Bank Leverage Regulation and Macroeconomic Dynamics *

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PRELIMINARY AND INCOMPLETE

Abstract

Regulatory constraints on bank leverage have been at the center of many policy discussions recently. One important question to emerge from these discussions has been whether these regulations should be time-dependent and how they would interact with the business cycle. We analyze this question using a dynamic stochastic general equilibrium model with banks and bank capital. In the model, bank capital emerges endogenously to solve an asymmetric information problem between banks and their creditors. The capital position of a bank thus affects its ability to attract loanable funds and, as a result, bank capital influences the business cycle through a bank capital channel of transmission. Government regulations on bank leverage interact with this channel. We use the model to conduct experiments on the strength of this interaction and find that regulations on bank leverage can have important effects on the dynamic responses of the economy to technology, financial and regulatory shocks.

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1 Introduction

Bank regulation is among the key policy issues to have emerged from the recent events in financial markets worldwide. One aspect of bank regulation that has generated intense interest is how regulatory constraints on bank leverage and balance sheets should be organized. Should they be strengthened? Should they vary with the cycle, perhaps tightening in good times and loosening when activity slows down? How should they interact with the conduct of monetary policy?

This paper addresses these important questions using the dynamic stochastic general equilibrium model with banks and bank capital developed in Meh and Moran (2010). In the model, bank capital emerges endogenously to solve an asymmetric information problem between bankers and their creditors. As a result, the capital position of a bank affects its ability to attract loanable funds and, at a macroeconomic level, bank capital influences the bank leverage, the transmission of shocks and business cycle through a bank capital channel of transmission. The model is embedded within a medium-scale version of the New Keynesian paradigm, in the spirit of Christiano et al. (2005) and Smets and Wouters (2007). Our paper thus enables this type of modeling, widely used for monetary policy analysis, to provide quantitative explorations on the likely impact of bank regulation on economic activity.

Government regulations on bank leverage naturally impact the bank capital channel, constraining what would otherwise be market-determined responses of bank leverage to macroeconomic factors. Our simulations reveal that this impact can have important quantitative implications. Our findings are as follows. We show that (i) the response of real variables to technology and monetary policy shocks is markedly dampened by regulation on bank leverage, while the response of prices and interest rates is more volatile; (ii) the response of the economy to financial (bank capital) shocks is more volatile under bank leverage regulation; (iii) a tightening in regulation standards has important economic consequences; and (iv) the time-series properties of the regulation on bank leverage can also have important consequences.

Our analysis contributes to a growing literature assessing the impact of bank regulation on economic fluctuations (Van den Heuvel, 2008; Angeloni and Faia, 2009; Covas and Fujita, 2009; Angelini et al., 2010; Dib, 2010). Contrary to much of this literature, however, the bank capital channel at the core of our contribution is motivated by the presence of an explicit moral hazard problem affecting the relationship between banks and their suppliers of loanable funds. In addition, in our model, the intensity with which bank monitor their borrowers, i.e. their investment in information gathering, constitutes a key margin of adjustment helping banks comply with statutory regulation.

The remainder of this paper is organized as follows. Section 2 describes the model and Section 3 discusses the model’s calibration. Section 4 presents our findings and Section 5
provides some concluding comments.

2 The Model

2.1 The environment

This section describes the structure of the model and the optimization problem of the economy's agents. Time is discrete, and one model period represents a quarter. There are three classes of economic agents: households, entrepreneurs, and bankers, whose population masses are $\eta^h$, $\eta^e$ and $\eta^b = 1 - \eta^h - \eta^e$, respectively. In addition there are firms producing intermediate and final goods, as well as a monetary authority.

There are three goods in the economy. First, intermediate goods are produced by monopolistically competitive firms facing nominal rigidities. Second, final goods are assembled by competitive firms using the intermediate goods. Third, capital goods are produced by entrepreneurs, with a technology that uses final goods as inputs and is affected by idiosyncratic uncertainty.

Two moral hazard problems affect the production of capital goods. First, entrepreneurs can influence their technology's probability of success and may choose projects with a low probability of success, to enjoy private benefits. Bank monitoring of entrepreneurs can lessen the severity of this moral hazard problem: the more intense bank monitoring is, the less severe the moral hazard problem becomes. As an alternative to monitoring, banks can require that entrepreneurs invest their own net worth when lending to them. The choice of banks regarding the intensity with which they monitor entrepreneurs will arise as a key variable in our analysis.

A second moral hazard problem is present in the model and occurs between banks and investors, their own source of funds. Investors lack the ability to monitor entrepreneurs so they deposit funds at banks and delegate the task of monitoring entrepreneurs to their bank. However, bank monitoring is private and costly, so that banks might be tempted to monitor less than agreed, because any resulting risk in their loan portfolio would be mostly borne by investors. As a result, investors require banks to invest their own net worth (their capital) in entrepreneurs' projects. Overall, our double moral hazard framework implies that over the business cycle, the dynamics of bank capital affects how much banks can lend, the dynamics of entrepreneurial net worth affects how much entrepreneurs can borrow, and banks' monitoring intensity impacts the strength of these two net worth channels.

In addition, we consider economies where regulatory requirements limits how much banks are allowed to lend. These requirements take the form of constraints on bank leverage, i.e the ratio of the size of banks' balance sheets to their capital, and can be time-dependent, loosening or tightening in response to economic activity. Such regulation naturally impact the effects that bank capital has on the propagation of shocks and the
business cycle and a key contribution of our analysis is to investigate quantitatively the strength of this impact.

2.2 Final good production

Competitive firms produce the final good by combining a continuum of intermediate goods indexed by $j \in (0, 1)$ using the standard Dixit-Stiglitz aggregator:

$$Y_t = \left( \int_0^1 \frac{\xi_p}{y_{jt}} \frac{d\theta}{\xi_p} \right)^\frac{1}{\xi_p - 1}, \quad \xi_p > 1,$$

(1)

where $y_{jt}$ denotes the time $t$ input of the intermediate good $j$, and $\xi_p$ is the constant elasticity of substitution between intermediate goods.

Profit maximization leads to the following first-order condition for the choice of $y_{jt}$:

$$y_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\xi_p} Y_t,$$

(2)

which expresses the demand for good $j$ as a function of its relative price $p_{jt}/P_t$ and of overall production $Y_t$. Imposing the zero-profit condition leads to the usual definition of the final-good price index $P_t$:

$$P_t = \left( \int_0^1 p_j^1 \frac{1}{1 - \xi_p} d\sigma \right)^\frac{1}{1 - \xi_p}.$$

(3)

2.3 Intermediate good production

Firms producing intermediate goods operate under monopolistic competition and nominal rigidities in price setting. The firm producing good $j$ operates the technology

$$y_{jt} = \begin{cases} z_t k_{jt}^\theta h_{jt}^\theta e_{jt}^\theta b_{jt}^\theta - \Theta & \text{if } z_t k_{jt}^\theta h_{jt}^\theta e_{jt}^\theta b_{jt}^\theta \geq \Theta \\ 0 & \text{otherwise} \end{cases}$$

(4)

where $k_{jt}$ and $h_{jt}$ are the amount of capital and labor services, respectively, used by firm $j$ at time $t$. In addition, $e_{jt}$ and $b_{jt}$ represent labor services from entrepreneurs and bankers.$^1$ Finally, $\Theta > 0$ represents the fixed cost of production and $z_t$ is an aggregate technology shock that follows the autoregressive process

$$\log z_t = \rho \log z_{t-1} + \varepsilon_{zt},$$

(5)

$^1$Following Carlstrom and Fuerst (1997), we include labor services from entrepreneurs and bankers in the production function so that these agents always have non-zero wealth to pledge in the financial contract described below. The calibration sets the value of $\theta_e$ and $\theta_b$ so that the influence of these labor services on the model’s dynamics is negligible.
where $p_z \in (0, 1)$ and $\varepsilon_{zt}$ is i.i.d. with mean 0 and standard deviation $\sigma_z$.

Minimizing production costs for a given demand solves the problem

$$\min_{(k_{jt}, h_{jt}, h^e_{jt}, h^b_{jt})} r_t k_{jt} + w_t h_{jt} + w^e_t h^e_{jt} + w^b_t h^b_{jt}$$

with respect to the production function (4). The (real) rental rate of capital services is $r_t$, while $w_t$ represents the real household wage. In addition, $w^e_t$ and $w^b_t$ are the compensation given entrepreneurs and banks, respectively, for their labor.

The first-order conditions of this problem with respect to $k_{jt}$, $h_{jt}$, $h^e_{jt}$ and $h^b_{jt}$ are respectively:

$$r_t = s_t z_t k_{jt}^{\theta_k - 1} h_{jt}^{\theta_h} h^e_{jt}^{\theta_e} h^b_{jt}^{\theta_b};$$

$$w_t = s_t z_t h_{jt}^{\theta_h} h^e_{jt}^{\theta_e} h^b_{jt}^{\theta_b};$$

$$w^e_t = s_t z_t h^e_{jt}^{\theta_e} h_{jt}^{\theta_h} h^b_{jt}^{\theta_b};$$

$$w^b_t = s_t z_t h^b_{jt}^{\theta_b}.$$  

In these conditions, $s_t$ is the Lagrange multiplier on the production function (4) and represents marginal costs. Combining these conditions, one can show that total production costs, net of fixed costs, are $s_t y_{jt}$.

The price-setting environment is as follows. Each period, a firm receives the signal to reoptimize its price with probability $1 - \phi_p$; with probability $\phi_p$, the firm simply indexes its price to last period’s aggregate inflation. After $k$ periods with no reoptimizing, a firm’s price would therefore be

$$p_{jt+k} = \prod_{s=0}^{k-1} \pi_{t+s} P_{jt},$$

where $\pi_t \equiv P_t/P_{t-1}$ is the aggregate (gross) rate of price inflation.

A reoptimizing firm chooses $\hat{p}_{jt}$ in order to maximize expected profits until the next reoptimizing signal is received. The profit maximizing problem is thus

$$\max_{\hat{p}_{jt}} E_t \sum_{k=0}^{\infty} (\beta \phi_p)^k \lambda_{t+k} \left[ \frac{p_{jt+k} Y_{jt+k}}{P_{t+k}} - s_{t+k} y_{jt+k} \right],$$

subject to (2) and (11).\(^2\)

The first-order condition for $\hat{p}_{jt}$ leads to

$$\hat{p}_t = \frac{E_t \sum_{k=0}^{\infty} (\beta \phi_p)^k \lambda_{t+k} s_{t+k} Y_{t+k} \pi_{t+k}^\xi}{E_t \sum_{k=0}^{\infty} (\beta \phi_p)^k \lambda_{t+k} Y_{t+k} \pi_{t+k}^{\xi-1}}.$$  

\(^2\)Time-$t$ profits are discounted by $\lambda_t$, the marginal utility of household income.
2.4 Capital good production

Each entrepreneur has access to a technology producing capital goods. The technology is subject to idiosyncratic shocks: an investment of $i_t$ units of final goods returns $R_i$ ($R > 1$) units of capital if the project succeeds, and zero units if it fails. The project scale $i_t$ is variable and determined by the financial contract linking the entrepreneur and the bank (discussed below). Returns from entrepreneurial projects are publicly observable.

A first moral hazard problem occurs because entrepreneurs may deliberately reduce the probability of success of their investment project in order to enjoy a private benefit. This moral hazard problem is formalized by assuming that entrepreneurs can choose from two classes of projects. First, the no private benefit project involves a high probability of success (denoted $\alpha$) and zero private benefits. Second, there exists a continuum of projects with private benefits. Projects from this class all have a common, lower probability of success $\alpha - \Delta \alpha$, but differ in the amount of private benefits they deliver to the entrepreneurs. The private benefit probabilities are denoted by $b_i$, where $i_t$ is the size of an entrepreneur’s project and $b \in [\underline{B}, \overline{B}]$. Among those, an entrepreneur will thus prefer the project with the highest private benefit $b$ possible, since they all produce the same low probability of success.

Throughout the analysis, it is assumed that only the project with no private benefit is economically productive:

$$q_t \alpha R_i - R_i^d i_t > 0 > q_t (\alpha - \Delta \alpha) R_i - R_i^d i_t + \overline{B} i_t,$$

where $q_t$ is the price of the capital goods produced by the entrepreneur’s technology and $R_i^d$ is the opportunity costs of the funds engaged in projects. A sufficient condition for this to hold is that $\overline{B} \leq \Delta \alpha R$: even the biggest private benefit generated by the second class of projects is smaller than the decrease in the probability of success it imposes.

Bank monitoring can reduce the private benefits associated with projects (ie. limit the ability of entrepreneurs to divert resources). In this framework, bank monitoring is interpreted as the inspection of cash flows and balance sheets, or the verification that firms conform with loan covenants (Holmstrom and Tirole, 1997). A bank monitoring at intensity $\mu_t$, say, limits the ability of an entrepreneur to divert resources to $b(\mu_t)$, where $b(0) = \overline{B}$, $b(\infty) = \underline{B}$, $b'(\cdot) < 0$ and $b''(\cdot) > 0$. Figure 1 illustrates the relationship between bank monitoring and entrepreneurial private benefits: a higher monitoring intensity, akin to a tighter bank-entrepreneur relationship, produces more information about the entrepreneur and thus reduces his ability to divert resources. By contrast, a lower monitoring intensity – a more “arms-lengths” relationship– generates less information, and thus more ability to divert and more severe moral hazard on the entrepreneur side.

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3This is in contrast with the costly state verification (CSV) literature, where bank monitoring is associated with bankruptcy-related activities.
Bank monitoring remains imperfect: even when monitored by his bank at intensity \(\mu_t\), an entrepreneur may still choose to run a project with private benefit \(b(\mu_t)\). A key component of the financial contract discussed below ensures that he has the incentive to choose the no-private benefit project instead.

Bank monitoring is also privately costly: monitoring an entrepreneur at intensity \(\mu_t\) entails a resource cost equal to \(z\mu_t\). Since monitoring is not publicly observable, a second moral hazard problem emerges in our environment, between banks and investors provide banks with loanable funds. A bank that invests its own capital in entrepreneur projects, however, lessens this problem, because this bank now has a private incentive to monitor as agreed the financed entrepreneurs. This reassures investors and allows the bank to attract more loanable funds.

The returns in the projects funded by each bank are assumed to be perfectly correlated. Correlated projects can arise because banks specialize (across sectors, regions or debt instruments) to become efficient monitors. The assumption of perfect correlation improves the model’s tractability and could be relaxed at the cost of additional computational requirements.

### 2.5 The Financial contract

An entrepreneur with net worth \(n_t\) undertaking a project of size \(i_t > n_t\) needs external financing (a bank loan) worth \(i_t - n_t\). The bank provides this funding with a mix of deposits it collects from investors (\(d_t\)) as well as its own net worth (capital) \(a_t\). Considering the costs of monitoring the project (\(= z\mu_t i_t\)), the bank thus lends an amount \(a_t + d_t - z\mu_t i_t\).

We concentrate on equilibria where the financial contract leads all entrepreneurs to undertake the project with no private benefits so that \(\alpha\) represents the probability of success of all projects. We also assume the presence of inter-period anonymity, which restricts the analysis to one-period contracts.

The financial contract under regulation is set in real terms and has the following structure. It determines an investment size (\(i_t\)), contributions to the financing from the bank (\(a_t\)) and the bank’s investors (\(d_t\)), and how the project’s return is shared among the entrepreneur (\(R^e_t > 0\)), the bank (\(R^b_t > 0\)) and the investors (\(R^h_t > 0\)). The contract also specifies the intensity \(\mu_t\) at which banks agree to monitor, to which corresponds an ability to divert resources \(b(\mu_t)\) on the entrepreneur side. Limited liability ensures that no agent earns a negative return.

The contract’s objective is to maximize the entrepreneur’s expected share of the return \(q_t \alpha R^e_t i_t\) subject to a number of constraints. These constraints ensure that entrepreneurs and bankers have the incentive to behave as agreed, that the funds contributed by the banker and the household earn (market-determined) required rates of return, and that the loan size respects the regulated leverage.
Formally, the contract is represented by the following optimization problem:

$$\max_{\{i, R^e_t, R^b_t, \alpha_t, d_t, \mu_t\}} q_t \alpha R^e_t i_t,$$  \(14\)

subject to

$$R = R^e_t + R^b_t; \quad (15)$$

$$q_t \alpha R^b_t i_t - z \mu_t i_t \geq q_t (\alpha - \Delta \alpha) R^b_t i_t; \quad (16)$$

$$q_t \alpha R^e_t i_t \geq q_t (\alpha - \Delta \alpha) R^e_t i_t + q_t b(\mu_t) i_t; \quad (17)$$

$$q_t \alpha R^b_t i_t \geq (1 + r^b_t) a_t; \quad (18)$$

$$q_t \alpha R^b_t i_t \geq (1 + r^b_t) d_t; \quad (19)$$

$$a_t + d_t - z \mu_t i_t \geq i_t - n_t. \quad (20)$$

$$i_t - n_t \leq \gamma^g_t a_t. \quad (21)$$

Equation (15) states that the shares promised to the three different agents must add up to the total return. Equation (16) is the incentive compatibility constraint for bankers, which must be satisfied in order for monitoring to occur at intensity \(\mu_t\), as agreed. This equation states that the expected return to the banker, net of the monitoring costs, must be at least as high as the expected return if not monitoring, a situation in which entrepreneurs would choose a project with the lower probability of success. Equation (17) is the incentive compatibility of entrepreneurs: given that bankers monitor at intensity \(\mu_t\), entrepreneurs can at most choose the project that gives them private benefits \(b(\mu_t)\). The constraint then ensures that they have an incentive to choose the project with no-private benefits and high probability of success instead. Equations (18) and (19) are the participation constraints of bankers and households, respectively. They state that these agents, when engaging their bank capital \(a_t\) and deposits \(d_t\), are promised a return that covers the (market-determined) required rates \(r^a_t\) and \(r^d_t\), respectively. Finally, equation (20) indicates that the loanable funds available to a banker (its own capital and the deposits it attracted), net of the monitoring costs, are sufficient to cover the loan given to the entrepreneur, while (21) specifies that the loan cannot be bigger than a regulated leverage \(\gamma^g_t > 1\) over bank capital.

In the next section, we will contrast the solution to (14)-(21) to another solution where the explicit government constraint on leverage (21) is absent. We will refer to the former solution as the Regulation case and to the latter as the No Regulation case. In both cases, however, the incentive-compatibility, participation, and budget constraints (15)-(20) continue to hold.

Imposing that the incentive-compatibility constraints (16) and (17), as well as the
budget constraint (15) hold with equality, we have

\[ R^e_t = \frac{b(\mu_t)}{\Delta \alpha}; \quad (22) \]
\[ R^b_t = \frac{z\mu_t}{q_t \Delta \alpha}; \quad (23) \]
\[ R^h_t = R - \frac{b(\mu_t)}{\Delta \alpha} - \frac{z\mu_t}{q_t \Delta \alpha}. \quad (24) \]

Note from (22) and (23) that the shares allocated to the entrepreneur and the banker are affected by the severity of the two moral hazard problems. More intense bank monitoring (an increase in \( \mu_t \)) reduces the project share that must be allocated to entrepreneurs, because the tighter link between the bank and the entrepreneur reduces the latter’s ability to divert resources (\( b(\mu_t) \) decreases). This effect is likely to be strong at first, as \( \mu_t \) is increased from relatively low values. However, more intense monitoring exacerbates the second moral hazard problem, between banks and their suppliers of loanable funds, because banks promise to allocate more costly resources to monitoring, even though this activity is private. On balance, (24) shows that the per-unit share of project return that can be credibly promised to the suppliers of loanable funds is limited by the two moral hazard problems. An increase in monitoring intensity, for example, lessens moral hazard on the entrepreneurial side, but exacerbates it on the bank side, the net effect on overall moral hazard depending on the efficiency of banks’ monitoring technology as measured by the schedule (\( \mu_t \)).

Introducing (24) in the participation constraint of households (19) holding with equality leads to the following:

\[ d_t = \frac{q_t \alpha}{1 + r^d_t} \left( R - \frac{b(\mu_t)}{\Delta \alpha} - \frac{z\mu_t}{q_t \Delta \alpha} \right) i_t. \quad (25) \]

This expression states that the share of outside loanable funds that goes into financing a given-size project, \( d_t/i_t \), is governed by two macroeconomic factors (the price of investment goods \( q_t \) and the cost of loanable funds \( r^d_t \)), as well as the overall extent of moral hazard in the financial market, represented by \( b(\mu_t) \) and \( \mu_t \).

Next (18) and (23) together can be used to deliver

\[ a_t = \alpha z \mu_t \frac{i_t}{(1 + r^d_t) \Delta \alpha}, \quad (26) \]

which states that for a given-size project, more intense monitoring of entrepreneurs require that banks invest more capital in the project. It also implies that all things equal, an increase in the rate of return on bank equity \( r^d_t \) (which reflects the aggregate scarcity of bank capital) reduce the amount of capital banks will invest in a given-size project.
Next, assume that the regulation constraint (21) binds. Using (20), it becomes

\[ a_t + d_t - z \mu_i = \gamma^0_t a_t. \]  

(27)

Using (25) and (26) to eliminate \( a_t \) and \( d_t \) from this expression yields a relation between the regulated leverage \( \gamma^0_t \) and the monitoring intensity \( \mu_t \) needed to achieve it while respecting all incentive and participation constraints:

\[ \gamma^0_t = 1 + \left( q_t (1 + r^0_t) \right) \left( \frac{\Delta \alpha R - b(\mu_t) - z \mu_t / q_t}{\mu_t} \right) - \frac{(1 + r^0_t) \Delta \alpha}{\alpha}. \]  

(28)

Expression (28) provides intuition about the way banks adjust their monitoring intensity \( \mu_t \) to comply with the regulatory requirements. The left-hand side of the expression is the leverage imposed by the regulator, while the right-hand side shows how banks’ monitoring decision and the resulting financial contract can achieve it. Consider first a bank monitoring at very low intensity, with \( \mu_t \to 0 \). Moral hazard on the entrepreneurial side worsens but eventually reaches its maximum extent \( B \). Meanwhile, very low intensity \( \mu_t \) decreases the moral hazard problem on the bank side considerably, and outside investors provide loanable funds very easily, leading leverage to rise dramatically. By contrast, a very high intensity of monitoring, \( \mu_t \to +\infty \), decreases moral hazard on the entrepreneurial size towards its lowest possible level \( \bar{B} \) but considerably increases moral hazard on the bank side, leading outside investors to withdraw loanable funds from banks and achievable leverage to fall. Between these two extremes, the assumed properties on the schedule \( b(\mu_t) \) ensure that a single value of \( \mu_t \) exists that achieves the regulated leverage. Figure 2 illustrates the situation by graphing regulated and achieved leverage as a function of \( \mu_t \) as well as the resulting choice for monitoring intensity.

What happens to the monitoring intensity of banks when economic conditions change? Although a complete analysis of this question requires the quantitative general equilibrium simulations we report in Section 4, intuition about three effects can be gained directly from (28) and Figure 2 to get the following results:

- **Result 1**: \( \frac{\partial \mu_t}{\partial q_t} > 0 \). Under the regulation solution, favorable shocks to the investment sector lead banks to increase their monitoring intensity.

A favorable shock to the demand for capital goods, like a technology shock to the intermediate-good production function, creates upwards pressure on \( q_t \), the price of the capital good relative to the resources used to produce it. This lessens moral hazard in the financial market, because it makes it easier for bank to credibly promise a sufficient return to the investors providing them loanable funds. A given intensity of bank monitoring is now more effective and keeping \( \mu_t \) unchanged would see leverage rise above the regulated level, as evidenced by the red dashed line in Figure 2. To counterbalance this upward
drift in leverage, banks increase their monitoring intensity, which requires a bigger capital position in the project and thus reduces leverage.

- **Result 2**: \( \frac{\partial \mu_t}{\partial r_t^d} < 0. \) Under the regulation solution, a decrease in the cost of loanable funds leads banks to increase their monitoring intensity.

A decrease in the cost of loanable funds (following a monetary policy easing) has similar consequences to the rise in \( q_t \) analyzed above: it lessens moral hazard in financial markets by making it easier to promise outside investors their required return (again, see the right-hand-side of (25)). Keeping \( \mu_t \) unchanged would once again lead leverage to increase above its regulated level. To compensated, banks increase their monitoring intensity, which is associated with a rise in their capital position in lending thus leverage eases back down to the regulated level.

- **Result 3**: \( \frac{\partial \mu_t}{\partial r_a} > 0. \) Under the regulation solution, a sudden scarcity of bank capital leads banks to increase their monitoring intensity.

An increase in \( r_a \), the market-required rate of return on bank equity (bank capital) signals a sudden scarcity of bank capital at the aggregate level. Banks thus have an incentive to economize on their use of bank capital and, all things equal, to rely more on loanable funds from depositors to finance entrepreneurs. This incentive would tend to push leverage upwards. Since leverage is fixed by regulation, banks compensate this tendency to increase leverage by increasing their monitoring intensity and their capital participation in entrepreneurial projects, which reduces their leverage and pushes it back down to its regulated level.

- **Result 4**: \( \frac{\partial \mu_t}{\partial \gamma_g} < 0. \) A tightening in the regulated leverage ratio leads banks to increase their monitoring intensity.

Tightening the regulation on bank leverage (ie. decreasing the allowed leverage \( \gamma_g \)) requires that banks engage more of their own capital in a given-size project. This requires that banks increase their monitoring intensity.

2.6 The financial contract in the absence of regulation

The solution to the financial contract in the absence of leverage regulation provides an important benchmark with which to study the solution just derived, where regulation is present and binding.

To arrive at the solution for the *no regulation* case, we keep expression (25) and (26), which are a consequence of incentive and participation constraints. Using the resource
constraint (20) to eliminate \(d_t\) from (25) yields:

\[
[(1 + z\mu_t)i_t - a_t - n_t] = \frac{q_t\alpha}{1 + r_d^t} \left( R - \frac{b(\mu_t)}{\Delta\alpha} - \frac{z\mu_t}{q_t\Delta\alpha} \right) i_t. 
\]  

(29)

and, using (26) to solve out for \(a_t\), enables us to characterize the project size as a function of entrepreneurial net worth \(n_t\), as follows:

\[
i_t = \frac{n_t}{1 + z\mu_t - \frac{\alpha z\mu_t}{(1 + r_d^t)\Delta\alpha} - \frac{q_t\alpha}{1 + r_d^t} \left( R - \frac{b(\mu_t)}{\Delta\alpha} - \frac{z\mu_t}{q_t\Delta\alpha} \right)} = \frac{n_t}{A_1(t)}. 
\]  

(30)

where \(1/A_1(t)\) is the leverage an entrepreneur can obtain over his net worth.

To obtain the solution for \(\mu_t\) in this case, rewrite the entrepreneur’s objective function in (14) using (22) and (30):

\[
\max_{\{i_t, R_t^e, R_t^d, R_t^a, a_t, \mu_t\}} q_t\alpha R_t^e i_t = q_t\alpha b(\mu_t) n_t A_1(t), 
\]  

(31)

Since the parameters \(\alpha\) and \(\Delta\alpha\) as well as the aggregate price of capital goods \(q_t\) are taken as given, this problem reduces to choosing \(\mu_t\) to maximize the ratio \(b(\mu_t) / A_1(t)\). The tradeoff is as follows: on the one-hand, higher monitoring intensity leads to decreases in \(b(\mu_t)\), reducing the entrepreneur’s return for a given-size project; on the other, the same increase in \(\mu_t\) allows for higher leverage and thus higher entrepreneurial projects, as \(1/A_1(t)\) increases.

The first-order condition leads to the following:

\[
b'(\mu_t) A_1(t) = A_1'(t)b(\mu_t); 
\]  

(32)

or, denoting the (non-constant) elasticity \(\epsilon_\mu \equiv -b'(\mu_t)\mu_t/b(\mu_t),\)

\[-\epsilon_\mu A_1(t) = A_1'(t)\mu_t.\]  

(33)

Developing (33) and using simple algebra leads to the following condition for monitoring intensity \(\mu_t\) under no regulation (see the Appendix for details):

\[
\frac{\epsilon_\mu}{1 + \epsilon_\mu} = z\mu_t \left[ 1 + r_d^t + \frac{\alpha}{\Delta\alpha} \left( \frac{r_d^a - r_d^d}{1 + r_d^d} \right) \right]. 
\]  

(34)

Our technical assumptions about the schedule \(b(\mu_t)\) ensure that a unique monitoring intensity can be identified using (34)(see Figure 3 for a graphical illustration). 4 Once \(\mu_t\) is

\[\text{The left-hand side of (34) is a concave function of } \mu_t \text{ that is equal to 0 for } \mu_t = 0. \text{ The right-hand side of the expression is a linear function of } \mu_t \text{ with a positive slope. This ensures that a unique value of } \mu_t \text{ exists that solves (34).} \]
identified, $i_t$ and $A_1(t)$ are as in (30), the required participation of the bank $a_t$ is found from (26) and the loanable funds used from outside depositors $d_t$ are from (25).

Expression (34) and Figure 3 can also be used to define intuition about the likely effects of macroeconomics shocks to bank monitoring intensity under the no regulation solution:

- **Result 1(M):** $\frac{\partial \mu_t}{\partial q_t} > 0$. Under the no regulation solution, favorable shocks to the investment sector lead banks to increase their monitoring intensity.

As discussed above, a favorable shock to the demand for capital goods lessens moral hazard in the financial market, because it makes it easier for bank to credibly promise a sufficient return to the depositors providing them loanable funds (see the right-hand-side of (25)). The high price of capital goods and their enhanced ability to attract loanable funds leads banks increase their leverage and the size of projects, which requires more intensive monitoring.

- **Result 2(M):** $\frac{\partial \mu_t}{\partial r_t} < 0$. Under the no regulation solution, a decrease in the cost of loanable funds leads banks to increase their monitoring intensity.

The decrease in the required rate on loanable funds associated with a monetary policy loosening has similar consequences to the rise in $q_t$ analyzed above: it improves the ability of banks to credibly promise a sufficient return to their suppliers of loanable funds, as evidenced by (25). Banks thus react by using more deposits and thus, all things equal, more financing to entrepreneurs and bigger projects, which requires increased monitoring.

- **Result 3(M):** $\frac{\partial \mu_t}{\partial r^{a}_t} < 0$. Under the no regulation solution, a sudden scarcity of bank capital leads banks to decrease their monitoring intensity.

A sudden scarcity of bank capital at the aggregate level manifests itself in an increase in $r^{a}_t$, the market-required rate of return on bank equity (bank capital). Recall that the financial contract requires banks to engage their own capital when financing entrepreneurs, in order for them to have the incentive to monitor entrepreneurs as much as they agreed. The rise in the required return on bank capital increases costs for banks and makes it harder to credibly commit to intense monitoring. As a result, $\mu_t$ decreases.

Table 1 synthesizes the movements in $\mu_t$ expected under both the no regulation and the regulation solution. Notice that the responses of bank monitoring to macroeconomic shocks can be qualitatively different depending on the solution analyzed. Interestingly, bank monitoring intensity responds to “standard” macroeconomic shocks (technology or monetary policy shocks) in similar ways under both types of solution, but in differentiated manner following financial (bank capital) shocks. Our simulations below present evidence on the quantitative strength of these reactions under general equilibrium.
### Table 1: Macroeconomic Shocks and Bank Monitoring

<table>
<thead>
<tr>
<th>Macroeconomic Shock</th>
<th>Effect on monitoring intensity $\mu$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Favorable shock to investment sector (positive technology shock, $q_t \uparrow$)</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>Increased scarcity of bank capital (negative shock in banking sector, $r^a_t \uparrow$)</td>
<td>$\downarrow$ $\uparrow$</td>
</tr>
<tr>
<td>Lower costs of loanable funds (monetary policy loosening, $r^d_t \downarrow$)</td>
<td>$\uparrow$ $\uparrow$</td>
</tr>
<tr>
<td>Tightening of bank regulation (lower leverage allowed $\gamma^g_t \downarrow$)</td>
<td>NA $\uparrow$</td>
</tr>
</tbody>
</table>

#### 2.7 Households

There exists a continuum of households indexed by $i \in (0, \eta^h)$. Households consume, allocate their money holdings between currency and bank deposits, supply units of specialized labor, choose a capital utilization rate, and purchase capital goods.

The wage-setting environment faced by households (described below) implies that hours worked and labor earnings are different across households. We abstract from this heterogeneity by referring to the results in Erceg et al. (2000) who show, in a similar environment, that the existence of state-contingent securities makes households homogeneous with respect to consumption and saving decisions. We assume the existence of these securities and our notation below reflects their equilibrium effect, so that consumption, assets and the capital stock are not contingent on household type $i$.

Lifetime expected utility of household $i$ is

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c^h_t - \gamma c^h_{t-1}, l_{it}, M^c_t / P_t),$$

where $c^h_t$ is consumption in period $t$, $\gamma$ measures the importance of habit formation in consumption, $l_{it}$ is hours worked, and $M^c_t / P_t$ denotes the real value of currency held.
The household begins period $t$ with money holdings $M_t$ and receives a lump-sum money transfer $X_t$ from the monetary authority. These monetary assets are allocated between funds invested at a bank (deposits) $D_t$ and currency held $M^c_t$ so that $M_t + X_t = D_t + M^c_t$. In making this decision, households weigh the tradeoff between the utility obtained from holding currency and the return from bank deposits, the risk-free rate $1 + r^d_t$.5

Households also make a capital utilization decision. Starting with beginning-of-period capital stock $k^h_t$, they can produce capital services $u_t k^h_t$ with $u_t$ the utilization rate. Rental income from capital is thus $r_t u_t k^h_t$, while utilization costs are $v(u_t) k^h_t$, with $v(.)$ a convex function whose calibration is discussed in Section 3 below. Finally, the household receives labor earnings $(W_{it}/P_t) l_t$, as well as dividends $\Pi_t$ from firms producing intermediate goods.

Income from these sources is used to purchase consumption, new capital goods (priced at $q_t$), and money balances carried into the next period $M_{t+1}$, subject to the constraint

$$c^h_t + q_t l^h_t + \frac{M_{t+1}}{P_t} = (1 + r^d_t) \frac{D_t}{P_t} + r_t u_t k^h_t - v(u_t) k^h_t + \frac{W_{it}}{P_t} l_t + \Pi_t + \frac{M^c_t}{P_t},$$

with the associated Lagrangian $\lambda_t$ representing the marginal utility of income. The capital stock evolves according to the standard accumulation equation:

$$k^h_{t+1} = (1 - \delta) k^h_t + i^h_t.$$

The first-order conditions associated with the choice of $c^h_t$, $M^c_t$, $u_t$, $M_{t+1}$, and $k^h_{t+1}$ are, respectively,

$$U_1(t) - \beta \gamma E_t U_1(t+1) = \lambda_t;$$

$$U_3(t) = r^d_t \lambda_t;$$

$$r_t = v'(u_t);$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} (1 + r^d_{t+1}) \left( \frac{P_t}{P_{t+1}} \right) \right\};$$

$$\lambda_t q_t = \beta E_t \left\{ \lambda_{t+1} \left[ q_{t+1} (1 - \delta) + r_{t+1} u_{t+1} - v'(u_{t+1}) \right] \right\},$$

where $U_j(t)$ represents the derivative of the utility function with respect to its $j^{th}$ argument in period $t$.

Wage Setting

5To be consistent with the presence of idiosyncratic risk at the bank level, we follow Carlstrom and Fuerst (1997) and Bernanke et al. (1999) and assume that households deposit money at a large mutual fund, which in turn deposits at a cross-section of banks, diversifying away bank-level risk.

15
We follow Erceg et al. (2000) and Christiano et al. (2005) and assume that each household supplies a specialized labor type \( l_{it} \), while competitive labor aggregators assemble all such types into one composite input using the technology

\[
H_t \equiv \left( \int_0^1 l_{it}^{\xi_w-1} l_{it}^{\xi_w} \, di \right)^{1/\xi_w}, \quad \xi_w > 1.
\]

The demand for each labor type is therefore

\[
l_{it} = \left( \frac{W_{it}}{W_t} \right)^{-\xi_w} H_t, \quad (42)
\]

where \( W_t \) is the aggregate wage (the price of one unit of composite labor input \( H_t \)). As was the case in the final-good sector, labor aggregators are competitive and make zero profits; imposing this result leads to the following determination for the economy-wide aggregate wage:

\[
W_t = \left( \int_0^1 W_{it}^{1-\xi_w} \, di \right)^{1/1-\xi_w}. \quad (43)
\]

Households set wages according to a variant of the mechanism used in the price-setting environment above. Each period, household \( i \) receives the signal to reoptimize its nominal wage with probability \( 1 - \phi_w \), while with probability \( \phi_w \) the household indexes its wage to last period’s price inflation, so that \( W_{i,t} = \pi_{t-1} W_{i,t-1} \). A reoptimizing worker takes into account the evolution of its wage and the demand for its labor (42) during the expected period with no reoptimization. The resulting first-order condition for wage-setting when reoptimizing \( \tilde{W}_{it} \) yields

\[
\tilde{W}_t = P_{t-1}^{-1} \frac{\xi_w}{\xi_w - 1} \frac{E_t \sum_{k=0}^{\infty} (\beta \phi_w)^k U_2(t+k)H_{t+k}^{\xi_w} \pi_{t+k}^{\xi_w}}{E_t \sum_{k=0}^{\infty} (\beta \phi_w)^k \lambda_{t+k} H_{t+k}^{\xi_w} \pi_{t+k}^{\xi_w - 1}},
\]

where \( w_t \equiv W_t/P_t \) is the real aggregate wage and \( U_2(\cdot) \) is the derivative of the utility function with respect to hours worked. Once the household’s wage is set, hours worked \( l_{it} \) is determined by (42).

### 2.8 Entrepreneurs and Bankers

There exists a continuum of risk neutral entrepreneurs and bankers. Each period, a fraction \( 1 - \tau^e \) of entrepreneurs and \( 1 - \tau^b \) of bankers exit the economy at the end of the period’s activities. Exiting agents are replaced by new ones with zero assets.\(^6\) Because of financing constraints, entrepreneurs and bankers have an incentive to delay consumption and accumulate net worth until they no longer need financial markets. Assuming a constant probability of death reduces this accumulation process and ensures that a steady state with operative financing constraints exists.

\(^6\)This follows Bernanke et al. (1999). Because of financing constraints, entrepreneurs and bankers have an incentive to delay consumption and accumulate net worth until they no longer need financial markets. Assuming a constant probability of death reduces this accumulation process and ensures that a steady state with operative financing constraints exists.
Entrepreneurs and bankers solve similar optimization problems: in the first part of each period, they accumulate net worth, which they invest in entrepreneurial projects later in that period. Exiting agents consume accumulated wealth while surviving agents save. These agents differ, however, with regards to their technological endowments: entrepreneurs have access to the technology producing capital goods, while bankers have the capacity to monitor entrepreneurs.

A typical entrepreneur starts period $t$ with holdings $k_{et}$ in capital goods, which are rented to intermediate-good producers. The corresponding rental income, combined with the value of the undepreciated capital and the small wage received from intermediate-good producers, constitute the net worth $n_t$ available to an entrepreneur:

$$n_t = (r_t + q_t(1 - \delta)) k_{et} + w_{et}. \quad (44)$$

Each entrepreneur then undertakes a capital-good producing project and invests all available net worth $n_t$ in the project. As described above, an entrepreneur whose project is successful receives a payment of $R_{et}i_t$ in capital goods; unsuccessful projects have zero return. Both the banker’s return $R_{bt}$ and the entrepreneur’s return $R_{et}$ depend on the choice of monitoring intensity described above. At the end of the period, entrepreneurs associated with successful projects but having received the signal to exit the economy use their returns to buy and consume final (consumption) goods. Successful surviving entrepreneurs save their entire return (retain all their earnings), which becomes their beginning-of-period real assets at the start of the subsequent period, $k_{et+1}$, as follows:

$$k_{et+1} = \begin{cases} R_{et}i_t, & \text{if surviving and successful} \\ 0, & \text{otherwise.} \end{cases} \quad (45)$$

This represents an optimal choice because of risk neutrality and the high return on internal funds, which induces them to postpone consumption. Unsuccessful agents neither consume nor save.

Similarly, a typical banker starts period $t$ with holdings of $k_{bt}$ capital goods (retained earnings from previous periods) and rents capital services to firms producing intermediate goods. We assume that the value of these retained earnings may be affected by a “valuation” shock $\kappa_t$, which breaks the tight link between retained bank earnings at time $t - 1$ and bank assets at time $t$. The occurrence of this shock is meant to represent episodes during which sudden deteriorations in the balance sheets of banks, caused by loan losses and asset writedowns, suddenly reduce bank equity and net worth.\(^7\) Inclusive of the valuation shock, a bank thus receives the income $a_t$

$$a_t = \kappa_t (r_t + q_t(1 - \delta)) k_{bt} + w_{bt}. \quad (46)$$

\(^7\)Similar valuation shocks to the financial position of the banking or entrepreneurial sectors have been proposed and analyzed by Goodfriend and McCallum (2007), Christiano et al. (2008) and Gertler and Karadi (2009).
which defines the bank’s net worth. We assume further that the valuation shock $\kappa_t$ follows the following AR(1) process:

$$\log \kappa_t = \rho \log \kappa_{t-1} + \epsilon_{t}^\kappa,$$

(47)

where $\rho \in (0, 1)$ and $\epsilon_{t}^\kappa$ is i.i.d. with mean 0 and standard deviation $\sigma_\kappa$.

Next, the bank also invests its own net worth $a_t$ in the projects of entrepreneurs (in addition to the funds $d_t$ invested by outside investors depositing at the bank). As above, a bank associated with successful projects but having received the signal to exit the economy consumes final goods, whereas successful surviving banks retain all their earnings to constitute their beginning-of-period real assets at the start of the subsequent period, $k_{t+1}^b$, so we have

$$k_{t+1}^b = \begin{cases} R_{t+1}^b, & \text{if surviving and successful} \\ 0, & \text{otherwise.} \end{cases}$$

(48)

2.9 Government policies

There are two government policies to specify in this economy: the regulation policy, which sets the constraints put on the leverage banks are allowed to have, and monetary policy which sets short-term rates.

Bank Leverage Regulation Policy

Regulation on bank leverage is assumed to follow the following rule:

$$\gamma_t^g = \gamma^g + \omega y_t + \epsilon_t^g,$$

(49)

where $\gamma^g$ is the steady-state leverage ratio allowed, $y_t$ represents output deviations from steady state, and $\epsilon_t^g$ is a shock to regulation that follows the AR(1) process

$$\epsilon_t^g = \rho_g \epsilon_{t-1}^g + \epsilon_{gt},$$

(50)

where $\rho_g \in (0, 1)$ and $\epsilon_{g}$ is i.i.d. with mean 0 and standard deviation $\sigma_g$.

The regulation rule (49) is specified at a general level to accommodate a series of different scenarios about regulation. A Time-invariant regulation, for example, corresponds to $\omega_y = \sigma_y = 0$. By contrast, regulation that loosens standards when economic activity weakens (a counter-cyclical regulation) sets $\omega_y < 0$, whereas a pro-cyclical regulation that tightens standards when economic activity decreases would have $\omega_y > 0$.

Monetary Policy

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The productivity ($z_t$), monetary ($\epsilon_t^{mp}$) and banking ($\epsilon_t^b$) shocks are realized.

Intermediate goods are produced, using capital and labor services; final goods are produced, using intermediates.

Households deposit savings in banks, who use these funds as well as their own net worth to finance entrepreneur projects $i_t$.

Entrepreneurs choose which project to undertake; bankers choose their intensity of monitoring.

Successful projects return $R_i$ units of new capital, shared between the three agents according to terms of financial contract. Failed projects return nothing.

Exiting agents sell their capital for consumption goods, surviving agents buy this capital as part of their consumption-savings decision.

All markets close.

Monetary policy sets $r^d_t$, the short-term nominal interest rate, according to the following rule:

$$r^d_t = (1 - \rho_r)r^d + \rho_r r^d_{t-1} + (1 - \rho_r) \left[ \rho_\pi (\pi_t - \bar{\pi}) + \rho_y \hat{y}_t \right] + \epsilon_t^{mp}, \quad (51)$$

where $r^d$ is the steady-state rate, $\bar{\pi}$ is the monetary authority’s inflation target, $\hat{y}_t$ represents output deviations from steady state, and $\epsilon_t^{mp}$ is an i.i.d monetary policy shock with standard deviation $\sigma^{mp}$.  

Table 2 below illustrates the sequence of events. The value of aggregate shocks are revealed at the beginning of the period. Intermediate goods are then produced, using capital and labor, and then final goods are produced, using the intermediates. Next, the production of capital goods occurs: households deposit funds in banks, who meet with entrepreneurs to arrange financing. Once financed, entrepreneurs choose which project to undertake and banks choose the intensity with which to monitor entrepreneurs, according to the double moral hazard environment described above. Successful projects return new units of capital goods that are distributed to households, banks, and entrepreneurs according to the terms of the financial contract. Exiting banks and entrepreneurs sell their share of capital good in exchange for consumption and households, and surviving banks and entrepreneurs make their consumption-savings decisions.

---

8The targeted rate for $r^d_t$ is achieved with appropriate injections in total money supply $X_t \equiv M_{t+1} - M_t$, where $M_t$ is the total money supply at time $t$. 19
2.10 Aggregation

As a result of the linear specification in the production function for capital goods, in the private benefits accruing to entrepreneurs, and in the monitoring costs facing banks, the distribution of net worth across entrepreneurs and bank capital across banks has no effects on bank’s decisions about their monitoring intensity $\mu_t$. We thus focus on the behavior of the aggregate levels of bank capital and entrepreneurial net worth.

Under the *Regulation Solution*, aggregate investment $I_t$ is given by the sum of individual projects $i_t$ from (21):

$$I_t = \gamma^b_t A_t + N_t,$$

where $A_t$ and $N_t$ denote the aggregate levels of bank capital and entrepreneurial net worth, respectively. $A_t$ and $N_t$ are found by summing (44) and (46) across all agents:

$$A_t = \kappa_t [r_t + q_t(1 - \delta)] K^b_t + \eta^b_t \omega^b_t;$$

$$N_t = [r_t + q_t(1 - \delta)] K^e_t + \eta^e_t \omega^e_t,$$

where $K^b_t$ and $K^e_t$ denote the aggregate wealth of banks and entrepreneurs at the beginning of period $t$. Recalling that $\eta^e$ and $\eta^b$ represent the population masses of entrepreneurs and banks, these are

$$K^b_t = \eta^b k^b_t; \quad K^e_t = \eta^e k^e_t.$$

As described above, banks and entrepreneurs survive to the next period with probability $\tau^b$ and $\tau^e$, respectively; surviving agents save all their wealth because of risk-neutral preferences and the high return on internal funds. Aggregate wealth at the beginning-of-period $t + 1$ is thus

$$K^b_{t+1} = \tau^b \alpha R^b_t I_t;$$

$$K^e_{t+1} = \tau^e \alpha R^e_t I_t.$$

Combining (52)-(56) yields the following laws of motion for $A_t$ and $N_t$:

$$A_{t+1} = \kappa_{t+1} [r_{t+1} + q_{t+1}(1 - \delta)] \tau^b \alpha R^b_t (\gamma^b_t A_t + N_t) + w^b_{t+1} \eta^b;$$

$$N_{t+1} = [r_{t+1} + q_{t+1}(1 - \delta)] \tau^e \alpha R^e_t (\gamma^e_t A_t + N_t) + w^e_{t+1} \eta^e.$$

Equation (57) illustrates the *bank capital channel* that is at play in the model: all things equal, an increase in aggregate investment $I_t$ increases earnings for the banking sector, and through a retained earnings mechanism serves to increase bank capital and thus further increases in lending and investment in the subsequent periods, which themselves increase bank earnings and bank capital, etc. This mechanism helps to propagate the effects of the initial shock several periods into the future. Further, one can see from (57)-(58) that bank capital $A_t$, through its effect on aggregate investment, also affects the evolution of
net worth of entrepreneurs, in an interrelated manner where entrepreneurial net worth \( N_t \) itself has an impact on future levels of bank capital.

Exiting banks and entrepreneurs consume the value of their available wealth. This implies the following for aggregate consumption of entrepreneurs and banks:

\[
C^b_t = (1 - \tau^b)q_t\alpha R^b_t I_t,
\]

\[
C^e_t = (1 - \tau^e)q_t\alpha R^e_t I_t.
\]

Finally, aggregate household consumption and capital holdings are

\[
C^h_t = \eta^h c^h_t; \quad K^h_t = \eta^h k^h_t,
\]

and the economy-wide equivalent to the participation constraint of banks (18) defines the aggregate equilibrium return on bank net worth:

\[
1 + r^a_t = \frac{q_t\alpha R^b_t I_t}{A_t}.
\]

### 2.11 The competitive equilibrium

A competitive equilibrium for the economy consists of (i) decision rules for \( c^h_t, i^h_t, W^h_t, k^h_{t+1}, u_t, M^h_t, D_t, \) and \( M_{t+1} \) that solve the maximization problem of the household, (ii) decision rules for \( \tilde{\beta}_{jt} \) as well as input demands \( k_{jt}, h_{jt}, h^e_{jt}, h^b_{jt} \) that solve the profit maximization problem of firms producing intermediate goods in (12), (iii) decision rules for \( i_t, R^e_t, R^h_t, R^b_t, \) \( a_t \) and \( d_t \) that solve the maximization problem associated with the financial contract, (iv) saving and consumption decision rules for entrepreneurs and banks, and (v) the following market-clearing conditions:

\[
K_t = K^h_t + K^e_t + K^b_t;
\]

\[
u_t K^b_t + K^e_t + K^b_t; = \int_0^1 k_{jt}dj;
\]

\[
H_t = \int_0^1 h_{jt}dj;
\]

\[
Y_t = C^h_t + C^e_t + C^b_t + I_t + \mu_t I_t;
\]

\[
K_{t+1} = (1 - \delta) K_t + \alpha RI_t ;
\]

\[
\eta^b d_t = \eta^h \frac{D_t}{P_t};
\]

\[
\overline{M_t} = \eta^b M_t.
\]

Equation (63) defines the total capital stock as the holdings of households, entrepreneurs and banks. Next, (64) states that total capital services (which depend on the utilization
rate chosen by households) equals total demand by intermediate-good producers. Equation (65) requires that the total supply of the composite labor input produced according to (42) equals total demand by intermediate-good producers. The aggregate resource constraint is in (66) and (67) is the law of motion for aggregate capital. Finally, (68) equates the aggregate demand of deposits by banks to the supply of deposits by households, and (69) requires the total supply of money $M_t$ to be equal to money holdings by households.

3 Calibration

The utility function of households is specified as

$$U(c_t^h - \gamma c_{t-1}^h, l_{i,t}, M_t^c/P_t) = \log(c_t^h - \gamma c_{t-1}^h) - \psi \frac{\mu^h t}{1 + \eta} + \zeta \log(M_t^c/P_t).$$

The weight on leisure $\psi$ is set in order that steady-state work effort by households is equal to 30% of available time. One model period corresponds to a quarter, so the discount factor $\beta$ is set at 0.99. Following results in Christiano et al. (2005), the parameter governing habits, $\gamma$, is fixed at 0.65, $\zeta$ is set in order for the steady state of the model to match the average ratio of $M1$ to $M2$, and $\eta$ is set to 1.

The share of capital in the production function of intermediate-good producers, $\theta^k$, is set to the standard value of 0.36. Recall that we want to reserve a small role in production for the hours worked by entrepreneurs and bankers. To this end, we fix the share of the labor input $\theta^h$ to 0.6399 instead of $1 - 0.36 = 0.64$, and then set $\theta^e = \theta^b = 0.00005$. The parameter governing the extent of fixed costs, $\Theta$, is chosen so that steady-state profits of the monopolists producing intermediate goods are zero. Following Meh and Moran (2010), the persistence of the technology shock, $\rho_z$, and its standard deviation, $\sigma_z$, are set to 0.95 and 0.0015, respectively. Finally, we set $\delta = 0.02$.

Price and wage-setting parameters are set following results in Christiano et al. (2005). The elasticity of substitution between intermediate goods ($\xi_p$) and the elasticity of substitution between labor types ($\xi_w$) are set to 6 and 21, which ensures that the steady-state markups are 20% in the goods market and 5% in the labor market. The probability of not reoptimizing for price setters ($\phi_p$) is 0.60 while for wage setters ($\phi_w$), it is 0.64.

To parameterize households’ capital utilization decision, we first require that $u = 1$ in the steady-state, and set $v(1) = 0$. This makes steady state computations independent of $v(.)$. Next, we set $\sigma_u = v''(u)/v'(u) = 0.25$ for $u = 1$. The monetary policy rule (51) is calibrated by appealing to the Taylor principle, fixing $\rho_\pi = 1.5$ as well as $\rho_r = 0.8$ and $\rho_y = 0$, in light of the evidence that policy makers smooth their actions through time but do not appear to strongly react to output deviations from trend. The trend rate of inflation $\pi$ is 1.005, or 2% on a net, annualized basis.
Table 3: Baseline Parameter Calibration

<p>| Household Preferences and Wage Setting |  |  |  |  |  |  |  |  |  |  |  |  |</p>
<table>
<thead>
<tr>
<th>γ</th>
<th>ζ</th>
<th>ψ</th>
<th>η</th>
<th>β</th>
<th>ξw</th>
<th>φw</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65</td>
<td>0.0018</td>
<td>9.05</td>
<td>1.0</td>
<td>0.99</td>
<td>21</td>
<td>0.64</td>
</tr>
</tbody>
</table>

<p>| Final Good Production |  |  |  |  |  |  |  |  |  |  |  |  |</p>
<table>
<thead>
<tr>
<th>θk</th>
<th>θh</th>
<th>θe</th>
<th>θb</th>
<th>ζ</th>
<th>ξp</th>
<th>φp</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>0.6399</td>
<td>0.00005</td>
<td>0.95</td>
<td>0.00005</td>
<td>6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

<p>| Capital Good Production and Financing |  |  |  |  |  |  |  |  |  |  |  |  |</p>
<table>
<thead>
<tr>
<th>B</th>
<th>B̄</th>
<th>α</th>
<th>R</th>
<th>τe</th>
<th>τb</th>
<th>Δα</th>
<th>χ</th>
<th>εb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1575</td>
<td>0.99</td>
<td>1.05</td>
<td>0.7</td>
<td>0.9</td>
<td>0.3</td>
<td>15.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

The schedule linking bank monitoring intensity $\mu_t$ and moral hazard on the entrepreneurial size $b(\mu_t)$ is specified as follows:

$$b(\mu_t) = B + B̄(1 + \chi\mu_t)^{-\varepsilon_b},$$

where we set $B = 0$, $B̄ = 0.5\Delta\alpha R$, $\chi = 15$, and $\varepsilon_b = 5$.

The parameters that remain to be calibrated are $\alpha$, $R$, $\Delta\alpha$, $\tau^e$, $\tau^b$ and $z$. We set $\alpha$ to 0.99, so that the (quarterly) failure rate of entrepreneurs is 1%, as in Carlstrom and Fuerst (1997). We use $z = 1$ as a benchmark. Further, we set $R = 1.05$ so that the steady-state (relative) price of capital is within a reasonable range. The parameter $\tau^b$ controls the rate of return on bank capital (bank equity) and is set to 0.9. The remaining parameters are $\Delta\alpha = 0.3$ and $\tau^e = 0.95$. Table 3 summarizes the numerical values of the model parameters. A solution to the model’s dynamics is found by linearizing all relevant equations around the steady state using standard methods.

Table 4 presents the steady-state characteristics of three economies, which differ in terms of leverage regulation. The first column of the table relates to an economy in the No Regulation case, where explicit government regulation on leverage is absent. This economy will serve to highlight the impact of regulation on macroeconomic outcomes.

Column 1 of the table indicates that under No Regulation, the leverage ratio of bank lending to bank capital is 10 or, said otherwise, the capital-asset ratio of banks is 10%. This rate is just under the 2002 average for US banks, on a risk-adjusted basis (Meh and Moran, 2010). Columns 2 and 3 illustrate the consequences on the economy’s steady state of imposing tighter regulatory requirements on leverage. Column 2 relates to a case where bank lending cannot be more than 5 times the bank capital base, whereas Column 3 relates
to an economy where regulation is even tighter and leverage cannot be more than 4 times the bank capital base.

Table 4: Characteristics of the Economy’s Steady State

<table>
<thead>
<tr>
<th>Variables</th>
<th>No Regulation</th>
<th>Regulation</th>
<th>Tighter Regulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank Leverage ($\gamma$)</td>
<td>10.0</td>
<td>5.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Capital-Asset Ratio</td>
<td>10.0%</td>
<td>20%</td>
<td>25%</td>
</tr>
<tr>
<td>Monitoring Intensity ($\mu$)</td>
<td>0.029</td>
<td>0.066</td>
<td>0.083</td>
</tr>
<tr>
<td>Entrepreneurs’ Moral hazard ($b(\mu)$)</td>
<td>0.025</td>
<td>0.0051</td>
<td>0.0028</td>
</tr>
<tr>
<td>Price of capital ($q$)</td>
<td>1.036</td>
<td>1.0618</td>
<td>1.080</td>
</tr>
<tr>
<td>GDP ($Y$)</td>
<td>0.800</td>
<td>0.7882</td>
<td>0.7807</td>
</tr>
<tr>
<td>Investment-Output Ratio ($I/Y$)</td>
<td>0.2221</td>
<td>0.2167</td>
<td>0.2130</td>
</tr>
<tr>
<td>Capital-Output Ratio ($K/Y$)</td>
<td>11.54</td>
<td>11.264</td>
<td>11.07</td>
</tr>
</tbody>
</table>

Table 4 shows that government regulation imposing lower ratios of bank leverage leads banks to monitor more intensively: $\mu_t$ is more than twice as high under regulation (Column 2) than it is in the absence of regulation (Column 1). As a corollary, moral hazard on the entrepreneurial ($b(\mu_t)$) is much smaller under regulation. Imposing even tighter regulation (Column 3) markedly extends this gap between the No Regulation and the Regulation cases. Tight regulation affects the ability of banks to lend and thus the capacity of entrepreneurs dependent on this lending to produce capital goods. The relative price of capital goods $q$ is thus higher under regulation, reflecting the more difficult intermediation process, and the investment-output and capital-output ratios are both lower. Finally, GDP itself falls as leverage tightens. The economic importance of these effects is significant: GDP, for example, is 1.5% lower under the first leverage rule than in the absence of regulation, a decline that reaches 2.4% under the second leverage rule.

4 Experiments

This section presents our impulse response experiments, which illustrate how the economy responds to various shocks. Throughout, we contrast the response of the economy with time-invariant regulation to sets of economic responses to shocks: one drawn from an environment where bank regulation is absent (the No Regulation economy) and one in which bank regulation is counter-cyclical, tightening when economic activity accelerates, and vice-versa. We show that there are important differences between the three sets of responses.

In addition, the section presents the results of an experiment where, starting from a
steady state with regulated leverage equal to 5 (as illustrated by Column 2 of Table 4), regulated leverage gradually declines towards a tighter regime where leverage can only be up to 4 times bank capital. The terminal steady-state is thus the one represented by Column 3 of Table 4. The experiment allows us to compute the transition paths for economic variables towards this new steady state with tighter regulation.

4.1 Impulse Responses to Shocks

Technology shocks

Figure 4 presents the effects of a one-standard deviation, positive technology shock on two economies. The first economy features the Time-invariant Regulation (i.e. $\omega_y = 0$) solution and its responses to the shock are in solid lines. The second economy is one where leverage regulation is absent: its responses to the shock are labeled No Regulation and are in dashed lines.

In both economies, the favorable technology shock induces a persistent increase in the productivity of the intermediate-good production function, which raises the expected return from physical capital in future periods. This translates into a positive shift in the demand for capital goods and puts upwards pressure on $q_t$, the relative price of these goods. However, Figure 4 shows that the two economies react differently to this shock: while output and investment increase sharply under the No Regulation solution, they experience more subdued but more persistent rises under Time-invariant Regulation. Moreover, the reaction of asset prices $q_t$, as well as those of inflation and interest rates are different across the two sets of responses, being more volatile under the regulatory solution. To explain why these differences occur, we start by analyzing the benchmark case where regulation plays no role in shaping bank behavior.

No Regulation

In the No Regulation economy, the favorable technology shock has important supply-side effects on the production of capital goods. To see this, recall expression (25) arising from the financial contract. Expressed with economy-wide variables, it reads:

$$D_t = \frac{q_t \alpha}{1 + r_t^d} R_t^d I_t = \frac{q_t \alpha}{1 + r_t^d} \left( R - \frac{b(\mu_t)}{\Delta \alpha} - \frac{z \mu_t}{q_t \Delta \alpha} \right) I_t. \quad (70)$$

As discussed above, the equation states that the reliance on outside funds $D_t$ in the financing of a given-size investment project is limited by the double moral hazard problem: banks need to credibly promise a sufficient return to outside investors, a promise that is
harder to make when moral hazard is more severe. The upward pressure on $q_t$ attenuates moral hazard, because it increases the value of the share of project return $R^h_t$ set aside for outside investors. It is now therefore possible for banks to attract more loanable funds for a given monitoring intensity: said otherwise, bank leverage will tend to rise, all things equal. In addition, the favorable macroeconomic shock leads banks to increase slightly their monitoring intensity, which alleviates moral hazard further and convinces outside investors to contribute more to investment projects. These two effects combine to induce a marked increase in bank leverage, which rises by 0.5% on impact. The favorable technology shock thus leads to easier financing conditions for entrepreneurs, acting as a favorable shift in the supply curve for capital goods. Bank lending and aggregate investment increase on impact, while the supply shift limits the increase in the price of capital.

The initial jump in aggregate investment increases earnings for banks and entrepreneurs. The modest rise in bank monitoring intensity contributes to this uptick in bank earnings because increased monitoring induces a rise in the share of project return set aside for banks. From that point on, the bank capital channel propagates the shock’s effects in subsequent periods: higher bank earnings translate into a higher stock of bank capital in the next period, which sets the stage for second-round, positive effects on bank lending and investment, as higher bank capital further facilitates the ability of banks to attract loanable funds and fund projects (recall the law of motion for bank capital in (57)). This propagation effect persists for several periods and as a result, investment rises gradually, peaking 7 periods after the onset of the shock at a maximum increase around 3.5%. Bank capital also experiences persistent increases, peaking at around 3.2% over its steady-state value, 8 periods after the onset of the shock.

**Time-invariant Regulation**

The responses to the shock are different in the *Time-invariant Regulation* economy (full lines). Under such regulation, banks must maintain an unchanged leverage ratio throughout the episode, even though the upward pressure on $q_t$ has created favorable conditions for financing entrepreneurs. Where banks to keep their monitoring intensity unchanged, leverage would rise over the regulated level. Banks are thus compelled by the regulation to increase their capital participation in investment projects, to adjust, and this severs to limit the rise in leverage. The expanded involvement of banks in financing entrepreneurs manifests itself in a sharp increase in their monitoring intensity, whose rise is more pronounced than in the economy where regulation was absent.

This combination, more pronounced monitoring and higher involvement of banks in financing, significantly modifies the macroeconomic impact of the technology shock. First, since bank capital is comprised of retained bank earnings from past periods, its ability
to change immediately after the shock is limited. In such circumstances, requiring more bank capital per unit of lending can only mean that the response of bank lending itself is limited. As a result, the immediate response of bank lending to the shock is indeed more subdued, relative to its reaction in the absence of regulation. A more subdued increase in bank lending also limits the increase in aggregate investment, and thus the propagating effects of the bank capital channel described above are thwarted. Moreover, the marked increase in monitoring decreases moral hazard on the entrepreneurial side significantly, to the point where the retained earnings of the entrepreneurial sector (entrepreneurial net worth) slightly decrease in the immediate aftermath of the shock, which serve to further mitigate the shock’s propagation in subsequent periods. Having constrained the expansion in bank lending, regulation thus prevents the favorable supply shift in the production of capital goods described above, and therefore their price \( q_t \) increases more than absent regulation.

Over the course of the subsequent periods, the expansion of banks’ involvement in financing firms and the increase in their monitoring intensity means that bank earnings and thus bank capital rise. By that time, however, the favorable macroeconomic conditions initiated by the technology shock have begun to subside. As a result, the rises in bank lending, investment, and output never reach the peaks attained under the \textit{No Regulation} case.

\textit{Counter-cyclical Regulation}

Several recent policy discussions on banking have centered around the time-series properties of regulatory leverage. Counter-cyclical regulatory, in which allowed leverage decreases as economic activity accelerates but increases when it weakens, has often been figured prominently in these discussions. To assess the likely effect of such a policy, Figure 5 contrasts the impact of the favorable technology shock on the \textit{Time-Invariant} economy analyzed above and its impact in an economy with Counter-cyclical regulation. The counter-cyclical aspect of policy is set using the regulation rule (49) with \( \omega_y = -5.0 \), so that allowed leverage declines by 0.05 for each percentage increase in GDP.\(^9\)

The presence of a counter-cyclical regulation on leverage exacerbates the effects described above when analyzing the time-invariant regulation. While banks would like to take advantage of the favorable technology shock to attract more outside funds and increase leverage, the regulation policy now requires instead that leverage decrease to accompany the rise in output. Complying with this regulation, leverage falls on impact and remains low throughout the episode. This is achieved by banks considerably increasing their cap-

\(^9\)Similar results are obtained when the counter-cyclical component of the regulation policy is specified with respect to bank lending rather than GDP.
ital involvement in given-sized projects, relative to what would occur in the absence of regulation. Reduced leverage means that less loanable funds are assembled by banks, which curtails their ability to arrange financing for entrepreneurs and strongly limits the rise in aggregate investment that follows the shock. Further, the favorable shift to the supply of capital goods is now severely limited, which means that $q_t$ increases more than previously.

The increased involvement of banks in financing entrepreneurs is associated with a sharp increase in their monitoring intensity. Increased involvement and higher monitoring play a similar dampening role on bank lending and aggregate investment as analyzed above under time-invariant regulation. The effect is now stronger however, because of the counter-cyclical nature of regulation. As a result, bank lending can only weakly increase following the technological shock.

Overall therefore, favorable technology shocks are associated, all things equal, with reduced moral hazard and easier access to outside funds. However, time-invariant (and at a stronger degree counter-cyclical) regulation prevents banks to tap these outside funds. Instead, it requires that bank keep an important capital position in lending. But since bank capital is comprised of retained bank earnings, it can only increase gradually after a shock: regulation thus acts as a break on the (efficient) rise in leverage and the economic expansion that would otherwise occur following the shock. Evidently, this effect would play out inversely following adverse technology shocks: regulation dampens technology shocks, be they favorable or adverse to the economy.

**Monetary Policy Shocks**

A monetary policy easing decreases the short term nominal interest rate $r^d_t$. This action lowers the opportunity cost of outside funds for banks and all things equal, will play a similar role as the favorable technology shock analyzed above: it will make it easier for banks to pay a sufficient return to outside investors for their loanable funds, thus moderating moral hazard and facilitating entrepreneurial financing. Regulation that prevents banks from taking advantage of this favorable macroeconomic environment to increase their leverage will therefore be expected to dampen the economic expansion that would otherwise follow the monetary policy easing. Figures 6 and 7 verify the quantitative importance of this effect, by contrasting again the impact of the shock under time-invariant regulation to two alternative economies, one where regulation is absent and the other where it is counter-cyclical. First, Figure 6 reports the impulse responses following the monetary policy easing for the *No Regulation* (dashed lines) and the *Time-invariant Regulation* (solid lines) economies.

*No Regulation*
In the No Regulation economy, the decrease in the cost of funds, by making it considerably easier to meet the participation constraint for outside investors (19), alleviates moral hazard and thus allows banks to increase their leverage. This effect is quite important and leverage increases by 4% on impact. A very modest increase of banks’ monitoring efforts also contributes to increase leverage. The strong rise in leverage allows banks to expand lending markedly and as a result aggregate investment also expands importantly.

In subsequent periods, the sharp initial rise in investment having increased earnings for both the banking and the entrepreneurial sectors, and retained earnings having increased net worth levels for these two sectors, the bank capital channel continues to propagate the effects of the shocks forward.

**Time-invariant Regulation**

As was the case following the technology shock, the responses to the monetary policy easing are different under Time-invariant Regulation (full lines). Banks must again maintain an unchanged leverage ratio throughout the episode, even though favorable conditions for financing entrepreneurs are present once more, this time arising from the decrease in \( r^d \). With unchanged monitoring intensity, bank leverage would rise way over the regulated level, as evidenced from the solution in the absence of regulation. Banks are thus compelled to increase their capital participation in investment projects to adjust, which stops the rise in leverage from happening but also strongly curtails the rises in bank lending and aggregate investment that are possible. A sharp increase in bank monitoring intensity is again associated with the shock, much over the modest increase noticed absent regulation.

The more limited rises in bank lending and aggregate investment prevent much of the bank capital channel’s propagating effects to materialize. In addition, and as was the case following the technology shock, the marked increase in monitoring has lowered moral hazard on the entrepreneurial side significantly so that entrepreneurial net worth decrease in the immediate aftermath of the shock, further limiting propagation of the shock in subsequent periods. The sharp increase in banks’ monitoring intensity does over time contribute to a rise in bank earnings and thus bank capital, but the monetary policy easing is short-lived, and thus the favorable macroeconomic conditions for financing entrepreneurs soon unwind and as a result, bank lending, investment, and output never reach the peaks attained under the No Regulation case.

**Counter-cyclical Regulation**

Figure 7 now contrasts the impact of the policy easing in the Time-Invariant economy with that arising under counter-cyclical regulation. As was the case for the technology
shock, counter-cyclical regulation exacerbates the effects described above: while banks
would like to take advantage of the favorable macroeconomic environment to attract more
outside funds and increase leverage, regulation now requires instead that leverage decrease
to accompany the rise in output. Complying with this regulation, leverage falls on impact
and remains low throughout the episode. This is achieved by banks increasing their
capital involvement in given-sized projects, relative to what they do under time-invariant
regulation and way above their behavior in the absence of regulation. Lower leverage
means that less loanable funds attracted by banks, reduced ability to arrange financing
for entrepreneurs and thus only a modest rise in aggregate investment.

As was the case following technology shocks, a monetary policy easing thus tends to
reduce moral hazard in financial markets and eases bank’s access to outside loanable funds.
However, time-invariant (and at a stronger degree counter-cyclical) regulation policies
prevent banks to tap these outside funds, requiring instead that banks keep an important
capital position in lending. Because bank capital is comprised of retained bank earnings,
it can only increase gradually after a shock and regulation limits the rises in leverage and
the economic expansion that would otherwise occur following the policy easing.

### Shocks to Bank Capital

We now consider the effects of shocks that lead to sudden declines in bank capital.
As described above, we study ‘valuation’ shocks which deteriorate the value of retained
earnings and thus declines in the capital position of banks. Figure 8 and 9 depict the
effects of such a negative shock to bank capital: Figure 8 contrasts the responses of the
*Time-invariant Regulation* economy (in solid lines) and its *No Regulation* counterpart
(dashed lines) to this shock, whereas Figure 10 allows to compare the responses under
time-invariant regulation to those arrived in the presence of counter-cyclical regulation.
The size of the shock has been chosen to set the initial decrease in bank capital at around
5%, a magnitude that appears in line with recent evidence on the likely effects of financial
distress episodes.\(^\text{10}\)

#### No Regulation

The sudden deterioration in aggregate bank capital signals it has now become much
scarcer. Figure 8 shows that in the *No Regulation* economy, banks react to this sudden
scarcity by relying relatively more on outside funds to finance a given-size project and
thus leverage ratios increase markedly. However, this important increase in leverage is

\(^{10}\)Following other authors analyzing shocks to net worth of the entrepreneurial or banking sectors (Good-
friend and McCallum, 2007; Christiano et al., 2008; Gertler and Karadi, 2009) we assume bank capital
shocks have moderate to high serial correlation. We thus set the autocorrelation \(\rho_\kappa\) to 0.9.
not sufficient to compensate for the reduced availability of bank capital and thus bank lending to entrepreneurs declines on impact and so does aggregate investment, which falls by around 1%. The reduced availability of bank capital also means that banks strive to economize on it while lending. As a result, they reduce their monitoring intensity, which falls by around 1% as well on impact.

The fall in aggregate investment, compounded by the decrease in monitoring intensity, imply that bank earnings drop markedly, which induces a subsequent drop in bank capital in subsequent periods. Through the bank capital channel, these reduced levels of bank capital then contribute to extend the negative effects of the shock, as they lead to further decreases in monitoring intensity and aggregate investment.

*Time-invariant Regulation*

The full lines of Figure 8 illustrate the responses of the Regulation Economy to the negative shock to bank capital. Leverage must then remain fixed throughout the episode, although as discussed above bank would like to economize on it and increase their leverage. Figure 8 shows that in this situation, the impact of regulation on leverage is to exacerbate the economic downturn created by the shock, relative to what occurred in the absence of regulation.

This results because banks, prevented from reducing their participation in the financing of a given-size project (increasing their leverage) must instead continue investing their own capital in bank lending. This continued involvement is associated with an important increase in monitoring intensity. Of course, without the ability to increase leverage, and at a time when bank capital has suffered a significant decline at the aggregate level, bank lending must go down importantly to adjust, and does so in a more pronounced manner than it does in the absence of regulation. In subsequent periods, the more depressed levels of investment have lead to more reduced bank and entrepreneurial earnings and thus bank capital and entrepreneurial net worth, the consequences of which being that bank lending decreases further.

*Counter-cyclical Regulation*

Figure 9 shows that following shocks to bank capital, the counter-cyclical regulation solution mimics, to some extent, the behavior of the economy in the absence of regulation: since output has decreased, it allows bank to increase their leverage, which enables them to alleviate somewhat the sudden scarcity of bank capital. As a result, bank lending does not decrease as much as it does under time-invariant regulation, and the economic downturn is not as pronounced. Counter-cyclical regulation therefore shields the economy from the worst of the negative effects following bank capital shocks.
4.2 Transition towards a tighter regulatory regime

Figure 10 presents the economy’s transition from the steady-state characterized by the regulation ratio of 5 to the steady-state corresponding to the lower regulated leverage ($\gamma^g = 4$) (See Table 4). The timing is as follows: an economy at rest at the steady-state with high leverage receives the news, at time $t$, that regulated, allowed leverage will gradually decrease from 5 to 4 over the course of the next 8 quarters, starting at time $t = 1$. Figure 10 presents the deterministic paths through which this economy will adjust to this new regulatory regime.

Although gradually incorporated, the regulatory tightening nevertheless starts affecting the economy right away. Reducing allowed leverage means that banks must increase their capital involvement in given-sized investment projects. But bank capital is largely fixed initially and therefore the tighter regulatory regime must be accommodated with a decrease in bank lending. Banks’ monitoring intensity also increases.

5 Conclusion

Many recent policy discussions within central banks and regulatory bodies have examined bank leverage regulation and possible changes to the structure of such regulation. Careful analysis of these issues will be greatly aided by quantitative macroeconomic models in which bank leverage plays a pivotal role in the transmission or the origination of shocks affecting economic dynamics.

This paper presents such a macroeconomic model and uses it to study the impact of different configurations of bank leverage regulation. The model emphasizes the bank capital channel of Meh and Moran (2010), in which bank capital, accumulated through banks’ retained earnings, mitigates moral hazard between banks and theirs suppliers of loanable funds and affects the ability of banks to lend and support economic activity.

Absent regulation, bank leverage (bank lending over bank capital) fluctuates as the macroeconomic environment changes, to accommodate the economy’s requirements for lending with the natural inertia in bank capital. Regulation that limits or directs movements in leverage can thus importantly affect the propagating impact of bank capital. We show that regulation can dampen the fluctuations associated with technology and monetary policy shocks, but augment those linked to bank capital shocks. In addition, regulation on leverage also has important quantitative impacts on the monitoring intensity of banks, because in the model this intensity is tied to the capital involvement of banks. These results point to a strong interconnection between monetary policy and bank regulation policy, suggesting that close cooperation between policy makers is likely to be fruitful.
References


Figure 1. Bank Monitoring and Entrepreneurs’ Private Benefits

\[ \text{Bank Monitoring Intensity} (\mu_t) \]

\[ \text{Resulting Private Benefits: } b(\mu_t) \]

\[ \Delta \alpha R \]

\[ B^{\max} \]
Figure 2. Choice of monitoring intensity $\mu_t$ under the Regulation Solution
Figure 3. Choice of monitoring intensity $\mu_t$ under no Regulation 
(Equation (34))

\[
\mu \cdot \left[ 1 + r_t^d + \left( \frac{\alpha^g}{\Delta \alpha} \right) \left( 1 - \frac{R^d}{R^a} \right) \right] / \left[ q \alpha^g R - R^d \right]
\]
Figure 4. Responses to a Positive Technology Shock

Time-invariant Regulation versus No Regulation
Figure 5. Responses to a Positive Technology Shock

*Time-invariant versus Counter-cyclical Regulation*

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**Output**

**Investment**

**Price of Capital**

**Leverage Ratio**

**Bank Capital**

**Bank Lending**

**Entrepreneurial Net Worth**

**Short Term Nominal Rate**

**Inflation**

**Borrowing Costs**

**Lending Spread**

**Monitoring Intensity**

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Figure 6. Responses to a Monetary Policy Easing

*Time-invariant Regulation versus No Regulation*
Figure 7. Responses to a Monetary Policy Easing

Time-invariant versus Counter-cyclical Regulation
Figure 8. Responses to a Negative Shock to Bank Capital

*Time-invariant Regulation versus No Regulation*
Figure 9. Responses to a Negative Shock to Bank Capital

Time-invariant versus Counter-cyclical Regulation

- Output
- Investment
- Price of Capital
- Leverage Ratio
- Bank Capital
- Bank Lending
- Entrepreneurial Net Worth
- Short Term Nominal Rate
- Inflation
- Borrowing Costs
- Lending Spread
- Monitoring Intensity

**Time-invariant Regulation**  
**Counter-cyclical Regulation**
Figure 10. Transition towards a Tighter Regulatory Regime
A Algebra for the market-based solution for $\mu_t$

Combining the first-order condition (33) with the definition of leverage $A_1(t)$ yields:

\[-\epsilon \mu \left[ 1 + z \mu_t - \frac{\alpha z \mu}{(1 + r_d^t) \Delta \alpha} - \frac{q_t \alpha}{1 + r_d^t} \left( R - \frac{b'(\mu_t)}{\Delta \alpha} - \frac{z \mu}{q_t \Delta \alpha} \right) \right] = \mu_t \left[ z - \frac{\alpha z}{(1 + r_d^t) \Delta \alpha} + \frac{q_t \alpha}{1 + r_d^t} \left( \frac{b'(\mu_t)}{\Delta \alpha} + \frac{z}{q_t \Delta \alpha} \right) \right], \tag{71}\]

Expanding and simplifying, this expression becomes:

\[-\epsilon \mu - \epsilon \mu z \mu_t \left[ 1 + \frac{\alpha}{\Delta \alpha} \left( \frac{1}{(1 + r_d^t)} - \frac{1}{(1 + r_d^t)} \right) \right] + \frac{\epsilon \mu q_t \alpha R}{(1 + r_d^t)} = \frac{\epsilon \mu q_t \alpha b'(\mu_t)}{\Delta \alpha (1 + r_d^t)}, \tag{72}\]

Notice that the last terms on both the left-hand and right-hand sides of the expression cancel out. Next, combining terms in $\mu_t$, the expression becomes:

\[\epsilon \mu \left( \frac{q_t \alpha R}{(1 + r_d^t)} - 1 \right) = z \mu_t (1 + \epsilon \mu) \left[ 1 + \frac{\alpha}{\Delta \alpha} \left( \frac{1}{(1 + r_d^t)} - \frac{1}{(1 + r_d^t)} \right) \right], \tag{73}\]

or

\[\frac{\epsilon \mu}{1 + \epsilon \mu} = z \mu_t \frac{1 + r_d^t + \frac{\alpha}{\Delta \alpha} \left( \frac{r_d^t - r_d^t}{1 + r_d^t} \right)}{q_t \alpha R - (1 + r_d^t)}, \tag{74}\]

as in the text.