Risky Mortgages

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June 8, 2010

Abstract

This paper develops a DSGE model with housing and risky mortgages. Housing is subject to idiosyncratic risk and some mortgages are defaulted in equilibrium. An unanticipated increase in the risk of mortgages produces a credit crunch where the rate of default on mortgages increases, lending is curtailed and aggregate demand drops. As a consequence of the credit crunch the economy experiences a recession. We compare economies that differ only in the riskiness of mortgages. We find that economies with lower volatility are characterized by lower rates of default on mortgages, higher loan-to-value ratios and leverage ratios. The macroeconomic effects of an unanticipated increase in the riskiness of mortgages are amplified in high-leverage economies. Monetary policy plays an important role in the transmission of risk shocks, as more inertial interest-rate rules deepen the short-run contraction of output.

Keywords: Financial crisis; Housing; Mortgage default

JEL Codes: E32, E44, G01, R31

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1 Introduction

The global financial crisis that began in August 2007 has its roots in increased mortgage delinquencies that put financial institutions into distress. The bursting of the housing bubble in the United States put many borrowers in a difficult financial position with mortgages they could not pay in the long run and larger than the value of the houses against which they were underwritten. As a result the rate of seriously delinquent mortgages increased from 2% in the third quarter of 2006 to 9.7% by the last quarter of 2009 – see Figure 1. Banks were forced to write down several hundred billion dollars in bad mortgages. These losses combined with a high degree of opacity surrounding mortgage-backed securities and a complicated web of interconnected obligations among financial institutions triggered a severe liquidity crisis in the interbank market. A credit crunch followed that caused failure of several financial institutions, brought many others close to it, raised interbank rates and drastically reduced household access to borrowing. The fall in housing prices and tightened credit conditions forced many borrowers to quickly reduce leverage and cut consumption and housing investment. The turmoil that started in the mortgage market amplified to the rest of the economy to spark the great recession.

Figure 2 shows the impulse responses from a VAR with detrended seriously delinquent mortgages (DELHP), nominal short-term interest rate (RR), change in the log of the implicit price deflator for the nonfarm business sector (DP), detrended real house prices (QQHP), detrended real per capita consumption (CCHP), and detrended real per capita GDP (YHP) from 1980Q1 to 2009Q4. The top part of Figure 2 illustrates the impulse responses of interest rate, inflation, housing prices, consumption and output to an innovation in serious delinquencies. It suggests a significant negative response of real house prices, private consumption and real GDP to a positive disturbance to mortgage delinquencies. This evidence points to a significant transmission mechanism of shocks from the mortgage market to the rest of the economy. The bottom part of Figure 2 shows the impulse responses of seriously delinquent mortgages to innovations to all other variables. These responses suggest that mortgage delinquencies respond significantly to developments in the macro-economy and our model should not treat them as an exogenous variable.

Our paper has two goals. The first is to develop a DSGE model with housing, non-durable

\footnote{See Appendix A for data definition and sources. Detrending is done using the Hodrick-Prescott filter.}
consumption and endogenous default on mortgages. To this end we assume that housing investment can be financed by mortgages that are subject to idiosyncratic risk and can be defaulted on. The second goal of the paper is to study how financial shocks originating in the mortgage market transmit to the rest of the economy. Driven by recent events, we focus on an unexpected increase in the risk of mortgages. We model this risk shock as an unexpected mean-preserving rise in the volatility of the idiosyncratic risk of housing.

Our model features two households that differ in terms of their discount factor. Savers have a higher discount factor than Borrowers and in equilibrium lend to Borrowers. Each household consists of many members. Each member starts with an ex-ante identical housing stock that is used as collateral for a mortgage. After the mortgage contract is finalized, each member experiences an idiosyncratic shock to the value of his housing stock that can be observed only by the member himself. However, the lender can pay a monitoring cost to observe the realized return. The introduction of idiosyncratic risk to housing has two effects. First, mortgage contracts must satisfy Savers’ participation constraint that guarantees a predetermined rate of return on aggregate loans. Hence, our model generates an endogenous borrowing constraint for Borrowers. Second, as in Bernanke, Gertler and Gilchrist (1999), the mortgage contract

Figure 1: Seriously Delinquent Mortgages: U.S.

Not seasonally adjusted: percentage of total loans.
Source: Mortgage Bankers Association, National Delinquency Survey
Impulse Responses to Cholesky One Standard Deviation Innovation to Delinquencies

Impulse Responses of Delinquencies to Cholesky One Standard Deviation Innovations in CCHP, DP, QQHP, RR, YHP

Figure 2: VAR Evidence

Notes: VAR estimated from 1980Q1 to 2009Q4. The dashed lines indicate the +/- one standard error bands. The Choleski ordering is DELHP, RR, DP, QQHP, CCHP, YHP. Vertical axis: percent deviation from baseline.
is truth-revealing. Borrowers experiencing low realization of the idiosyncratic shock default on their mortgages. On the other hand, Borrowers who repay their mortgages pay a state-contingent rate that is above the predetermined one. Hence, our model is characterized by default on mortgages and an external finance premium.

Within this model we study the effects of an unanticipated increase in the idiosyncratic risk to housing. According to our results, a risk shock produces a credit crunch that reduces demand in the non-durable sector. Mortgage default rates as well as the spread between the adjustable mortgage rate and the risk-free rate soar while lenders’ profits drop significantly. Households with mortgages are particularly hurt. Tighter credit conditions force borrowers to reduce their mortgages and, as a result, cut their non-durable consumption. Aggregate non-durable consumption and total output in the economy fall. An increase in idiosyncratic risk of housing generates an economic recession.

We compare two economies that differ only in the risk of mortgages, namely in the volatility of the idiosyncratic risk underlying mortgages. The economy with the lower volatility has a lower steady-state rate of default on mortgages and, as a result, mortgages are larger and the economy is more leveraged. When the volatility of idiosyncratic risk unexpectedly increases, borrowers are hurt more and the effects of the credit crunch are amplified. Hence, more leveraged economies suffer deeper slumps in response to a worsening of the distribution of mortgages.

We analyze the response of our model to an increase in the risk of mortgages under alternative specifications of the interest-rate rule that governs monetary policy. We find that more inertial rules are characterized by less aggressive interest-rate reductions in response to a risk shock that ultimately lead to deeper output contractions.

The rest of the paper is organized as follows. Section 2 discusses the related literature and section 3 presents the model. In section 4 we perform our benchmark calibration and in Section 5 we analyze the transmission mechanism in response to monetary and technology shocks. Section 6 analyzes the transmission mechanism in response to a idiosyncratic mortgage risk shock and compares the macroeconomic effects of a credit crunch in high- and low-leverage economies. Section 7 analyzes the robustness of our results to different monetary policy rules and different parameter values. Section 8 concludes.
2 Related Literature

A growing literature has been incorporating housing in economic models. Iacoviello (2005) builds on Kiyotaki and Moore (1997) to model two households with different discount factors and housing as a durable good that can be used as collateral in borrowing. To ensure the existence of an equilibrium, this model features an exogenous borrowing constraint according to which impatient agents can borrow a fraction of the expected discounted future value of their houses. Iacoviello and Neri (2010) expand the work of Iacoviello (2005) and write a DSGE model with housing that is estimated using U.S. data for the period 1965:1 to 2006:3. Calza, Monacelli and Stracca (2009) analyze how the transmission mechanism of monetary shocks in a housing model à la Iacoviello is affected by alternative values of the down-payment rate and the interest rate mortgage structure. Monacelli (2009) documents positive co-movement in durable and nondurable consumption in response to a monetary policy shock and argues that a DSGE model with an exogenous borrowing constraint is consistent with the empirical evidence. Our housing model shares a number of features with the contributions above. The novelty of our paper is to introduce idiosyncratic housing risk and endogenous default on mortgages. Another important difference is that our credit market friction generates an endogenous borrowing constraint.

The literature on the financial accelerator is vast. Starting with Bernanke et al. (1999) and then Carlstrom and Fuerst (1997), many papers have introduced this credit friction in DSGE models to analyze its effect on the transmission of shocks. We do not present an exhaustive review of this literature here but rather focus on some recent applications. Christiano, Motto and Rostagno (2009) augment a standard monetary DSGE model to include financial markets and fit the model to European and U.S. data. Cohen-Cole and Martinez-Garcia (2008) consider a model with a financial accelerator as in Bernanke, Gerlter and Gilchrist and introduce systemic risk, namely an aggregate variable that affects the variance of idiosyncratic risk, and banking regulation. Our paper is the first, to our knowledge, to introduce a financial accelerator with idiosyncratic shocks in a model with housing.

Dellas, Diba and Loisel (2010) add a banking sector to an otherwise standard New-Keynesian model and consider some financial shocks, among them an exogenous increase in the exogenous rate of default of firms on bank loans. They argue that price stability is all that matter for monetary policy, even when financial factors are present. Our work differs from Dellas et al.
Our model features the housing sector but no financial sector; default on loans is endogenous in our model while it is completely exogenous in Dellas et al. (2010). Iacoviello (2010) introduces a banking sector in a model with housing and studies an exogenous shock to how much borrowers repay. This repayment shock is exogenous and different from default because borrowers do not lose their houses following a negative repayment shock.

3 The Model

Our starting point is a model with patient and impatient households that consume non-durable goods and housing service and work. Many features of our model draw from the housing model of Iacoviello (2005), Iacoviello and Neri (2010) and Monacelli (2009). Our focus, however, is on the mortgage contract and on how its features matter for the transmission of shocks. Hence, we do not rely on an exogenous borrowing constraint but rather derive it endogenously from the lenders’ participation constraint after explicitly introducing idiosyncratic risk and default. We follow the literature on matching and assume that there is perfect insurance among household members so that consumption of non-durable goods and housing services are ex-post equal across all members. Thanks to this assumption our model can feature idiosyncratic risk while retaining a tractable two-representative households (patient and impatient) setup.

3.1 Households

The economy is populated by a continuum of households distributed over the [0, 1] interval. A fraction $\psi$ of identical households has discount factor $\beta$ while the remaining fraction $1 - \psi$ has discount factor $\gamma > \beta$. We are going to refer to the households with the lower discount factor as Borrowers, as these households value current consumption relatively more than the other agents and therefore want to borrow. We are going to refer to households with the higher discount factor as Savers.
Borrowers

Borrowers have a lifetime utility function given by

\[
\max \sum_{t=0}^{\infty} \beta^t E_0 \{U(X_t, N_{C,t}, N_{H,t})\} \quad 0 < \beta < 1
\]  (1)

where \(N_{C,t}\) is hours worked in the non-durable sector, \(N_{H,t}\) is hours worked in the housing sector, and \(X_t\) is an index of non-durable and durable consumption services defined as

\[
X_t \equiv \left[ \left(1 - \alpha\right) \frac{1}{\eta} C_t^{\frac{\eta-1}{\eta}} + \alpha \frac{1}{\eta} H_{t+1}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},
\]  (2)

where \(C_t\) denotes consumption of non-durable goods, \(H_t\) denotes consumption of housing services, \(\alpha\) is the share of housing in the consumption index and \(\eta \geq 0\) is the elasticity of substitution between housing and non-durable services. We assume that housing services in period \(t\) are equal to the housing stock at the beginning of period \(t\). Assuming that services are a fraction of the stock is not going to change our results qualitatively. Borrowers are subject to the sequence of budget constraints:

\[
P_{C,t}C_t + P_{H,t}H_{t+1} + \left[1 - F(\bar{\omega}_t)\right](1 + R_{Z,t})L_t = L_{t+1} + W_{C,t}N_{C,t} + W_{H,t}N_{H,t} + \]

\[
(1 - \delta) \left[1 - G(\bar{\omega}_t)\right] P_{H,t}H_t,
\]  (3)

where \(P_{C,t}\) is the price of non-durable goods, \(P_{H,t}\) is the price of housing, \(L_{t+1}\) are the loans taken from Savers at \(t\) to be repaid in period \(t + 1\). \(R_{Z,t}\) is the state-contingent interest rate that non-defaulting Borrowers pay at \(t\) on the loans \(L_t\) taken at time \(t - 1\). This rate is determined at time \(t\) after the realization of shocks and in order to satisfy the participation constraint of lenders, which we explain in details later. Hence, our mortgage contracts are characterized by adjustable interest rates. In equilibrium some loans are going to be defaulted on. The term \([1 - G(\bar{\omega}_t)]\) represents Borrowers’ housing stock at the end of period \(t\) after a fraction of the loans has been defaulted on; \([1 - F(\bar{\omega}_t)]\) is the fraction of loans that is repaid to lenders. We explicitly derive and explain these two terms later. The housing stock depreciates at the rate \(\delta\). \(W_{j,t}\) is the nominal wage in sector \(j = C, H\). Each household decides non-durable good consumption, housing investment (and consumption), working hours in the two sectors and
loans.

Each household consists of many members. The household decides total housing investment $H_{t+1}$, the state-contingent mortgage rates to be paid next period on the contracts signed this period, and then assigns the housing stock $H_{i,t+1}$ to the $i$-th member, where $\int_i H_{i,t+1} di = H_{t+1}$. The $i$-th member finalizes the mortgage contract connected to the housing stock $H_{i,t+1}$ following the instructions of the household and manages his housing stock. All members are ex-ante identical. After the mortgage contract is finalized, the $i$-th household member experiences an idiosyncratic shock $\omega_{i,t+1}$ such that ex-post housing is $\omega_{i,t+1}H_{i,t+1}$. The random variable $\omega_{i,t+1}$ is i.i.d. across members of the same household with mean equal to one, i.e. $E_t(\omega_{i,t+1}) = 1$, and its cumulative distribution function obeys standard regularity conditions. This implies that while there is idiosyncratic risk at the household-member level, there is no risk at the household level and $E_t(\omega_{i,t+1}H_{i,t+1}) = H_{t+1}$.

After idiosyncratic shocks are realized, the household decides which loans to repay and which to default. Loans connected to housing stocks that experienced high realizations of the idiosyncratic shock $\omega_{i,t+1}$ are repaid while loans connected to housing stocks with low realizations are defaulted on. Let $\bar{\omega}_{t+1}$ be the threshold value of the idiosyncratic shock for which the member is able to repay the loan at the contractual rate $R_{Z,t+1}$, namely

$$\bar{\omega}_{t+1}(1 - \delta)P_{H,t+1}H_{t+1} = (1 + R_{Z,t+1})L_{t+1}. \quad (4)$$

Loans connected to $\omega_{i,t+1} \in [\bar{\omega}_{t+1}, \infty]$ are repaid and lenders receive $(1 + R_{Z,t+1})L_{t+1}$. On the other hand, loans connected to $\omega_{i,t+1} \in [0, \bar{\omega}_{t+1}]$ are defaulted. For these loans lenders pay a monitoring cost to assess and seize the collateral. It is exactly the presence of monitoring that induces Borrowers to truthfully reveal their idiosyncratic shock and justifies the incentive compatibility constraint (4). As in Bernanke et al. (1999), we assume that the monitoring cost is equal to the fraction $\mu$ of the housing value, $\omega_{i,t+1}P_{H,t}H_{t+1}$ and that the defaulting household member is left with nothing.

Following Bernanke et al. (1999) we consider a mortgage contract that guarantees lenders a

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2 The c.d.f is continuous, at least once-differentiable and it satisfies

$$\frac{\partial \omega h(\omega)}{\partial \omega} > 0,$$

where $h(\omega)$ is the hazard rate.
pre-determined rate of return on their total loans. At time $t$ Savers make total loans $L_{t+1}$ to Borrowers and demand the gross rate of return $1 + R_{L,t}$. This rate of return is pre-determined at $t$ and non-state contingent. Hence, the time $t$ participation constraint of lenders is given by:

$$(1 + R_{L,t})L_{t+1} = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1 - \mu)(1 - \delta)P_{H,t+1}H_{t+1}f(\omega)d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} (1 + R_{Z,t+1})L_{t+1}f(\omega)d\omega. \quad (5)$$

The return on total loans is equal to the housing stock net of monitoring costs and depreciation of defaulting Borrower members and the repayment of non-defaulting members. After idiosyncratic and aggregate shocks have realized, the threshold value $\bar{\omega}_{t+1}$ and the state-contingent mortgage rate $R_{Z,t+1}$ are determined so as to satisfy the participation constraint above. Other things equal, an aggregate shock that raises $\bar{\omega}_{t+1}$ and the rate of default on mortgages generates an increase in the adjustable rate $R_{Z,t+1}$ paid by non-defaulting members in order to satisfy the participation constraint $(5)$. This implies that shocks that periods characterized by rising mortgage default rates are also accompanied by rising mortgage interest rates.

We now simplify the Borrower problem as follows. Let

$$G(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}f(\omega)d\omega \quad (6)$$

be the expected value of the idiosyncratic shock conditional on the shock being less than or equal to the threshold value $\bar{\omega}_{t+1}$ and let

$$\Gamma(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega)d\omega + G(\bar{\omega}_{t+1}) \quad (7)$$

be the expected gross share of housing value that goes to lenders. Then the participation constraint in terms of non-durable consumption prices can be written more compactly as

$$(1 + R_{L,t})l_{t+1} = (\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))(1 - \delta)p_{H,t+1}\pi_{C,t+1}H_{t+1}, \quad (8)$$

where $p_{H,t+1}$ is the relative price of houses in terms of non-durable consumption and $l_{t+1}$ are real loans. The loan-to-value ratio is given by

$$\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}), \quad (9)$$
and it measures the size of the loan (principal plus interests) as a fraction of the housing value net of depreciation. The loan-to-value ratio also measures the net share of the housing value that goes to the lender for repayment.

Following the decision to default at time $t$, Borrowers are left with the following stock of housing:

$$\int_{\bar{\omega}_{t+1}}^{\infty} \omega_{t+1} f(\omega) d\omega = [1 - G(\bar{\omega}_{t+1})] (1 - \delta) P_{H,t+1}, \quad (10)$$

where the second equality makes use of the fact that $E_t(\bar{\omega}_{t+1}) = 1$. This is the expression used in the Borrowers budget constraint (3).

Using the relationship between $\bar{\omega}_{t+1}$ and $R_{Z,t+1}$ we can eliminate $R_{Z,t+1}$ from the Borrower budget constraint and rewrite it in real terms as

$$C_t + p_{H,t} H_{t+1} + (1 + R_{L,t-1}) \frac{l_t}{\pi_{C,t}} = l_{t+1} + (1 - \delta) [1 - \mu G(\bar{\omega}_t)] P_{H,t} H_t + w_{C,t} N_{C,t} + w_{H,t} N_{H,t}, \quad (11)$$

where $\pi_{C,t}$ is non-durable-good inflation and $w_{C,t}, w_{H,t}$ are real wages in the $C$ and $H$ sector, respectively, in terms of $P_{C,t}$. Borrowers maximize (1) subject to the budget constraint (11) and the participation constraint (8) with respect to the variables $C_t, H_{t+1}, N_{C,t}, N_{H,t}, l_{t+1}, \bar{\omega}_{t+1}$. The respective first-order conditions are:

$$U_{C,t} - \lambda_{BC,t} = 0, \quad (12)$$

$$U_{H,t+1} - \lambda_{BC,t} p_{H,t} + \beta (1 - \delta) E_t \left( [1 - \mu G(\bar{\omega}_{t+1})] P_{H,t+1} \lambda_{BC,t+1} + \lambda_{PC,t+1} p_{H,t+1} \pi_{C,t+1} \right) = 0, \quad (13)$$

$$\lambda_{PC,t+1} p_{H,t+1} \pi_{C,t+1} \left[ \Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}) \right] = 0, \quad (14)$$

$$U_{N_j,t} + \lambda_{BC,t} w_{j,t} = 0 \quad j \in [C, H]$$

$$\lambda_{BC,t} - (1 + R_{L,t}) E_t \left[ \lambda_{PC,t+1} + \beta \lambda_{BC,t+1} \right] = 0, \quad (15)$$

$$-\beta \lambda_{BC,t+1} \mu G'(\bar{\omega}_{t+1}) + \lambda_{PC,t+1} \pi_{C,t+1} \left[ \Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}) \right] = 0, \quad (16)$$

where $\lambda_{BC,t}$ is the Lagrangian multiplier on Borrowers budget constraint and $\lambda_{PC,t+1}$ is the Lagrangian multiplier on the participation constraint (8). Notice that the first-order condition with respect to $\bar{\omega}_{t+1}$ is state-by-state and not in expected terms.
Savers

We denote Savers’ variables with a ~, except for loans. Savers maximize lifetime utility

$$\max_{t=0}^{\infty} \sum_{t=0}^{\infty} \gamma^t E_0 \left\{ U(\tilde{X}_t, \tilde{N}_{C,t}, \tilde{N}_{H,t}) \right\} \quad 0 < \beta < \gamma < 1$$ (17)

where ~\(X_t\) is defined similarly to (2) and subject to the sequence of budget constraints:

$$\tilde{C}_t + p_{H,t} \tilde{H}_{t+1} + \tilde{l}_{t+1} = (1 - \delta) p_{H,t} \tilde{H}_t + (1 + R_{L,t-1}) \frac{\tilde{l}_t}{\pi_{C,t}} + w_{C,t} \tilde{N}_{C,t} + w_{H,t} \tilde{N}_{H,t} + \tilde{\Delta}_t,$$ (18)

where ~\(\Delta_t\) are profits in the intermediate goods sector, which are taken as given.

Savers maximize (17) subject to the budget constraint (18) with respect to ~\(C_t, \tilde{H}_{t+1}, \tilde{N}_{C,t}, \tilde{N}_{H,t}, \tilde{l}_{t+1}\). The first-order conditions, respectively, are

$$U_{\tilde{C}_t} - \tilde{\lambda}_{BC,t} = 0,$$ (19)

$$U_{\tilde{H}_{t+1}} - \tilde{\lambda}_{BC,t} p_{H,t} + \gamma (1 - \delta) E_t \left[ \tilde{\lambda}_{BC,t+1} p_{H,t+1} \right] = 0,$$ (20)

$$U_{\tilde{N}_{j,t}} + \tilde{\lambda}_{BC,t} w_{j,t} = 0 \quad j \in [C, H],$$ (21)

$$-\tilde{\lambda}_{BC,t} + \gamma (1 + R_{L,t}) E_t \left[ \frac{\tilde{\lambda}_{BC,t+1}}{\pi_{C,t+1}} \right] = 0,$$ (22)

where ~\(\lambda_{BC,t}\) is the Lagrangian multiplier on Savers budget constraint.

3.2 Firms and Technology

Both the non-durable C and the housing H sector have intermediate and final good producers.

Final Good Producers

Final good producers are perfectly competitive and produce \(Y_{j,t}, j = C, H\). The technology in the \(j\)-th final good sector is given by

$$Y_{j,t} = \left( \int_0^1 Y_{j,t}(i) \frac{\varepsilon_j - 1}{\varepsilon_j} di \right)^{\varepsilon_j / \varepsilon_j - 1},$$ (23)
where $\varepsilon_j > 1$ is the elasticity of substitution among intermediate goods in sector $j$. Standard profit maximization implies that the demand for intermediate good $i$ is given by

$$Y_{j,t}(i) = \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon_j} Y_{j,t}, \forall i$$

(24)

where the price index is

$$P_{j,t} = \left( \int_0^1 P_{j,t}(i)^{1-\varepsilon_j} di \right)^{\frac{1}{1-\varepsilon_j}}.$$

### Intermediate Good Sectors

There are two intermediate good sectors $j \in [C, H]$ and in each intermediate sector there is a continuum of firms each producing a differentiated good $i \in [0, 1]$. These firms are monopolistically competitive. We assume that intermediate good firms face a quadratic cost proportional to output given by

$$\frac{\theta_j}{2} \left( \frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 Y_{j,t},$$

where $\theta_j$ measures the degree of price rigidity in sector $j$. Intermediate good firm $i$ uses labor to produce according to the linear production function

$$Y_{j,t}(i) = A_{j,t} L_{j,t}(i),$$

(25)

where $A_{j,t}$ is the stochastic level of technology in sector $j$.

Firm $i$ chooses labor and its nominal price so as to maximize expected nominal profits. The maximization problem for firm $i$ is given by

$$\max_{P_{j,t}(i), L_{j,t}(i)} E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_t \left[ P_{j,t}(i) Y_{j,t}(i) - W_{j,t} L_{j,t}(i) - \frac{\theta_j}{2} \left( \frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 P_{j,t} Y_{j,t} \right] + m c_{j,t}(i) P_{j,t} [A_{j,t} L_{j,t}(i) - Y_{j,t}(i)] \right\},$$

(26)

where the demand is given in (24) and

$$\Lambda_t = \frac{\gamma^f \tilde{\lambda}_{BC,t}}{\tilde{\lambda}_{BC,0}}$$
is the stochastic discount factor for Savers.

The first-order condition relative to labor is

\[-W_{j,t} + mc_{j,t}(i)P_{j,t}A_{j,t} = 0,\]  (27)

which states that the nominal marginal cost equals the ratio of the nominal wage to the marginal product of labor. Since the marginal productivity of labor and wages are the same across all firms, \(mc_{j,t}(i) = mc_{j,t}.\) The first-order condition relative to the price is given by

\[Y_{j,t} [1 - \varepsilon_j + \varepsilon_j mc_{j,t} - \theta_j \pi_{j,t} (\pi_{j,t} - 1)] + \theta_j \gamma E_t \left[\frac{\tilde{\lambda}_{BC,t+1} \pi_{j,t+1}^2 (\pi_{j,t+1} - 1)}{\tilde{\lambda}_{BC,t} \pi_{j,t+1}^2 (\pi_{j,t+1} - 1)}\right] = 0,\]  (28)

where \(\pi_{j,t}\) denotes gross inflation in sector \(j\) prices.

### 3.3 Monetary Policy

We assume that monetary policy follows a simple Taylor-type rule for the nominal interest rate:

\[\frac{1 + R_{L,t}}{1 + R_L} = A_{M,t} \pi_{C,t}^{\phi_{\pi}}, \quad \phi_{\pi} > 1,\]  (29)

where \(R_L\) is the steady-state nominal interest rate and \(A_{M,t}\) is a monetary policy shock. We have assumed that monetary policy targets only inflation in the non-durable goods sector.\(^3\)

### 3.4 Market Clearing

Equilibrium in the non-durable goods market requires that production of the final non-durable good net of adjustment costs equals aggregate demand:

\[Y_{C,t} = \psi C_t + (1 - \psi) \tilde{C}_t + \frac{\theta_C}{2} (\pi_{C,t} - 1)^2 Y_{C,t}.\]  (30)

\(^3\)Assuming that monetary policy targets inflation in both sectors does not affect our results.
Similarly, equilibrium in the housing market requires

\[ Y_{H,t} = \psi [H_{t+1} - (1 - \delta)(1 - \mu G(\tilde{\omega}_t))H_t] + (1 - \psi) \left( \tilde{H}_{t+1} - (1 - \delta)\tilde{H}_t \right) + \frac{\theta_H}{2} (\pi_{H,t} - 1)^2 Y_{H,t}. \]

(31)

Equilibrium in the labor market requires

\[ \int_0^1 L_{C,t}(i) di = \psi N_{C,t} + (1 - \psi)\tilde{N}_{C,t}, \quad (32) \]

\[ \int_0^1 L_{H,t}(i) di = \psi N_{H,t} + (1 - \psi)\tilde{N}_{H,t}, \quad (33) \]

while the equilibrium in the credit market requires:

\[ \psi l_t = (1 - \psi)\tilde{l}_t. \]

(34)

We define total output as

\[ Y_t = Y_{C,t} + p_{h,t}Y_{H,t}, \quad (35) \]

and net housing investment as

\[ NIH_t = \psi (H_{t+1} - H_t) + (1 - \psi)(\tilde{H}_{t+1} - \tilde{H}_t). \]

(36)

Net housing investment measures the investment in the housing sector net of depreciation and monitoring costs.

4 Functional Forms, Calibration and Steady State

We assume the following utility function:

\[ U(X_t, N_t) \equiv \ln X_t - \frac{\nu}{1 + \varphi} \left[ N_{C,t}^{1+\varphi} + N_{H,t}^{1+\varphi} \right], \quad \varphi > 0 \]

(37)

where \( \varphi \) is the inverse of the elasticity of labor supply to wages. We follow Horvath (2000), Calza et al. (2009), and Iacoviello and Neri (2010) in assuming that hours in the non-durable and housing sector are imperfect substitutes. As a result, wages are not equalized in the two
sectors and substitution of hours across sectors in response to changes in the relative price are reduced.

There are three exogenous shocks in our model that evolve according to the following first-order autoregressive processes

\[
\ln A_{C,t} = \rho_C \ln A_{C,t-1} + \epsilon_{C,t}, \quad \rho_C \in (-1, 1), 
\]

\[
\ln A_{H,t} = \rho_H \ln A_{H,t-1} + \epsilon_{H,t}, \quad \rho_H \in (-1, 1), 
\]

\[
\ln A_{M,t} = \rho_M \ln A_{M,t-1} + \epsilon_{M,t}, \quad \rho_M \in (-1, 1), 
\]

where \(\epsilon_C, \epsilon_H, \epsilon_M\) are i.i.d. innovations with mean zero and standard deviation \(\sigma_C, \sigma_H, \sigma_M\), respectively.

The parameters values for our calibration are specified in Table 1. The steady-state values are reported in Table 2. For the Borrower and Saver discount factors, the rate of depreciation for housing, the elasticity of substitution between non-durable goods and housing services and the share of housing in the consumption bundle we use the values in Monacelli (2009). The Saver discount factor pins down the steady-state interest rate at \(R_L = 0.0101\) on a quarterly basis. This implies an annual interest rate equal of four percentage points. For the degree of price stickiness, we assume that housing prices are fully flexible and set \(\theta_H = 0\), which is in line with the empirical estimation of Iacoviello and Neri (2010) and the empirical evidence on price stickiness for durable goods. For non-durable goods, we assume \(\theta_C = 40\), which is standard in the literature. Given our elasticity of substitution across non-durable goods, this implies the same slope of the Phillips curve that would emerge in the typical Calvo-Yun model with a probability of not changing prices equal to 0.67 and firms changing their price on average every nine months. We set the parameter in the interest rate rule \(\phi_\pi = 1.5\). Section 7.1 analyzes the robustness of our results to different specifications of the interest rate rule.

As for the idiosyncratic risk in the housing sector, we follow Bernanke et al. (1999) and assume that \(\omega\) is distributed log-normally:

\[
\ln \omega \sim N\left(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2\right). 
\]
We choose the variance of the distribution of $\ln \omega$ to match the pre-crisis delinquency rates for prime loans. In the first quarter of 2006 the seasonally adjusted annual delinquency rate on prime adjustable rate mortgages (ARMs) was 2.3 percentage points. We choose $\sigma_\omega = 0.2$ and obtain an annual default rate of 2.36 percentage points. We follow Bernanke et al. (1999) and set $\mu = 0.12$ so that monitoring costs are 12% of the housing value. At the steady state, the loan-to-value ratio in our benchmark calibration is equal to 60 percentage points. This is lower than 75.7 percentage points, which is the average U.S. loan-to-value ratio between 1973 and 2008. We can raise the steady-state loan-to-value ratio by reducing $\sigma_\omega$, the riskiness of loans; however this is going to reduce the steady-state rate of default. For this reason, we prefer to match the delinquency rate but have a lower loan-to-value ratio than suggested by the data.

At the steady state, the quarterly mortgage rate rate paid by non-defaulting Borrowers $R_Z = 0.0111$, which corresponds to an annual rate of 0.0453. We define the external finance premium at $t$ as $R_{Z,t} - R_{L,t}$, namely the difference between the ex-post state-contingent rate paid by non-defaulting household members and the risk-free interest rate, which in our setting is equivalent

---

4 Source: Several Press Releases of the Mortgage Bankers Association.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Steady State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption, Borrowers*</td>
<td>0.4558</td>
</tr>
<tr>
<td>Housing Demand, Borrowers*</td>
<td>0.4066</td>
</tr>
<tr>
<td>Hours Worked in $C$ Sector, Borrowers*</td>
<td>0.5442</td>
</tr>
<tr>
<td>Hours Worked in $H$ Sector, Borrowers*</td>
<td>0.5442</td>
</tr>
<tr>
<td>$p_h$ Output $H$/Total Output</td>
<td>0.0819</td>
</tr>
<tr>
<td>Loans</td>
<td>2.2427</td>
</tr>
<tr>
<td>Loan-to-Value Ratio</td>
<td>0.5917</td>
</tr>
<tr>
<td>Leverage Ratio</td>
<td>0.8012</td>
</tr>
<tr>
<td>Default Rate on Mortgages†</td>
<td>0.0059</td>
</tr>
<tr>
<td>External Finance Premium†</td>
<td>0.0010</td>
</tr>
<tr>
<td>Mortgage Interest Rate†</td>
<td>0.0111</td>
</tr>
</tbody>
</table>

* Expressed as percentage of aggregate level, e.g. Consumption, Borrowers = $\psi C/(\psi C + (1 - \psi)\hat{C})$.
†Quarterly
The Leverage Ratio is calculated as $L/[L + (W_H/P_C)N_H + (W_C/P_C)N_C]$.

Table 2: Steady State under the Benchmark Calibration

This premium captures the additional cost that Borrowers must pay for their risky mortgages relative to risk-free borrowing. At the steady state, the external finance premium is equal to 0.43 percentage points on an annual basis. We calculate the empirical counterpart to our external finance premium as the difference between the 30-Year Conventional Mortgage Rate and the interest rate on the U.S. Treasury 30-Year bonds. The average difference between these two interest rates between 1977 and 2009 was 1.5 percentage points. This makes the finance premium of our benchmark model one percentage point lower than its empirical counterpart.

5 Dynamic Response to Monetary and Technology Shocks

Figure 3 illustrates the impulse responses of the model under the benchmark calibration in response to a monetary shock, namely a 25 basis point increase in the nominal interest rate $R_{L,t}$. Savers, who are consumption smoothers, reduce consumption of non-durable goods in response to higher interest rates. Borrowers are affected in two ways. First, they experience

5Our definition of external finance premium differs somewhat from that in Bernanke et al. (1999), where the premium is the difference between the costs of funds raised externally and the opportunity costs of funds internal to the firm.
6See the H-15 Release of the Federal Reserve Economic Data
7See the Economic Report to the President, Table B73.
an increase in the cost of borrowing. Second, deflation for non-durable goods raises their real debt via the Fisher effect. As a result, they reduce mortgage loans and consumption of non-durable goods and housing services. Borrowers raise internal funds by increasing their labor supply in the housing sector, which makes the real wage in the housing sector fall and depresses the relative price of houses. As a result $\bar{\omega}$, the threshold value of the idiosyncratic shock below which household members do not repay their mortgages, goes up and with it the default rate and monitoring costs. Since a lower fraction of households repays the loans, the adjustable interest rate on mortgages $R_{Z,t}$ increases on impact. The interest rate $R_{L,t}$ on outstanding mortgages to be repaid at time $t$ was set at time $t-1$ and cannot change on impact. As a result the external financial premium jumps up on impact. The negative wealth effect stemming from higher default, higher mortgage rates and lower housing value makes Borrowers further cut their consumption of non-durable goods. Aggregate consumption and production of non-durable goods fall. Monitoring costs reduce the existing stock of housing, thereby raising housing demand. As a result, production in the housing sector increases slightly. However, the contraction in the non-durable sector more than compensates the expansion in the construction sector and total output falls.

Figure 4 illustrates the impulse responses of the model under the benchmark calibration in response to a negative technological shock in the non-durable sector, namely a one percentage point decrease in $A_{C,t}$. Output falls and prices increase in the $C$ sector, which in turn raise the nominal interest rate via the interest rate rule. As a result, Savers reduce non-durable consumption. Borrowers face higher costs of borrowing and therefore reduce loans, consumption and housing but increase hours worked in both sectors. Aggregate consumption falls. The increase in non-durable consumption prices and the fall in real wages, both in the $C$ and the $H$ (not shown in the diagram) sectors, reduce the relative price of housing, which makes housing investment more attractive for Savers. Net housing investment and output in the housing sector slightly increase. The increase in the nominal interest rate stemming from a negative technological shock affects the mortgage market much along the same lines of a monetary shock: the default rate, monitoring costs and the loan-to-value ratio all go up. The fall in housing value raises default, which in turn raises the adjustable mortgage rate, the external finance premium and monitoring costs. Loans are reduced and total output in the economy falls.

Figure 5 illustrates the impulse responses of the model under the benchmark calibration in
Figure 3: Impulse Responses to a 25 basis points Monetary Shock
The vertical axis is the % deviation from steady state; the horizontal axis are quarters after the shock. Net housing investment is shown as change from last period divided by steady-state output in the housing sector.
Figure 4: Impulse Responses to a -1% Technology Shock in the Consumption-Good Sector
Figure 5: Impulse Responses to a 1% Technology Shock in the Housing Sector
response to an increase in the level of technology in the housing sector, namely a one percentage point increase in $A_{H,t}$. Production increases and prices fall in the housing sector so that the relative housing price falls. Since housing is cheaper, Savers increase their demand. The fall in housing prices affects Borrowers in two ways. First, they take fewer loans as the value of their houses is now lower. Second, a reduced housing value makes them default more. Both effects induce Borrowers to reduce non-durable consumption and housing demand. Total output goes up because the increase in production in the housing sector more than compensates the fall in output in the non-durable goods sector stemming lower aggregate demand. Lower aggregate consumption reduces prices in the non-durable sector and the nominal interest rate. As a consequence, loans are slightly reduced. The fall in housing value affects the mortgage market in the same way as before. The default rate, the external finance premium, monitoring costs and the loan-to-value ratio increase, although the change is quantitatively small.

6 Credit Crunch

This section analyzes the dynamic response of the model to an unexpected increase in the standard deviation of the distribution of $\ln \omega$, the idiosyncratic shock to Borrowers’ housing stock. Intuitively, we want to capture the situation in which loans are made on the basis of an expected distribution for idiosyncratic risk but the actual distribution turns out to be characterized by a higher standard deviation. To do this, we assume that the standard deviation of $\ln \omega$ is itself an exogenous shock subject to a first-order autoregressive process:

$$\ln \frac{\sigma_{\omega,t}}{\sigma_{\omega}} = \rho_\sigma \ln \frac{\sigma_{\omega,t-1}}{\sigma_{\omega}} + \epsilon_{\sigma_{\omega,t}}, \quad (42)$$

where $\epsilon_{\sigma_{\omega}}$ is an i.i.d. shock with mean zero and finite standard deviation $\sigma_{\sigma_{\omega}}$. We set $\rho_\sigma = 0.9$. Notice that an increase in $\sigma_{\omega}$, the standard deviation of the distribution of $\ln \omega$, has two effects on the distribution itself: it increases the variance and lowers the mean, which implies a leftward shift of the distribution. See Figure 41. Since the log-normal distribution does not take negative values, a fall in the mean implies a thicker lower tail of the distribution. Thus, for the same value of $\bar{\omega}$, a higher standard deviation implies a higher cumulative distribution function and therefore a higher default rate on mortgages.
Other papers model an increase in risk. Christiano et al. (2009) introduce a financial accelerator as in Bernanke et al. (1999) in a model of the financial sector and model a risk shock as an increase in the standard deviation of idiosyncratic risk in loans to entrepreneurs. In our setting, idiosyncratic risk is in mortgage loans. Cohen-Cole and Martinez-Garcia (2008) also consider idiosyncratic risk in loans to entrepreneurs and introduce systemic risk as a correlation between the mean of the idiosyncratic risk and the aggregate shock, as in Faia and Monacelli (2007). We do not consider systemic risk but allow for unexpected shocks to the standard deviation of the distribution of idiosyncratic risk.

6.1 Dynamic Response to a Shock to $\sigma_{\omega,t}$

An increase in $\sigma_{\omega}$ affects the economy through two channels. First, the loan-to-value ratio, which is the net share of the housing value going to Savers for repayment, falls. This makes the participation constraint more binding. Second, since a lower fraction of Borrowers repays their debts, monitoring costs increase sharply and the housing stock of Borrowers falls.

Figure 6 shows the impulse responses of the model to a 10% increase in the standard deviation $\sigma_{\omega}$. An increase in the standard deviation implies a leftward shift of the distribution that raises the density of household members in the left tail of the distribution. The threshold $\bar{\omega}_t$ below which Borrower household members default barely changes but, because of the shift in the distribution, the rate of default on mortgages jumps up considerably. As a result, monitoring costs also increase. Borrowers experience a reduction in their housing stock because more loans are defaulted on and the corresponding houses seized as collateral by Savers. At the same time loans and the loan-to-value ratio drop, thereby tightening credit conditions. Borrowers cut non-durable consumption and housing demand and increase hours worked. Aggregate consumption of non-durable goods falls as the reduction in Borrowers consumption more than compensate the increase of Savers, thereby reducing output and inflation in the non-durable sector. Lower inflation reduces the nominal interest rate and Savers raise current consumption of housing and non-durable goods and reduce loans. Net housing investment, which measures the change in housing production net of depreciation and monitoring costs, falls. Nevertheless, the relative

---

8From the first quarter of 2006 to the fourth quarter of 2009 delinquency rates on prime real estate loans increased almost threefold in the United States, from 2.3% to 6.84%. To obtain a similar increase in the rate of default in our model we need a forty percentage points increase in the standard deviation of the distribution of idiosyncratic risk.
Figure 6: Impulse Responses to a 10\% Increase in $\sigma_\omega$
price of houses increases on impact. The increase in the first period is due to the large increase in monitoring costs, which reduces the stock of housing and whose replacement raises housing production and nominal housing prices on impact – see (31). After the first period nominal housing price start falling but not fast enough to compensate the deflation in non-durable sector. Total output in the economy falls driven by the reduction in output in the non-durable sector, which represents more than 80% of the total economy. Because more Borrowers default, the adjustable mortgage rate \( R_{Z,t} \) must increase to satisfy the participation constraint of Savers. This, coupled with a fall in the risk-free rate \( R_{L,t} \), brings a substantial increase in the external finance premium faced by impatient agents.

### 6.2 Credit Crunch: High- and Low-Leverage Economies

This section compares two economies characterized by different steady-state standard deviations of the distribution of the idiosyncratic housing risk. In the first economy (the “High-Leverage Economy”) the standard deviation at the steady state is \( \sigma_\omega = 0.2 \), which is the value used for our benchmark economy. In the second economy (the “Low-Leverage Economy”) \( \sigma_\omega = 0.6 \). We show that higher leverage amplifies the macroeconomic effects associated with an unexpected increase in idiosyncratic risk in housing.

Table 3 reports the steady-state levels of a number of endogenous variables in the two economies. The last column of Table 3 reports the percentage point difference between High- and Low-Leverage economies. Intuitively, a lower standard deviation of the distribution of idiosyncratic risk is associated with a lower rate of default on mortgages, a lower external finance premium and an higher loan-to-value ratio. In this scenario Borrowers demand more residential housing, which can be used as collateral, and borrow more substituting out consumption for the non-durable goods and leisure. We calculate the leverage ratio as the fraction of total expenses financed by loans, namely consumption of \( C \) and \( H \) plus loan repayment over loans. The leverage ratio captures the dependence of Borrowers from external funding. Because loans are larger when idiosyncratic volatility is lower, steady-state loans are 173 percentage points higher in the High-Leverage economy, the loan-to-value ratio is 143 percentage points higher and the leverage ratio is 34 percentage points higher.

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9All the other parameters are set according to the values in Table 1.
Loans are measured as changes from the steady state.
<table>
<thead>
<tr>
<th>Variable</th>
<th>High Leverage</th>
<th>Low Leverage</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output C</td>
<td>0.5418</td>
<td>0.5411</td>
<td>0.13</td>
</tr>
<tr>
<td>Output H</td>
<td>0.1618</td>
<td>0.1574</td>
<td>2.8</td>
</tr>
<tr>
<td>Consumption, Borrowers</td>
<td>0.4938</td>
<td>0.5015</td>
<td>-1.55</td>
</tr>
<tr>
<td>Consumption, Savers</td>
<td>0.5897</td>
<td>0.5806</td>
<td>1.56</td>
</tr>
<tr>
<td>Housing Demand, Borrowers</td>
<td>12.9464</td>
<td>11.7881</td>
<td>9.85</td>
</tr>
<tr>
<td>Housing Demand, Savers</td>
<td>18.9016</td>
<td>19.1091</td>
<td>-1.09</td>
</tr>
<tr>
<td>Hours Worked, Borrowers in C Sector</td>
<td>0.5897</td>
<td>0.5806</td>
<td>1.56</td>
</tr>
<tr>
<td>Hours Worked, Borrowers in H Sector</td>
<td>0.1761</td>
<td>0.1689</td>
<td>4.28</td>
</tr>
<tr>
<td>Hours Worked, Savers in C Sector</td>
<td>0.4938</td>
<td>0.5015</td>
<td>-1.54</td>
</tr>
<tr>
<td>Hours Worked, Savers in H Sector</td>
<td>0.1475</td>
<td>0.1459</td>
<td>1.10</td>
</tr>
<tr>
<td>Loans</td>
<td>2.2427</td>
<td>0.8189</td>
<td>173.86</td>
</tr>
<tr>
<td>Loan-to-Value Ratio</td>
<td>0.5917</td>
<td>0.2437</td>
<td>142.80</td>
</tr>
<tr>
<td>Leverage Ratio</td>
<td>0.8012</td>
<td>0.6001</td>
<td>33.51</td>
</tr>
<tr>
<td>Default Rate on Mortgages</td>
<td>0.0059</td>
<td>0.0205</td>
<td>-71.22</td>
</tr>
<tr>
<td>Monitoring Cost</td>
<td>0.0004</td>
<td>0.0005</td>
<td>-20.12</td>
</tr>
<tr>
<td>External Finance Premium</td>
<td>0.0010</td>
<td>0.0059</td>
<td>-83.05</td>
</tr>
<tr>
<td>Mortgage Interest Rate</td>
<td>0.0111</td>
<td>0.0160</td>
<td>-30.63</td>
</tr>
</tbody>
</table>

The Leverage Ratio is calculated as $L/[L + (W_H/P_C)N_H + (W_C/P_C)N_C]$

Table 3: High- and Low-Leverage Economies: Steady-state Comparison

Next we analyze how differences in steady-state leverage ratios translate into different responses over the business cycle. Figure 7 plots the impulse responses of the endogenous variables to a ten percentage points increase in $\sigma_\omega$ for the High-Leverage (red starred line) and the Low-Leverage (blue line with diamonds) economy. The effects of an increase in the risk of mortgages are amplified in the High-Leverage economy, namely the economy with the lower steady-state standard deviation. In particular, the credit crunch is deeper and the adverse effects on Borrowers stronger. Loans fall more in the High-Leverage economy, which implies that Borrowers must de-leverage more in these economies. As a result, Borrowers consumption of non-durable goods and housing services fall more in the economy with high leverage; aggregate consumption and prices of non-durable goods also fall more. The deeper contraction in non-durable demand generates a deeper reduction in total output.
7 Robustness Analysis

This section analyzes how our results are sensitive to the specification of monetary policy, adjustment costs and the serial correlation of the shock to the riskiness of mortgages.

7.1 Monetary Policy

Our benchmark model features an interest rate rule that responds only to consumption-good inflation. In this section we consider a more general class of rules that allows for a response to output and for interest-rate smoothing:

\[
\frac{1 + R_{Lt}}{1 + R_L} = A_{M,t} \left[ \pi_{C,t} \left( \frac{Y_t}{Y} \right)^{\phi_y} \right]^{1 - \phi_r} \left[ \frac{1 + R_{Lt-1}}{1 + R_L} \right]^{-\phi_r}, \quad \phi_r > 1, \quad \phi_y \geq 0, \quad \phi_r < 1.
\]

(43)

Figure 8 shows the impulse responses to a ten percentage points increase in the standard deviation \(\sigma_\omega\) under alternative parameter values for \(\phi_y\) and \(\phi_r\) in the rule above. For all specifications we keep the coefficient on inflation constant and equal to \(\phi_\pi = 1.5\). The starred red line is the benchmark specification where \(\phi_y = \phi_r = 0\); the blue line with diamonds and the light blue with circles allow for interest-rate smoothing with \(\phi_r = 0.9\), with the latter featuring also a response to output with \(\phi_y = 0.125\). The brown line with triangles features a response to output with \(\phi_y = 0.125\) but no interest-rate smoothing.

The negative contemporaneous response of the risk-free interest rate is dampened down under interest-rate smoothing. Higher idiosyncratic risk raises the rate of default on mortgages, monitoring costs, the mortgage interest rate, the external finance premium and the loan-to-value ratio under all specifications and almost identically. The smaller reduction in the nominal interest rate under interest-rate smoothing has important implications for non-durable consumption. Borrowers cut their consumption more and Savers increase it by less, thereby making the negative response of aggregate consumption stronger. As a result, output in the non-durable sector as well as total output display a much stronger contraction under interest-rate smoothing. Interestingly, without interest rate smoothing output targeting amplifies the effect of a risk shock on consumption and total output.
Figure 8: Impulse Responses to a 10% Increase in $\sigma_\omega$ and Monetary Policy
7.2 Housing Adjustment Costs

We add housing adjustment costs to our model to capture the fact that changing the stock of housing entails some costs. For example, selling, buying or renovating a house requires a considerable amount of time spent in searching, fees, etc. More precisely, we assume that Borrowers face the following costs to changing their housing investment:

\[ \Psi_t \equiv \frac{\chi}{2} \frac{P_{H,t} H_t}{H} \left( \frac{I_{H,t}}{H} - \frac{I_H}{H} \right)^2, \quad \chi \geq 0, \]

where \( I_{H,t} = H_{t+1} - (1-\delta)(1-\mu G(\bar{\omega}_t))H_t \). Savers face a similar housing investment adjustment cost.

Figure 9 compares the impulse responses of the model to a ten percentage points increase in the standard deviation \( \sigma_\omega \) when \( \chi = 0 \) and there are no adjustment costs (red starred line) to the responses when \( \chi = 2 \) (blue line with diamonds). The main effect of housing adjustment costs is to dampen out the response of housing demand of the two groups of agents without changing its sign. The negative response of net housing investment is stronger. In order to reduce housing demand less, Borrowers cut substantially more their consumption of non-durable goods. Aggregate consumption and production in the \( C \) sector falls deeper as well as total output in the economy. The variables related to the lending contract display stronger autocorrelation in the presence of adjustment costs but their overall pattern is similar to the case where adjustment costs are absent.

8 Conclusions

This paper introduces default on mortgages in a model with housing. Our model features amplification because mortgage default rates increase in response to adverse shocks and because non-defaulting Borrowers face higher adjustable mortgage rates. To capture the events at the heart of the recent financial crisis, the paper considers an unexpected increase in mortgage risk. Our model predicts a substantial increase in the default rate and the external financial premium faced by Borrowers and a credit crunch that curtails loans and consumption in the non-durable sector for many periods. A shock in the housing sector transmits to the rest of the economy and generates a recession. Economies characterized by different mortgage risk have different
Figure 9: Impulse Responses to a 10% Increase in $\sigma_\omega$ and Adjustment Costs
leverage ratios in equilibrium. The macroeconomic effects of a credit crunch are amplified in highly-leveraged economies.

Our model predicts strong, positive output responses in the housing sector following adverse shocks. This happens because monitoring costs cause a reduction in the housing stock that agents want to replenish quickly. A model featuring a rental market is likely to eliminate these strong output responses. Our model features a simple loan market whereby patient agents lend to impatient ones and financial intermediation plays a limited role. Introducing a more formal banking sector is likely to amplify the effects of a risk shock. We leave these extensions to future work.
References


A Data and Sources


Y : Gross Domestic Product. Real Gross Domestic Product (seasonally adjusted, billions of chained 2005 dollars, Table 1.1.6), divided by CNP16OV. Source: BEA.


RR : Nominal Short-term Interest Rate. 3-month Treasury Bill Rate (Secondary Market Rate), expressed in quarterly units. (Series ID: H15/RIFSGFSM03_NM). Source: Board of Governors of the Federal Reserve System.


DEL : Seriously delinquent mortgages, not seasonally adjusted, percentage of total mortgages. Source: Mortgage Bankers Association, National Delinquency Survey.