinants of bank interest spreads. We also find that differentials in bank productivity and in the productivity of the industrial sector explain most of the variability across countries of bank rates on loans and bank interest margins.

Idiosyncratic shocks hitting the productivity of the banking industry produce impacts that are always negligible on the rest of the economy. Banks use too small a share of resources to influence their prices significantly. Furthermore, interest costs are not large enough to induce large reallocation of resources in industrial firms, even when shocks hitting banks are very large. The role of banks in this model is in fact rather limited, as they exclusively provide short-term loans to finance the working capital need of industrial firms. But a striking implication of our results is that to the extent that banks fulfill this limited traditional role, they exert a rather limited influence on the productive sector.
This simple model helps to understand why, if bank-specific issues such as size are to matter at a macro level, it would have to be because of broader market effects of banking-industry-specific events not accounted for by the simple interactions between banks and firms borrowing to finance working capital. For example, this is the case of most of the recent models that have analyzed the role of collateral constraints, and the potential endogenous response of the price of collateral to aggregate lending. These models not only describe processes that feed back to create “double-whammy” effects on the banks, but these interactions ultimately generate sizable general equilibrium effects because they produce large wealth effects on households.

We also find that positive productivity shocks hitting the industrial sector that are not matched by a similar shock on the productivity of banks induce sizable increases of the rate on loans, and sharp increases in the total hours worked in the banking system. The first effect is in part produced by the higher resource costs that banks face, but it is largely driven by the increase in the rates on deposits caused by the shock. The effect on hours worked is produced by the increased demand for loans due to the higher wages and cost of capital, and it implies that the overall number of hours worked in the banking system is much more volatile over the business cycle than in the industrial sector.

A symmetric productivity shock generates a decline of the spread at impact, which is reversed in subsequent periods as the it overshoots its long run value. The pattern obtained from the model replicates very closely the empirical response of bank interest margins to the productivity shock obtained from a structural VAR analyzing data for the United States during the period 1956-2011.

Appendix

6.1 Steady State

The interest rate on deposits is easily obtained as the inverse subjective discount factor

\[ r^d = \frac{1}{\beta} \]
Similarly, it is easy to obtain the rental rate of capital from equation (2):

\[ r = \frac{1 - \beta (1 - \delta)}{\beta} \]

Using equations (8) and (14), we can write the capital intensities of the two sectors as

\[ \frac{k^b}{h^b} = \left( \frac{r}{(r^l - r^d) \beta z \kappa} \right)^{\frac{1}{\kappa - 1}} \]  \hspace{1cm} (19)

and

\[ \frac{k^f}{h^f} = \left( \frac{\beta r^l \mu + r (1 - \mu)}{\alpha a} \right)^{\frac{1}{\alpha - 1}} \]  \hspace{1cm} (20)

Using firms’ labor and capital costs\(^9\) one gets the loans to output ratio\(^10\)

\[ \frac{l}{y} = \frac{\mu}{(\beta r^l \mu + 1 - \mu)} \]  \hspace{1cm} (21)

Since \( \frac{l}{y} \) is equal to \( \frac{z (k^b)^{\alpha} (k^b)^{1-\alpha}}{a(k^f)^{\alpha} (h^f)^{1-\alpha}} \), we can write the ratio of hours worked in banks to hours worked in firms as

\[ \frac{h^b}{h^f} = \frac{l \left( \frac{k^f}{k^b} \right)^{\alpha}}{y \left( \frac{k^b}{k^f} \right)^{\kappa}} \]  \hspace{1cm} (22)

Using equations (19) to (21), we can rewrite (22) as

\[ \frac{h^b}{h^f} = \frac{\mu}{(\beta r^l \mu + 1 - \mu)} \frac{a \left[ \left( \frac{\beta r^l \mu + r (1 - \mu)}{\alpha a} \right)^{\frac{1}{\kappa - 1}} \right]^{\alpha}}{z \left[ \left( \frac{r}{(r^l - r^d) \beta z \kappa} \right)^{\frac{1}{\kappa - 1}} \right]^{\kappa}} \]  \hspace{1cm} (23)

\(^9\)From the factor demands of firms one gets: \( wh^f = \frac{(1-\alpha)a(k^f)^{\alpha} (h^f)^{1-\alpha}}{(\beta r^l \mu + 1 - \mu)} \) and \( rk^f = \frac{\alpha a(k^f)^{\alpha} (h^f)^{1-\alpha}}{(\beta r^l \mu + 1 - \mu)} \). From this, it follows that \( \frac{a(k^f)^{\alpha} (h^f)^{1-\alpha}}{(a(k^f)^{\alpha} (h^f)^{1-\alpha})} = \frac{l^d}{y} \).

\(^10\)Here we consider the loans market clearing condition and set \( l = l^d = l^u \).
Since we set total worked hours to 0.3 in steady state, we can use the fact that $h^b + h^f = 0.3$, to obtain $h^f$ from (23) as

$$h^f = \frac{0.3}{1 + \frac{a \left( \frac{\beta r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\alpha - 1}{\alpha}}}{1 + \left( \frac{\beta r l + 1 - \mu}{\beta r l + 1 - \mu} \right) \frac{a \left( \frac{\beta r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\mu - 1}{\mu}}}{z \left( \frac{r}{(r - r^l) \beta z} \right)^{\frac{1}{\beta}}}.$$ 

Hence, hours worked in banks are

$$h^b = 0.3 \left[ 1 - \frac{1}{1 + \frac{a \left( \frac{\beta r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\mu - 1}{\mu}}}{1 + \left( \frac{\beta r l + 1 - \mu}{\beta r l + 1 - \mu} \right) \frac{a \left( \frac{\beta r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\mu - 1}{\mu}}}{z \left( \frac{r}{(r - r^l) \beta z} \right)^{\frac{1}{\beta}}}} \right].$$

The capital stock employed by the two sectors can now be easily obtained from (19) and (20):

$$k^b = \left( \frac{r}{(r^l - r^d) \beta z k} \right)^{\frac{\alpha - 1}{\alpha}} 0.3 \left[ 1 - \frac{1}{1 + \frac{a \left( \frac{\beta r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\mu - 1}{\mu}}}{1 + \left( \frac{\beta r l + 1 - \mu}{\beta r l + 1 - \mu} \right) \frac{a \left( \frac{\beta r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\mu - 1}{\mu}}}{z \left( \frac{r}{(r - r^l) \beta z} \right)^{\frac{1}{\beta}}}} \right].$$

$$k^f = \left( \frac{\beta r^f r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\alpha - 1}{\alpha}} 0.3 \left[ 1 + \frac{a \left( \frac{\beta r^f r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\mu - 1}{\mu}}}{1 + \left( \frac{\beta r^f r l + 1 - \mu}{\beta r^f r l + 1 - \mu} \right) \frac{a \left( \frac{\beta r^f r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\mu - 1}{\mu}}}{z \left( \frac{r}{(r - r^l) \beta z} \right)^{\frac{1}{\beta}}}} \right].$$

Equation (13) can be used to obtain the wage rate:

$$w = \frac{(1 - \alpha) a \left( \frac{k^f}{\beta r^f} \right)^{\alpha}}{\left( \beta r^f r l + 1 - \mu \right)}$$

which, using (20) reads as

$$w = \frac{(1 - \alpha) a \left( \frac{\beta r^f r l + r (1 - \mu)}{\alpha a} \right)^{\frac{\alpha - 1}{\alpha}}}{\left( \beta r^f r l + 1 - \mu \right)}$$

(24)
Equation (7) in steady state reads as

$$\beta z (1 - \kappa) \left( \frac{k^b}{h^b} \right)^\kappa \left( r^l - r^d \right) = w$$

Using (24) and (19), the latter equation can be rewritten as

$$\beta z (1 - \kappa) \left( \frac{r}{(r^l - r^d) \beta z K} \right)^{\frac{\alpha - 1}{\alpha}} \left( r^l - r^d \right) =$$

$$(1 - \alpha) a^0 \left( \frac{3 r^l \mu + r(1 - \mu)}{\alpha a} \right)^{\frac{\alpha}{\alpha - 1}} \left( \beta r^l \mu + 1 - \mu \right)$$

We can obtain the ratio of banks’ capital intensity to the economy capital intensity as:

$$\frac{k^b}{h^b} / \frac{k}{h} = \frac{k^b}{h^b} / \frac{k}{h} = \left( \frac{r}{(r^l - r^d) \beta z K} \right)^{\frac{\alpha - 1}{\alpha - 1}}$$

$$= \left( \frac{r}{(r^l - r^d) \beta x K} \right)^{\frac{\alpha - 1}{\alpha - 1}}$$

We use (26) to impose the restriction

$$\frac{k^b}{h^b} / \frac{k}{h} = BK/BW$$  (27)

Notice that both (25) and (27) only depend on $z$, $\kappa$ and $r^l$. Considering also the restriction

$$z = \frac{A/\text{BW}}{\text{TFP}^2}$$

one gets a system of three unknowns in three equations and $z$, $\kappa$ and $r^l$ can be obtained.
6.2 Loglinear Model

The competitive equilibrium in log-linear form is defined by the following system of equations\[11\]

\[-\sigma \tilde{c}_t = -\sigma \tilde{c}_{t+1} + \frac{r}{r + 1 - \delta} \tilde{r}_{t+1},\]

\[\tilde{w}_t = \gamma \frac{h}{1-h} \tilde{h}_t + \sigma \tilde{c}_t,\]

\[-\sigma \tilde{c}_t = -\sigma \tilde{c}_{t+1} + \tilde{r}^d_t,\]

\[-\sigma \tilde{c}_{t+1} + \tilde{z}_t + \kappa \left( \tilde{k}^b_t - \tilde{h}^b_t \right) + \frac{\rho^l}{\rho^l - \rho^d} \tilde{r}^l_t - \frac{\rho^d}{\rho^l - \rho^d} \tilde{r}^d_t = -\sigma \tilde{c}_t + \tilde{w}_t,\]

\[-\sigma \tilde{c}_{t+1} + \tilde{z}_t + (\kappa - 1) \left( \tilde{k}^b_t - \tilde{h}^b_t \right) + \frac{\rho^l}{\rho^l - \rho^d} \tilde{r}^l_t - \frac{\rho^d}{\rho^l - \rho^d} \tilde{r}^d_t = -\sigma \tilde{c}_t + \tilde{r}_t,\]

\[-\sigma \tilde{c}_t + \frac{(1 - \alpha) a \left( \frac{k^f_t}{h^f_t} \right)^\alpha}{(1 - \alpha) a \left( \frac{k^f_t}{h^f_t} \right)^\alpha - w (1 - \mu)} \left( \tilde{a}_t + \alpha \tilde{k}^f_t - \alpha \tilde{h}^f_t \right) - \frac{w (1 - \mu)}{(1 - \alpha) a \left( \frac{k^f_t}{h^f_t} \right)^\alpha - w (1 - \mu)} \tilde{w}_t = -\sigma \tilde{c}_{t+1} + \tilde{r}_t + \tilde{w}_t,\]

\[-\sigma \tilde{c}_t + \frac{\alpha a \left( \frac{k^f_t}{h^f_t} \right)^{\alpha - 1}}{\alpha a \left( \frac{k^f_t}{h^f_t} \right)^{\alpha - 1} - r (1 - \mu)} \left( \tilde{a}_t + (\alpha - 1) \tilde{k}^f_t - (\alpha - 1) \tilde{h}^f_t \right) - \frac{r (1 - \mu)}{\alpha a \left( \frac{k^f_t}{h^f_t} \right)^{\alpha - 1} - r (1 - \mu)} \tilde{r}_t = -\sigma \tilde{c}_{t+1} + \tilde{r}_t + \tilde{r}_t,\]

\[11\text{Variables without the time subscript are steady state values. For any generic variable } x, \tilde{x} \text{ represents its log-deviation from the steady state.}\]
\[
\frac{c}{y} \tilde{c}_t + \frac{k}{y} \tilde{k}_t - (1 - \delta) \frac{k}{y} \tilde{k}_{t-1} = \bar{a}_t + \alpha \tilde{k}^f_t + (1 - \alpha) \tilde{h}^f_t, \\
\tilde{k}_{t-1} = \frac{k^f}{k} \tilde{k}^f_t + \frac{k_b}{k} \tilde{k}^b_t, \\
\tilde{h}_t = \frac{h^f}{h} \tilde{h}^f_t + \frac{h_b}{h} \tilde{h}^b_t, \\
\frac{w h^f}{w h^f + r k^f} (\bar{w}_t + \tilde{h}^f_t) + \frac{r k^f}{w h^f + r k^f} (\bar{r}_t + \tilde{k}^f_t) = \bar{z}_t + (1 - \kappa) \tilde{h}^b_t + \kappa \tilde{k}^b_t.
\]

### 6.3 Loan Supply Function

Bank revenues in the model are given by net interest revenues, \((r^d_t - r^d_t) l_t\), while bank costs are \(w_t h^b_t + r_t k^b_t\); the representative bank’s cost minimization problem is thus the following:

\[
\min E_0 \sum_{t=0}^{\infty} \beta^t c^{-\sigma}_t (w_t h_t + r_t k_t), \\
\text{s.t.} \\
\tilde{l}_t = \bar{l}_t (h^b_t)^{1-\kappa} (k^b_t)^{\kappa},
\]

The first order conditions of the problem are

\[
w_t = \phi_t (1 - \kappa) \left( \frac{k^b_t}{h^b_t} \right)^{\kappa},
\]

and

\[
r_t = \phi_t \kappa \left( \frac{k^b_t}{h^b_t} \right)^{\kappa-1},
\]

where \(\phi\) is the Lagrange multiplier on the production function. Combining the two first order conditions, we obtain the efficiency condition

\[
\frac{w_t}{r_t} = \frac{1 - \kappa}{\kappa} \frac{k^b_t}{h^b_t}.
\]
Equations (30), (31), and (29) allow to get the labor and capital direct demand functions:

$$h^b_t = \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa} \frac{t_t^b}{z_t},$$  

(33)

$$k^b_t = \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa-1} \frac{t_t^b}{z_t}.$$  

(34)

Substituting (33) and (34) in (28), we obtain the cost function

$$C(l_t) = E_0 \sum_{t=0}^{\infty} \beta^t c_t^{-\sigma} \left\{ \frac{t_t^b}{z_t} \left[ w_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa} + r_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa-1} \right] \right\}.$$ 

The marginal cost of producing one unit of loans at time $t$ is

$$\frac{\partial C(l_t)}{\partial l_t} = c_t^{-\sigma} \frac{w_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa} + r_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa-1}}{z_t}.$$  

(35)

Imposing that under perfect competition marginal costs equal the discounted interest margin, we get:

$$\beta E_t c_t^{-\sigma} \left( r_t^l - r_t^d \right) = c_t^{-\sigma} \frac{w_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa} + r_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa-1}}{z_t}.$$ 

Using the equation $r_t^d = \frac{1}{\beta} E_t c_t^{-\sigma}$, we get:

$$\frac{r_t^l - r_t^d}{r_t^d} = w_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa} + r_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa-1} \frac{1}{z_t}.$$ 

The loans supply function is

$$r_t^l = \left[ w_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa} + r_t \left( \frac{r_t \frac{1 - \kappa}{w_t}}{w_t} \right)^{\kappa-1} \frac{1}{z_t} \right] r_t^d.$$
In steady state, the loans supply function becomes

\[ r^l = \left[ w \left( \frac{1-\kappa}{\kappa} \right)^k + r \left( \frac{1-\kappa}{\kappa} \right)^{k-1} \right] r^d. \]

### 6.4 Loan Demand Function

The cost minimization problem of the representative firm is the following:

\[
\min_{E_0} \sum_{t=0}^{\infty} \beta^t c_t^{-\gamma} r_t \left( w_{t-1} h_{t-1}^f + r_{t-1} k_{t-1}^f \right) \mu + w_t h_t^f + r_t k_t^f (1 - \mu),
\]

s.t.

\[ \overline{y}_t = a_t \left( \frac{k_t^f}{h_t^f} \right)^{1-\alpha} \left( \frac{k_t^f}{h_t^f} \right)^\alpha. \tag{36} \]

The first order conditions are:

\[ q_t (1 - \alpha) a_t \left( \frac{k_t^f}{h_t^f} \right)^\alpha c_t^{-\gamma} = w_t (1 - \mu) c_t^{-\gamma} + \beta E_t c_{t+1}^{-\gamma} r_t^i w_i \mu, \]

and

\[ q_t \alpha a_t \left( \frac{k_t^f}{h_t^f} \right)^{\alpha - 1} c_t^{-\gamma} = r_t (1 - \mu) c_t^{-\gamma} + \beta E_t c_{t+1}^{-\gamma} r_t^i r_i \mu, \]

where \( q \) is the Lagrange multiplier on the production function. Combining the two first order conditions we get:

\[ \frac{w_t (1 - \mu) c_t^{-\gamma} + \beta E_t c_{t+1}^{-\gamma} r_t^i w_i \mu}{(1 - \alpha) a_t \left( \frac{k_t^f}{h_t^f} \right)^\alpha} = \frac{r_t (1 - \mu) c_t^{-\gamma} + \beta E_t c_{t+1}^{-\gamma} r_t^i r_i \mu}{\alpha a_t \left( \frac{k_t^f}{h_t^f} \right)^{\alpha - 1}}. \tag{37} \]

Using (36) and (37), we obtain the labor and capital direct demand functions

\[ h_t^f = \left( \frac{1 - \alpha \ r_t (1 - \mu) c_t^{-\gamma} + \beta E_t c_{t+1}^{-\gamma} r_t^i r_i \mu}{\alpha w_t (1 - \mu) c_t^{-\gamma} + \beta E_t c_{t+1}^{-\gamma} r_t^i w_i \mu} \right)^\alpha \overline{y}_t / a_t. \]
and
\[ k^f_t = \left( \frac{1 - \alpha}{\alpha} \frac{r_t (1 - \mu) c^{-\sigma}_t + \beta E_t c^{-\sigma}_{t+1} r^1_t r_t \mu}{w_t (1 - \mu) c^{-\sigma}_t + \beta E_t c^{-\sigma}_{t+1} r^1_t w_t \mu} \right)^{\alpha-1} \frac{\mu}{a_t}. \]

Since \( l_t^d = \mu \left( w_t h^f_t + r_t k^f_t \right) \), the loan demand function can be written as
\[ l^d_t = \mu \left\{ \frac{w_t \left( 1 - \alpha \frac{r_t (1 - \mu) c^{-\sigma}_t + \beta E_t c^{-\sigma}_{t+1} r^1_t r_t \mu}{w_t (1 - \mu) c^{-\sigma}_t + \beta E_t c^{-\sigma}_{t+1} r^1_t w_t \mu} \right)^\alpha}{\alpha} \right\} \frac{\mu}{a_t}. \] (38)

In steady state it becomes
\[ l^d = \mu \left\{ \frac{w \left( 1 - \alpha \frac{r (1 - \mu) c^{-\sigma} + \beta c^{-\sigma} r^1 r \mu}{\alpha} \right)^\alpha}{\alpha} \right\} \frac{\mu}{a}. \] (39)

References


Curdia, V., Woodford, M., September 2010. Credit spreads and monetary policy. Journal of Money, Credit and Banking 42 (s1), 3–35.


