Optimal Exchange Rate Policy Under Collateral Constraints and Wage Rigidity

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Abstract

Existing literature on small open economies has studied separately two opposite effects of currency depreciation during crises: in the presence of nominal wage rigidity, exchange rate depreciation reduces unemployment; in the presence of collateral constraints that link external debt to the value of income, exchange rate depreciation tightens the collateral constraint and leads to higher consumption adjustment. This paper shows that in a model that includes both frictions, exchange rate policy faces a “credit access – unemployment trade-off,” i.e., a trade-off between reducing involuntary unemployment and relaxing the external credit limit. A quantitative study of this model shows that during financial crisis episodes, optimal policy features large nominal and real exchange rate depreciation. This is because, while containing real exchange rate depreciation can have welfare gains related to second moments (lower consumption volatility) its costs are related to first-moments (higher average unemployment rate). The optimal policy implies a lower currency depreciation than that necessary to achieve full employment, which is consistent with “managed-floating” exchange rate policy, typically observed during financial crises in emerging economies. Sudden Stops (or large current account adjustments) are part of the endogenous response under the optimal exchange rate policy to large negative shocks.

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1 Introduction

During external crises, exchange rate policy in emerging economies seems to imply choosing “between a rock and a hard place.” Typical examples are the policy debates during the East Asian and Latin American crises in the late 1990s (see Fischer, 1998; Calvo, 2001; Stiglitz, 2002) and the peripheral European crises that started in 2008 (see, for example, Krugman, 2010; Feldstein, 2011). On the one hand, it is argued that devaluing the exchange rate reduces unemployment and speeds up the recovery for the economy. On the other hand, it is claimed that, given that many of these economies have liabilities denominated in foreign currency, an exchange rate depreciation would produce sizable debt revaluation in terms of domestic income, and lead to financial destabilization.

I will call this the “credit access - unemployment trade-off” of exchange rate policy. The objective of this paper is to determine the optimal exchange rate policy under this trade-off by combining two branches of the literature that have studied the two phenomena separately. The first branch is the literature on downward nominal wage rigidity, as in Schmitt-Grohe and Uribe (2011, 2012a, 2012b). This literature has shown the high costs, in terms of unemployment and welfare, of fixing the nominal exchange rate during external crises. In particular, the combination of two nominal rigidities – the factor price rigidity and the fixed exchange rate – creates a real wage rigidity that generates an inefficient and costly adjustment to negative shocks, with involuntary unemployment (as argued in Friedman, 1953). The authors show that currency pegs can quantitatively account for the high increase in unemployment observed during episodes such as the Argentinean crisis of 2001 or the peripheral Europe crisis of 2008. In this framework, nominal exchange rate depreciations during external crisis periods can reduce the real value of wages and restore full employment in the economy.

The second branch of the literature is that in which external borrowing is limited by the value of collateral in the form of tradable and nontradable output (as in Mendoza, 2002, 2005; Arellano and Mendoza, 2003; Durdu, Mendoza, and Terrones, 2009; Korninek, 2011; Bianchi, 2011; Benigno et al., 2011, 2012a, 2012b, 2012c). Part of this literature has stressed the costs, in terms of deleveraging and consumption adjustment, associated with real exchange rate depreciations during external crisis periods in the presence of this form of credit limit. In particular, in an economy with debt denominated in the international unit of account, real exchange rate depreciations are associated with revaluations of external debt in terms of domestic income, and thus may imply a tightening in the collateral constraint and trigger an endogenous “Sudden Stop”\(^1\), generating a large cost in terms of consumption adjustment. For instance, Mendoza (2005) concludes that policy-induced changes in relative prices can lead to a downward spiral by the combination of balance-sheet effects and Fisher’s (1933) debt-deflation. Similarly, Benigno et al. (2012b, 2012c)

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\(^1\)Calvo (1998) labeled “Sudden Stops” episodes of large and abrupt reversals in external credit flows that characterize emerging economies.
study the welfare gains from supporting, i.e. not depreciating, the real exchange rate in times of crises.

This paper combines these two branches of the literature and determines the optimal exchange rate policy in a standard two sector (tradable and nontradable) small open economy model that incorporates both frictions: downward nominal wage rigidity and a collateral constraint in the form of tradable and nontradable output.

The contribution of this paper is threefold. First, it constructs an environment that provides a theoretical justification for the credit access – unemployment trade-off. In particular, it shows that in this economy, credit access and unemployment are two separate and conflicting factors hitting welfare, justifying the exchange rate policy debate that is typically observed during financial crises in emerging economies. The tension between the two policy objectives stems from the fact that, for typical parameter values of the intratemporal elasticity of substitution between tradable and nontradable goods, the price effect of increasing employment (real exchange rate depreciation) dominates the quantity effect, and thus the value of collateral is decreasing in the level of employment. Hence, the policy maker might need to choose between a lower level of unemployment with a tighter collateral constraint, and higher unemployment with a looser collateral constraint.

The second contribution of this paper is to show that, even in the presence of this trade-off, currency depreciations are in general optimal during external crises. The welfare costs of unemployment that result from fixing the nominal exchange rate constitute first-order costs, that, for a calibrated version of the model, significantly outweigh the second-order consumption smoothing welfare gains from relaxing the collateral constraint during crises. As a consequence, fixing the nominal exchange rate is an inefficient way to deal with the credit constraint and results in high welfare costs, both in regular business cycle fluctuations (0.8 percent on average) and in periods of binding collateral constraints (1.8 percent on average).

Third, a calibrated version of the model shows that, under binding credit constraints, the optimal currency depreciation is large (52 percent on average), but smaller than that which would achieve full employment (71 percent on average). Optimal exchange rate policy is thus consistent with a “managed-floating” exchange rate policy, so often observed in emerging economies during financial crises (see, for example, Calvo and Reinhart, 2002; Klein and Shambaugh, 2009; Calvo, 2013).

The presence of the credit access – unemployment trade-off is a key difference between this paper and previous studies on exchange rate policy that combine nominal rigidities and financial frictions. On the one hand, in several of these studies, credit access is not an objective in itself, but rather is relevant insofar as it affects employment and output. This is the case for instance in Aghion, Bacchetta, and Banerjee (2001), Cespedes, Chang, and Velasco (2004), Cook (2004), Devereux, Lane, and Xu (2006), and Braggion, Christiano, and Roldos (2009). In this literature the discussion is about whether exchange rate
depreciations are expansionary or contractionary. On the other hand, often in previous literature, currency depreciations do not affect or can even improve the borrowing capacity in the economy, a case parallel to that of the “divine coincidence” in New Keynesian models, where stabilizing inflation also stabilizes the output gap (Blanchard and Gali, 2005). This is the case, for instance, in Cespedes et al. (2004), in which the behavior of the risk premium is independent of the exchange rate regime, and in Fornaro (2013), in which the net effect of exchange rate depreciations is to help stabilize the economy by containing the contraction in asset prices and collateral values. In the setup of the present paper, the form of the collateral constraint, together with the nominal wage rigidity, implies that credit access and employment are two factors that display opposite welfare effects.

The empirical literature shows that both sides of the trade-off are supported by evidence. Cross-country regressions for emerging economies tend to show both that fixing the exchange rate during financial crisis episodes is associated with larger output contractions (see, for example, Ortiz et al., 2009) and that balance-sheet effects play a key role in determining the access to international credit markets (see, for example, Calvo, Izquierdo, and Mejia, 2008). These findings highlight the relevance of evaluating this policy trade-off with a structural model that allows the construction of counterfactuals and welfare comparisons of different exchange rate regimes.

Moreover, the policy choice under this trade-off is a long-standing and still lively policy debate. Keynes, for instance, was first actively opposed to the return of Britain to the gold standard after World War I, arguing that it would have a very high cost in terms of unemployment (Keynes, 1925). However, when the Great Depression started, Keynes recommended against devaluation, claiming that now the costs in terms of debt revaluation and financial destabilization would outweigh the benefits (Irwin, 2011). In the same line, Diaz-Alejandro (1965), analyzing Argentina’s exchange rate policy in the 1950s, highlighted the possibility that devaluations lead to negative wealth effects and adjustment in consumption from income distribution and balance-sheet effects. This policy debate was triggered again by the crisis of peripheral Europe that started in 2008, in which there are, simultaneously, high unemployment and high debt levels denominated in euros.

The rest of the paper is organized as follows. Section 2 presents the model economy. Section 3 defines three possible exchange rate regimes in this setup (optimal, full-employment and fixed exchange rate policies) and describes the exchange rate policy trade-off that emerges in this economy. Section 4 presents the quantitative analysis comparing the aggregate dynamics and welfare under the three exchange rate regimes. Section 5 compares exchange rate devaluations to fiscal devaluations. Section 6 concludes.
2 The Model Economy

This section describes the model economy used to conduct exchange rate policy analysis. It extends the two sector (tradable and nontradable), dynamic, stochastic, small open economy model with a downward nominal wage rigidity of Schmitt-Grohe and Uribe (2011, 2012a), to include a collateral constraint in the form of tradable and nontradable output. The economy only has access to a one-period, non-state-contingent debt instrument, denominated in units of tradable goods, capturing liability dollarization. The model then features a nominal rigidity and two financial frictions that will interact to determine the exchange rate policy trade-off.

Tradables are endowed to the economy, and their price is determined by the law of one price. Nontradables are produced by the economy, and their price is determined by domestic demand and supply. Fluctuations in the small open economy are driven by exogenous shocks to the value of tradable endowment (which can be interpreted as shocks to terms of trade or to productivity in the tradable sector) and to the interest rate on external debt, two sources of business cycle fluctuations that have been widely studied in emerging economies (Mendoza, 1995; Neumeyer and Perri, 2005; Uribe and Yue, 2006).

2.1 Households

Households’ preferences over consumption are described by the expected utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t),$$

where $c_t$ denotes consumption, the function $U(.)$ is assumed to be strictly increasing and concave, the subjective discount factor $\beta \in (0, 1)$, and $\mathbb{E}_t$ denotes expectation conditional on the information set available at time $t$.

The consumption good is assumed to be a composite of tradables and nontradable goods, with a CES aggregation technology:

$$c_t = A(c^T_t, c^N_t) = \left[a \left(c^T \right)^{1-\xi} + (1-a) \left(c^N \right)^{1-\xi}\right]^{\frac{\xi}{1-\xi}},$$

where $c^T_t$ denotes tradable consumption and $c^N_t$ denotes nontradable consumption.

Each period, households receive a stochastic endowment ($y^T_t$) and profits from the ownership of firms producing nontradable goods ($\Pi_t$). They supply inelastically $h_t$ hours of work to the labor market. Due to the presence of the wage rigidity (discussed in detail in the next sections), households will only be able to sell $h_t \leq \bar{h}$ hours in the labor market.

$^2$Liability dollarization or “original sin” is a phenomenon that has been widely documented to lie at the heart of balance-sheet effects in emerging economies (see Eichengreen, Hausmann, and Panizza, 2005).
The level of actual hours worked \((h_t)\) is determined by firms and is taken as given by the households.

Households have access to a one-period, non-state-contingent bond denominated in units of tradable goods that can be traded internationally paying an exogenous and stochastic gross interest rate \(R_t\). The model therefore assumes full liability dollarization. It is assumed that the vector of exogenous states, \(s_t^X \equiv [y_t^T, R_t]\), follows a first-order Markov process. Debt acquired in period \(t\) is taxed at rate \(\tau_t^d\). Households’ sequential budget constraint is therefore given by:

\[
dt+1 + R_t (1 - \tau_t^d) = dt + c_t^T + pt c_t^N - (y_t^T + w_t h_t + \Pi_t) - T_t,
\]

where \(dt+1\) denotes the level of debt assumed in period \(t\) and due in period \(t+1\), \(pt \equiv p_t^N\) denotes the relative price of nontradables in terms of tradables, \(w_t\) denotes the wage rate in terms of tradable goods, and \(T_t\) denotes a lump sum transfer in period \(t\).

It is assumed that households face a collateral constraint by which external debt cannot exceed a fraction \(\kappa\) of income:

\[
dt+1 \leq \kappa (y_t^T + w_t h_t + \Pi_t),
\]

where \(\kappa > 0\). This form of collateral constraint has been extensively used in the literature on small open economies to capture the effect of currency mismatch on external credit market access (e.g. Mendoza, 2002, 2005; Arellano and Mendoza, 2003; Durdu, Mendoza, and Terrones, 2009; Korninek, 2011; Bianchi, 2011; Benigno et al., 2011, 2012a, 2012b, 2012c): while collateral includes income from both tradable and nontradable sectors, external debt is fully denominated in units of tradables. The credit market frictions from which this constraint arises are not modeled here explicitly, but this form of collateral constraint can be seen as describing an environment in which lenders manage default risk by imposing a debt limit linked to households current income, as is typically the case of lending criteria in mortgage or consumer credit markets.\(^3\) It is consistent with empirical evidence that current income is a significant determinant of credit market access (Jappelli, 1990). Moreover, this formulation of the credit limit has been shown to account for the main features of business cycles and Sudden Stop episodes in emerging economies (Mendoza, 2002; Bianchi, 2011).

In addition, households are assumed to face a no-Ponzi game constraint of the form:

\[
dt+1 \leq d^N,
\]

where \(d^N\) denotes the natural debt limit. As in Aiyagari (1994), this is defined as the maximum value of external debt that the household can repay almost surely starting from

\(^3\)Korninek (2011) shows that this form of the collateral constraint can be rationalized as a renegotiation-proof form of debt contract in an imperfect credit market in which households can renegotiate external debt and lenders can extract at most a fraction of the current income of borrowers if debt is renegotiated.
that period, assuming that its tradable consumption is zero forever. Formally, denoting \( y^T \) as the minimum possible level of tradable endowment and \( R \) as the maximum possible level of external interest rate, the natural debt limit is defined as \( d^N \equiv \frac{R}{R-1} y^T \). Since the collateral value in the credit limit (4) depends on relative prices which can be affected by policy variables, imposing constraint (5) in addition to (4) is in order to prevent Ponzi schemes induced by the policy maker (Mendoza, 2005; Benigno et al., 2012b).

The household problem is to choose state-contingent plans for \( c_t, c^T_t, c^N_t, \) and \( d_{t+1} \) in order to maximize the expected utility (1) subject to the consumption aggregation technology (2), the sequential budget constraint (3), the collateral constraint (4), and the no-Ponzi game constraint (5), for a given value of initial debt level \( d_0 \), for the given sequence of prices \( w_t \) and \( p_t \), for the given sequence of hours worked \( h_t \) and profits \( \Pi_t \), stochastic tradable endowment \( y^T_t \) and interest rate \( R_t \), and for the given sequence of policies \( \tau^d_t \) and \( T_t \).

Denoting \( \lambda_t \), the Lagrange multiplier associated with the budget constraint (3), \( \mu_t \) the Lagrange multiplier associated with the collateral constraint (4), and \( \eta_t \) the Lagrange multiplier associated with the no-Ponzi game constraint (5), the optimality conditions are (2), (3), (4), (5), the first order conditions:

\[
\lambda_t R^{-1}_t \left( 1 - \tau^d_t \right) = \beta E_t \lambda_{t+1} + \mu_t + \eta_t, \tag{6}
\]

\[
U_c A_T(\tilde{c}^T_t, c^N_t) = \lambda_t, \tag{7}
\]

\[
\left( \frac{1-a}{a} \right) \left( \frac{c_T^T}{c_T^N} \right)^{**} = p_t, \tag{8}
\]

and the complementary slackness conditions:

\[
\mu_t \geq 0, \mu_t \left( \kappa (y^T + w_t h_t + \Pi_t) - d_{t+1} \right) = 0, \tag{9}
\]

\[
\eta_t \geq 0, \eta_t \left( d^N - d_{t+1} \right) = 0. \tag{10}
\]

### 2.2 Firms

Each period, operating in competitive labor and product markets, firms hire labor to produce the nontradable good, \( y^N_t \). Profits each period are given by:

\[
\Pi_t = p_t F'(h_t) - w_t h_t,
\]

where the production function \( F(\cdot) \) is assumed to be increasing and concave.

The firms’ problem is to choose \( h_t \) to maximize profits given prices \( p_t \) and \( w_t \). The first-order condition of this problem is:

\[
p_t F'(h_t) = w_t. \tag{11}
\]

This condition implicitly defines the demand of labor from firms.
2.3 The Labor Market

Nominal wages \( (W_t) \) are assumed to be downwardly rigid as in Schmitt-Grohe and Uribe (2011, 2012a, 2012b):\(^4\)

\[
W_t \geq \gamma W_{t-1},
\]

for \( \gamma > 0 \).

It is assumed that the *law of one price* holds for tradable goods, implying that \( P_T^t = E_t P_T^T \), where \( E_t \) is the nominal exchange rate and \( P_T^T \) is the foreign currency price of tradable goods. Assuming that \( P_T^T \) is constant and normalized to one, wages in terms of tradable goods \( (w_t) \) can be expressed as:

\[
w_t = \frac{W_t}{E_t}.
\]

From this, the wage rigidity can be expressed as follows:

\[
w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t},
\]

where \( \epsilon_t \) is the gross depreciation rate of the nominal exchange rate, defined as \( \epsilon_t \equiv \frac{E_t}{E_{t-1}} \).

Actual hours worked cannot exceed the inelastically supplied level of hours:

\[
h_t \leq \bar{h}.
\]

When the nominal wage rigidity binds, the labor market can exhibit involuntary unemployment, given by \( \bar{h} - h_t \). This means the following slackness condition must hold at all dates and states:

\[
\left( w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) (\bar{h} - h_t) = 0.
\]

This condition means that when the nominal wage rigidity is not binding, the labor market must exhibit full employment, and if it exhibits unemployment, it must be the case that the nominal wage rigidity is binding.

2.4 The Government

The government imposes a proportional tax (subsidy) on debt \( \tau_t^d \) that is rebated lump sum to households \( (T_t) \), to balance its budget each period:

\[
\frac{d_{t+1}}{R_t} \tau_t^d = T_t.
\]

Section 3 discusses how this capital control tax is determined.

\(^4\)Schmitt-Grohe and Uribe (2011) show that this form of wage rigidity is supported both by macro evidence during external crises in emerging economies, and by micro evidence for the US (Barattieri, Basu and Gottschalk, 2010).
2.5 General Equilibrium Dynamics

The market for nontradable goods clears at all times:

\[ c_t^N = F(h_t). \]  \hfill (16)

Combining the equilibrium price equation, (8), with condition (16), the firms’ optimality condition, (11), can be expressed as follows:

\[ w_t = \left(1 - \frac{a}{\bar{a}}\right) \left(\frac{1}{\gamma} c_t^T\right)^{\frac{1}{2}} F(h_t)^{-\frac{1}{2}} F'(h_t). \]  \hfill (17)

Combining condition (16) with households’ budget constraint, (3), the definition of firms’ profits, and the government’s budget constraint, (15), the resource constraint of the economy becomes:

\[ \frac{d_{t+1}}{R_t} = d_t + c_t^T - y_t^T. \]  \hfill (18)

Using the definition of firms’ profits, the equilibrium price equation, (8), and the market clearing condition for nontradables, (16), the collateral constraint, (4), can be re-expressed as follows:

\[ d_{t+1} \leq \kappa \left(y_t^T + \left(1 - \frac{a}{\bar{a}}\right) \left(\frac{1}{\gamma} c_t^T\right)^{\frac{1}{2}} F(h_t)^{-\frac{1}{2}} F'(h_t)\right) \equiv d(h_t, c_t^T, y_t^T, \bar{a}, y_t^T). \]  \hfill (19)

The general equilibrium dynamics are then given by stochastic processes \(c_t^N, c_t^T, h_t, p_t, w_t, d_{t+1}, \lambda_t, \mu_t, \eta_t, T_t\) satisfying the set of equations (GE):

(5): \(d_{t+1} \leq d^N\),
(6): \(\lambda_t R_t^{-1} (1 - \tau_t^d) = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t + \eta_t\),
(7): \(U_c (c_t^T, c_t^N) A_T (c_t^T, c_t^N) = \lambda_t\),
(8): \(p_t = \left(\frac{1-a}{\bar{a}}\right) \left(\frac{1}{\gamma} c_t^T\right)^{\frac{1}{2}}\),
(9): \(\mu_t \geq 0, \mu_t \left(\kappa \left(y_t^T + p_t F(h_t)\right) - d_{t+1}\right) = 0\),
(10): \(\eta_t \geq 0, \eta_t \left(d^N - d_{t+1}\right) = 0\),
(12): \(w_t \geq \gamma \frac{w_{t-1}}{\alpha_t}\),
(13): \(h_t \leq \bar{h}\),
(14): \(\left(w_t - \gamma \frac{w_{t-1}}{\alpha_t}\right) \left(\bar{h} - h_t\right) = 0\),
(15): \(T_t = \tau_t^d d_{t+1} R_t^{-1}\),
(16): \(c_t^N = F(h_t)\),
(17): \(w_t = \left(\frac{1-a}{\bar{a}}\right) \left(\frac{1}{\gamma} c_t^T\right)^{\frac{1}{2}} F(h_t)^{-\frac{1}{2}} F'(h_t)\),
(18): \(d_{t+1} R_t^{-1} = d_t + c_t^T - y_t^T\),
(19): \(d_{t+1} \leq \kappa \left(y_t^T + \left(\frac{1-a}{\bar{a}}\right) \left(\frac{1}{\gamma} c_t^T\right)^{\frac{1}{2}} F(h_t)^{-\frac{1}{2}} F'(h_t)\right)\),

given the exogenous processes \(y_t^T\) and \(R_t\), an exchange rate policy \(\epsilon_t\), a capital control tax policy \(\tau_t^d\) and initial conditions \(w_{-1}\) and \(d_0\).
3 Exchange Rate Regimes

3.1 Definition of Exchange Rate Regimes

I consider three possible exchange rate regimes: the optimal, the full-employment, and the fixed exchange rate policies. Exchange rate regimes will be evaluated conditional on an optimal capital control tax policy. Both the collateral constraint in the form of tradable and nontradable output and the downward wage rigidity used in this paper create a pecuniary externality that may induce overborrowing (Bianchi, 2011; Schmitt-Grohe and Uribe, 2012a). A tax (subsidy) on debt $\tau^d_t$ will be used to eliminate the inefficiency stemming from the collateral constraint. A similar strategy is often adopted in the New Keynesian literature where imposing a tax (subsidy) eliminates the distortion stemming from monopolistic competition (see, for example, Woodford, 2003). Evaluating different exchange rate policies without using the optimal capital control tax remains for future research. However, the overborrowing result found in the previous related literature probably makes conservative the devaluation rate under the optimal exchange rate policy and optimal capital control taxes: without optimal capital control taxes, external debt, and thus wages, would increase more during booms, and thus for a given negative shock and exchange depreciation, unemployment would be larger.

Optimal Exchange Rate Policy

The optimal exchange rate policy with optimal capital control taxes is defined as follows:

Definition 1 (Optimal exchange rate policy with optimal capital control taxes): Processes $\{\epsilon_t, \tau^d_t\}$ that maximize households’ expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (GE).

To characterize the allocation under optimal exchange rate and capital control tax policy, I set up the Ramsey problem dropping constraints (6)-(10), (12), and (14)-(17). Appendix A shows that any $\{d_{t+1}, c^T_t, h_t\}$ that satisfy (5), (13), (18), and (19) also satisfy (GE). The Ramsey problem is then to maximize (1) with respect to $\{d_{t+1}, c^T_t, h_t\}$, subject to (5), (13), (18), and (19). The dynamics under the optimal exchange rate policy with optimal capital control taxes can be thus expressed with the following Bellman equation:

$^{5}$Overborrowing refers to a situation in which the social planner’s optimal average level of external debt is lower than that of the decentralized equilibrium.
\[ V^{OP} (s^X_t, d_t) = \max_{d_{t+1}, c^T_t, h_t} \left[ U \left( A(c^T_t, F(h_t)) \right) + \beta \mathbb{E}_d V^{OP} (s^X_{t+1}, d_{t+1}) \right] \]  
\text{s.t. } \frac{d_{t+1}}{R_t} = d_t + c^T_t - y^T_t,
\[ d_{t+1} \leq \kappa \left( y^T_t + \left( 1 - \frac{a}{a} \right) \left( c^T_t \right)^{\frac{1}{\xi}} F(h_t) \right) \]
\[ d_{t+1} \leq d^N, \]
\[ h_t \leq \overline{h}, \]

where \( V^{OP} (s^X_t, d_t) \) denotes the value function of households under optimal exchange rate and capital control tax policies. This formulation will be used in the quantitative analysis.

**Full-Employment Exchange Rate Policy**

In the second place, consider an exchange rate policy aimed at maintaining full employment at all states and dates. Under the “full-employment” policy:

\[ h_t = \overline{h}, \]  
(21)

for every \( t \). The full-employment policy with optimal capital control taxes can be defined as follows:

**Definition 2** (Full-employment exchange rate policy with optimal capital control taxes): Processes \( \{ \epsilon_t, \tau^d_t \} \) that maximize households’ expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (GE), and the full-employment constraint (21).

To characterize the optimal allocation under the full-employment policy, I follow the same strategy as in the optimal exchange rate policy and drop constraints (6)-(10) and (12)-(17). Appendix A shows that any \( \{ d_{t+1}, c^T_t, h_t \} \) that satisfy (5), (18), (19), and (21), also satisfy (GE) and (21). Thus, the dynamics under the full-employment exchange rate policy with optimal capital control taxes can be expressed with the following Bellman equation:

\[ V^{FE} (s^X_t, d_t) = \max_{d_{t+1}, c^T_t} \left[ U \left( A(c^T_t, F(h)) \right) + \beta \mathbb{E}_d V^{FE} (s^X_{t+1}, d_{t+1}) \right] \]  
\text{s.t. } \frac{d_{t+1}}{R_t} = d_t + c^T_t - y^T_t,
\[ d_{t+1} \leq \kappa \left( y^T_t + \left( 1 - \frac{a}{a} \right) \left( c^T_t \right)^{\frac{1}{\xi}} F(h) \right) \]
\[ d_{t+1} \leq d^N, \]
where $V^{FE}(s_t^X, d_t)$ denotes the value function of households under the full-employment exchange rate policy with optimal capital control taxes.

**Fixed Exchange Rate Policy**

Finally, consider a policy aimed at keeping fixed the exchange rate at all states and dates. Under the fixed exchange rate policy or currency peg:

$$\epsilon_t = 1,$$  
(23)

for every $t$. The fixed exchange rate policy with optimal capital control taxes can be defined as follows:

**Definition 3** (Fixed exchange rate policy with optimal capital control taxes): Processes $\{\epsilon_t, \tau_d^t\}$ that maximize households’ expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (GE), and currency peg constraint (23).

To characterize the allocation under the currency peg with optimal capital control taxes, I follow a similar strategy to that of the optimal exchange rate policy and drop constraints (6)-(10) and (15)-(16). Appendix A shows that any $\{d_{t+1}, c_t^T, h_t, w_t, \epsilon_t\}$ that satisfy (5), (12)-(14), (17)-(19), and (23), also satisfy (GE) and (23). Thus, the dynamics under the currency peg with optimal capital control tax policy can be expressed with the following Bellman equation:

$$V^{PEG}(s_t^X, d_t, w_{t-1}) = \max_{d_{t+1}, c_t^T, h_t, w_t} \left[ U \left( A \left( c_t^T, F(h_t) \right) \right) + \beta E_t V^{PEG}(s_{t+1}^X, d_{t+1}, w_t) \right]$$  
(24)

s.t. $d_{t+1} \leq d^N$,

$$w_t \geq \gamma w_{t-1},$$

$$h_t \leq \bar{h},$$

$$(w_t - \gamma w_{t-1}) (\bar{h} - h_t) = 0,$$

$$w_t = \left( \frac{1 - a}{a} \right) (c_t^T)_{\frac{1}{2}} F(h_t)^{-\frac{1}{2}} F'(h_t),$$

where $V^{PEG}(s_t^X, d_t, w_{t-1})$ denotes the value function of households under the currency peg and optimal capital control taxes.
3.2 Optimal Exchange Rate Policy, Unemployment and Credit Limit: Analytical Results

This section studies the relationship between unemployment and the credit limit under the optimal exchange rate policy. Although, given the complexity of the model, a numerical solution is required for a full characterization, some analytical results can be obtained to show the trade-off involved in exchange rate policy. These results will be relevant for understanding the numerical solution for the dynamics of the economy under the optimal exchange rate policy in the next section. The allocation under the optimal exchange rate policy, defined in the previous section, can be characterized as follows:

**Proposition 1.** Under the optimal exchange rate policy with optimal capital control taxes (Definition 1) the following conditions hold at all dates and states:

- if \( \xi < 1 \), \( (\bar{h} - h_t) (\bar{d}(h_t, c^T_t, y^T_t) - d_{t+1}) = 0 \),
- if \( \xi \geq 1 \), \( h_t = h_\star \).

*Proof.* See Appendix B. ■

Thus, if the intratemporal elasticity of substitution is greater than or equal to one, optimal exchange rate policy always features full employment. If the intratemporal elasticity of substitution is less than one, a slackness condition is established between unemployment and the collateral constraint under the optimal exchange rate policy: if the collateral constraint is not binding, the labor market must exhibit full employment, and if there is unemployment, it must be the case that the collateral constraint is binding. As discussed at the end of this section, empirical evidence for emerging economies provides wide support for the intratemporal elasticity of substitution being less than one. To understand the role of the intratemporal elasticity of substitution and the interaction between unemployment and the collateral constraint under the optimal exchange rate policy, a discussion is in order regarding the trade-off faced by exchange rate policy in this economy.

**The Credit Access – Unemployment Trade-off**

Parallel to the traditional inflation-unemployment trade-off in the New Keynesian literature, the exchange rate policy in this economy may face a “credit access – unemployment trade-off.” First, in this setup, employment and credit market access are two independent welfare objectives. Under a binding credit constraint, the tighter the constraint for a given debt level, the higher the debt repayment and the adjustment required of the household’s consumption in order to repay debt. Thus, credit market access directly affects tradable consumption and welfare (as Mendoza, 2005, and Bianchi, 2011 study for endowment economies). This is a difference from most of the previous literature on exchange rate
policy that integrates nominal rigidities and financial frictions, where credit market access does not directly affect welfare, but rather is relevant only insofar as it affects employment and output. This is the case, for instance, in Cespedes et al. (2004), Cook (2004), and Devereux et al. (2006), which study an economy featuring the financial amplificator (Bernanke, Gertler, and Gilchrist, 1999), or in Aghion et al. (2001) and Braggion et al. (2009), which analyze economies with credit-constrained firms. In this literature, the central stabilization policy objective is employment or output, and the discussion is about whether exchange rate depreciations are in fact expansionary or contractionary.

Second, in the economy of the present paper, exchange rate policy might imply a tension between these two objectives. Under binding nominal downward wage rigidity, a depreciation of the nominal exchange rate decreases real wages and, thus, helps achieve the objective unemployment reduction. But it is also associated with a real exchange rate depreciation, which decreases the value of nontradable output in units of tradables. Recall that the collateral in this economy is given by the value, in units of tradables, of tradable and nontradable output. Thus, if the price effect (real exchange rate depreciation) dominates the quantity effect (employment increase), an exchange rate depreciation can decrease the collateral value and make the credit limit tighter, which goes against the credit access objective. The price effect dominates the quantity effect if the intratemporal elasticity of substitution between tradables and nontradables is less than one ($\xi < 1$).

As discussed in the next section, this assumption is widely supported by empirical evidence on emerging economies. Under this assumption, the following proposition can be established:

**Proposition 2.** If $\xi < 1$, given an initial state $(s_t^X, d_t)$, for any debt level $d_{t+1}^*$ with associated tradable consumption $c_t^T = (d_{t+1}^* R_t^{-1} - d_t + y_t^T) > 0$ for which $d_{t+1}^* > \overline{d}(\overline{h}, c_t^T, y_t^T)$, there exists a level of employment $h_t^* \in (0, \overline{h})$ for which $d_{t+1}^* = \overline{d}(h_t^*, c_t^T, y_t^T)$.

**Proof.** See Appendix B. ■

This shows that for any debt level that does not satisfy the credit limit under full employment, there exists a level of employment below full employment, for which the real exchange rate is sufficiently appreciated to ensure the credit limit is satisfied for that debt level. This result stems from the fact that if the intratemporal elasticity of substitution is less than one ($\xi < 1$), the collateral constraint is decreasing in the level of employment. Therefore, as previously mentioned, this provides a theoretical justification for the existence of the exchange rate policy debate that is typically observed during financial crises in emerging economies, which weighs the two policy objectives.

If the intratemporal elasticity of substitution is greater than or equal to one ($\xi \geq 1$), the credit access – unemployment trade-off vanishes, as implied by Proposition 1. In particular, if the intratemporal elasticity of substitution is equal to one ($\xi = 1$), employment...
does not influence the collateral constraint. This would be similar to the result obtained by Cespedes et al. (2004), where, in the framework of the financial amplificator (Bernanke et al., 1999), the behavior of the risk premium ends up being independent of the exchange rate regime. If the intratemporal elasticity of substitution is greater than one \((\xi > 1)\), the credit access – unemployment trade-off overturns and a decrease in unemployment also helps relax the collateral constraint. This would be a case parallel to that of the “divine coincidence” in New Keynesian models, where inflation stabilization also helps output stabilization (Blanchard and Gali, 2005). Fornaro (2013) obtains this result using a financial friction in the form of a collateral constraint limiting the size of external debt to a fraction of the market value of an asset (e.g. land). Exchange rate depreciations increase employment and help stabilize the economy by preventing asset prices from falling too much. Therefore, the presence of the credit access – unemployment trade-off is a major difference between the present paper and previous literature and provides a stronger alternative to flexible exchange rates.

Empirical Evidence on the Intratemporal Elasticity of Substitution

As shown in Propositions 1 and 2, there exists a tension between credit access and unemployment only if the elasticity of substitution between tradable and nontradable goods is less than one \((\xi < 1)\). If this is the case, tradable and nontradable goods are gross complements, and the price effect (real exchange rate depreciation) associated with increasing employment dominates the quantity effect. As a result, exchange rate depreciation can decrease the collateral value and make the credit limit tighter.

There is wide support from the empirical literature for the intratemporal elasticity of substitution being less than one. In a sample of developed and emerging economies, Stockman and Tesar (1995) estimate a value of the elasticity of substitution of 0.44. Separating the samples of developed and emerging economies, Mendoza (1995) finds values of the elasticity of 0.74 and 0.43, respectively. In studies for emerging economies, Gonzalez-Rozada et al. (2004) found estimates in the range between 0.4 and 0.48 for Argentina and Lorenzo, Aboal, and Osimani (2005), found estimates in a range between 0.46 and 0.75 for Uruguay.\(^6\)

Moreover, following this empirical literature, all the studies referenced in the present paper that calibrate a two sector small open economy model, use a parameter value of the elasticity of substitution in the range between 0.44 and 0.83.

\(^6\)Ostry and Reinhart (1992) found evidence inconclusive in this respect with estimates between 0.66 and 1.44 depending on the emerging region and the instrumental variable considered. For a survey on the methodologies used to estimate the elasticity of substitution between tradable and nontradable goods see Akinci (2011).
4 Quantitative Analysis

The objective of this section is to quantitatively characterize the aggregate dynamics of the model economy under the optimal exchange rate policy and compare its performance in terms of welfare to that under the full-employment and fixed exchange rate policies, both during periods of financial crises and under regular business cycle fluctuations.

4.1 Calibration and Computation

To characterize the aggregate dynamics under different exchange rate regimes, I solve a calibrated version of the functional problems defined in Section 3. In particular the functional equations (20), (22) and (24) are solved numerically to approximate the dynamics under the optimal exchange rate policy, the full-employment policy, and the currency peg respectively. Due to the presence of occasionally binding constraints, I resort to the method of value function iteration over a discretized state space to compute the numerical solutions.

As mentioned in Section 2, the consumption aggregator is assumed to be a CES aggregator. I further assume a CRRA period utility function and an isoelastic form for the production function:

\[ U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \]
\[ F(h) = h^\alpha. \]

The model is calibrated at the annual frequency, to match Argentinean data. Argentina is used as a benchmark to conduct this exercise because it is an emerging economy whose exchange rate regimes and financial crises have been widely studied, in particular in the two branches of the literature that this paper combines.

All parameter values used in the benchmark calibration are shown in Table 1. The inverse of the intertemporal elasticity of substitution is set to \( \sigma = 2 \), a standard value in the business cycle literature for small open economies (see, for example, Mendoza 1991). The intratemporal elasticity of substitution is set to \( \xi = 0.44 \), using the estimates of Gonzalez-Rozada et al. (2004) for Argentina (see Section 3.2 for a review of the literature on this parameter). I set \( \alpha = 0.75 \), following the evidence in Uribe (1997) on the labor share in the nontradable sector in Argentina, and \( \gamma = 0.96 \), following the evidence in Schmitt-Grohe and Uribe (2011) on downward nominal wage rigidity. The mean level of tradable output and the labor endowment (\( \bar{h} \)) are normalized to one.

The parameters \( \{\beta, a, \kappa\} \) are used to match three key moments in the ergodic distributions of the model under the optimal exchange rate policy to the ones observed in
Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Inverse of intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.44</td>
<td>Intratemporal elasticity of substitution</td>
</tr>
<tr>
<td>$a$</td>
<td>0.295</td>
<td>Share of tradables</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.8</td>
<td>Annual subjective discount factor</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.263</td>
<td>Share of income used as collateral</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>Labor share in nontradable sector</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.96</td>
<td>Degree of downward nominal rigidity</td>
</tr>
</tbody>
</table>

historical Argentinean data (for the period 1975-2011). The three data moments that are considered are typically targeted in related literature (for example, Bianchi, 2011 and Benigno et al., 2012a): an average level of external debt-to-GDP ratio of 21 percent, a share of tradable output in GDP of 32.9 percent, and a frequency of “Sudden Stops” of 5.5 percent. A Sudden Stop in the model is defined as a period in which the economy exhibits a change in the current account larger than one standard deviation, following Eichen green, Gupta and Mody (2006), from which the frequency of Sudden Stops is obtained for a sample of emerging economies. The average debt-to-GDP ratio and the share of tradable output in GDP for Argentina are computed using Lane and Milesi-Ferreti (2007) and WDI datasets, respectively. For the latter, I follow the conventional definition in the literature and include agriculture and manufacturing sectors as tradable sectors in the economy. The parameter values obtained from this calibration are $\beta = 0.8$, $a = 0.295$ and $\kappa = 0.263$. Section 4.4 studies the sensitivity of the optimal policy to this calibration.

It is assumed that the two exogenous driving forces, tradable endowment and interest rate, follow a first-order VAR of the form:

$$
\begin{bmatrix}
\ln(y_t) \\
\ln(R_t)
\end{bmatrix}
= \Phi
\begin{bmatrix}
\ln(y_{t-1}) \\
\ln(R_{t-1})
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_t^y \\
\varepsilon_t^R
\end{bmatrix},
$$

where $\begin{bmatrix}\varepsilon_t^y & \varepsilon_t^R\end{bmatrix} \sim i.i.d.N(\emptyset, \Omega)$ and $R$ denotes the interest rate mean level.

The parameters of this stochastic process are estimated using Argentinean data since 1983. Tradable endowment is measured with the cyclical component of value added in agriculture and manufacturing, using the WDI dataset, as mentioned above. Interest rates on external debt are measured as the sum of EMBI spreads and the Treasury-Bill rate. Since data on EMBI spreads for Argentina is available since 1994 the series were extended back to 1983, using Neumeyer and Perri (2005) dataset, which uses a measure similar to the one considered here. The interest rate series is then deflated with a measure

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7 The frequency for emerging economies is similar to the frequency in Argentina during this period, and to other empirical estimates, such as Calvo et al. (2008).
of expected dollar inflation.\textsuperscript{8} The years 2002-2005, in which Argentina defaulted and was excluded from international markets (Cruces and Trebesch, 2013), were not included in the estimation. The following OLS estimates are obtained:

\[ \hat{\Phi} = \begin{bmatrix} 0.42 & -0.28 \\ 0.32 & 0.93 \end{bmatrix}, \quad \hat{\Omega} = \begin{bmatrix} 0.002 & -0.001 \\ -0.001 & 0.001 \end{bmatrix}, \quad \hat{R} = 1.113. \]

This process is approximated with a Markov-chain, setting a grid of 15 equally spaced points for both \( \ln(y^T_t) \) and \( \ln(R_t/R_t) \), yielding 225 exogenous states. To estimate the transition-probability matrix, I use the method proposed by Terry and Knotek (2011) extending Tauchen (1986).\textsuperscript{9}

Finally, to approximate the aggregate dynamics of the economy under the optimal and the full-employment policies, I discretize the endogenous state space \((d_t)\) using 1001 equally spaced points. To approximate the dynamics under a currency peg, I use 251 equally spaced points for debt \((d_t)\) and 250 equally spaced points for the log of previous period wage \((w_{t-1})\). The next sections present the results for the quantitative analysis.

### 4.2 Optimal Exchange Rate Policy during Financial Crises

Under no binding collateral constraints, the optimal exchange rate policy always consists of depreciating the nominal exchange rate in response to negative shocks to achieve full employment, as implied by Proposition 1. This section characterizes the optimal exchange rate policy under periods of binding collateral constraints, or financial crises, and compares the dynamics of the economy under the different exchange rate regimes.

To do this, the calibrated version of the model is simulated for 2 million quarters, identifying periods in which the collateral constraint is binding under the optimal exchange rate policy. The beginning of a financial crisis episode \((t = 0)\) is defined as the first period in which the collateral constraint binds. The response of the variables during all episodes of financial crises is then averaged.

Figure 1 depicts the average external shock during a financial crisis episode. In the two years that precede a financial crisis episode, tradable endowment contracts and interest rate increases. At the crisis trough \((t = 0)\), tradable output is 10 percent below its mean, and the annual interest rate is 16 percent, 4 percentage points above its mean. In the three years following the trough, tradable output and the interest rate recover their pre-crisis levels.

\textsuperscript{8}In particular, \(R_t\) is measured as \(R_t = (1 + i_t) E_t \left( \frac{1}{1 + \pi_{t+1}^*} \right)\), where \(i_t\) denotes the interest rate on Argentinian external debt in US dollars, and \(\pi_{t+1}^*\) denotes US CPI. \(E_t \left( \frac{1}{1 + \pi_{t+1}^*} \right)\) is obtained as the one step ahead forecast of an estimated AR(1).

\textsuperscript{9}I am grateful to Stephen J. Terry and Edward S. Knotek II for sharing the codes for Markov-Chain approximations of vector autoregressions, which were used in this paper to estimate the transition probability matrix of the stochastic process.
The average response of the nominal exchange rate and endogenous variables under the different exchange rate regimes are shown in Figure 2. Optimal and full-employment exchange rate policies display striking similarities and a sharp contrast with the response under a currency peg. Even under binding collateral constraints \((t = 0)\), the optimal exchange rate policy is not to fix but to substantially depreciate the nominal exchange rate, 52 percent on average. This depreciation is smaller than that under the full-employment policy (71 percent), and as a result, some involuntary unemployment emerges under binding collateral constraints (1.6 percent on average at the crisis trough). However, unemployment under the optimal exchange rate policy is significantly lower than that observed under the currency peg (6.2 percent on average at the crisis trough).

The large, but still contained, optimal currency depreciation during periods of financial crises is consistent, for instance, with the typical behavior observed in emerging economies in the global financial turbulence of 2008 (see Figure 3). During this episode, emerging economies considerably depreciate the exchange rate (24 percent on average), but also contain the depreciation, as can be observed in the fall in international reserves. Calvo (2013) shows that this pattern of large nominal depreciation (more than 20 percent on average) with simultaneous exchange rate intervention is the typical policy observed in emerging economies during periods of Sudden Stops since 1980.
It is important to note that, in periods of binding collateral constraints, the large real exchange rate depreciation under the optimal exchange rate policy (the relative price of nontradable goods is 39 percent below its mean at the crisis trough), implies a large adjustment of external debt and tradable consumption. Under the optimal policy, the
contraction of tradable consumption is much larger than the contraction in nontradable consumption: at the crisis trough, tradable consumption is 20.8 percent below its mean while nontradable consumption is only 1.2 percent below its mean. To understand this result, note that by market clearing of nontradables, a contraction in nontradable consumption implies a contraction in production and income, while a contraction in tradable consumption only implies an effect on consumption smoothing. I will come back to this in the next section when studying welfare. In this sense, Sudden Stops (understood as large current account adjustments), are in fact part of the endogenous response under the optimal exchange rate policy to large negative external shocks, to prevent greater unemployment. This is again in sharp contrast to the behavior under the currency peg, where, for the same exogenous shock, external debt continues increasing and the current account deficit expands; at the crisis trough, tradable consumption is 7.6 percent below its mean and nontradable consumption 4.2 percent below its mean.
This section compares welfare under the different exchange rate regimes. The welfare costs of an exchange rate regime $i$ with respect to an exchange rate regime $j$ are computed as the percentage increase in the consumption stream under exchange rate regime $i$ that will make the representative household indifferent between that consumption stream and that under the exchange rate regime $j$. Formally, the compensation rate under the regime $i$ with respect to regime $j$, $\lambda_{i,j}$, in a state $s^i_t$ is implicitly defined by:

$$E \left\{ \sum_{k=0}^{\infty} \beta^k \left( \frac{c^i_{t+k} (1 + \lambda_{i,j} (s^i_t))}{1 - \sigma} - 1 \right) \right\} | s^i_t = E \left\{ \sum_{k=0}^{\infty} \beta^k \left( \frac{c^j_{t+k}}{1 - \sigma} - 1 \right) \right\} | s^i_t,$$

where $i, j \in \{OP, FE, PEG\}$, $s^i_t = (s^X_t, d_t)$ if $i \in \{OP, FE\}$ and $s^i_t = (s^X_t, d_t, w_{t-1})$ if $i = \{PEG\}$.

It follows that:

$$\lambda_{i,j} (s^i_t) = \left[ \frac{V^j \left( s^i_t \right) (1 - \sigma) + (1 - \beta)^{-1}}{V^i \left( s^i_t \right) (1 - \sigma) + (1 - \beta)^{-1}} \right]^{\frac{1}{1-\sigma}} - 1.$$

The distribution of the compensation $\lambda_{i,j} (s^i_t)$ depends on the distribution of the state $s^i_t$, which in turn depends on the exchange rate regime. Thus, as noted in Schmitt-Grohe and Uribe (2011), to compute the distribution of $\lambda_{i,j} (s^i_t)$, it is convenient to evaluate $V^i \left( s^i_t \right)$ and $V^j \left( s^i_t \right)$, sampling from the ergodic distribution of the state under the same exchange rate regime.\(^{10}\)

With this methodology, I construct the distribution of the welfare costs of the full employment policy and the currency peg with respect to optimal policy, and the distribution of welfare costs of the currency peg with respect to full-employment policy. Results are presented in Table 2 and indicate that the average welfare costs of the full-employment policy with respect to the optimal policy (0.006 percent) are significantly lower than welfare costs of the currency peg with respect to the optimal policy (0.8 percent).

To understand this result, Table 3 compares the unconditional first and second moments of consumption, real exchange rate, unemployment and external debt under the three exchange rate regimes. It can be seen that the only gain of the optimal exchange rate policy with respect to the full-employment policy is a lower volatility of tradable and total consumption. This indicates that the welfare costs of the full-employment exchange rate policy imposes a maximum level of debt which is lower than in the other two exchange rate regimes. Sampling from the distribution of states under the optimal policy, for example, would imply that for some levels of debt it would not be possible to respect the collateral constraint and non-negative consumption under the full-employment exchange rate policy.
Table 2: Welfare Costs by Exchange Rate Regime

<table>
<thead>
<tr>
<th>Welfare Costs of:</th>
<th>Full-Employment Policy with respect to:</th>
<th>Currency Peg Optimal Policy</th>
<th>Currency Peg Full-Employment Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.006</td>
<td>0.833</td>
<td>0.827</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.024</td>
<td>0.694</td>
<td>0.695</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.0</td>
<td>7.115</td>
<td>7.113</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.001</td>
<td>0.001</td>
<td>−1.71</td>
</tr>
</tbody>
</table>

Note: welfare costs expressed in percent. The welfare costs of an exchange rate regime $i$ with respect to an exchange rate regime $j$, are defined as the percentage increase in the consumption stream under exchange rate regime $i$ that will make the representative household indifferent between that consumption stream and that under the exchange rate regime $j$ in a given state.

Table 3: First and Second Moments by Exchange Rate Regime

<table>
<thead>
<tr>
<th></th>
<th>OP</th>
<th>FE</th>
<th>CP</th>
<th>OP</th>
<th>FE</th>
<th>CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption $(c_t)$</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.0286</td>
<td>0.0289</td>
<td>0.030</td>
</tr>
<tr>
<td>Tradable Consumption $(c^T_t)$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.95</td>
<td>0.083</td>
<td>0.084</td>
<td>0.058</td>
</tr>
<tr>
<td>Nontradable Consumption $(c^N_t)$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.004</td>
<td>0.000</td>
<td>0.023</td>
</tr>
<tr>
<td>Real Exchange Rate $(p_t)$</td>
<td>2.09</td>
<td>2.09</td>
<td>2.16</td>
<td>0.407</td>
<td>0.414</td>
<td>0.253</td>
</tr>
<tr>
<td>Unemployment $(\bar{h} - h_t)$, in percent</td>
<td>0.04</td>
<td>0.0</td>
<td>0.8</td>
<td>0.5</td>
<td>0.0</td>
<td>2.9</td>
</tr>
<tr>
<td>External Debt $(d_t)$</td>
<td>0.64</td>
<td>0.64</td>
<td>0.52</td>
<td>0.04</td>
<td>0.04</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Note: Unconditional moments were computed in respective ergodic distributions. OP, FE and CP denote optimal exchange rate policy, full-employment exchange rate policy and currency peg respectively, as defined in Section 3.

rate policy with respect to the optimal policy are only related to consumption smoothing, akin to those initially studied by Lucas (1987), typically in the order of magnitude of one-tenth of one percent.

In contrast, the welfare costs of currency pegs are costs affecting first moments: the currency peg displays a higher average unemployment rate (0.8 percent on average). As studied in Schmitt-Grohe and Uribe (2011), the welfare effects related to first moments tend to be much larger than those related to second moments. Moreover, currency pegs do not even generate a welfare gain from the point of view of volatility: the volatility of consumption of tradables is lower, but the volatility of nontradable consumption is higher, resulting in a higher volatility of consumption with respect to the other two regimes. As a consequence, on average, the full-employment policy is superior to the currency peg in terms of welfare.
Figure 4 shows welfare costs during financial crisis episodes (as defined in Section 4.2). It can be observed that financial crises are periods in which the welfare costs of both the full-employment policy and the currency peg increase. The size of the increase in the welfare costs of the full-employment policy are, again, much smaller than the increase in the welfare costs of currency pegs: at the crisis trough the welfare costs of the full-employment and currency peg with respect to the optimal exchange rate policy are 0.06 percent and 1.83 percent respectively. As a consequence, the welfare costs of the currency peg with respect to the full-employment exchange rate regime also rise during financial crises, reaching 1.77 percent at the crisis trough, meaning that, currency pegs are particularly costly in terms of welfare during periods of binding collateral constraints.

Figure 4: Welfare Costs By Exchange Rate Regime During Financial Crises

Note: the welfare costs of an exchange rate regime $i$ with respect to an exchange rate regime $j$, are defined as the percentage increase in the consumption stream under exchange rate regime $i$ that will make the representative household indifferent between that consumption stream and that under the exchange rate regime $j$ in a given state. See Section 4.2 for the definition of a financial crisis episode.
4.4 Sensitivity Analysis

This section studies how the characterization of the optimal exchange rate policy during financial crises is affected by the choice of alternative parameter values. In particular, Figure 5 shows the average values of nominal exchange rate depreciation, real exchange rate, unemployment, and tradable consumption under the optimal exchange rate policy in periods in which the collateral constraint binds for alternative parameter values. The focus is on financial crises since, as shown in Proposition 1, periods of non-binding collateral constraint are always characterized by full employment under the optimal exchange rate policy, independent of parameter values.

I begin by studying alternative values for the intratemporal elasticity of substitution, considering values in the range used in the literature, between $\xi = 0.4$ and $\xi = 0.83$ (see Section 3.2 for a survey). The value of this parameter used in the baseline calibration is $\xi = 0.44$, following the estimates of Gonzalez-Rozada et al. (2004) for Argentina. As explained in Section 3.2, this parameter determines the extent to which exchange rate depreciations decrease collateral values. If $\xi \geq 1$, there is no negative effect of a currency depreciation on collateral values, and the optimal policy is always to achieve full employment. As expected from this, Figure 5 indicates that the higher the value of the intratemporal elasticity of substitution, the lower the unemployment rate under the optimal exchange rate policy. In this sense, the conclusions obtained in the baseline calibration are conservative with respect to this parameter value: a higher intratemporal elasticity of substitution would imply an optimal policy even closer to full-employment.

In the second place, I study alternative values for $\kappa$, the parameter that governs the collateral constraint. To the best of my knowledge, there is no empirical estimate available of this parameter. In the baseline calibration this parameter was set to $\kappa = 0.263$ to match the probability of Sudden Stops. I now consider alternative values for the collateral parameter ranging from $\kappa = 0.2$ (average debt-to-GDP ratio in Argentinean data) to $\kappa = 0.645$ (maximum debt-to-GDP ratio in Argentinean data). Results in Figure 5 indicate that, in this range of parameter values, the higher the collateral parameter, the lower the depreciation rate under the optimal exchange rate policy, and the higher resulting unemployment and tradable consumption. The difference is non-trivial: for instance, with a value of $\kappa = 0.645$ the average depreciation rate in a period of financial crisis is 10.3 percent and resulting unemployment, 5.3 percent (which compares to 1.6 percent in the baseline scenario). The intuition for this result is that, the higher the collateral parameter, the higher the effect that containing real exchange rate depreciation has on collateral values, and thus the higher the benefits of containing depreciation. However, even in this case it can be observed that the optimal policy features a large real exchange rate depreciation during financial crises (24 percent fall in the relative price of nontradable goods), implying that it is optimal to contract tradable consumption more than nontradable consumption and employment.
In the third place, it worth mentioning that $\gamma$, the degree of wage rigidity, does not affect the allocation under the optimal exchange rate policy, except for the value of the optimal nominal depreciation rate, $\epsilon_t$. This can be seen from the fact that the parameter $\gamma$ does not enter in the formulation of the Bellman equation (20) that describes the dynamics of $\{d_{t+1}, c^T_t, h_t\}$ under the optimal exchange rate policy. It follows that the real exchange rate depreciation under the optimal exchange rate policy does not depend on the parameter $\gamma$ either. The only variable under the optimal exchange rate policy that is affected by the parameter $\gamma$ is the nominal exchange rate depreciation: the lower the $\gamma$, the lower average nominal exchange rate depreciation required to implement the optimal allocation during a financial crisis episode, as illustrated in Figure 5.

As a summary, the main findings regarding optimal exchange rate during financial crises in previous sections are robust with respect to alternative values of structural parameters: the optimal exchange rate policy implies, on average, large real exchange rate depreciations during financial crises; this is achieved by depreciating the nominal exchange rate, and implies a relatively small increase in unemployment rate compared to that of tradable consumption.

Figure 5: Sensitivity of Optimal Policy during Financial Crises

Note: Figures denote the value of each variable at the trough ($t = 0$) of the average financial crisis episode (see Section 4.2 for the definition of a financial crisis episode).
5 Exchange Rate Policy and Fiscal Devaluations

So far, this paper has considered the nominal exchange rate as a possible instrument available to the policy maker. Motivated by the peripheral European crises that started in 2008 in which, in principle, countries cannot use the exchange rate as a policy instrument, a part of the related literature has focused on fiscal alternatives to exchange rate devaluations, or fiscal devaluations (see, for example, Farhi, Gopinah, Itskhoki, 2011 and Schmitt-Grohe and Uribe, 2011). A key result, in the case of an economy with a collateral constraint in the form of tradable and nontradable income, is the one obtained in Benigno et al (2012b): using taxes on nontradable or tradable consumption, the Ramsey planner can achieve the unconstrained first-best.

A similar result can be shown in the setup of the present paper with the presence of a downward nominal wage rigidity, and even under a currency peg. I begin by expanding the setup of the model economy, by assuming that the policy maker, in addition to the capital control tax, has access to a tax (subsidy) on nontradable consumption, \( \tau_N^t \). Households’ sequential budget constraint is now given by:

\[
d^{t+1}_t + \frac{1}{R_t} \left( 1 - \tau^d_t \right) = d_t + c^T_t + p_t \left( 1 + \tau^N_t \right) c^N_t - (y^T_t + w_t h_t + \Pi_t) - T_t.
\]

(25)

From this, the four equilibrium conditions that are modified in the system of equations (GE), defining the general equilibrium dynamics (Section 2), are: the equilibrium price of nontradables, (8), the equation that defines the government balanced budget each period, (15), the firms’ optimality condition in equilibrium, (17), and the collateral constraint, (19); these equations are replaced, respectively, by:

\[
p_t = \frac{1}{1 + \tau^N_t} \left( \frac{1-a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{1 \xi}.
\]

(26)

\[
d^{t+1}_t + \frac{1}{R_t} \tau^d_t + p_t \tau^N_t c^N_t = T_t
\]

(27)

\[
w_t = \frac{1}{1 + \tau^N_t} \left( \frac{1-a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{1 \xi} F(h_t)^{-1} F'(h_t),
\]

(28)

\[
d^{t+1} \leq \kappa \left( y^T_t + \frac{1}{1 + \tau^N_t} \left( \frac{1-a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{1 \xi} F(h_t)^{-1} F'(h_t) \right).
\]

(29)

Thus, the general equilibrium dynamics in this expanded setup is given by stochastic processes \( c^N_t, c^T_t, h_t, p_t, w_t, d_{t+1}, \lambda_t, \mu_t, \eta_t, T_t \) satisfying the set of equations (GE’):

\{(5), (6), (7), (9), (10), (12), (13), (14), (16), (18), (26), (27), (28), (29)\}, given the exogenous processes \( y^T_t \) and \( R_t \), an exchange rate policy \( \epsilon_t \), a capital control tax policy \( \tau^d_t \), a tax policy on nontradable consumption \( \tau^N_t \), and initial conditions \( w_{-1} \) and \( d_0 \).

\footnote{The optimal allocation attained under a tax (subsidy) on nontradable consumption can alternatively be attained with a tax (subsidy) on tradable consumption.}
The following proposition shows that in an economy with a currency peg, with a tax on nontradable consumption, and a capital control tax, the Ramsey planner can achieve the unconstrained first-best of the economy:

**Proposition 3.** Define the unconstrained first-best solution as processes \( \{d^*_t, c^*_t, h^*_t\} \) that solve

\[
\max \left\{ d_{t+1}, c^*_T, h^*_t \right\}
\]

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U \left( A(c^*_t, F(h_t)) \right)
\]

s.t. \( \frac{d_{t+1}}{R_t} = d_t + c^*_t - y^*_t \),
\( h_t \leq \bar{h} \),
\( d_{t+1} \leq d^N \).

In an economy defined by (GE') and (23), there exists a tax policy \( \{\tau^N_t, \tau^d_t\} \) that decentralizes the unconstrained first-best solution.

**Proof.** See Appendix B.

This result implies that the credit access – unemployment trade-off analyzed in this paper is a particular feature of exchange rate policy, not necessarily present in some forms of fiscal alternatives to exchange rate policy that have been studied in the literature as fiscal devaluations. This is because, under binding wage rigidity, while an exchange rate depreciation decreases real wages and expands the supply of nontradables (equation (11)), which is associated with an increase in employment and a decrease in the price of nontradable goods (real exchange rate depreciation), an increase in a subsidy to nontradable consumption expands the demand of nontradables for every level of tradable consumption (equation (26)), which is associated with both an increase in employment and an increase in the relative price of nontradable goods. For this reason, while an exchange rate devaluation decreases collateral values, a fiscal devaluation implemented with a subsidy on nontradable consumption implies an increase in collateral values, a no trade-off between credit access and unemployment. Thus, with fiscal devaluations, the policy maker can simultaneously achieve full employment and make the collateral constraint not binding, as implied by Proposition 3.

The implications of this analysis are twofold. First, from a theoretical perspective, this shows that the conclusions obtained from analyzing taxes on nontradable goods cannot in general be interpreted as conclusions for exchange rate policy, as it is frequently done in related literature. Second, from a policy perspective, this analysis suggests that in the presence of collateral constraints in the form of tradable and nontradable output, fiscal devaluations might be a better instrument for responding to large negative shocks than exchange rate devaluations.\(^{12}\)

\(^{12}\)The idea of fiscal devaluations as an alternative to exchange rate devaluations to avoid both financial
6 Conclusions

This paper fills the gap in the literature between downward nominal wage rigidity and collateral constraints in the form of tradable and nontradable income, by studying the optimal exchange rate policy in a model economy that includes both frictions simultaneously. In the presence of downward nominal wage rigidity, allowing for nominal exchange rate depreciations can help attenuate an increase in unemployment. In the presence of collateral constraints in terms of tradable and nontradable output, fighting real exchange rate depreciation alleviates consumption adjustment. The “credit access – unemployment” trade-off that exchange rate policy faces in the presence of these two mechanisms captures a central discussion of the policy debate typically observed in financial crises since the Great Depression.

The main finding, in a quantitative analysis of this model, is that the optimal exchange rate policy always allows for large real exchange rate depreciations (39 percent fall in the relative price of nontradable goods on average). The main reason for this is that while the welfare gains of resisting real exchange rate depreciation are related to second moments (a lower consumption volatility), its welfare costs are related to first moments (a higher average unemployment rate). This means that “Sudden Stops” understood as large current account adjustments, are an optimal reaction of exchange rate policy to negative external shocks. In other words, while exchange rate policy could prevent Sudden Stops by resisting real exchange rate depreciation, its associated costs in terms of unemployment make this policy suboptimal.

It is also shown that during financial crises, while the optimal nominal exchange rate depreciation is large (52 percent on average), it is smaller than that which achieves full-employment (71 percent on average). This finding can help rationalize “managed-floating” exchange rate regimes, widely used by emerging economies in periods of Sudden Stops.

During periods in which the collateral constraint does not bind, optimal exchange rate policy always achieves full-employment. This highlights the role of prudential policies: to reduce the probability of hitting the collateral constraint is to reduce the probability of experiencing any involuntary unemployment under the optimal exchange rate policy.

destabilization and unemployment goes back at least to Keynes (1931), who, on these grounds, recommended in Britain, when the Great Depression started, the use of a tax on imports instead of abandoning the gold standard (Irwin, 2011).
7 Appendices

7.1 Appendix A: Omitted Constraints in Ramsey Problems

In Section 3.1, to characterize the allocation under the different exchange rate regimes, I follow the strategy of setting up the Ramsey problem dropping constraints. This Appendix shows that the omitted constraints are satisfied.

7.1.1 Optimal Exchange Rate Policy

This section shows that any \( \{d_{t+1}, c^T_t, h_t\} \) that satisfy (5), (13), (18), and (19) also satisfy (GE). Pick \( c^N_t = F(h_t) \) to satisfy (16). Pick \( p_t = \left( \frac{1-a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{\frac{1}{\gamma}} \) to satisfy (8). Pick \( \mu_t = \eta_t = 0 \). This makes (9) and (10) hold. Pick \( \lambda_t = U_c(c^T_t, c^N_t) A_T(c^T_t, c^N_t) \) to satisfy (7). Choose \( \tau^d_t = 1 - R_t \beta \frac{E_t \eta a + \mu + \eta}{\lambda_t} \) to satisfy (6). Choose \( T_t \) to satisfy (15) as: \( T_t = \tau^d_t d_{t+1} R_t^{-1} \). Pick \( w_t = \left( \frac{1-a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{\frac{1}{\gamma}} F(h_t) \) to satisfy (17). Given \( w_{t-1} \), pick \( \epsilon_t \) to satisfy (12) with equality: \( \epsilon_t = \gamma \frac{w_{t-1}}{w_t} \). Finally, since (12) holds with equality, (14) always holds: \( \left( w_t - \gamma \frac{w_{t-1}}{w_t} \right)(h_t - h_t) = 0 \).

7.1.2 Full-Employment Exchange Rate Policy

This section shows that any \( \{d_{t+1}, c^T_t, h_t\} \) that satisfy (5), (18), (19), and (21), also satisfy (GE) and (21). Pick \( c^N_t = F(h_t) \) to satisfy (16). Pick \( p_t = \left( \frac{1-a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{\frac{1}{\gamma}} \) to satisfy (8). Pick \( \mu_t = \eta_t = 0 \). This makes (9) and (10) hold. Pick \( \lambda_t = U_c(c^T_t, c^N_t) A_T(c^T_t, c^N_t) \) to satisfy (7). Choose \( \tau^d_t = 1 - R_t \beta \frac{E_t \eta a + \mu + \eta}{\lambda_t} \) to satisfy (6). Choose \( T_t \) to satisfy (15) as: \( T_t = \tau^d_t d_{t+1} R_t^{-1} \). Pick \( w_t = \left( \frac{1-a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{\frac{1}{\gamma}} F(h_t) \) to satisfy (17). Given \( w_{t-1} \), pick \( \epsilon_t \) to satisfy (12) with equality: \( \epsilon_t = \gamma \frac{w_{t-1}}{w_t} \). Finally, by (21) and (13), (14) always holds: \( \left( w_t - \gamma \frac{w_{t-1}}{w_t} \right)(h_t - h_t) = 0 \).

7.1.3 Fixed Exchange Rate Policy

This section shows that any \( \{d_{t+1}, c^T_t, h_t, w_t, \epsilon_t\} \) that satisfy (5), (12)-(14), (17)-(19), and (23), also satisfy (GE) and (23). Pick \( c^N_t = F(h_t) \) to satisfy (16). Pick \( p_t = \left( \frac{1-a}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{\frac{1}{\gamma}} \) to satisfy (8). Pick \( \mu_t = \eta_t = 0 \). This makes (9) and (10) hold. Pick \( \lambda_t = U_c(c^T_t, c^N_t) A_T(c^T_t, c^N_t) \) to satisfy (7). Choose \( \tau^d_t = 1 - R_t \beta \frac{E_t \eta a + \mu + \eta}{\lambda_t} \) to satisfy (6). Choose \( T_t \) to satisfy (15) as: \( T_t = \tau^d_t d_{t+1} R_t^{-1} \).

7.2 Appendix B: Proof of Propositions

7.2.1 Proof of Proposition 1

The Ramsey problem of optimal exchange rate policy under an optimal capital control tax is to maximize (1) with respect to \( \{d_{t+1}, c^T_t, h_t\} \), subject to (5), (13), (18) and (19).
The Lagrangian of the Ramsey problem is then given by:

\[
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ U \left( A \left( c_t^T, F(h_t) \right) \right) + \phi^F \left[ \frac{d_{t+1}}{\bar{h}_t} - d_t + y_t^T - c_t^T \right] + \phi^\mu \left[ \kappa \left( y_t^T + \left( \frac{1-a}{a} \right) \left( c_t^T \right)^{1-\frac{1}{\xi}} F(h_t) \right) - d_{t+1} \right] + \phi^W \left[ \bar{h} - h_t \right] \right\},
\]

where \(\phi^F, \phi^\mu, \phi^\eta,\) and \(\phi^W\) are Lagrange multipliers.

The optimality conditions associated with this problem are (5), (13), (18), (19), the first order conditions:

\[
\frac{\phi^F_t}{R_t} = \beta \mathbb{E}_t \phi^F_{t+1} + \phi^\mu_t + \phi^\eta_t, \quad (30)
\]

\[
\phi^F_t = U_c A_T \left( c_t^T, F(h_t) \right) + \phi^\mu_t \kappa \left( \frac{1}{\xi} \right) \left( \frac{1-a}{a} \right) \left( \frac{c_t^T}{F(h_t)} \right)^{\frac{1}{\xi} - 1}, \quad (31)
\]

\[
\phi^W_t = F^\prime(h_t) \left[ U_c A_N \left( c_t^T, F(h_t) \right) + \phi^\mu_t \left( \frac{\xi-1}{\xi} \right) \kappa \left( \frac{1-a}{a} \right) \left( \frac{c_t^T}{F(h_t)} \right)^{\frac{1}{\xi}} \right], \quad (32)
\]

and the complementary slackness conditions:

\[
\phi^\mu_t \geq 0; \quad \phi^\mu_t \left[ \kappa \left( y_t^T + \left( \frac{1-a}{a} \right) \left( c_t^T \right)^{\frac{1}{\xi}} F(h_t) \right) - d_{t+1} \right] = 0, \quad (33)
\]

\[
\phi^\eta_t \geq 0; \quad \phi^\eta_t \left[ d^N - d_{t+1} \right] = 0, \quad (34)
\]

\[
\phi^W_t \geq 0; \quad \phi^W_t \left( \bar{h} - h_t \right) = 0. \quad (35)
\]

First, consider the case with \(\xi < 1\). Assume, contrary to the statement of the proposition, that under the optimal exchange rate policy, at some date \(t, h_t < \bar{h}\) and \(d_{t+1} < \bar{d}(h_t, c_t^T, y_t^T)\). By (35) it follows that \(\phi^W_t = 0\). By (32), and since \(c_t^T > 0, h_t > 0, F^\prime(h_t) > 0, U_c A_N \left( c_t^T, F(h_t) \right) > 0\) and \( \left( \frac{\xi-1}{\xi} \right) < 0\), this implies that \(\phi^\mu_t > 0\), which contradicts (33), which requires \(\phi^\mu_t \left( \bar{d}(h_t, c_t^T, y_t^T) - d_{t+1} \right) = 0\).

Second, consider the case with \(\xi \geq 1\). Assume, contrary to the statement of the proposition, that under the optimal exchange rate policy, at some date \(t, h_t < \bar{h}\). By (32), and since \(c_t^T > 0, h_t > 0, F^\prime(h_t) > 0, U_c A_N \left( c_t^T, F(h_t) \right) > 0\), \( \left( \frac{\xi-1}{\xi} \right) \geq 0\) and \(\phi^\mu_t \geq 0\), this implies that \(\phi^W_t > 0\), which contradicts (35), which requires \(\phi^W_t \left( \bar{h} - h_t \right) = 0\).

7.2.2 Proof of Proposition 2

Given the initial state \((s^*_t, d_t)\) and a debt level \(d_{t+1}^*\) with associated tradable consumption \(c_t^{*,T} = (d_{t+1}^* R_t^{-1} - d_t + y_t^T)\), pick \(h_t^*\) such that \(d_{t+1}^* = \bar{d}(h_t^*, c_t^{*,T}, y_t^T)\). This implies setting \(h_t^*\) as:

\[
h_t^* = F^{-1} \left( \left( \frac{1-a}{a} \right) \left( d_{t+1}^* R_t^{-1} - d_t + y_t^T \right)^{\frac{1}{\xi}} \left( d_{t+1}^* \kappa^{-1} - y_t^T \right)^{-\frac{1}{\xi}} \right). \]

By assumption \(d_{t+1}^* > \bar{d}(\bar{h}, c_t^{*,T}, y_t^T)\). Since \(\xi < 1\), it follows that
Finally, since \((d_{t+1}^* R_t^{-1} - d_t + y_T^T)^{1/2} > 0\), and \(F(\overline{h}) > 0\), by the assumption that 
\(d_{t+1}^* > \overline{d}(\overline{h}, c_t^T, y_t^T)\), it also follows that \(d_{t+1}^* > \kappa y_t^T\), and thus \(h_t^* > 0\).

### 7.2.3 Proof of Proposition 3

The Lagrangian of the unconstrained first-best problem is given by:

\[
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{U(A(c_t^T, F(h_t)))}{R_t} + \phi^F_t \left[ \frac{d_{t+1}^*}{R_t} - d_t + y_T^T - c_t^T \right] + \phi^\eta_t \left[ d_t^N - d_{t+1} \right] + \phi^W_t \left[ \overline{h} - h_t \right] \right\},
\]

where \(\phi^F_t\), \(\phi^\eta_t\), and \(\phi^W_t\) are Lagrange multipliers.

The optimality conditions associated with this problem are (5), (13), (18), the first
order conditions:

\[
\frac{\phi^F_t}{R_t} = \beta \mathbb{E}_t \phi^F_{t+1} + \phi^\eta_t,
\]

\[
\phi^F_t = U_c A T (c_t^T, F(h_t)),
\]

\[
\phi^W_t = F'(h_t) U_c A N (c_t^T, F(h_t)),
\]

and the complementary slackness conditions:

\[
\phi^\eta_t \geq 0; \quad \phi^\eta_t \left[ d_t^N - d_{t+1} \right] = 0,
\]

\[
\phi^W_t \geq 0; \quad \phi^W_t \left[ \overline{h} - h_t \right] = 0.
\]

I now show that there exists a tax policy \(\{\tau_t^{N*}, \tau_t^{d*}\}\) such that allocations \(\{d_{t+1}^*, c_t^T, h_t^*\}\) that satisfy the optimality conditions of the unconstrained first-best, also satisfy \((GE')\) and (23). Pick \(\epsilon_t = 1\) for all \(t\) to satisfy (23). Note that the unconstrained first-best allocation always features full employment: by (38), and since \(F'(h_t) > 0\), and \(U_{c,t} A N, t(c_t^T, F(h_t)) > 0\), it follows that \(\phi^W_t > 0\), and thus, by (40), \(h_t^* = \overline{h}\). This implies that (14) always holds. Choose \((1 + \tau_t^{N*}) = \min \left\{ \left(1 - \frac{1}{\gamma} \right) \frac{\gamma t^{N*}}{\gamma t^{N*+1}} F(\overline{h}) \right\} \frac{1}{(d_{t+1} - y_t^T)^{1/2}}\), and \(u_t^*\) to satisfy (28). This implies that (29) and (12) always hold. Pick \(c_t^{N*} = F(\overline{h})\) to satisfy (16). Pick \(p_t^* = \frac{1}{1+\tau_t^{d*}} \left(1 - \frac{1}{\gamma} \right) \left(\frac{c_t^{d*}}{c_t^{d*}}\right)^{1/2}\), to satisfy (26). Pick \(\mu_t^* = \eta_t^* = 0\) for all \(t\). This makes (9) and (10) to hold. Pick \(\lambda_t^* = U_c (c_t^{d*}, c_t^{N*}) A T (c_t^{d*}, c_t^{N*})\) to satisfy (7). Choose \(\tau_t^{d*}\) to satisfy (6) as: \(\tau_t^{d*} = 1 - R_t \beta \frac{E_t N_{t+1}^{d*} + p_t^* \gamma \tau_t^{N*} t^{N*}}{\lambda_t^*}\). Choose \(T_t^* = \frac{d_{t+1}^*}{R_t} \tau_t^{d*} + p_t^* \tau_t^{N*} c_t^{d*}\) to satisfy (27).
References


[38] Klein Michael W. and Jay C. Shambaugh (2009), Exchange Rate Regimes in the Modern Era, MIT Press.


