Large banks, loan rate markup and monetary policy∗

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Abstract

Despite the financial sector in many advanced economies is dominated by a few big intermediaries, general equilibrium models with banking — even the latest generation flourished after the financial crisis — have typically assumed the presence of atomistic banks, thereby neglecting potential interactions between policy and banking decisions. This paper studies the implications of introducing large monopolistic banks that can affect macroeconomic outcomes and so the response of monetary policy to inflation in a model with collateral constraints that links the borrowers’ credit capacity to the value of their durable assets. First, we find that the optimal loan markup is positively related to the level of entrepreneurs’ leverage and to the degree of inflation aversion of the central bank in the long run. Second, in the short run large banks generate endogenous countercyclical movements of the bank loan markup, which amplify the impact of monetary and technology shocks on the real economy. This type of financial accelerator adds-up to the standard one — due to the presence of borrowing constraints — and is crucially related to the existence of non-atomistic banks. Moreover, this new financial accelerator is increasing in the central bank’s aggressiveness in stabilizing inflation.

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1 Introduction

In the wake of the financial crisis of 2008, theoretical work aimed at incorporating financial intermediation in modern dynamic general equilibrium models (DSGE) has flourished (see Brunnermeier et al., 2012, for a review). This literature assumes that the market structure of the banking sector is either perfectly competitive (e.g. Christiano et al., 2010; Gertler and Karadi, 2011) or monopolistically competitive with atomistic banks (e.g. Gerali et al., 2010; Andrés and Arce, 2012; Andrés et al., 2013), i.e. small banks whose individual decisions do not have effects on aggregate outcomes. Yet, a distinctive feature of the banking sector in many advanced economies is the presence of very large players: as an example, the average asset share held by the 5 biggest banks in the euro area countries over the period 2005-2010 was around 50 per cent. This paper is intended to bridge this gap, by providing the first formal analysis of non-atomistic banks in a stochastic dynamic general equilibrium New Keynesian environment.

Basic microeconomic theory predicts that the degree of competition in a given market depends on the size and the number of competing operators. In particular, when there are few large players who understand that their individual decisions can affect aggregate outcomes and so other players’ behavior, strategic complementarities may arise. In this setup, the economic outcomes might significantly deviate from those obtained under perfect competition or monopolistic competition with atomistic agents.

The impact of bank size on the cost and quantity of bank lending is an important issue. A number of empirical studies pointed out how concentration in the banking sector is relevant for determining the spread between lending rates and deposit rates in a group of advanced countries.\(^1\) Of course, the existence of large and complex financial institutions not only affects competition in the banking sector. The literature assessing the influence of big banks has also focused on issues of systemic risk, interconnectedness, and too-big-to-fail. Here, we offer an alternative model for the study of large banks based on the new macro literature emerged after the crisis which have made significant progress in terms of incorporating loan spreads and studying loan rate setting behavior by banks (e.g. Curdia and Woodford (2010); Andrés and Arce (2012); Gerali et al. (2010)).

\(^1\)See e.g. Maudos and Fernandez de Guevara (2004); Corvoisier and Gropp (2002); Dick and Lehnert (2010); Corbae and D’Erasmo (2013).
In this paper we start from a baseline New Keynesian model with borrowing constraints and a monopolistically competitive banking sector, and relax the assumption that there is an infinite amount of atomistic banks in the economy. We assume instead that the economy is populated by a small number $n$ of intermediaries, each with a size of $1/n$, so that atomistic banks are embedded as a special case. In this framework, large banks understand that their choice about the (individual) loan rate level affects aggregate outcomes in the economy; they also realize that this choice interacts with the choices of the other banks and with the response of the central bank. We develop a strategic interaction framework which is similar in spirit to the one developed in the literature on non-atomistic wage setters in the labor markets.\(^2\) In particular, we assume that banks have perfect rationality and internalize the aggregate effects of their individual loan rate decisions. As a consequence, the equilibrium (aggregate) level of the loan spread is affected by the number of banks, which proxy for the level of concentration in the economy and determine the degree of strategic interaction (for $n \to \infty$ banks are atomistic and the strategic effects disappear). In addition, in this context of strategic complementarity, the loan spread is a function of the elasticity of the loan demand and of the policy rate to changes in (a given) bank’s loan rate.

Two important results regarding the determination of the loan spread arise. First, the spread is positively related to the entrepreneurs’ leverage, reflecting the fact that a higher leverage implies a greater elasticity of the policy rate to changes in loan rates, which in turn increases banks’ market power. Entrepreneurs’ leverage (which is proportional to the ratio of borrowing from the banks to net worth) is countercyclical, as net worth tends to fall (increase) more than borrowing after shocks negatively (positively) affecting output.\(^3\) Therefore loan margins also move countercyclically, amplifying the impact of exogenous shocks on the economy. This mechanism unveils a new type of financial accelerator, which is crucially related to the presence of non-atomistic banks and adds up to the standard financial accelerator discussed in the literature on the credit channel (Bernanke and Gertler, 1989; Bernanke et al., 1999; Kiyotaki and Moore, 1997), which is also at work in

\(^2\)See e.g. Cukierman and Lippi (2001); Lippi (2003); Soskice and Iversen (2000); Lawler (2000); Guzzo and Velasco (1999).

\(^3\)This results is in line with what is observed in the data (see e.g. Chugh, 2012; Levy and Hennessy, 2007). In particular, Levy and Hennessy (2007) argue that leverage is mostly countercyclical for highly-constrained firms.
the model due to the presence of borrowing-constrained agents.

Second, we find that the loan spread depends on the design of the monetary policy rule. For simplicity, we limit the analysis to the simple case in which monetary policy sets the short-term interest rate based on a rule that only responds to deviations of inflation from the steady state. We find that the spread is higher the more aggressive the response to inflation is, as measured by the parameter determining the systematic response in the simple rule. In addition, the degree of inflation aversion also interacts with the financial accelerator described above, increasing its effectiveness the more aggressive the central bank is.

As an intuition for the results above, consider the case in which a (large) bank decides to increase the loan rate, with the aim of increasing its profits (the story is symmetric in the case of a reduction of the loan rate). The bank anticipates that such increase would (proportionally) augment the aggregate interest rate on loans and, as a consequence, reduce the amount of credit that borrowers can obtain (because of the collateral constraint). Because of the credit restriction, entrepreneurs reduce investment and capital accumulation, pushing down the price of capital and the marginal cost of goods-producing firms. This effect is stronger the higher the initial leverage of the borrowers. Moreover, due to the optimal price-setting behavior in the goods market, the decline in marginal costs reduces inflation, triggering a loosening of the monetary stance proportional to the degree of the monetary authority's inflation aversion. As the policy rate in the model corresponds to the deposit rate paid to households by banks, both a higher leverage and a higher inflation aversion will offer greater incentives to the bank to raise the loan rate by reducing its marginal costs to a larger extent.

The mechanism described here requires strong rationality assumptions: banks must be able to internalize the general equilibrium effects of their initial interest rate change. These assumptions, however, are perfectly consistent with the rational expectation setup typically underlying DSGE models; moreover, the paper uses a type of strategic interaction that has been extensively used in a labor market context, where similar rationality hypotheses apply to the wage-setting unions. In this sense, the main contribution of the paper is to show that neglecting the role of large banks in a general equilibrium framework might lead to misleading conclusions, at least as regards the implications for loans market competition. We highlight the importance of the interaction between large banks and monetary policy but our framework could easily be extended to study other types of policies, such as credit or
macroprudential policy.

Our results recall a number of different papers that have studied the role of bank margins in business cycle fluctuations. Andrés and Arce (2012) find that, in the long run, stronger banking competition increases output; in the short-run, output, credit and housing prices are more responsive on impact to shocks in an environment of highly competitive banks. Olivero (2010), besides providing empirical support for countercyclical bank loan margins, shows that these are associated with deeper recessions. Gilchrist (2004) finds that a countercyclical cost of credit is the main driving force for the international transmission of business cycles. Finally, our work is related to the recent literature on macroeconomic models that study how financial frictions amplify shocks near the steady state of the system, such as Christiano et al. (2005), Christiano et al. (2008), Curdia and Woodford (2010), and Gertler and Karadi (2011).

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 shows the implications of the strategic interaction between large banks and monetary policy and its impact on the model’s steady-state. Section 4 illustrates the dynamic properties due to the link between the endogenous behavior of banks and the general equilibrium properties of the economy. Section 5 concludes.

2 The model

The structure of the model is a simplified version of Gerali et al. (2010), who build a DSGE model with collateral constraints à la Kiyotaki and Moore (1997) that links the borrowers’ debt accumulation to the value of their durable assets, in the spirit of Iacoviello (2005), and monopolistic competition with atomistic banks in the loan market. The main departure from their framework is that we allow for fully flexible rates and banks are assumed to be non-atomistic, so that their number \(n\) and size \((1/n)\) are finite.

The economy is populated by two groups of agents of equal mass, households and entrepreneurs. Households work, consume and deposit resources into a bank. Entrepreneurs need collateral, namely physical capital, to take a loan from the banking sector. Thus, they accumulate physical capital and rent capital services to firms. Entrepreneurs discount the future at rate \(\beta_E\) while the discount factor of households is \(\beta_P > \beta_E\).\(^4\)

\(^4\)As standard in these models, a smaller \(\beta_E\) ensures than in the steady state and its
Monetary policy is conducted according to a simple rule, whereby the nominal interest rate is set in response to endogenous variations in inflation, \( \pi_t = \log P_t - \log P_{t-1} \), as follows:

\[
R_{ib}^b = R_{ib}^b \phi_\pi \exp(\varepsilon_{R_{ib}}^b), \quad \phi_\pi \geq 0
\]

where \( R_{ib}^b \) is the gross nominal interest rate, \( R_{ib}^b \) is the steady state level of \( R_{ib}^b \), and \( \varepsilon_{R_{ib}}^b \) is a (white noise) monetary policy innovation with zero mean and variance \( \varsigma_{R_{ib}}^b \).

Large banks collect deposits from households (i.e. patient savers) and lend to entrepreneurs (i.e. impatient borrowers). The banking sector is perfectly competitive in the deposit market (i.e. the interest rate on deposits equals the policy rate \( R_{ib}^b \)) and monopolistically competitive in the loan market.

In addition, the economy is populated by two types of firms: capital goods producers and consumption goods producers. The former operate in a perfectly competitive market. The consumption producers operate instead in monopolistic competition: they transform entrepreneurial capital and household labor into an intermediate good using a constant-returns-to-scale technology and sell them to retailers. Retailers brand the goods, thus differentiating them, and sell the differentiated good at a price which includes a markup over the purchasing cost; retailers face quadratic price adjustment costs, implying the existence of a New Keynesian Phillips curve.\(^5\)

2.1 Producers

In the goods sector wholesale producers use the following Cobb-Douglas technology

\[
y_t^E(i) = A_t^E (k_{t-1}^E(i))^\alpha (l_t(i))^{1-\alpha},
\]

neighborhood the borrowing constraint is always binding while households are net lenders and entrepreneurs are net borrowers (Iacoviello, 2005). In this model \( \beta_P > \beta_E \) is only a necessary condition; in order to have a binding borrowing constraint at the steady state, a further restriction has to be imposed on \( \beta_E \). We will derive it below.

\(^5\)A Calvo-Yun approach in which firms face an endogenous probability of changing prices would be an alternative way of modeling sticky prices. However, under the assumption of symmetric equilibria, a quadratic cost of adjusting prices produces an aggregate New Keynesian Phillips curve which is observational equivalent to the one arising under the Calvo-Yun approach.
where \( l_t(i) \) is aggregate households’ labor supplied to producer \( i \) and \( k_{t-1}^E(i) \) is physical capital at the beginning of period \( t \). \( A_t^E \) is a productivity shock to the neutral technology that follows a process represented by \( \log(A_t^E) = \log(A_0^E) + \varepsilon_t^A \) where \( \varepsilon_t^A \) is white noise with zero mean and variance \( \varsigma^A \).

The retail goods market is assumed to be monopolistically competitive as in Bernanke et al. (1999). Retailers purchase the intermediate good at the wholesale price \( P_t^W \) and choose \( P_t(i) \) so as to maximize

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} P_t(i)y_t^E(i) - P_t^W y_t^E(i) - \frac{k_p}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 P_t y_t^E,
\]

subject to the demand derived from consumers’ maximization,

\[
y_t^E(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_y} y_t^E,
\]

where \( \epsilon_y > 1 \) denotes the elasticity of substitution across brand types, \( \Lambda_{0,t} \equiv \beta_P c_0^P / c_t^P \) is the households’ stochastic discount factor, \( c_t^P \) is current consumption, and \( y_t = (\int_0^1 y_t(i)^{(\epsilon_y - 1)/\epsilon_y} d_i)^{\epsilon_y/(\epsilon_y - 1)} \). From cost minimization we obtain labor and capital demands:

\[
w_t = (1 - \alpha) \frac{y_t^E(i)}{P_t^E(i)x_t},
\]

\[
r_t^k = \alpha \frac{y_t^E(i)}{k_{t-1}^E(i)x_t},
\]

where \( w_t \) is real wage and \( r_t^k \) is the rental rate of physical capital. Hence, the first order conditions for \( P_t(i) \) yields

\[
1 - \frac{mk^y}{mk^y - 1} + \frac{mk^y}{mk^y - 1} mc_t^E - \kappa_p (\pi_t - 1) \pi_t
\]

\[
+ \beta P \mathbb{E}_t \left[ \frac{c_t^P}{c_{t+1}^P} \kappa_p (\pi_{t+1} - 1) \pi_{t+1} \frac{y_{t+1}^E}{y_t^E} \right] = 0,
\]

where \( mk^y \equiv \epsilon_y / (\epsilon_y - 1) \), \( mc_t^E = 1 / x_t \) is the real marginal cost and \( x_t \equiv P_t / P_t^W \).

In the capital sector, perfectly competitive firms buy last-period capital at price \( Q_t^k \) from entrepreneurs (owners of these firms) and \( I_t \) units of final goods.
from retailers at price $P_t$. The transformation of the final good into new capital is subject to adjustment costs. Letting $\Delta \bar{z}_t = k_t^E - (1 - \delta_k) k_{t-1}^E$ denote the increased stock of effective capital $\bar{z}$, firms choose $\bar{z}_t$ and $I_t$ so as to maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^E (q_t^k \Delta \bar{z}_t - I_t)$ subject to $\bar{z}_t = \bar{z}_{t-1} + \left[ 1 - \frac{\kappa_i}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t,$

where $q_t^k \equiv Q_t^k / P_t$ is the real price of capital, $\Lambda_{0,t}^E \equiv \beta E_{c_t}^E / c_t^E$ is the entrepreneurs’ stochastic discount factor and $c_t^E$ the current consumption. The firm’s first-order condition can be written as:

$$1 = q_t^k \left[ 1 - \frac{\kappa_i}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa_i \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right]$$

$$+ \beta E_{c_t^E} \left[ c_t^E q_{t+1}^k \kappa_i \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right]. \quad (7)$$

### 2.2 Households and entrepreneurs

Household $i$ solves the following problem

$$\max \{ c_t^P(i), l_t^P(i), d_t^P(i) \} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t} \left[ \log(c_t^P(i)) - \frac{l_t^P(i)^{1+\phi}}{1+\phi} \right] \quad (8)$$

subject to the budget constraint:

$$c_t^P(i) + d_t^P(i) \leq w_t l_t^P(i) + R_{t-1}^{ib} d_{t-1}^P(i) + J_t^f(i) + J_{t-1}^b(i) \quad (9)$$

where $d_t^P(i)$ is bank deposits in real terms, $J_t^f(i)$ and $J_{t-1}^b(i)$ are dividends (profits in real terms) remitted to patient households respectively by the good and banking sectors.\(^6\) The relevant first-order conditions are the Euler equation

$$\frac{1}{c_t^P(i)} = \beta_P \mathbb{E}_t \frac{R_t^{ib}}{c_{t+1}^P(i)} \quad (10)$$

\(^6\)Though it is not critical to our central message here, credits and debts are assumed to be indexed to current inflation; this removes the so called ‘nominal credit/debt-channel’ from the model. This channel, which implies that changes in the price level have real effects on the aggregate economy because they redistribute real resources between borrowers and lenders, is quite important in the Gerali et al. (2010) and in many papers with a collateral channel (e.g. Iacoviello, 2005); however, it is possible to show that introducing the nominal credit/debt-channel would not affect the key strategic mechanisms at work in this paper.
and the labor-supply decision

$$l_t^p(i)^\phi = \frac{w_t}{c_t^p(i)}.$$  \hspace{1cm} (11)

Combining (10) and (11) with the labor demand (4), we obtain the condition for equilibrium in the labor market

$$(1 - \alpha)g_t^E(i)mc_t\beta_p\mathbb{E}_t\frac{R_t^b}{c_{t+1}^i(i)} = [l_t^p(i)]^{1+\phi}.$$  \hspace{1cm} (12)

Entrepreneurs’ optimization problem is given by

$$\max \{c_t^E(i), k_t^E(i), b_t^E(i)\} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t^E \log(c_t^E(i))$$  \hspace{1cm} (13)

subject to a budget constraint,

$$c_t^E(i) + R_{t-1}^b b_{t-1}^E(i) + w_t l_t^p(i) + q_t^k k_t^E(i) \leq \frac{y_t^E(i)}{x_t} + b_t^E(i) + q_t^k (1 - \delta^k) k_{t-1}^E(i),$$  \hspace{1cm} (14)

and a borrowing constraint,

$$R_t^b l_t^E(i) \leq \mathbb{E}_t m^E q_t^{k} k_{t-1}^E(i)(1 - \delta^k)$$  \hspace{1cm} (15)

where $b_t^E(i)$ indicates the amount of bank lending taken by entrepreneurs, $m^E$ is a parameter that can be interpreted as the value of the loan-to-value (LTV) ratio chosen by the banks (i.e., the ratio between the amount of loans issued and the discounted next-period value of entrepreneurs’ assets) and $R_t^b$ is the aggregate interest rate on bank loans.

Like in Gerali et al. (2010), we assume a Dixit-Stiglitz framework for the loan market. Therefore, an entrepreneur seeking an amount of loans $b_t^E(i)$ has to purchase a composite basket of slightly differentiated financial products, each supplied by a bank $u$, with elasticity of substitution equal to $\varepsilon^b$ (with $\varepsilon^b > 1$). This constraint can be expressed as:

$$\left[ b_t^E(i, j) \frac{\varepsilon_{b-1}^b}{\varepsilon_t^b} \text{d}j \right]^{\varepsilon_t^b}_{\varepsilon_{b-1}^b} \geq b_t^E(i)$$  \hspace{1cm} (16)
Let \( \int_0^1 R^b_t(j) b^E_t(i, j) \, dj \) denote the total repayment due to the continuum of financial products demanded by entrepreneur \( i \). Demand for real loans \( b^E_t(i) \) from entrepreneur \( i \) is obtained from minimizing the total repayment over \( b^E_t(i, j) \), subject to the constraint (16). Aggregating over symmetric entrepreneurs yields a familiar loan demand function

\[
b^E_t(j) = \left( \frac{R^b_t(j)}{R^b_t} \right)^{-\epsilon^b} b^E_t.
\]  

(17)

with \( R^b_t \) defined as

\[
R^b_t = \left[ \int_0^1 R^b_t(j)^{1-c^b} \, dj \right]^{\frac{1}{1-c^b}}.
\]  

(18)

Define by \( \lambda^E_t \) and \( \lambda^E_t s^E_t \) the multipliers on the constraints (14) and (15) respectively. The first-order conditions for the entrepreneur’s problem read:

\[
\lambda^E_t - \beta_E \mathbb{E}_t R^b_t \lambda^E_t = \lambda^E_t s^E_t,
\]  

(19)

\[
\lambda^E_t s^E_t m^E \mathbb{E}_t \frac{q^k_{t+1}(1 - \delta^k)}{R^b_t} + \beta_E \mathbb{E}_t \lambda^E_t [r^k_{t+1}(1 - \delta^k) + r^k_{t+1}] = q^k_t \lambda^E_t,
\]  

(20)

where

\[
\lambda^E_t = 1/c^E_t.
\]  

(21)

The intertemporal choice of an entrepreneur (19) is distorted when the credit constraint is binding, i.e. when \( s^E_t > 0 \). Equation (20) equates the marginal cost of one unit of capital \( q^k_t \lambda^E_t \) to its (expected discounted) marginal benefit. The latter has three components: i) the expected future price of capital, since capital acquired today can be resold tomorrow to the capital sector at \( q^k_{t+1}(1 - \delta^k) \); ii) the marginal product of capital purchased today that can be rented in the good sector tomorrow, \( r^k_{t+1} \); iii) the shadow value of borrowing, since capital acquired today can be used as collateral in borrowing.

Define the entrepreneurs’ net worth as follows:

\[
nw^E_t \equiv r^k_t k^E_{t-1} + q^k_t (1 - \delta^k) k^E_{t-1} - R^b_t b^E_t.
\]  

(22)

• At the end of period \( t \)
1. Entrepreneurs hold $nw_t^E$;
2. Banks lend funds to entrepreneurs $b_t^E$ to purchase new capital $q_t^k k_t^E$ in the capital sector.

- At the beginning of period $t + 1$
  1. $k_t^E$ is rent to the goods sector at $r_{t+1}$ by entrepreneurs;
  2. Entrepreneurs sell $q_{t+1}^k (1 - \delta^k) k_t^E$ to the capital sector;
  3. Entrepreneurs pay back $R_t^b b_t^E$ to the banking sector.

Thus, in equation (22) $r_t^k k_{t-1}^E + q_t^k (1 - \delta^k) k_{t-1}^E$ denotes the entrepreneur’s capital return at time $t$ while $R_{t-1}^b b_{t-1}^E$ is the effective cost of borrowing.

Drawing on Andrés et al. (2013), we use (22) into (20) and (19) so that entrepreneurs’ aggregate consumption, $c_t^E$, can be rewritten as a constant fraction of their net wealth, $nw_t$, (see Appendix A)

$$c_t^E = (1 - \beta_E) nw_t^E. \quad (23)$$

Similarly entrepreneurs’ aggregate capital in the next period, $k_t^E$, is also a function of net wealth

$$q_t^k k_t^E = \frac{\beta_E}{1 - b_t^E/(q_t^k k_t^E)} nw_t^E. \quad (24)$$

Before turning to the derivation of the optimal loan interest rate, it is convenient to define the entrepreneurs’ debt-to-capital ratio,

$$V_t^E \equiv \frac{b_t^E}{q_t^k k_t^E} \quad (25)$$

and the gross expected change of capital price,

$$\Delta_{t+1} \equiv E_t q_{t+1}^k / q_t^k. \quad (26)$$

From equation (24) we derive the following relation between the debt-to-capital ratio and the entrepreneurs’ leverage ($LV_t^E$):

$$LV_t^E \equiv \frac{q_t^k k_t^E}{nw_t^E} = \frac{\beta_E}{1 - V_t^E}. \quad (27)$$
2.3 Banks

The economy is populated by a finite number of banks (with \( n \geq 2 \)), which collect time deposits from households and issue loans to entrepreneurs. We assume that the deposit market is perfectly competitive while (as mentioned above) the loan market is modelled along Gerali et al. (2010), with a Dixit-Stiglitz type of competition. Before lending funds to entrepreneurs, each bank observes the entrepreneur’s net wealth (22) and takes it as given. Loan types are equally distributed across banks. Loan interest rates are fully flexible and set independently and simultaneously. In particular, the representative bank \( u \), where \( u \in \{1, \ldots, n\} \), \( n \geq 2 \), sets the same interest rate \( R_b^b(u) \) on all loans provided to entrepreneurs \( j \in u \) so as to maximize the following profits

\[
J^b_t = \int_{j \in u} \left[ R_b^b(j) - R_{ib}^b \right] b^E_t(j) dj
\]  

subject to the loan demand (17), the budget constraints (9), (14), the borrowing constraint (15), the New Keynesian Phillips curve (6), the equilibrium condition for the labor market (12), and the interest rate rule (1).

The solution to the banks’ problem reads:

\[
R^b_t = \frac{e^b(n-1) + \Sigma_{b,t} + \Sigma_{R^b,t} R^b_t}{e^b(n-1) + \Sigma_{b,t} - n} \equiv M^b_t R^b_t,
\]  

where \( \Sigma_{b,t} \) and \( \Sigma_{R^b,t} \) are respectively the elasticity (in absolute value) of aggregate loans, \( b^E_t \), and the elasticity of policy interest rate, \( R^b_t \), to the aggregate loan rate, \( R^b_t \).

The first order condition (29) is the key equation for our results.\(^7\) It shows that banks set the loan interest rate as a markup (\( M^b_t \)) over the policy interest rate. In standard models with monopolistic competition this markup (and thus the loan rate) is typically time-invariant and depends only on the elasticity of substitution among varieties. In this case, instead, due to the assumption of non-atomistic banks, it depends on the number of banks in the economy and is time-varying, according to the elasticities of aggregate loans and of the policy rate to the aggregate loan rate.

The reason why \( M^b_t \) is endogenously determined by \( n \), \( \Sigma_{b,t} \) and \( \Sigma_{R^b,t} \) is the strategic interaction that the presence of large banks induces among

\(^7\)For a complete derivation of this expression and of its components see Appendix B.
banks and between banks and the central bank.

The number of banks $n$ is relevant because the size of banks is inversely proportional to their number. In turn, the bank’s size determines the impact of a change in bank $u$’s loan rate on the aggregate loan rate $R^b_t$, as shown in Appendix B by

$$\frac{\partial R^b_t}{\partial R^b_t(u)} = \frac{1}{n}. \tag{30}$$

Note that when the bank’s size tends to zero, i.e. $n$ tends to infinity, the effects of strategic interactions disappear and the markup converges to the value it assumes in standard models of monopolistic competition:

$$\lim_{n \to \infty} M^b_t = \frac{e^b}{e^b - 1}. \tag{31}$$

$\Sigma_{b,t}$ and $\Sigma_{R^b,t}$ appear in the expression of the markup because they affect the incentives of the banks to strategically change the loan rate, which in turn depend on the impact that such change has on the different components of banks’ profit (28): loan demand $b^E_t$ and the cost of deposits $R^b_t$. To understand the intuition, consider the case of an increase in the loan rate (a symmetric argument could be used for the case of reduction in the loan rate). If credit constraints are binding, the increase in $R^b_t$ reduces entrepreneurs’ borrowing, according to equation (15). In turn, the reduction in loans lowers banks’ profits (for given levels of the interest rates) thus reducing the incentive to increase the interest rate in the first place. The intensity of the reduction in borrowing is proportional to $\Sigma_{b,t}$, which is therefore negatively related to $M^b_t$. The algebraic expression for $\Sigma_{b,t}$ reveals that, in turn, the intensity of loan reduction is proportional to the level of firms’ leverage (as implied by the borrowing constraint):

$$\Sigma_{b,t} \equiv -\frac{\partial b^E_t}{\partial R^b_t} \frac{R^b_t}{b^E_t} = 1 + \Sigma_{LV,t}, \tag{32}$$

where

$$\Sigma_{LV,t} \equiv -\frac{\partial LV^E_t}{\partial R^b_t} \frac{R^b_t}{LV^E_t} = \frac{LV^E_t}{\beta^E_t} - 1 \tag{33}$$

---

8We assume that credit constraints always bind in the steady state (and a neighborhood of it), namely $s^E > 0$. Note that this implies, from equation (19), that $1 - \beta^E R^b > 0$. 13
denotes the elasticity of entrepreneurs’ leverage (27) to the aggregate loan rate $R_b^t$.

The relation between the markup $M_b^t$ and $\Sigma_{Rb,t}$ is somewhat less direct and relies on the impact that a rise in the loan rate has on aggregate demand, via the reduction in borrowing. Indeed as (borrowers’) leverage reduces, entrepreneurs are forced to reduce capital expenditure (through (24)) and consumption. The fall in aggregate demand puts downward pressure on marginal costs and on inflation (via the Phillips curve (6)) prompting a response by the central bank which, as mentioned, is assumed to follow a simple rule targeting deviations of inflation from its (zero) steady state. Banks anticipate that the ensuing cut in the policy rate will lower their financing cost, offering incentives to increase the loan rate in the first place. This effect is proportional to $\Sigma_{Rb,t}$, which therefore displays a positive correlation with the bank’s markup. The expression for $\Sigma_{Rb,t}$ is:

$$\Sigma_{Rb,t} \equiv -\frac{\partial R_b^t}{\partial R_b^t} R_b^t = \frac{q_t^k k_t^F m c_t \phi_{\pi}}{c_t^P \phi_{\pi} m c_t + y_t^E \Psi (mk^y - 1) \kappa_p + mk^y m c_t \phi_{\pi}} \Sigma_{LV,t},$$

(34)

where $\Psi \equiv (1 - \alpha)/[mk^y (\alpha + \phi)]$.

Two things are worth stressing. First, borrowers’ leverage plays a significant role also in this case: the elasticity of the policy rate is positively correlated with $LV_t^E$, reflecting the fact that — other things being equal — the fall in aggregate demand and the ensuing policy response is stronger the higher entrepreneurs’ leverage. Second, $\Sigma_{Rb,t}$ also depends on the degree of central bank’s inflation aversion $\phi_{\pi}$, which determines the intensity of monetary policy response for a given reduction in aggregate inflation. This result underscores the potential importance of the strategic interaction between large banks and the central bank, something which is completely overlooked by models with atomistic banks. In particular, it shows how the design of monetary policy may interact with market power in the banking sector and have an impact on banks’ interest rate decisions.

---

9 This is reminiscent of the non-atomistic wage setter literature result where big unions internalize the effects of their wage policy on inflation. For a description of the main strategic effects analyzed in this strand of literature both in a closed and open framework see Cuciniello (2011).

10 As mentioned in the introduction, here we limit the analysis to the interaction between large banks and monetary policy, which is certainly easier to understand. Our framework could however be extended to study the interaction with other types of policies, such as
3 The steady state

What are the implications of the mechanism described in the previous section? Here we first provide an analysis of the steady-state properties of the model. In the next section we focus instead on the dynamic properties of the model with large banks.

The non-stochastic steady state of the model is derived by setting the shocks to their mean value. We normalize the technology parameter \( A^E \) so that \( y^E = 1 \) and assume that \( 1 - \beta^ER^b > 0 \), which guarantees that the collateral constraint is binding in steady state (see footnote 8). With a gross inflation rate equal to one, the Phillips curve (6) implies that \( mc = 1/mk^y \).

From equation (7) steady state \( q^k = 1 \). The bank’s first order condition then reads:

\[
R^b = \frac{M^b}{\beta_P}, \tag{35}
\]

where the markup in the banking sector is given by

\[
M^b = \frac{\epsilon^b(n - 1) + \Sigma_b + \Sigma_{R^b}}{\epsilon^b(n - 1) + \Sigma_b - n} > 1, \tag{36}
\]

and

\[
\Sigma_{R^b} = \frac{k^E mc\phi_{\pi}}{c^P \phi_{\pi} mc + \Psi[(mk^y - 1)\kappa_P + \phi_{\pi}]} \left(\frac{LV^E}{\beta^E} - 1\right) \tag{37}
\]

\[
\Sigma_b = \frac{LV^E}{\beta^E}. \tag{38}
\]

Steady state variables affecting policy rate elasticity (37) are given by (see Appendix C)

\[
ce^P = 1 - \frac{\alpha(1 - \beta^E + \delta^k LV^E)}{[1 - (1 - \delta^k)LV^E(1 - m^E)]mk^y}, \tag{39}
\]

\[
k^E = \frac{\alpha LV^E}{[1 - (1 - \delta^k)LV^E(1 - m^E)]mk^y}. \tag{40}
\]

credit or macroprudential policy, which could deliver additional interesting results.

\footnote{Steady state \( R^b \), and hence the markup, is derived by solving simultaneously for \( R^b \), \( c^P \), \( k^E \), and \( LV^E \) equations (35), (39), (40), and (41).}
\[ LV^E = \frac{\beta_E R^b_t}{R^b - (1 - m^E) \delta^k}. \] (41)

Figure 1 depicts graphically the relation between the steady-state level of the markup and the level of entrepreneurs’ leverage \((LV^E_t)\), under different assumptions regarding the level of competition in the banking sector. In particular, we study the cases in which the number of banks operating in the market equals 3 (blue line), 5 (red) and 10 (black). The figure is obtained by setting \(\phi = 1.5\) and the remaining parameters as in Table 1.

A number of considerations are in order. First, the markup is positively related with the level of borrowers’ leverage. In the previous section we commented how the effect of \(LV^E\) on \(\mathcal{M}^b\) was in principle ambiguous, as it was positively related with both \(\Sigma_b\) and \(\Sigma_{R^b}\) which had opposite effects on the markup. The graphical result suggests that in our calibration the impact of \(LV^E\) on \(\Sigma_{R^b}\) prevails. Second, as the number of banks grows, \(\mathcal{M}^b\)
decreases — for any given value of $LV^E$ — gradually converging to 1.007, which corresponds to the value of $\frac{\epsilon^b}{\epsilon^b - 1}$, that is, the value of the markup with atomistic banks. Moreover, as $n$ increases, the positive relation with leverage also disappears, in line with the irrelevance of strategic interactions.

Figure 2 shows the relation among $\phi_{\pi}$, $n$, and the bank’s markup $M^b$ in a tridimensional plot (with the the remaining parameters still calibrated as in Table 1). The value of entrepreneurs’ leverage underlying the figure is 3. In this case, we note that the degree of central bank’s inflation aversion is positively related to the markup. Also in this case, note that this result holds as far as the number of banks is not too big (and therefore their size is non-negligible): symmetrically to the previous figure, for $n = 10$ we find that $\phi_{\pi}$ has no longer any correlation with the markup, which converges to $\frac{\epsilon^b}{\epsilon^b - 1} = 1.007$. These results underline a potential trade-off for the central bank regarding the choice of the appropriate degree of aggressiveness towards inflation: a higher $\phi_{\pi}$ stabilizes inflation to a larger extent, but it induces an increase in the degree of monopolistic power of banks in the long run.

Table 1: Baseline calibration

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_P$</td>
<td>0.997  household discount factor</td>
</tr>
<tr>
<td>$\beta_E$</td>
<td>0.99  entrepreneurial discount factor</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1  inverse Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.3  product elasticity with respect to physical capital</td>
</tr>
<tr>
<td>$m^E$</td>
<td>0.8  entrepreneurs LTV ratio</td>
</tr>
<tr>
<td>$\epsilon^b$</td>
<td>151  elasticity of substitution of loans</td>
</tr>
<tr>
<td>$\epsilon^g$</td>
<td>6  elasticity of substitution of goods</td>
</tr>
<tr>
<td>$\kappa_p$</td>
<td>30  price stickiness</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>0.9  investment adjustment cost</td>
</tr>
<tr>
<td>$\delta^k$</td>
<td>0.025  depreciation rate of physical capital</td>
</tr>
<tr>
<td>$\varsigma_{AE}$</td>
<td>0.01  TFP standard deviation innovation</td>
</tr>
<tr>
<td>$\varsigma^{R^b}$</td>
<td>0.0025  monetary policy standard deviation innovation</td>
</tr>
</tbody>
</table>
4 The financial acceleration markup and the propagation of shocks

We now turn to studying the dynamic responses of the model, showing the impact of different calibration for $n$ and $\phi_\pi$. We again refer to Table 1 for the calibration of the other parameters. However, to isolate the steady-state effects of banking competition on the endogenous banking market structure, we assume the existence of a subsidy $\Upsilon$ that fully offsets the static distortion from monopolistic competition in the banking sector, i.e. $M^b/(1+\Upsilon) = 1$.

Figure 3 reports the response to a temporary monetary restriction (defined as a shock to $\varepsilon_{R}^{ib}$ in equation (1)), calibrating the inflation coefficient $\phi_\pi$ at 1.5. The blue lines correspond to the case of atomistic banks, while the red lines correspond to the case of large banks ($n = 3$).

Following the shock inflation and output drop, reflecting the contrac-
tion in consumption and the fall in investment, associated to a lower entrepreneurs’ borrowing capacity. The price of capital also falls and entrepreneurs’ net wealth is hit, more than compensating the reduction in loans, so that leverage increases. Note that the negative contemporaneous correlation between leverage and output is in line with the findings in Levy and Hennessy (2007) and Chugh (2012).

When banks are atomistic, there is one-to-one relationship between changes in the loan rate moves and changes in the policy rate, as the markup is fixed. When banks are large, instead, the loan interest rate increases by more, as banks’ markup is positively related to leverage. As a consequence, all the negative dynamics in the model get accelerated, bringing about a stronger contraction in output. This mechanism unveils the existence, in this context, of a new type of financial accelerator, which is crucially related to the presence of large banks. This effect is different in nature to the standard financial accelerator discussed in the literature on the credit channel (Bernanke and Gertler, 1989; Bernanke et al., 1999; Kiyotaki and Moore, 1997) and adds-up to that channel, which is also at work in the model due to the presence of borrowing-constrained agents.¹²

Figure 4 displays the response of the model to a positive productivity shock (see equation (2)), again comparing the case of atomistic banks and large banks. As is standard, the positive technology shock reduces inflation and increases output. Also in this case, the size of the response of output (and inflation) is magnified by the presence of large banks, that induces a countercyclical movement in bank’s markup.

In the previous two exercises, we kept the value of the central bank’s inflation aversion \( \phi_\pi \) fixed at 1.5. However, the value of this parameter also has an impact on the dynamic response of the model to the shocks. Figure 5 compares the response of the banks’ markup to a negative technology shock when \( \phi_\pi = 30 \), which can be considered as the case of monetary policy following a strict inflation targeting, and when \( \phi_\pi = 1.1 \), i.e., the case of a ”weak” inflation aversion. In both cases, the number of banks \( n \) is set at 3. The figure shows that the (countercyclical) response of the markup is magnified in the case of aggressive inflation targeting, bringing about a stronger accelerating effect than in the case of weak inflation targeting.

¹²Under standard calibration and linearized solution methods, the introduction of credit constraints per se does not seem to have a sizeable amplification effect (e.g. Cordoba and Ripoll, 2004) or important implications for monetary policy conduct (e.g. Bernanke et al., 1999; Iacoviello, 2005).
Figure 3: Impulse response to a monetary contraction standard deviation. Comparing the models with atomistic and large banks. Percent deviation from steady state.

5 Conclusions

This paper extends a New-Keynesian model with banks and financial frictions to allow for the presence of large banks, i.e. intermediaries that internalize the aggregate effects of their individual decisions. This framework also generates a strategic interaction between the banks and the central bank, because the policy rate corresponds to the banks’ marginal cost.

In this framework, we find a number of interesting insights. First, when intermediaries are large, the steady-state markup that they charge on loans does not only depend on the (exogenous) elasticity of substitution among loan varieties. It is also positively related to the degree of borrowers’ leverage.
in the economy and to the degree of inflation aversion of the central bank. The reason for this is that these two items affect the elasticity of the banks’ marginal cost (the policy rate) to the price that they set (the loan rate). Second, in terms of the dynamic properties of the model, our framework generates countercyclical movements of the loan rate markup. This is due to the fact that borrowers’ leverage — to which the markup is correlated — falls (rises) during good (bad) times, in turn reflecting the dynamics of net worth. The countercyclical movement of the loan markup amplifies the impact of aggregate shocks on the real economy, generating a kind of financial accelerator so far — to the best of our knowledge — not emphasized in the literature. This amplifying effect adds-up to the standard one, which is also

Figure 4: Impulse response to a productivity increase standard deviation. Comparing the models with atomistic and large banks. Percent deviation from steady state.
Figure 5: Effect of different inflation coefficients with a negative technological shock and $n = 3$. Percent deviation from steady state.
at work in the model due to the presence of the borrowing constraint, and is crucially related to the existence of non-atomistic banks. Moreover, we find that the magnitude of the amplification effect is increasing in the central bank’s aggressiveness in stabilizing inflation.

The results identified in this paper are likely to have significant implications, both for the appropriate conduct of monetary policy and for financial stability considerations. For example, optimal monetary policy prescriptions may change once the strategic interaction between the central bank and large financial institutions is taken into account. Moreover, the effectiveness of various monetary and macro-prudential policy settings may depend on the interaction of these policies with the behavior of non-atomistic banks. This analysis is left for future research.

References


Christiano, Lawrence, Roberto Motto, and Massimo Rostagno. (2008)


Appendices

A The derivation of equations (23) and (24)

From (19) and (20), we obtain

\[ q^k_t - m^E q^k_{t+1} / R^b_t = \beta_E c^E_t / c^E_{t+1} \left[ r^k_{t+1} + q^k_{t+1} (1 - m^E) (1 - \delta^k) \right]. \tag{A.1} \]

Using the definition of entrepreneurs’ net worth in text (22), the entrepreneurs’ budget constraint can be rewritten as

\[ c^E_t = n w^E_t - q^k_t k^k_t + b^E_t. \tag{A.2} \]

Now, we guess that entrepreneurs’ consumption is a fraction \(1 - \beta_E\) of net worth as follows:

\[ c^E_t = (1 - \beta_E) n w^k_t. \tag{A.3} \]

Thus, plugging the guess into equation (A.1) yields

\[ q^k_t k^E_t - b^E_t = \beta_E n w^E_t \tag{A.4} \]

which corresponds to equation (24) in the text. Finally, in order to verify our initial guess and so equation (23), combine (A.4) and (A.2).

B The bank’s \( u \) problem solution

An impact of bank loan rate on aggregate loan rate

The loan rate set by a representative bank \( u \) is the same for all the types of loan supplied. We assume that each bank simultaneously sets the loan rate, \( R^b_t(u) \), taking the other banks’ loan rate as given. Thus, from the aggregate loan index,

\[ R^b_t = \left[ \int_0^1 R^b_t(j)^{1-c^b} \, dj \right]^{1-c^b}, \tag{B.1} \]
we have that in a symmetric equilibrium, i.e. when $R_b^b(u) = R_t^b$, 

$$
\frac{\partial R_t^b}{\partial R_t^b(u)} = \frac{\partial}{\partial R_t^b(u)} \left[ \int_{j \in u} R_t^b(j)^{1-\epsilon_b} dj + \int_{j \notin u} R_t^b(j)^{1-\epsilon_b} dj \right]^{1 \over 1-\epsilon_b} = \frac{1}{n} \left[ \frac{R_t^b(u)}{R_t^b} \right]^{\epsilon_b} = \frac{1}{n}. \tag{B.2}
$$

Note that, because of symmetry, it is also true that

$$
\frac{\partial R_t^b}{\partial R_t^b(u)} \frac{R_t^b(u)}{R_t^b} = \frac{\partial R_t^b}{\partial R_t^b(u)} = \frac{1}{n}. \tag{B.3}
$$

**Loans demand and policy rate elasticities to aggregate loan rate index**

Define by

$$
\Xi_{Z,t} \equiv \frac{\partial Z_t}{\partial R_t^b} \frac{R_t^b}{Z_t}
$$

the elasticity of variable $Z_t$ with respect to $R_t^b$. Bank’s elasticities are computed *taking as given expectations* about variables in the next period.

When the borrowing constraint (15) is binding, we can use equations (25) and (27) and rewrite it as follows

$$
b_t^E = V_t^E L V_t^E n w_t^E. \tag{B.4}
$$

As banks set the interest rate after having observed the entrepreneurs’ net wealth (22), they also take the rental rate and price of capital as given. Thus, we can derive the following (perceived) relation

$$
\Xi_{b,t} = \Xi_{V,t} + \Xi_{LV,t} = -1 + \Xi_{LV,t} \tag{B.4}
$$

between the elasticity of loans demand and borrowers’ leverage, which corresponds to $-\Sigma_{b,t}$ in the text (32).

Similarly, from the equilibrium condition for the labor market (12), the interest rate rule (1), and the production function (2) we obtain

$$
\Xi_{y,t} + \Xi_{mc,t} + \Xi_{Rb,t} = (1 + \phi) \Xi_{l,t}. \tag{B.5}
$$
and
\[ \Xi_{\pi,t} = \phi_{\pi} \Xi_{\pi,t}. \] (B.6)

Now, combining the budget constraint for households (9) and for entrepreneurs (14) yields the clearing condition in the final goods market
\[ y_t^E \left[ 1 - \frac{\kappa_p}{2} (\pi_t - 1)^2 \right] = \frac{c_{t+1}^P}{\beta P R^b_t} + (1 - \beta_E) n w_t^E + LV_t^E n w_t^E - q_t k_{t-1} (1 - \delta^k). \]

Differentiate with respect to \( R^b_t \) and evaluate at zero net inflation, \( \pi_t = 1 \), the above resource constraint; using \( \frac{\partial Z_t}{\partial R^b_t} = \Xi_{Z,t} \frac{Z_t}{R^b_t} \), it reads
\[ y_t \Xi_{y,t} = n w_t^E LV_t^E \Xi_{LV,t} - c_t^P \Xi_{R^b,t}. \] (B.8)

and the New Keynesian Phillips curve (6) leads to the following expression
\[ \kappa_p (mk^y - 1) \Xi_{\pi,t} = mk^y mc_t \Xi_{mc,t}. \] (B.9)

Finally, taking logs of the entrepreneurs’ leverage (27) and differentiating with respect to \( R^b_t \) yields
\[ \Xi_{LV,t} = -\frac{V_t^E}{1 - V_t^E} = 1 - \frac{LV_t^E}{\beta_E}, \] (B.10)

which corresponds to \( -\Sigma_{LV,t} \) in the text (33). Expression (34) is derived by solving the system of equations (B.5)-(B.9) for \( \Xi_{Z,t} \) where \( Z \in \{y, mc, R^b, l^b, \pi\} \).

**Banks’ first-order condition**

Taking derivative of (28) with respect to \( R^b_t(u) \) and using (30) yields at the symmetric equilibrium, \( R^b_t(j) = R^b_t \),
\[ R^b_t = \frac{(n - 1) c^b (R^b_t - R^{ib}_t)}{n} + \left( R^b_t - R^{ib}_t \right) \frac{\partial b^E}{\partial R^b_t} \frac{R^b_t}{v_t^E} - \frac{\partial R^b_t}{\partial R^b_t} \frac{R^b_t}{n} \] (B.11).
Substituting for $\frac{\partial b^E}{\partial R^t} = -\Sigma b^t R^E_t$ and $\frac{\partial R^b}{\partial R^t} = -\Sigma R^b_t R^E_t$ yields expression (29) in the text.

C The steady state

Without loss of generality we normalize the technology parameter $A^E$ so that $y^E = 1$ in steady state. From the Euler equation (10) and the firms’ optimal condition in the capital good sector (7), we have that $R^b = 1/\beta P$ and $q = 1$. Thus, in steady state equations (25), (27), and (24) read:

\[ V^E = b^E/k^E, \quad \text{(C.1)} \]
\[ LV = k^e/nw^E, \quad \text{(C.2)} \]

and
\[ LV = \beta_E/(1 - V^E). \quad \text{(C.3)} \]

At the zero inflation the New Keynesian Phillips curve yields $mc = 1/mk^y$ and the resource constraint is given by
\[ c^P + nw^E(LV^E - 1) - R^b b^E + \alpha/mk^y + nw^E(1 - \beta_E) = 1. \quad \text{(C.4)} \]

From equations (12), (15), and (22) we have that
\[ (P^e)^{1+\phi} = \frac{1 - \alpha}{c^P mk^y} \quad \text{(C.5)} \]

and
\[ b^E R^b = k^E m^E (1 - \delta^k), \quad \text{(C.6)} \]
\[ nw^E + R^b b^E - \alpha/mk^y - k^E (1 - \delta^k) = 0. \quad \text{(C.7)} \]

Equations (39), (40), and (41) are derived by solving the system of equations (C.1)-(C.7) for $c^P$, $k^E$, $LV^E$, $b^E$, $nw^E$, $p^P$ and $V^E$. 

29