

# Growth, Employment Uncertainty and Phillips Curve Tradeoffs

Mewael F. Tesfaselassie\* and Dennis Snower†

Preliminary Version  
June 2010

## Abstract

The paper studies the effects of trend growth and employment uncertainty on the long-run Phillips curve. It shows that employment uncertainty leads to a more favorable long-run tradeoff between inflation and output—steady state output increases with trend inflation for low to moderate rates of inflation. It also shows that trend growth may lead to a more favorable long-run tradeoff between inflation and output if consumption and work are complementary in household utility.

JEL Classification:

Keywords: trend growth, employment uncertainty, nominal and real rigidities, long-run Phillips curve.

## 1 Introduction

The evidence concerning the long-run Phillips curve, which relates steady-state output to trend inflation, is mixed and heavily colored by theoretical preconceptions. Mankiw (2001) wrote that “if one does not approach the data with a prior favoring long-run neutrality, one would not leave the data with that posterior. The data’s

---

\*Corresponding author: Kiel Institute for the World Economy, Hindenburgufer 66, 24105 Kiel, Germany. E-mail: mewael.tesfaselassie@ifw-kiel.de, Tel: +49 431 8814 273.

†Kiel Institute for the World Economy, Hindenburgufer 66, 24105 Kiel, Germany. E-mail: dennis.snower@ifw-kiel.de

best guess is that monetary shocks leave permanent scars on the economy.” Still, it is well known that in New Keynesian models with nominal rigidities, the long-run Phillips curve is downward sloping, except for inflation rates near zero. That is, higher trend inflation is associated with lower steady-state output (see e.g., King and Watson (1996), Ascari (2004), Graham and Snower (2004) and Yun (2005)). The explanation for this result is that a high enough positive trend inflation increases the average markup in the economy, reducing output demand and in turn output. Furthermore, under price (wage) staggering high enough trend inflation exacerbates the distortionary effect of price (wage) dispersion on output.

This paper reexamines the nature of the long-run Phillips curve by incorporating two important features into the New Keynesian model, namely, trend growth and employment uncertainty. It then shows that, owing to these two features, the long-run Phillips curve is upward sloping for trend inflation significantly larger than zero (in particular for empirically relevant inflation rates).

Our analysis shares the following conceptual building blocks: (i) general equilibrium analysis with microfoundations based on intertemporal optimization, (ii) rational expectations, (iii) no money illusion, (iv) imperfect competition, and (v) temporary nominal rigidities. However, the introduction of employment uncertainty (a real rigidity) in the analysis of the long-run Phillips curve is novel. Employment uncertainty is shown to have an important effect on the wage setting decision of households—the higher is employment uncertainty the less likely it becomes that households will receive the currently set wages that are to accrue in the future. Thus, when setting wages, households attach less weight to future remuneration when there is employment uncertainty. If one proxies employment uncertainty by the average quarterly unemployment rate, which in the euroarea has exceeded 10 percent for much of the past two decade. The introduction of trend growth is similar to Amano (2009) but our model allows for potential non-separable utility in household preferences, in line with some recent empirical evidence on the consumption Euler equation, in which parameter restrictions are imposed consistent with balanced growth facts (see e.g., Basu and Kimball (2002) and Kiley (2007)).

In incorporating trend growth into a New Keynesian model, we use insights from the literature on balanced growth and business cycles (e.g., King, Plosser and Rebelo (1988a,b)) as well as the literature on long-run labor supply and consumption Euler equation (e.g., Basu and Kimball (2002), Kiley (2007)) to make sure that the business cycle model has implications that are consistent with balanced growth facts.

We provide an intuitive explanation for the non-monotonic nature of the long-run Phillips curve by identifying several channels that imping on it, some positive and some negative. The model has both price and wage staggering, and while employment uncertainty affects wage setting, trend growth affects both wage and price

setting.

A positive tradeoff between inflation and output rests on the following effects:

1. When setting wages, workers discount the future more heavily due to employment uncertainty.
2. When setting prices, firms discount the future more heavily due to declining marginal utility from consumption on account of growth.

A negative tradeoff between inflation and output arises on the following effects:

1. The greater the rate of inflation, the greater is the instability of prices and wages (i.e. relative wages and prices are more variable at higher rates of inflation).
2. But the instability of wages is mitigated through the complementarity between consumption and employment.

In short, the slope of the Phillips curve depends on the relative strengths of the positive and negative effects. The negative effects depend on the magnitude of inflation (i.e. the higher the rate of inflation, the larger are the negative effects). But the positive effects do not depend on the magnitude of inflation. It follows that, at low inflation rates, the positive effects dominate, whereas at higher inflation rates the negative effects dominate. We would like to remark that the nature of the long-run Phillips curve is an empirical matter, and its investigation clearly lies beyond the scope of this theoretical paper. All we do here is to derive a Phillips curve in the presence of trend growth and employment uncertainty and calibrate the results. In particular, Section 2 constructs a basic model with trend growth but without employment uncertainty. Then Section 4 extends this model to allow for employment uncertainty and Section 5 checks the robustness of the main results when partial wage and price indexation is allowed. Finally, Section 6 gives concluding summary.

## 2 Model with growth

The model features Calvo nominal wage and price rigidities, where in any give period a fraction of households (firms) can not reset their wages (prices) optimally. In order to keep the model as simple as possible, we assume labor to be the only input in the production function. Furthermore, productivity follows a deterministic trend. The model has the property that, in the presence of productivity growth and in a

balanced growth path aggregate output, consumption, real wages grow at the same rate as productivity, while employment is constant.

## 2.1 Households

We start by specifying household utility, defined over consumption  $C_t$  and hours worked  $N_t$ , under balanced growth. As is shown by King, Plosser and Rebelo (1988a,b), for a business cycle model to be consistent with balanced steady state growth (and constancy of the great-ratios)<sup>1</sup>, household utility must be of the form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} V(N_t)}{(1-\sigma)}; \sigma > 0 \quad (1)$$

The utility function may be potentially non-separable in both arguments.<sup>2</sup> Consistent with available empirical estimates (see for e.g., Basu and Kimball (2002), Kiley (2007) and Guerron-Quintana (2008)) and recent theoretical analyses (e.g., Monacelli and Perotti Monacelli and Perotti (2009) and Bilbiie (2009)) we assume  $U_{C,N} > 0$ .<sup>3</sup> In order to ensure concavity of  $V(N)$ ,  $v'(N) > 0$  is required. The specific functional form of  $V(N_t)$  is such that the elasticity of intertemporal substitution in consumption (holding  $N$  constant) should be independent of the level of consumption, consistent with balanced growth. As in Basu and Kimball (2002) and Kiley (2007) we assume  $V(N_t) = e^{-(1-\sigma)v(N_t)}$  so that the elasticity of intertemporal substitution in consumption is given by  $1/\sigma$ . Moreover, without loss of generality we let  $v(N_t) = N_t^{1+\eta}/(1+\eta)$ , where  $\eta > 0$ .

As is well known in the business cycle literature, non-separable utility makes the model intractable under wage staggering, as consumption would depend on the entire history of wages (complicating aggregation of consumption decisions (see e.g., Erceg, Henderson and Levin (2001))). In order to avoid this problem, we follow Schmidt-Grohe and Uribe (2005) and assume a large representative household with a continuum of members, each supplying a differentiated labor service. The household cares about per capita consumption  $C_t = \int_0^1 C_{j,t} dj$  and per capita hours worked  $N_t = \int_0^1 N_{j,t} dj$  (where  $j$  indexes household members) and sets wages for each of its members.

---

<sup>1</sup>See for e.g., Solow (1956).

<sup>2</sup>A limiting case of the utility function is when  $\sigma = 1$ , so that  $u(C_t, N_t) = \log C_t - V(N_t)$ .

<sup>3</sup>Note that  $U_{C,N} > 0$  does not necessarily imply that consumption and work are complementary (i.e., consumption and leisure are substitutes), in sense that the demand for  $C$  increases if real wages (the price of leisure) increases (see e.g., Bilbiie (2009)).

The household consumes a continuum of differentiated goods, indexed by  $k$ , which are transformed into a Dixit-Stiglitz composite good  $C_t$  as follows

$$C_t = \left( \int_0^1 C_{k,t}^{1/\mu_p} dk \right)^{\mu_p} \quad (2)$$

where  $\mu_p = \frac{\theta_p}{\theta_p - 1}$  and  $\theta_p$  is the elasticity of substitution between any two differentiated goods. We first solve for the household's consumption allocation across all goods for a given level of  $C_t$ . Minimizing total expenditure  $\int_0^1 P_{k,t} C_{k,t} dk$  subject to (2) gives the demand for each good  $k$

$$C_{k,t} = \left( \frac{P_{k,t}}{P_t} \right)^{-\theta_p} C_t \quad (3)$$

where  $P_t$  is the aggregate price index (or the price level), which is defined as

$$P_t = \left( \int_0^1 P_{k,t}^{1-\theta_p} dk \right)^{\frac{1}{1-\theta_p}} \quad (4)$$

Next we derive the optimal decisions regarding the paths of  $C_t$ ,  $N_t$  and  $W_t^j$ . The household maximizes

$$E_t \sum \beta^i U(C_{t+i}, N_{t+i}) \quad (5)$$

subject to the budget constraint

$$C_{t+i} + \frac{B_{t+i}}{P_{t+i}} = \int_0^1 \frac{W_{t+i}^j}{P_{t+i}} N_{t+i}^j dj + (1 + I_{t-1+i}) \frac{B_{t-1+i}}{P_{t+i}} + \frac{D_{t+i}}{P_{t+i}} \quad (6)$$

and the resource constraint

$$N_{t+i} = \int_0^1 N_{t+i}^j dj = N_{t+i}^d \int_0^1 \left( \frac{W_{t+i}^j}{W_{t+i}} \right)^{-\theta_w} dj \quad (7)$$

Here,  $\beta$  is the discount factor,  $I_t$  is the nominal interest rate on bond holdings  $B_t$ ,  $N_t^j$  is the number of hours worked of labor type  $j$ ,  $W_t^j$  is the nominal wage of labor type  $j$ ,  $W_t$  is the aggregate wage level,  $D_t$  is the nominal profit income from all firms and  $N_{t+i}^d$  is the aggregate labor demand, details of which are given below when solving each firms' labor demand.

Let  $\lambda^c$  and  $\lambda^n$  be, respectively, the lagrange multipliers associated with the budget constraint (6) and the resource constraint (7). Then, the maximization problem in lagrangian form is

$$\begin{aligned} \ell &= E_t \sum \beta^i \left\{ U(C_{t+i}, N_{t+i}) + \lambda_{t+i}^n \left[ N_{t+i} - N_{t+i}^d \int_0^1 \left( \frac{W_{t+i}^j}{W_{t+i}} \right)^{-\theta_w} dj \right] \right. \\ &+ \lambda_{t+i}^c \left[ C_{t+i} + \frac{B_{t+i}}{P_{t+i}} - N_{t+i}^d \int_0^1 \frac{W_{t+i}^j}{P_{t+i}} \left( \frac{W_{t+i}^j}{W_{t+i}} \right)^{-\theta_w} dj \right. \\ &\left. \left. - (1 + I_{t-1+i}) \frac{B_{t-1+i}}{P_{t+i}} - \frac{D_{t+i}}{P_{t+i}} \right] \right\} \end{aligned}$$

and the first-order conditions with respect to  $C_{t+i}$  and  $N_{t+i}$  are, respectively,  $U_{C,t+i} = -\lambda_{t+i}^c$  and  $U_{N,t+i} = -\lambda_{t+i}^n$ ; moreover,  $W_{t+i}^j = W_{t+i}^*$  if set optimally and  $W_{t+i}^j = W_{t+i-1}^j$  otherwise. For now we abstract from considerations of nominal wage indexation (see Section 5).

The assumption of Calvo wage staggering implies that in any period only a fraction  $1 - \omega_w$  of labor markets are allowed to reset wages, so that the household's lagrangian that is relevant for resetting wages is

$$\ell_W = E_t \sum (\beta \omega_w)^i \left\{ \lambda_{t+i}^n \left[ -N_{t+i}^d \left( \frac{W_t^j}{W_{t+i}} \right)^{-\theta_w} \right] + \lambda_{t+i}^c \left[ -N_{t+i}^d \frac{W_t^j}{P_{t+i}} \left( \frac{W_t^j}{W_{t+i}} \right)^{-\theta_w} \right] \right\} \quad (8)$$

As all labor types whose wages are reset face an identical optimization problem, all set an identical wage, denoted by  $W_t^*$ . The first-order condition with respect to  $W_t^*$  gives

$$E_t \sum (\beta \omega_w)^i N_{t+i|t} \left( \frac{W_t^*}{P_{t+i}} U_{C,t+i} + \mu_w U_{N,t+i} \right) = 0 \quad (9)$$

where we made use of the first-order conditions for  $C_{t+i}$  and  $N_{t+i}$  to substitute out the lagrangian multipliers. The parameter  $\mu_w = \frac{\theta_w}{\theta_w - 1}$  represents the wage markup while

$$N_{t+i|t} = \left( \frac{W_t^*}{W_{t+i}} \right)^{-\theta_w} N_{t+i}^d \quad (10)$$

is the demand for labor in period  $t + i$  whose wages was last reset in period  $t$ .

The first order condition (9) shows that trend growth in consumption affects wage decisions via the marginal rate of substitution between consumption and work.

The functional form of the utility function implies that,  $U_{C,t+i} = C_{t+i}^{-\sigma}V(N_{t+i})$  and  $U_{N,t+i} = -C_{t+i}^{1-\sigma}N_{t+i}^\eta V(N_{t+i})$ . As long as  $\sigma > 1$  there are two countervailing effects of trend growth. On the one hand the marginal utility of consumption declines faster with trend consumption growth pushing newly set wages up. On the other hand, due to complementarity in the utility function consumption growth decreases the marginal disutility of work (i.e., work becomes less costly) and this moderates wage increases.

Solving (9) for  $W_t^*$ , making use of (10) and dividing through by  $W_t$  we get

$$\frac{W_t^*}{W_t} = \mu_W \frac{E_t \sum (\beta \omega_w)^i C_{t+i}^{1-\sigma} V(N_{t+i}) N_{t+i}^\eta N_{t+i}^d \left(\frac{W_{t+i}}{W_t}\right)^{\theta_w}}{\frac{W_t}{P_t} E_t \sum (\beta \omega_w)^i C_{t+i}^{-\sigma} V(N_{t+i}) N_{t+i}^d \left(\frac{W_{t+i}}{W_t}\right)^{\theta_w} \frac{P_t}{P_{t+i}}} \quad (11)$$

where  $W_t^*/W_t$  is the relative wage.

We incorporate trend growth via deterministic growth in labor productivity  $A_t$ , namely,  $A_{t+i} = \gamma A_{t+i-1} = \gamma^i A_t$ , where  $\gamma > 1$  is one plus the growth rate of productivity. Then, along the balanced growth path we have  $C_{t+i} = \gamma C_{t+i-1}$ . Under trend growth, equation (11) can be rewritten as

$$\frac{W_t^*}{W_t} = \mu_W \frac{E_t \sum (\beta \omega_w)^i (\gamma^i)^{1-\sigma} c_{t+i}^{1-\sigma} V(N_{t+i}) N_{t+i}^\eta N_{t+i}^d \left(\frac{W_{t+i}}{W_t}\right)^{\theta_w}}{\frac{W_t}{A_t P_t} E_t \sum (\beta \omega_w)^i (\gamma^i)^{1-\sigma} c_{t+i}^{-\sigma} V(N_{t+i}) N_{t+i}^d \left(\frac{W_{t+i}}{W_t}\right)^{\theta_w} \frac{A_t P_t}{A_{t+i} P_{t+i}}} \quad (12)$$

where  $c_{t+i} = C_{t+i}/A_{t+i}$  is a stationary variable. Note that the term  $\beta \omega_w \gamma^{1-\sigma}$  can be thought of as the effective discount factor, which decreases with  $\gamma$  provided  $\sigma > 1$ .

## 2.2 Firms

Firms' pricing decisions in the goods market are analogous to the wage setting decisions of households in the labor market. There is a continuum of monopolistically competitive firms over the unit interval. Let firm  $k$  has a production function of the form

$$Y_{k,t} = A_t N_{k,t} \quad (13)$$

where  $A_t$  is productivity and  $N_{k,t}$  is labor services.  $N_{k,t}$  is a composite made of a continuum of differentiated labor services

$$N_{k,t} = \left( \int_0^1 N_{k,t}^{j \frac{1}{\mu_w}} dj \right)^{\mu_w} \quad (14)$$

where  $\mu_w \equiv \frac{\theta_w}{\theta_w - 1}$ . Minimizing firm  $k$ 's total wage bill  $\int_0^1 W_t^j N_{k,t}^j dj$  subject to (36) leads to the demand for labor of type  $j$

$$N_{k,t}^j = \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} N_{k,t} \quad (15)$$

where  $\theta_w$  is the elasticity of substitution between any two different labor types. Aggregate labor demand is then given by  $N_t^d = \int_0^1 N_{k,t} dk$ .

$$W_t = \left( \int_0^1 W_t^{j(1-\theta_w)} dj \right)^{\frac{1}{1-\theta_w}} \quad (16)$$

While firms choose prices optimally, output is demand determined, which in turn pins down labor demand.

Firm  $k$  maximizes its profit

$$E_t \sum \omega_p^i Q_{t,t+i} \left( \frac{P_{k,t}}{P_{t+i}} C_{kt+i} - \phi_{t+i} C_{kt+i} \right) \quad (17)$$

where  $Q_{t,t+i} = \beta^i \left[ \frac{C_{t+i}^{-\sigma} V(N_{t+i})}{C_t^{-\sigma} V(N_t)} \right]$  is the stochastic discount factor and  $\phi_t = \phi_{k,t} = \frac{W_t}{A_t P_t}$  is the real marginal cost. Note here that the stochastic discount factor depends negatively on trend consumption growth.

Using the demand function for good  $k$  in the profit function

$$E_t \sum \omega_p^i Q_{t,t+i} C_{t+i} \left[ \left( \frac{P_{k,t}}{P_{t+i}} \right)^{1-\theta_p} - \phi_{t+i} \left( \frac{P_{k,t}}{P_{t+i}} \right)^{-\theta_p} \right] \quad (18)$$

The first order condition is

$$\frac{P_t^*}{P_t} = \mu_p \frac{E_t \sum (\beta \omega_p)^i \left( \frac{C_{t+i}}{C_t} \right)^{1-\sigma} V(N_{t+i}) \phi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta_p}}{E_t \sum (\beta \omega_p)^i \left( \frac{C_{t+i}}{C_t} \right)^{1-\sigma} V(N_{t+i}) \left( \frac{P_{t+i}}{P_t} \right)^{\theta_p - 1}} \quad (19)$$

where  $P_t^*/P_t$  is the optimal relative price, identical for all optimizing firms. De-trending consumption analogous to the households' wage setting problem, equation (19) can be rewritten as

$$\frac{P_t^*}{P_t} = \mu_p \frac{E_t \sum (\beta \omega_p)^i (\gamma^i)^{1-\sigma} c_{t+i}^{1-\sigma} V(N_{t+i}) \phi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta_p}}{E_t \sum (\beta \omega_p)^i (\gamma^i)^{1-\sigma} c_{t+i}^{1-\sigma} V(N_{t+i}) \left( \frac{P_{t+i}}{P_t} \right)^{\theta_p - 1}} \quad (20)$$

Similar to the wage setting problem, as long as  $\sigma > 1$ , consumption growth affects pricing decisions. However, unlike the wage setting equation, higher trend growth unequivocally reduces the stochastic discount factor, pushing newly set price down.

### 2.3 Aggregation and market clearing

The aggregate wage index can be rewritten as a weighted average of optimized and non-optimized wages

$$W_t = \left( (1 - \omega_w) W_t^{*(1-\theta_w)} + \omega_w W_{t-1}^{(1-\theta_w)} \right)^{1/(1-\theta_w)} \quad (21)$$

Similarly, the aggregate price index can be rewritten as a weighted average of optimized and non-optimized prices

$$P_t = \left[ (1 - \omega_p) (P_t^*)^{1-\theta_p} + \omega_p P_{t-1}^{1-\theta_p} \right]^{\frac{1}{1-\theta_p}} \quad (22)$$

Moreover, using the aggregate resource constraint and imposing goods and labor market clearing, we get a relationship between aggregate employment  $N_t$  and aggregate output  $Y_t$ . To see this

$$N_t = \int_0^1 N_t^j dj = \int_0^1 \left( \int_0^1 N_{k,t}^j dj \right) dk = \Delta_{w,t} \Delta_{p,t} y_t \quad (23)$$

where  $\Delta_{w,t}$  denotes wage dispersion

$$\Delta_{w,t} = \int_0^1 \left( \frac{W_{jt}}{W_t} \right)^{-\theta_w} dj \quad (24)$$

and  $\Delta_{p,t}$  denotes price dispersion

$$\Delta_{p,t} = \int_0^1 \left( \frac{P_{k,t}}{P_t} \right)^{-\theta_p} dk \quad (25)$$

and  $y_t = Y_t/A_t$ .

Using backward recursion, the wage and price dispersion equations can be rewritten in dynamic forms (see e.g., Schmidt-Grohe and Uribe (2005))

$$\Delta_{w,t} = (1 - \omega_w) \left( \frac{W_t^*}{W_t} \right)^{-\theta_w} + \omega_w \left( \frac{W_{t-1}}{W_t} \right)^{-\theta_w} \Delta_{w,t-1} \quad (26)$$

$$\Delta_{p,t} = (1 - \omega_p) \left( \frac{P_t^*}{P_t} \right)^{-\theta_p} + \omega_p \left( \frac{P_{t-1}}{P_t} \right)^{-\theta_p} \Delta_{p,t-1} \quad (27)$$

### 3 The long run tradeoff between output and inflation

Having derived the key aggregate equations, we are now in a position to analyze the long run tradeoff between steady state output and trend inflation. Let  $\Pi_w$  and  $\Pi_p$  denote, respectively, trend wage inflation and trend price inflation. In a balanced growth path, real wages grow at the same rate as productivity (i.e.,  $\gamma > 1$ ) implying  $\Pi_w = \Pi_p \gamma$ . Note that a given value of  $\gamma$  ties down only the ratio  $\frac{\Pi_w}{\Pi_p}$ . Thus the model with productivity growth is consistent with continued price inflation, so that nominal wages rise faster than productivity, or with a constant price level, so that nominal wages rise pari passu with productivity.<sup>4</sup>

Let  $z_{w,t} = W_t^*/W_t$  and  $z_{p,t} = P_t^*/P_t$ . In steady state the relative wage setting equation (noting that  $\Pi_w = \Pi_p \gamma$ ) is

$$z_w = \mu_w \left( \frac{cN^\eta}{\phi} \right) \frac{1 - \beta\omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w - 1}}{1 - \beta\omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w}} \quad (28)$$

and the relative price setting equation is

$$z_p = \mu_p \phi \frac{1 - \beta\omega_p \gamma^{1-\sigma} \Pi_p^{\theta_p - 1}}{1 - \beta\omega_p \gamma^{1-\sigma} \Pi_p^{\theta_p}} \quad (29)$$

The following points recap how productivity growth affects wage and price decision making:

1. wage setting equation (12): there are three effects—1) the marginal utility of consumption declines faster with trend consumption growth, pushing newly set wages up. 2) due to complementarity in the utility function consumption growth decreases the marginal disutility of work (i.e., work becomes less costly) and this pushes newly set wages down. 3) trend wage inflation increases with productivity growth (given trend price inflation), exacerbating the negative effect of wage dispersion on output. The net effect depends on the relative magnitudes of  $\sigma$  and  $\theta_w$ .
2. price setting equation (20): the stochastic discount factor is a function of the marginal utility of consumption; future profits are discounted more heavily the higher is trend consumption growth.

---

<sup>4</sup>The model is also consistent with constant nominal wages and price deflation equal to productivity growth.

In steady state, the goods market clearing condition is  $c = y$  and the aggregate resource constraint implies  $N = \int_0^1 N_j dj = \int_0^1 \int_0^1 N_{k,j}^d dk dj = \Delta_w \Delta_p y$ . Using both equation to substitute for  $c$  and  $N$  leads to

$$z_w = \mu_w \frac{(\Delta_w \Delta_p)^\eta y^{1+\eta} (1 - \beta \omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w - 1})}{\phi (1 - \beta \omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w})} \quad (30)$$

Solving for  $y$  in terms of  $\Pi_p$  we get

$$y = \left( \frac{\mu_w^{-1} (\Delta_w \Delta_p)^{-\eta} \phi z_w (1 - \beta \omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w})}{1 - \beta \omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w - 1}} \right)^{\frac{1}{1+\eta}} \quad (31)$$

where  $\Delta_w, \Delta_p, \phi$  and  $z_w$  are all functions of  $\Pi_p$ . First  $z_w$  can be inferred from the aggregate wage index so that

$$z_w = z_w(\Pi_p, \gamma, \omega_w, \theta_w) = \left( \frac{1 - \omega_w (\Pi_p \gamma)^{\theta_w - 1}}{1 - \omega_w} \right)^{1/(1-\theta_w)} \quad (32)$$

which in turn determines the steady state wage dispersion,  $\Delta_w$

$$\Delta_w = \frac{(1 - \omega_w) z_w^{-\theta_w}}{1 - \omega_w (\Pi_p \gamma)^{\theta_w}} \quad (33)$$

Likewise from the aggregate price index  $z_p$  is given by

$$z_p = z_p(\Pi_p, \omega_p, \theta_p) = \left( \frac{1 - \omega_p \Pi_p^{\theta_p - 1}}{1 - \omega_p} \right)^{1/(1-\theta_p)} \quad (34)$$

which in turn determines the steady state price dispersion,  $\Delta_p$

$$\Delta_p = \frac{(1 - \omega_p) z_p^{-\theta_p}}{1 - \omega_p \Pi_p^{\theta_p}} \quad (35)$$

Finally the steady state optimal price setting and the price index pin down the steady state real marginal cost  $\phi$ . Using the above substitutions we may interpret the relationship between  $y$  and  $\Pi_p$  as the *long-run Phillips curve*: As steady-state inflation varies, so does aggregate steady-state output. Importantly, the nature of the curve depends on the magnitude of productivity growth,  $\gamma$ .

While we have given intuitions for the various channels whereby trend growth affect aggregate relationships, one difficulty is that the long run Phillips curve is highly non-linear and is thus analytically intractable. Therefore, following the literature (see e.g., Ascari (2004), Graham and Snower (2004) and Amano (2009)), we study its properties numerically. We calibrate the model using values that are quite standard in the business cycle literature (see e.g., Ascari (2004), Graham and Snower (2004), Schmidt-Grohe and Uribe (2005) and Amano (2009)).

**Parameter configuration**

Parameter		Calibrated values
$\beta$	quarterly discount factor	0.99
$\omega_p$	fraction of firms not resetting prices	0.75
$\omega_w$	fraction of workers not resetting wages	0.8
$\theta_p$	elast. of subst. in goods	10
$\theta_w$	elast. of subst. in labor services	5
$\gamma$	productivity growth	{1, 1.005, 1.01}
$\sigma$	degree of non-separability	{2, 4, 6}
$\eta$	disutility from work	{0.2, 0.6, 1}

We vary the three key parameters of the model: (1) productivity growth,  $\gamma \in \{1, 1.005, 1.01\}$ , which implies annual growth rates of 0%, 2%, 4%, respectively; (2) the degree of nonseparability in utility  $\sigma \in \{2, 4, 6\}$  (note that  $\sigma$  as high than 6 is consistent with recent empirical estimates, e.g., Guerron-Quintana (2008)) and (3) the curvature of the disutility of work  $V(N)$ ,  $\eta \in \{0.2, 0.6, 1\}$ .<sup>5</sup>

In order to help compare our results with those from previous studies (e.g., Ascari (2004)) in the figures that follow we plot the percentage deviation of steady state output from its level at zero steady state price inflation, denoted by  $\tilde{y}$ , against trend price inflation  $\Pi_p$  expressed in quarterly terms.

Figure 1 shows the relation between  $\tilde{y}$  and  $\Pi_p$  for alternative values of trend productivity growth  $\gamma$ . The case of constant productivity  $\gamma = 1$  is shown by the dotted line, and it is clear that here steady state output and trend price inflation are negatively related even at very low inflation rates. In general, the figure shows that the steady state effect of productivity maybe positive or negative depending on the degree of nonseparability between consumption and work in the utility function (captured by the parameter  $\sigma$ ) and the curvature of the disutility function from work (captured by the parameter  $\eta$ ). Evidently, the positive effect of trend growth on the slope of the Phillips curve dominates the negative effect when the degree of nonseparability

<sup>5</sup>Estimates of  $\eta$  based on nonseparable utility may differ from those based on separable utility. [reference would be helpful here]

is large enough and the (constant-consumption) marginal disutility of work is small enough (e.g., bottom-right panel of figure 1).

[Figure 1 Here]

## 4 Extensions: employment uncertainty

We incorporate employment uncertainty analogous to the nominal wage and price staggering. In particular, suppose in any given period, only a fraction  $\rho$  of all households are allowed to work (provide labor services) and of those who are allowed to work, only a fraction  $1 - \omega_w$  can set their wages optimally. Thus, in any given period, a household's member is in one of three states: (1) unemployed with probability  $1 - \rho$ , (2) employed and non-optimizing with probability  $\rho\omega_w$ , (3) working and optimizing with probability  $\rho(1 - \omega_w)$ . If a household member  $j$  who reenters the labor market after being unemployed for at least one period is not allowed to set a new wage then  $W_t^j = W_{t-1}$ .

Let  $\Omega \subset [0, 1]$  be the set of employed people randomly chosen every period from the entire population. Total number of hours worked is  $N_{t+i} = \int_{\Omega} N_{t+i}^j dj$ . For simplicity let the employed people lie in the interval  $[0, \rho]$ ,  $\rho < 1$  measures the fraction of employed workers. Note that the composition of employed workers changes from one period to another.

Here,  $N_{k,t}$  is a composite made of a continuum of differentiated labor services of size  $\rho$ .

$$N_{k,t} = \rho^{1-\mu_w} \left( \int_0^{\rho} N_{k,t}^{j \frac{1}{\mu_w}} dj \right)^{\mu_w} \quad (36)$$

where  $\rho^{1-\mu_w}$  is a normalization term, included so as to shutdown the effect of the number of varieties on aggregate behavior (what is called the “love of varieties” see e.g. Benassy (1996) and Bergin and Corsetti (2007)). The normalized aggregator has the property that in the event of a common wage  $W^*$ , the aggregate wage index is equal to the common wage.

As in the case with no employment uncertainty firm  $k$  minimizes total wage bill  $\int_0^{\rho} W_t^j N_{k,t}^j dj$  subject to (36). Since all firms face identical wage profiles and run identical production function, we drop the indexing variable  $k$ . The demand for labor of type  $j$  is

$$N_t^j = \frac{1}{\rho} \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} N_t \quad (37)$$

The aggregate wage index

$$W_t = \left( \int_0^\rho \frac{1}{\rho} W_t^{j(1-\theta_w)} dj \right)^{\frac{1}{1-\theta_w}} \quad (38)$$

Note here that from the set of workers employed in period  $t$  a fraction  $1 - \rho$  were not employed in period  $t - 1$ . While a fraction  $(1 - \omega_w)$  of these newly employed workers will set a new wage, a fraction  $\omega_w$  are assumed to receive  $W_{t-1}$ . Under this assumption, the aggregate wage index can be rewritten in terms of optimized and non-optimized wages

$$\begin{aligned} W_t^{1-\theta_w} &= \int_0^\rho \frac{1}{\rho} W_t^{j(1-\theta_w)} dj \\ &= (1 - \omega_w) W_t^{*(1-\theta_w)} + \rho \omega_w \int_0^\rho \frac{1}{\rho} W_{t-1}^{j(1-\theta_w)} dj \\ &\quad + (1 - \rho) \omega_w W_{t-1}^{(1-\theta_w)} \\ &= (1 - \omega_w) W_t^{*(1-\theta_w)} + \rho \omega_w W_{t-1}^{(1-\theta_w)} + (1 - \rho) \omega_w W_{t-1}^{(1-\theta_w)} \\ &= (1 - \omega_w) W_t^{*(1-\theta_w)} + \omega_w W_{t-1}^{(1-\theta_w)} \end{aligned} \quad (39)$$

which takes the same form as in the case with no employment uncertainty.

The aggregate resource constraint for labor can be rewritten as

$$N_t = \int_0^\rho N_t^j dj = N_t^d \Delta_{w,t} \quad (40)$$

where  $\Delta_{w,t}$ , which denotes wage dispersion, is given by

$$\begin{aligned} \Delta_{w,t} &= \int_0^\rho \frac{1}{\rho} \left( \frac{W_{jt}}{W_t} \right)^{-\theta_w} dj \\ &= (1 - \omega_w) \left( \frac{W_t^*}{W_t} \right)^{-\theta_w} + \rho \omega_w \left( \frac{W_{t-1}}{W_t} \right)^{-\theta_w} \Delta_{w,t-1} \\ &\quad + (1 - \rho) \omega_w \left( \frac{W_{t-1}}{W_t} \right)^{-\theta_w} \\ &= (1 - \omega_w) z_{w,t}^{-\theta_w} + \rho \omega_w \Pi_{w,t}^{\theta_w} \Delta_{w,t-1} + (1 - \rho) \omega_w \Pi_{w,t}^{\theta_w} \end{aligned} \quad (41)$$

Here in deriving the dynamics of  $\Delta_{w,t}$  using backward recursion, one has to take into account that the composition of workers changes every period

The household's utility maximization problem will be subject to

$$N_{t+i} = \int_0^\rho N_{t+i}^j dj \quad (42)$$

and

$$C_{t+i} + \frac{B_{t+i}}{P_{t+i}} = \int_0^\rho \frac{W_{t+i}^j}{P_{t+i}} N_{t+i}^j dj + (1 + I_{t-1+i}) \frac{B_{t-1+i}}{P_{t+i}} + \frac{D_{t+i}}{P_{t+i}} \quad (43)$$

Here  $N_{t+i}^j$  is total hours of of employment in market  $j$ .

When setting  $W_t^*$  the household will take into account the fact that  $W_t^*$  will not be effective in those future periods when the worker is unemployed, leading to further discounting of future payoffs. With this modification the first-order condition (9) becomes

$$E_t \sum (\beta \rho \omega)^i N_{t+i|t} \left( \frac{W_t^*}{P_{t+i}} U_{C,t+i} + \mu_w U_{N,t+i} \right) = 0 \quad (44)$$

where  $\beta \rho \omega < \beta \omega$ . The upshot is that under employment uncertainty the optimal reset wage depends less on expectations about future payoffs. The optimal relative wage is

$$\frac{W_t^*}{W_t} = \mu_w \frac{E_t \sum (\beta \rho \omega_w)^i \left( \frac{A_{t+i}}{A_t} \right)^{1-\sigma} c_{t+i}^{1-\sigma} V(N_{t+i}) N_{t+i}^\eta N_{t+i}^d \left( \frac{W_{t+i}}{W_t} \right)^{\theta_w}}{\frac{W_t}{A_t P_t} E_t \sum (\beta \rho \omega_w)^i \left( \frac{A_{t+i}}{A_t} \right)^{-\sigma} c_{t+i}^{-\sigma} V(N_{t+i}) N_{t+i}^d \left( \frac{W_{t+i}}{W_t} \right)^{\theta_w} \frac{P_t}{P_{t+i}}} \quad (45)$$

#### 4.1 Long-run tradeoff under employment uncertainty

Following the same steps as in the case with no employment uncertainty, steady state output is given by

$$y = \left( \frac{\mu_w^{-1} (\Delta_w \Delta_p)^{-\eta} \phi z_w \left( 1 - \beta \rho \omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w} \right)}{1 - \beta \rho \omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w - 1}} \right)^{\frac{1}{1+\eta}} \quad (46)$$

where  $\Delta_p, \phi$  and  $z_w$  are the same as in previous section while  $\Delta_w$  is now

$$\Delta_w = \frac{(1 - \omega_w) z_w^{-\theta_w} + (1 - \rho) \omega_w (\Pi_p \gamma)^{\theta_w}}{1 - \rho \omega_w (\Pi_p \gamma)^{\theta_w}} \quad (47)$$

We think the degree of employment uncertainty  $\rho$  can be proxied by the long run average rate of unemployment so as to capture the fraction of the labor force which is unemployed in any given quarter independent of the business cycle. The long run average rate of unemployment differs across countries (see e.g., Blanchard and Gali (2010)). We show results for  $\rho \in \{1, 0.9\}$  where  $\rho = 1$  refers to the case with no employment uncertainty.

[Figure 2 Here]

Figure 2 shows the individual effects of employment uncertainty and trend productivity, as well as the joint effects. All panels show that employment uncertainty leads to more favorable tradeoff, independent of the degree of nonseparability in utility.

## 4.2 Complementarity between growth and employment uncertainty

We examine whether the effects of trend growth and unemployment are complementary, in the sense that given trend inflation, the combined effects of growth and employment uncertainty on steady state output is larger than the *sum* of the individual effects. Figure 3 summarizes the results.

[Figure 3 Here]

All panels of Figure 3 show that there is complementarity between trend growth and unemployment.

## 5 Sensitivity to partial indexation

Let  $0 \leq \varphi_w \leq 1$  and  $0 \leq \varphi_p \leq 1$  denote the degree of indexation to wage inflation and price inflation respectively. Then it can be shown that the steady state output is given by

$$y = \left( \frac{\mu_w^{-1} (\Delta_w \Delta_p)^{-\eta} \phi z_w \left( 1 - \beta \rho \omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{\theta_w (1-\varphi_w)} \right)}{(1 - \beta \rho \omega_w \gamma^{1-\sigma} (\gamma \Pi_p)^{(\theta_w - 1)(1-\varphi_w)})} \right)^{\frac{1}{1+\eta}} \quad (48)$$

where the steady state aggregate wage index takes the form

$$z_w = \left( \frac{1 - \omega_w (\Pi_p \gamma)^{(\theta_w - 1)(1-\varphi_w)}}{1 - \omega_w} \right)^{1/(1-\theta_w)} \quad (49)$$

and the steady state wage dispersion is given by

$$\Delta_w = \frac{(1 - \omega_w)z_w^{-\theta_w} + (1 - \rho)\omega_w(\Pi_p\gamma)^{\theta_w(1-\varphi_w)}}{1 - \rho\omega_w(\Pi_p\gamma)^{\theta_w(1-\varphi_w)}} \quad (50)$$

Furthermore, the steady state optimal relative price, which as before determines  $\phi$ , is given by

$$z_p = \mu_p\phi \frac{1 - \beta\omega_p\gamma^{1-\sigma}\Pi_p^{(\theta_p-1)(1-\varphi_p)}}{1 - \beta\omega_p\gamma^{1-\sigma}\Pi_p^{\theta_p(1-\varphi_p)}} \quad (51)$$

while the aggregate price index  $z_p$  is given by

$$z_p = \left( \frac{1 - \omega_p\Pi_p^{(\theta_p-1)(1-\varphi_p)}}{1 - \omega_p} \right)^{1/(1-\theta_p)} \quad (52)$$

which in turn determines the steady state price dispersion

$$\Delta_p = \frac{(1 - \omega_p)z_p^{-\theta_p}}{1 - \omega_p\Pi_p^{\theta_p(1-\varphi_p)}} \quad (53)$$

Lacking firm empirical estimate for the degree of indexation, we calibrate partial price and wage indexation as  $\varphi_w = \varphi_p = 0.5$ . Comparing Figure 5 with Figure 2, it is clear that partial indexation improves the tradeoff between output and inflation (in particular, in the case with unemployment  $\rho = 0.9$  and productivity growth  $\gamma = 1.01$ ,  $\tilde{y}$  remains positive for  $\Pi_p$  as large as 6 %, annualized).

As shown in Figure 4 and Figure 5, introducing partial indexation reinforces the positive effect of productivity and employment uncertainty.

[Figure 4 and Figure 5 Here]

On the other hand, under partial indexation there are cases where there is no clear result of complementarity between growth and employment uncertainty, especially for low to moderate rates of trend inflation (see Figure 6, lower-right panels).

[Figure 6 Here]

## 6 Summary

This paper reexamines the nature of the long-run Phillips curve by incorporating two important (and empirically relevant) features into an otherwise standard New Keynesian model, namely, trend growth and employment uncertainty. It then shows that, owing to these mechanisms, the long-run Phillips curve is upward sloping even for trend inflation that is significantly larger than zero. Previous research on the matter seems to conclude that the long-run Phillips curve is downward sloping except for near zero trend inflation. Our results shows that such a conclusion is not warranted.

Moreover, we show that the effects of trend growth and employment uncertainty are complementary, in the sense that given trend inflation, the combined effects of growth and employment uncertainty on steady state output is larger than the *sum* of the individual effects. Finally, our main results are strengthened by the presence of nominal indexation schemes.

## References

- Amano, R. and Moran, K. and Murchison, M. and Rennison, A. (2009), Trend Inflation, Wage and Price Rigidities, and Productivity Growth, *Journal of Monetary Economics*, 56, 353–364.
- Ascari, G. (2004), Staggered Prices and Trend Inflation: Some Nuisances, *Review of Economic Dynamics*, 7, 642–667.
- Basu, S. and Kimball, M. (2002), Long-Run Labor Supply and the Elasticity of Intertemporal Substitution for Consumption, *Unpublished Manuscript*.
- Benassy, J.-P. (1996), Taste for Variety and Optimum Production Patterns in Monopolistic Competition, *Economics Letters*, 52, 41–47.
- Bergin, P. and Corsetti, G. (2007), The Extensive Margin and Monetary Policy, *Journal of Monetary Economics*, 55, 1222–1237.
- Bilbiie, F. (2009), Non-Separable Preferences and Frisch Labor Supply: One Solution to a Fiscal Policy Puzzle, *CEPR Discussion Papers*, 7484.
- Blanchard, O. and Gali, J. (2010), Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment, *American Economic Journal: Macroeconomics*, 2, 1–30.
- Erceg, C. and Henderson, D. and Levin, A. (2001), Optimal Monetary Policy with Staggered Wage and Price Contracts, *Journal of Monetary Economics*, 46, 281–313.
- Graham, L. and Snower, D. (2004), The Real Effects of Money Growth in Dynamic General Equilibrium, *ECB Working Paper Series*, 412.
- Guerron-Quintana, P. (2008), Refinements on Macroeconomic Modeling: The Role of Non-separability and Heterogeneous Labor Supply, *Journal of Economic Dynamics and Control*, 32, 3613–3630.
- Kiley, L. (2007), Habit Persistence, Non-separability Between Consumption and Leisure, or Rule-of-Thumb Consumers: Which Accounts for the Predictability of Consumption Growth?, *Finance and Economics Discussion Series*, Federal Reserve Board.
- King, R. and Plosser, C. and Rebelo, S. (1988a), Production Growth and Business Cycles: I The Basic Neoclassical Model, *Journal of Monetary Economics*, 21, 195–232.

- King, R. and Plosser, C. and Rebelo, S. (1988b), Production Growth and Business Cycles: II New Directions, *Journal of Monetary Economics*, 21, 309–341.
- King, R. and Watson, A. (1996), Inflation Targeting in a St. Louis Model of the 21st Century, *Federal Reserve of St. Louis Review*, 78, 83–107.
- Mankiw, G. (2001), The Inexorable and Mysterious Tradeoff between Inflation and Unemployment, *The Economic Journal*, 111, 45–61.
- Monacelli, T. and Perotti, R. (2005), Fiscal Policy, Wealth Effects, and Markups, *NBER Working Papers*, 14584.
- Schmidt-Grohe, S. and Uribe, M. (2005), Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model, *NBER Working Papers*, 11854.
- Solow, R. (1956), A Contribution to The Theory of Economic Growth, *Quarterly Journal of Economics*, 70, 65–94.
- Yun, T. (2005), Optimal Monetary Policy with Relative Price Distortion, *American Economic Review*, 70, 65–94.

## Figures

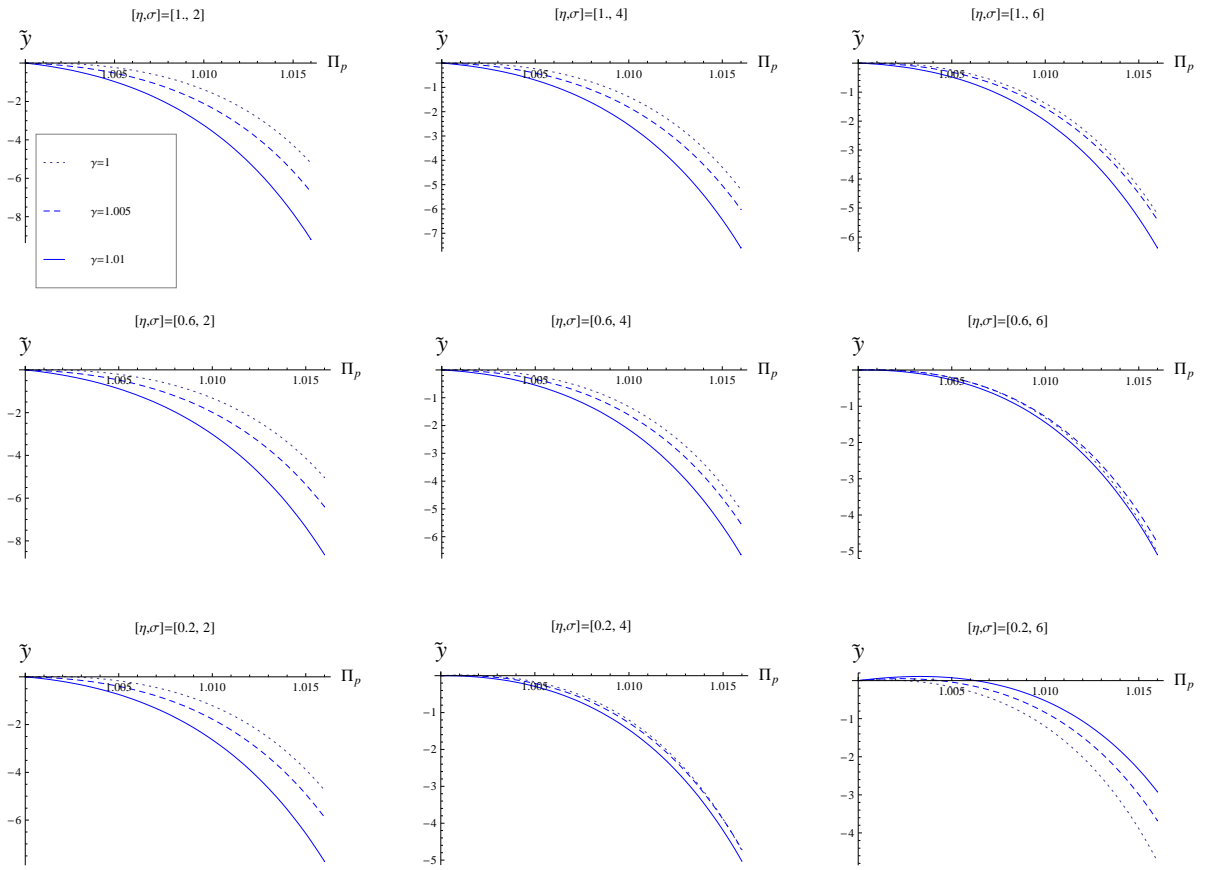


Figure 1: Output and price inflation under alternative productivity growth rates:  $\gamma = 1$  (dotted line),  $\gamma = 1.005$  (dashed line),  $\gamma = 1.01$  (solid line).

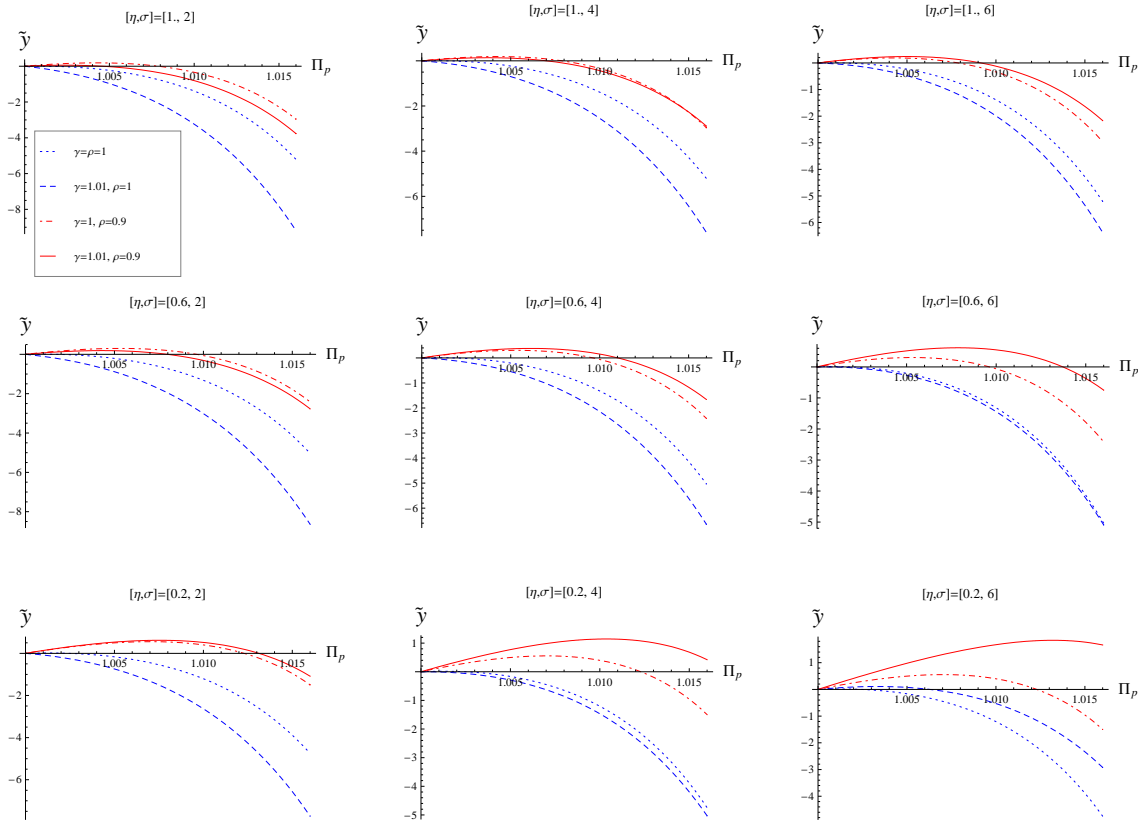


Figure 2: The individual and joint effects of growth and employment uncertainty:  $\gamma = 1, \rho = 1$  (dotted line),  $\gamma = 1.01, \rho = 1$  (dashed line),  $\gamma = 1, \rho = 0.9$  (dot-dashed line); joint effect  $\gamma = 1.01, \rho = 0.9$  (solid line).

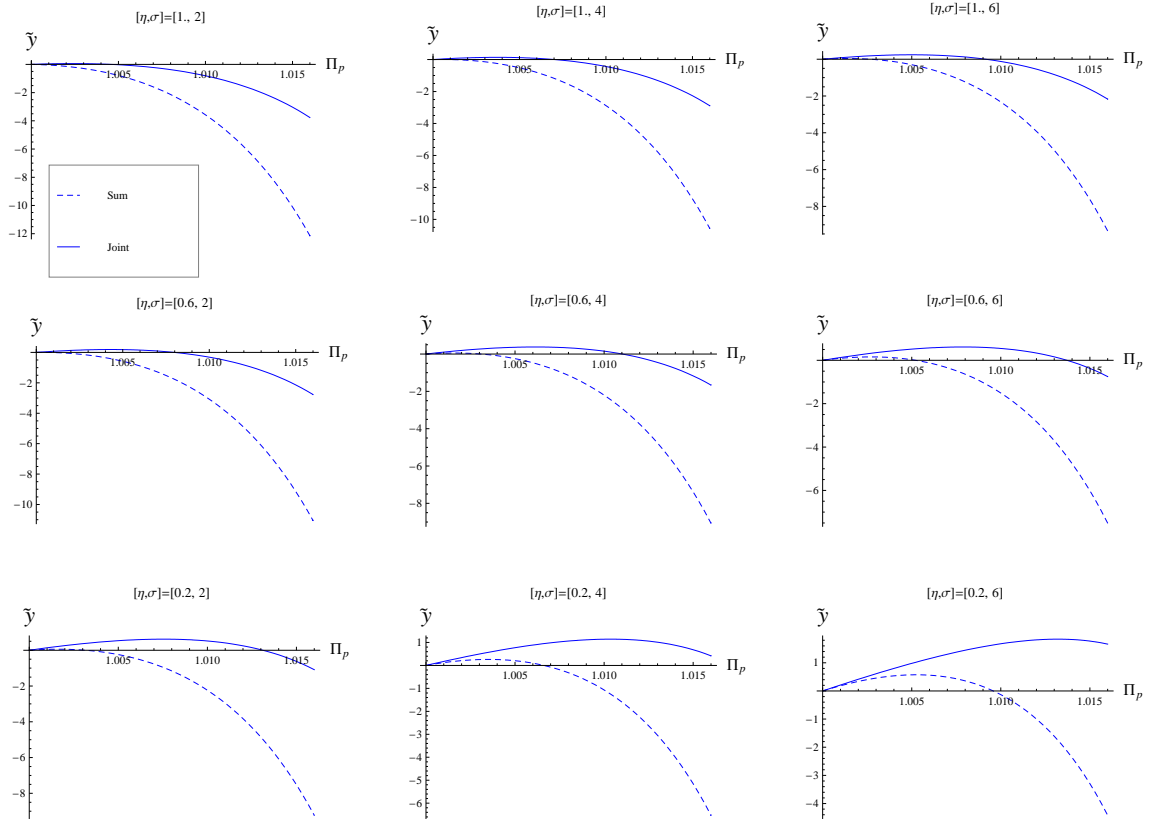


Figure 3: Complementarity between growth and employment uncertainty, comparing individual and joint effects: sum of the individual effects (dashed line); joint effect  $\gamma = 1.01, \rho = 0.9$  (solid line).

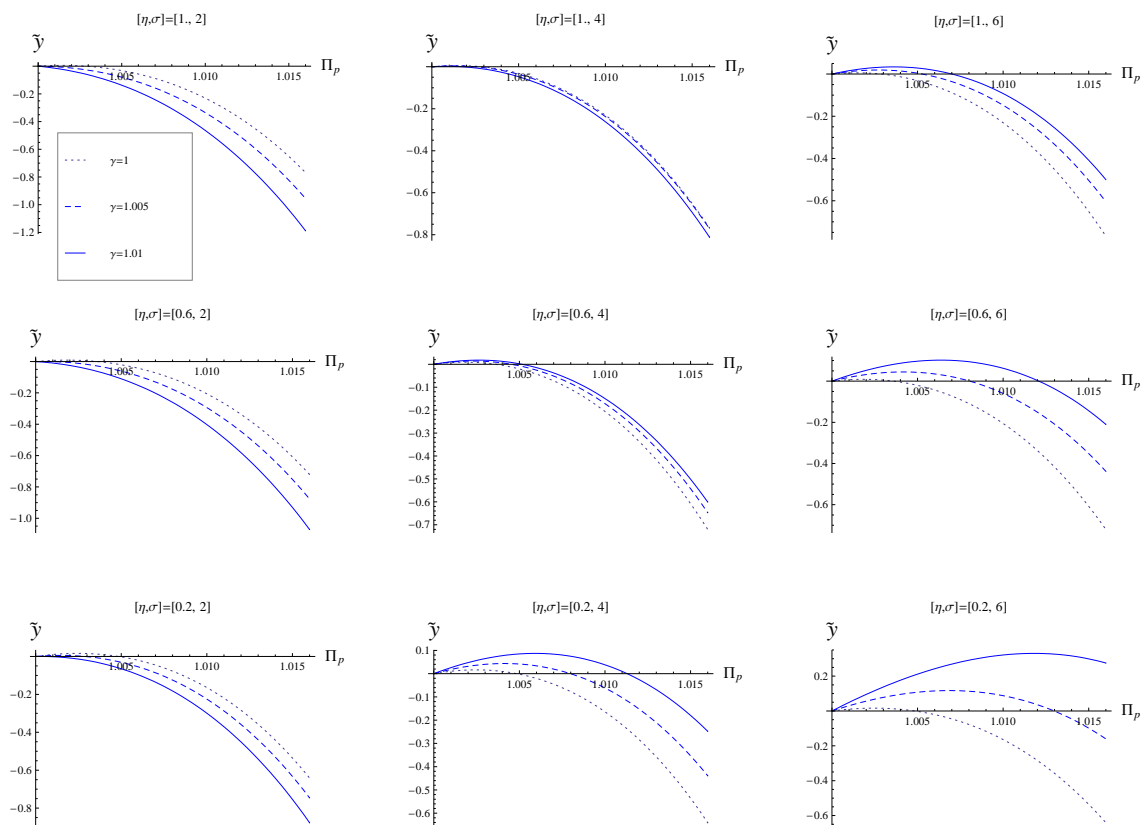


Figure 4: Output and price inflation under alternative productivity growth rates with partial indexation ( $\varphi_w = \varphi_p = 0.5$ ):  $\gamma = 1$  (dotted line),  $\gamma = 1.005$  (dashed line),  $\gamma = 1.01$  (solid line).

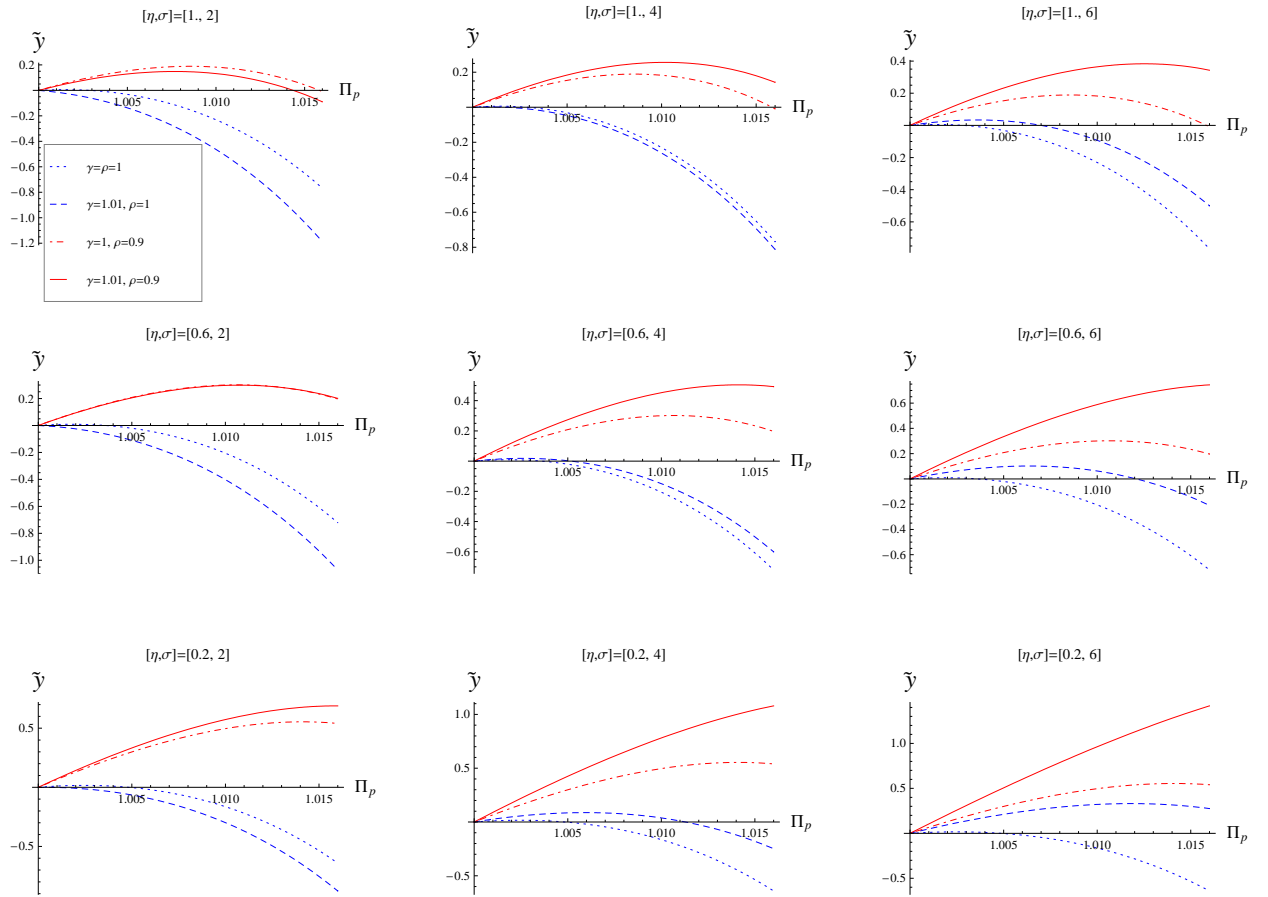


Figure 5: The individual and joint effects of growth and employment uncertainty with partial indexation ( $\varphi_w = \varphi_p = 0.5$ ):  $\gamma = 1, \rho = 1$  (dotted line),  $\gamma = 1.01, \rho = 1$  (dashed line),  $\gamma = 1, \rho = 0.9$  (dot-dashed line); joint effect  $\gamma = 1.01, \rho = 0.9$  (solid line).

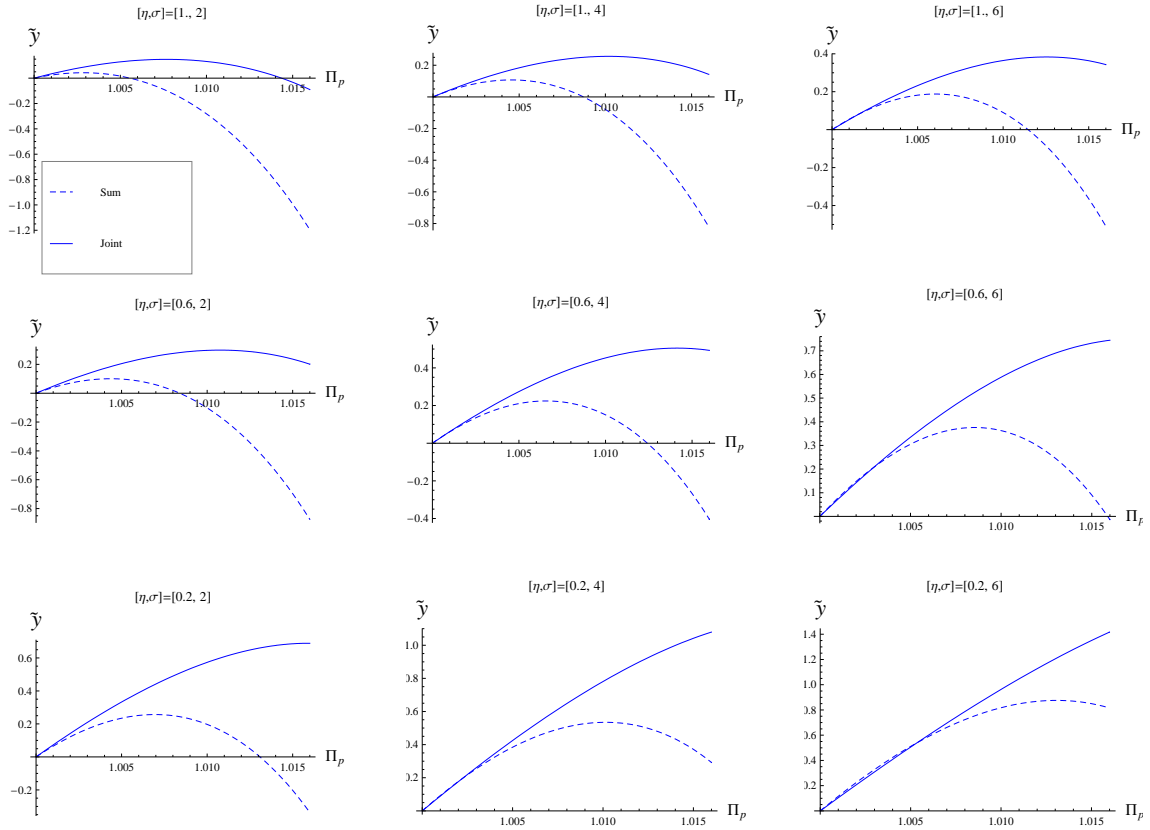


Figure 6: Complementarity between growth and employment uncertainty, comparing individual and joint effects, with partial indexation ( $\varphi_w = \varphi_p = 0.5$ ): sum of the individual effects (dashed line); joint effect  $\gamma = 1.01, \rho = 0.9$  (solid line).