

# Investigating Zero Lower Bound Effects under Financial Instability: Fiscal Policy\*

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## Abstract

The aim of this paper is to investigate the effects of a fiscal stimulus in a deep recession scenario, in which credit market imperfections are important and the nominal interest rate is constrained by its zero lower bound. We simulate a severe recession by a sudden decrease in the business sector net worth and a shift from spending to saving by households. We then explore the effects of different fiscal policies for government spending and the income tax rate. The latter may include a concern for public debt consolidation or implementation delays. We find that the *impact* government spending multiplier can be greater than one for more than a year, due to the reinforced effect of zero nominal interest rates and credit market frictions. In contrast, the zero lower bound constraint decreases the value of the income tax multiplier, while financial frictions have a positive impact. Additionally, we find that, when both financial frictions and the zero lower bound are present, small implementation delays may imply big spending multipliers in the short-run, contrary to the case in which financial imperfections are not important.

Keywords: Zero Lower Bound, Financial Accelerator, Fiscal Policy

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# 1 Introduction

The severe recession of 2008-2010 has prompted monetary and fiscal authorities to undertake drastic policies. One of them corresponds to the reduction of the target short-run nominal interest rate by different central banks to record low levels. In the United States, for instance, the fed funds rate has been set to a range of 0 and 0.25 per cent since December 2008. In addition, different governments have implemented important fiscal packages aimed at stimulating their economies. For the U.S. case, the *American Recovery and Reinvestment Act* (ARRA) has planned to distribute 787 billion dollars (around 5.5 per cent of 2009 GDP) in the course of the next few years. These noteworthy policy measures have led to a renewed interest in the assessment of fiscal stimulus efficiency under the zero lower bound on the nominal interest rate. However, in a context of a financial turmoil, the role of financial frictions might be crucial in the analysis. Indeed, few studies suggest that financial accelerator mechanisms – in which firms’s balance sheet positions affect the cost of external finance – are relatively more substantial during recessions than during booms.<sup>1</sup>

In the case where financial frictions matter, and in which the interest rate is almost zero, we ask how powerful is a fiscal package at stimulating output. The analysis of government spending expansions in a zero nominal interest rate environment has been widely carried out in the literature. For instance, Christiano *et al.* (2009a) argue that the impact of government spending on output can be large when the zero lower bound on the nominal interest rate is binding. Indeed, the well-known crowding-out of investment by government spending is neutralized when the interest rate stays close to zero for an extended period of time. The reason behind this result lies on the fact that the rise in inflation, leaded by the fiscal stimulus, reduces the real interest rate when the nominal rate remains fixed. Erceg and Lindé (2010) reach the same conclusion but they also show that the size of the fiscal stimulus matters. Precisely, they argue that the fiscal multiplier is lower for massive government spending expansions.<sup>2</sup> Furthermore, Fernández-Villaverde (2010) assesses the impact of a fiscal stimulus in an economy featuring financial frictions. He concludes that credit market imperfections magnify the impact of government spending on output. The latter holds since,

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<sup>1</sup>For instance, Christiano *et al.* (2003) use a financial accelerator model to study the Great depression in the U.S. Gertler *et al.* (2007) employ a similar strategy for the Korean economy during the Asian financial crisis of 1997. Finally, Peersman and Smets (2005), using European industry data, show that the financial accelerator mechanism may explain the asymmetric effects of monetary policy during booms and recessions.

<sup>2</sup>Corsetti *et al.* (2010) show that the positive impact of the so-called “spending reversal” on output can be large when the zero lower bound is binding, once it is undertaken sufficiently late on the recovery path. Drautzburg and Uhlig (2010) shows that the size of the fiscal multiplier depends on the duration of the zero lower bound. Focusing on changes in policy regimes, Eggertsson (2008) argues that monetary and fiscal policy coordination can induce a shift in expectations, that helps the economy to recover when the short-term nominal interest rate is close to zero.

by increasing inflation, government spending indirectly ameliorates the balance sheet positions of firms, which in turn reduces the cost of credit.

Given these results, one may therefore suspect that the presence of financial frictions and of zero short-term interest rates would reinforce the final effects of a fiscal stimulus. The contribution of our paper is precisely to stress the role of credit market imperfections and a liquidity trap in amplifying the effects of a fiscal stimulus. To do so, we proceed in two steps. First, we evaluate the effects of a stimulus package on an economy featuring financial frictions and a zero nominal interest rate. The fiscal stimulus package that we consider consists of either an increase in government spending, or a cut in distortionary income taxes. The importance of financial imperfections and the zero-interest-rate floor on the efficiency of the fiscal policy (defined by the fiscal multiplier) is further investigated. Second, we focus on the role of several fiscal policy regimes on the spending multiplier. Precisely, we consider that there is a concern for public debt consolidation, characterized by different taxation rules that are set to pay back the debt. We also stress the impact of implementation lags in government spending on the fiscal policy efficiency.

To pursue our analysis, we use a financial accelerator model with nominal and real rigidities, plus a zero lower bound constraint on the nominal interest rate. Precisely, we refer to the financial frictions model of Bernanke, Gertler, and Gilchrist (2000), but we introduce nominal-denominated debt contracts as Christiano *et al.* (2009b) and Fernández-Villaverde (2010).<sup>3</sup> The latter introduces a new transmission device, which is similar to the debt-deflation channel that Fisher (1933) used to explain the worsening of economic conditions during the Great Depression.

Using this framework, a hypothetical recession is generated from a negative financial shock that drop the net worth of entrepreneurs and a shift from spending to saving by households (i.e., a preference shock). These shocks guarantee that the nominal interest rate hits the zero lower bound on impact.<sup>4</sup> After characterizing the economic dynamics under the zero lower bound constraint, we explore the role of different fiscal policies aimed at either: expanding government purchases, or cutting distortionary income taxes. We also pay attention to the size of the spending multiplier

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<sup>3</sup>The financial accelerator model *à la* Bernanke *et al.* (2000) have been extensively employed to emphasize the amplification effects of financial frictions (Gilchrist and Leahy, 2002; Meier and Müller, 2006; Faia, 2007; Christensen and Dib, 2008; among many others). More recently, some authors have investigated the relative importance of financial shocks over the business cycle by estimating the financial accelerator model (Christiano *et al.*, 2003-2009b; Fuentes-Albero, 2009; Gilchrist *et al.*, 2009; Nolan and Thoenissen, 2009; Queijo von Heideken, 2009).

<sup>4</sup>The combination of shocks allow us to bind the zero lower bound on impact without assuming unreasonable size of individual shocks. Reifschneider and Williams (2000), Schmitt-Grohe and Uribe (2007), Amano and Shukayev (2009), among others, argue that in order to bind the zero lower bound constraint, single shocks would have to be quite big with respect to what is usually estimated. Thus, the probability that the zero lower bound constraint binds with a single shock is very low.

according to different taxation policies and implementation lags in government spending.

We obtain several results. First, we find that an increase in government purchases has a fiscal multiplier greater than one for at least one year when the economy suffers from an important decrease in the business sector net worth and a shift from spending to saving. This is not only due to the duration of the zero lower bound for a sizable amount of time, like in Christiano *et al.* (2009a), but also to the presence of financial frictions. Indeed, government spending has an important impact on the external finance premium and investment, which originates from the debt-deflation channel (as in Fernández-Villaverde, 2010).

Second, we show that an income tax cut does not allow for a fiscal multiplier higher than one. Even worse, the fiscal multiplier can be negative in the short-run when financial frictions are omitted. This result can be explained as follows. Unlike a government spending expansion, a income tax cut is not characterized by a direct increase in aggregate demand. Therefore, its effect on output is smaller. Additionally, the income tax cut positively affects entrepreneurs' disposable income and gives them more incentives to invest. Consequently, the presence of financial frictions magnify the effect of an income tax cut on investment and output.

Third, our results highlight the importance of financial frictions when a government spending expansion is financed by distortionary taxes. In such a case, increasing government spending rises the tax burden, which negatively affects investment decisions and dampens the expansionary effects of fiscal policy. In the absence of financial frictions, the spending multiplier may become negative in the long-run. However, when credit market imperfections are important, the negative role of distortionary taxes may be overcome due to the financial accelerator mechanism and the debt-deflation channel .

And fourth, we emphasize the importance of lag implementation of a government spending policy in an economy featuring credit market imperfections. In a model without financial frictions, the implementation delays in fiscal policy decreases the size of the spending multiplier. However, the presence of the financial accelerator mechanism inverse this result, at least for the short-run. The reason behind this result is that, when the fiscal stimulus is announced in advance, economic agents expect higher inflation in the future, which reduces the risk premium and boost investment. The latter result depends again in whether the zero lower bound constraint is binding, that financial imperfections are important, and that the debt-deflation channel is active.

The rest of the paper is as follows. Section 2 introduces the model. Section 3 elaborates on its calibration and explains the solution method when the interest rate is equal to zero. Section 4 characterizes the responses of certain macroeconomic variables to the recessionary shocks that

depress the interest rate towards zero. Section 5 assess the importance of financial frictions and zero-interest-rate floor on the fiscal multipliers. Section 6 focus on the role of distortionary tax rules and implementation delays in the spending multiplier. Finally, some concluding remarks are offered.

## 2 The Model

The framework is based on a standard New-Keynesian model with real and nominal rigidities, which is enriched with frictions in the credit market *à la* Bernanke *et al.* (2000). The model is composed of households, firms, entrepreneurs, capital producers, a financial intermediary, a central bank, and a government.

Frictions in the credit sector arise from asymmetric information between the representative financial intermediary (lender) and entrepreneurs (borrowers). The lender must pay a monitoring cost in order to observe an entrepreneur's realized return, while the latter observe it for free. This arrangement results in a moral hazard problem, which can be partially solved by collateralizing the loan using the entrepreneur's net worth. Therefore, an increasing relationship between the external finance premium and net worth emerges, since relatively low levels of equity increases the entrepreneur's probability of default on a loan. During an economic downturn, asset prices fall and the balance sheet of entrepreneurs deteriorates, leading to an increase in the external finance premium which in turn discourages investment, decreasing assets prices even further. This shock amplification effect of the credit market corresponds to the financial accelerator mechanism. The model also features the so-called Fisher's debt-deflation channel, since we assume that debt contracts are denominated in nominal terms. Thus, an unexpected decrease in prices, driven by a recession, increases the value of real debt and reduces the net worth of entrepreneurs, which reinforces even further the decline in economic activity.

Following the New Keynesian literature, the model also features several real and nominal rigidities which have been found to be empirically relevant. For instance, we assume that there are habits in the consumption behavior of households, adjustment costs in investment, variable capital utilization, and that prices and wages are rigid and indexed to past inflation. These assumptions allows to mimic the observed inertia of inflation and output. Finally, our framework differs from the widespread monetary dynamic, stochastic general equilibrium model in one important way: the central bank's target nominal interest rate is leaded by the Taylor rule, but it is also constraint by its zero lower bound.

## 2.1 Households

**Preferences** The economy is inhabited by a continuum of differentiated households, indexed by  $i \in [0, 1]$ . A typical household selects a sequence of consumption, wages, and savings that are invested in a financial intermediary that pays the riskless rate of return. Households differ by the specific labor type they are endowed with, which gives them monopolistic power to set their own wage. Household  $i$ 's objective is to maximize her expected sequence of present and future utility flows given by

$$\mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \varepsilon_T \mathbb{U}(c_T - bc_{T-1}) - \mathbb{V}(\ell_{i,T}^h) \right\},$$

subject to the sequence of constraints

$$c_t + \frac{d_{t+1}}{R_t} \leq (1 - \tau_t)[w_{i,t} \ell_{i,t}^h + \text{div}_t] + \frac{d_t}{1 + \pi_t} + \Upsilon_t, \quad (1)$$

where  $\mathbb{E}_t$  is the expectation operator conditional to the information available in period  $t$ .  $\beta \in (0, 1)$  is the subjective discount factor,  $b \in [0, 1)$  is the habit parameter and  $\tau_t$  corresponds to the time-varying income taxes;  $c_t$  denotes real consumption;  $P_t$  is the price of final goods;  $w_{i,t} \equiv W_{i,t}/P_t$  and  $\ell_{i,t}^h$  denote the real wage and the labor supply of type- $i$  household's at period  $t$ ;  $1 + \pi_t = P_t/P_{t-1}$  represents the gross inflation rate;  $d_t \equiv D_t/P_{t-1}$ , where  $D_t$  denotes the nominal deposits carried over from period  $t - 1$ , and maturing in period  $t$ ;  $R_t$  denotes the riskless gross nominal interest rate;  $\text{div}_t$  denotes real profits redistributed by monopolistic firms; and  $\Upsilon_t$  denotes real lump-sum taxes adjusted according to a rule specified below.

In addition,  $\varepsilon_t$  denotes a preference shock which follows an autorregressive process of the form

$$\log(\varepsilon_t) = \rho_\varepsilon \log(\varepsilon_{t-1}) + \epsilon_{\varepsilon,t},$$

where  $\rho_\varepsilon \in (0, 1)$ , and  $\epsilon_{\varepsilon,t} \sim \text{iid}(0, \sigma_\varepsilon)$ .

The first order conditions with respect to  $c_t$  and  $d_{t+1}$  are

$$\varepsilon_t \mathbb{U}_c(c_t - bc_{t-1}) - \beta b \mathbb{E}_t \{ \varepsilon_{t+1} \mathbb{U}_c(c_{t+1} - bc_t) \} = \lambda_t. \quad (2)$$

$$\frac{\lambda_t}{R_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{1 + \pi_{t+1}} \right\}. \quad (3)$$

where  $\mathbb{U}_c(\cdot)$  denotes the derivative of  $\mathbb{U}(\cdot)$  w.r.t.  $c_t$ , and  $\lambda_t$  is the Lagrangian multiplier associated to the budget constraint. Equation (2) defines the marginal utility of consumption. Equation (3) is the risk free bond equation which establishes that the ratio of the marginal utility of future and current consumption is equal to the inverse of the real interest rate.

**Wage Setting** A typical household  $i$  acts as a monopoly supplier of type- $i$  labor. Following Erceg *et al.* (2000), we assume that the set of differentiated labor inputs, indexed by  $i \in [0, 1]$ , are aggregated into a single labor input  $\ell_t^h$  by a competitive labor intermediary. The latter produces the aggregate labor input according to the following CES technology

$$\ell_t^h = \left( \int_0^1 \left[ \ell_{i,t}^h \right]^{(\theta_w - 1)/\theta_w} di \right)^{\theta_w / (\theta_w - 1)}, \quad (4)$$

where  $\theta_w > 1$  is the elasticity of substitution between any two labor types. The maximization of profits by the representative labor intermediary yields typical labor demand functions

$$\ell_{i,t}^h = \left( \frac{W_{i,t}}{W_t} \right)^{-\theta_w} \ell_t^h, \quad (5)$$

where  $W_{i,t}$  denotes the nominal wage rate associated to type- $i$  labor.

Following Calvo (1983), it is assumed that at each point in time only a fraction  $1 - \alpha_w$  of the households can set a new wage, which will remain fixed until the next time period the household is drawn to reset its wage. Wages that are not allowed to be reoptimized at time  $t$  are indexed the most recently available inflation measure at degree  $\gamma_w \in (0, 1)$ . Since the household is a monopoly supplier, it internalizes the demand for its labor given by Equation (5) when setting its wage. Let  $W_{i,t}^*$  denote the nominal wage rate chosen by type- $i$  household at time  $t$ , and  $\ell_{i,t,T}^{h*}$  the hours worked in period  $T$  if type- $i$  household last re-optimized its wage in period  $t$ . Type- $i$  household selects  $W_{i,t}^*$  in order to maximize her expected lifetime utility. The first order condition is described by

$$\mathbb{E}_t \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} \ell_{i,t,T}^{h*} \left\{ \lambda_T \delta_{t,T}^w w_{i,t}^* - \mu_w \nabla_{\ell} \left( \ell_{i,t,T}^{h*} \right) \right\} = 0, \quad (6)$$

where  $\nabla_{\ell}(\cdot)$  denotes the derivative of  $\mathbb{V}(\cdot)$  w.r.t.  $\ell^h$ ,  $w_{i,t}^* \equiv W_{i,t}^*/P_t$  is the time  $t$  optimal real wage,  $\mu_w = \theta_w / (\theta_w - 1)$  denotes the wage mark-up and  $\delta_{t,T}^w$  equals  $\prod_{j=t}^{T-1} (1 + \pi)^{1 - \gamma_w} (1 + \pi_j)^{\gamma_w}$  when  $T > t$  and 1 otherwise. Let  $1 + \pi$  denote the steady-state inflation rate

## 2.2 Entrepreneurs

**Optimal Financial Contract** There are a continuum of risk neutral entrepreneurs, indexed by  $e \in [0, 1]$ . At time  $t$ , type- $e$  entrepreneur purchases at price  $Q_t$  the stock of capital  $\tilde{k}_{e,t+1}$  for use in  $t + 1$ . She finances her capital expenditures by her own internal resources and debt. Let  $n_{e,t+1}$  be the available real net worth of type- $e$  entrepreneur at the end of period  $t$  and  $b_{e,t+1}$  the amount of real debt asked to the financial intermediary (or lender). Accordingly,

$$q_t \tilde{k}_{e,t+1} = b_{e,t+1} + n_{e,t+1}, \quad (7)$$

where  $q_t \equiv Q_t/P_t$ . The lender is also risk neutral and obtains funds from households, who are promised to receive back their deposits plus interest earnings according to the riskless gross rate of return, which in real terms is given by  $r_t = R_t/E_t(1 + \pi_{t+1})$ . Following Bernanke *et al.* (2000), it is assumed that the ex-post gross return on capital for type- $e$  entrepreneur,  $r_{e,t+1}^k$ , is affected by an idiosyncratic disturbance, denoted by  $\omega_{e,t+1}$ . The latter is an i.i.d. random variable across time and types, with a continuous and once-differentiable c.d.f.,  $F(\omega)$ , over a non-negative support. Also, it is considered that  $\omega_{e,t}$  is unknown to both the entrepreneur and the lender prior to the investment decision, with  $E(\omega) = 1$  and  $V(\omega) = \sigma_\omega^2$ .

In the spirit of Townsend (1979), lenders bear a fixed monitoring cost in order to observe the borrowers' realized return, while borrowers observe it for free. This particular setting raises the problem of moral hazard, with borrowers possibly misreporting their realized returns to fake a bankruptcy. The monitoring cost is a proportion  $\mu \in [0, 1]$  of the realized gross payoff to the entrepreneur's capital, i.e.,  $\mu\omega_{e,t+1}r_{e,t+1}^kq_t\tilde{k}_{e,t+1}$ .

The type- $e$  entrepreneur chooses the value of his project's capital,  $q_t\tilde{k}_{e,t+1}$ , and the associated level of borrowing,  $b_{e,t+1}$ , prior to the realization of  $\omega_{e,t+1}$ . The optimal contract is characterized by a gross non-default loan rate,  $r_{e,t+1}^g$ , and a threshold value of the idiosyncratic shock, say  $\bar{\omega}_{e,t+1}$ , such as for values of  $\omega_{e,t+1}$  greater or equal than  $\bar{\omega}_{e,t+1}$ , the entrepreneur repays its debt at rate  $r_{e,t+1}^g$ . Thus,  $\bar{\omega}_{e,t+1}$  and  $r_{e,t+1}^g$  are defined by

$$\bar{\omega}_{e,t+1}r_{e,t+1}^kq_t\tilde{k}_{e,t+1} = r_{e,t+1}^gb_{e,t+1}.$$

In the case where  $\omega_{e,t+1} < \bar{\omega}_{e,t+1}$ , the entrepreneur declares bankruptcy and the lender pays the monitoring cost to audit the entrepreneur, and keeps all of the borrower's realized returns. The lender participates in the contract as long as he is assured to receive an expected loan return equal to the opportunity costs of its funds. Since it is assumed that the lender can perfectly diversify the risk associated with the loan, its relevant opportunity cost is represented by the economy riskless rate  $r_t$ .

A detailed description of the financial contract is provided in the technical appendix, available upon request. Bernanke *et al.* (2000) also offer a detailed treatment of the lender problem. However, two assumptions need to be highlighted. First, since the lender and all entrepreneurs are risk neutral and households are risk averse, all systematic risk along with the expected defaulting cost is passed through the lenders and absorbed by entrepreneurs. And second, it is assumed that entrepreneurs have a linear utility in consumption and are subject to similar aggregate shocks, which allows for the aggregation of the financial contract terms, such as  $r_{e,t+1}^k = r_{t+1}^k$  for all  $e$ , and  $\tilde{k}_{t+1} = \int \tilde{k}_{e,t+1}de$ .

More importantly, the aggregation allows for an identification of a common value for the financial threshold  $\bar{\omega}$ , such as  $\bar{\omega}_{e,t+1} = \bar{\omega}_{t+1}$  for all entrepreneurs.

Let  $\check{r}_t = E_t\{r_{t+1}^k/r_t\}$  be the expected discounted return on capital. The optimal lending contract consists in choosing  $\tilde{k}_{t+1}$  and  $\bar{\omega}_{t+1}$  in order to maximize the entrepreneurs expected returns subject to the participation constraint of the lender. The first order conditions of the above problem imply that, in equilibrium, the discounted return on capital will be equal to the marginal cost of external finance, i.e.

$$\check{r}_t = x\left(\frac{q_t \tilde{k}_{t+1}}{n_{t+1}}\right), \quad (8)$$

with  $x'(\cdot) > 0$ , for  $n_{t+1} < q_t \tilde{k}_{t+1}$ . The intuition behind function  $x(\cdot)$  is quite simple: All else equal, the cost of external finance should increase whenever the leverage ratio,  $q_t \tilde{k}_{t+1}/n_{t+1}$ , increases. This is so, since the entrepreneurs' net worth (or collateral) is relatively low, which in turn increases the probability of default. Consequently, the lender ask to be compensated with a higher cost of borrowing in order to participate in the contract. This is the key feature of the financial accelerator model, and allow us to treat the expected discounted return on capital,  $\check{r}_t$ , as the external finance premium.

**Entrepreneurs in General Equilibrium** Type-*e* entrepreneur, that owns the stock of capital  $\tilde{k}_{e,t}$ , provides capital services  $k_{e,t}$  to intermediate firms, according to

$$k_{e,t} = u_{e,t} \tilde{k}_{e,t},$$

where  $u_{e,t} > 0$  is the individual rate of capital utilization. At the beginning of period  $t$ , after observing all the shocks, entrepreneurs choose how intensively to use their capital. They rent capital services to intermediate firms, and once goods have been produced, they sell to the capital producer the remaining un-depreciated stock of capital. Thus, the gross return to holding a unit of capital from  $t - 1$  to  $t$  can be written as

$$r_{e,t}^k \equiv \frac{(1 - \tau_t)u_{e,t}z_t + (1 - \delta(u_{e,t}))q_t}{q_{t-1}}. \quad (9)$$

where  $z_t$  is the real payment for capital services taxed at a rate of  $\tau_t$  per cent at date  $t$ , and  $\delta(u) \in [0, 1]$  is a convex depreciation function around the steady-state. Similar to Queijo von Heideken (2009), we consider a function with  $\delta(0) = 0$ ,  $\lim_{u \rightarrow \infty} \delta(u) = 1$ , and in the steady-state  $\delta(1) = \delta$ . Entrepreneurs choose the rate of capital utilization by maximizing Equation (9) with

respect to  $u_{e,t}$ . Notice that, since  $z_t$  and  $q_t$  are aggregate prices, all entrepreneurs will choose exactly the same rate of capital utilization, independently of their own capital holdings. Thus,

$$r_{e,t}^k = r_t^k \text{ and } u_{e,t} = u_t \quad \forall e,$$

such as

$$z_t = \delta'(u_t)q_t.$$

Following Bernanke *et al.* (2000) and Carlstrom and Fuerst (1997), entrepreneurs participate in the general labor market by supplying one unit of labor every period, earning the nominal wage  $W_t^e$ . Finally, each entrepreneur has a random probability of exit the economy of  $1 - \gamma_t$ . This captures the idea that entrepreneurs cannot accumulate enough wealth to be fully self-financed.<sup>5</sup> A decrease in the value of  $\gamma_t$  reduces the aggregate net worth, since more entrepreneurs leave the economy. The aggregate net worth falls in any case, because new entrepreneurs begin with a zero net worth. We assume that the parameter  $\gamma_t$  follows the process

$$\log(\gamma_t) = \rho_\gamma \log(\gamma_{t-1}) + (1 - \rho_\gamma) \log(\gamma) + \epsilon_{\gamma,t},$$

where  $\rho_\gamma \in (0, 1)$ , and  $\epsilon_{\gamma,t} \sim \text{iid}(0, \sigma_\gamma)$ . Christiano *et al.* (2009b) interpret variations in  $\gamma_t$  as movements in the value of assets that are not obviously linked to movements in fundamentals. Nolan and Theonissen (2009) appeal to the former Federal Reserve chairman Alan Greenspan's remark about "irrational exuberance", concerning the stock market boom in the U.S in 1996: a drop in  $\gamma_t$ , then, can be viewed as a decrease in the the value of entrepreneurs' assets, which will have spill over effects on the credit market, the external finance premium, and the capital market. The aggregate real net worth of entrepreneurs at the end of period  $t$ ,  $n_{t+1}$ , is given by

$$n_{t+1} = \gamma_t v_t + (1 - \tau_t) w_t^e \tag{10}$$

where  $(1 - \tau_t)w_t^e$  denotes the after-tax real wage earned by entrepreneurs and  $v_t$  equals gross revenues from capital holdings from  $t - 1$  to  $t$  less borrowing repayments (i.e. the entrepreneurs' equity)

$$v_t = r_t^k q_{t-1} \tilde{k}_t (1 - \mu G(\bar{\omega}_t)) - r_{t-1} (q_{t-1} \tilde{k}_t - n_t) \tag{11}$$

Entrepreneurs that fail in  $t$ , consume the residual net worth such as  $c_t^e = (1 - \gamma_t)\varrho v_t$ , where the complementary fraction  $(1 - \varrho)$  is transferred, in lump-sum taxes to households.

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<sup>5</sup>It is assumed, though, that the rate of birth of entrepreneurs equals its mortality rate, in order to keep constant the number of entrepreneurs.

### 2.3 Capital Producer

At the end of period  $t$ , a competitive capital producer combines the existing capital stock of period  $t$  with final goods, which denotes aggregate investment  $i_t$ , in order to produce the stock of capital to be used in period  $t + 1$ , i.e.  $\tilde{k}_{t+1}$ . The capital producer buy the available capital stock at the end of each period, and he sell it back, after producing the new capital, to the entrepreneurs at price  $Q_t$ . Following Christiano *et al.* (2009b), we assume that the capital producer faces increasing investment adjustment costs denoted by  $\Phi(i_t/i_{t-1})$ , where  $\Phi(\cdot)$  is an increasing and concave function and  $\Phi(0) = 0$ . The problem of the capital producer is as follows

$$\max_{i_t} \mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \lambda_T \left\{ Q_T \left[ (1 - \delta(u_T)) \tilde{k}_T + \left[ 1 - \Phi \left( \frac{i_T}{i_{T-1}} \right) \right] i_T \right] - Q_T (1 - \delta(u_T)) \tilde{k}_T - P_T i_T \right\}.$$

The inclusion of adjustment costs allow for a variable price of capital, which in turn will contribute to the volatility in entrepreneurial net worth. In equilibrium, the relative price of capital,  $q_t \equiv Q_t/P_t$ , is given by

$$q_t = \left[ 1 - \Phi \left( \frac{i_t}{i_{t-1}} \right) + \mathbb{E}_t \left\{ \Phi' \left( \frac{i_{t+1}}{i_t} \right) \left[ \frac{i_{t+1}}{i_t} \right]^2 \right\} \right]^{-1}, \quad (12)$$

and the aggregate capital stock evolves according to

$$\tilde{k}_{t+1} = (1 - \delta(u_t)) \tilde{k}_t + \left[ 1 - \Phi \left( \frac{i_t}{i_{t-1}} \right) \right] i_t. \quad (13)$$

### 2.4 Final Goods Producers

The final good,  $y_t$ , used for consumption and investment, is produced in a competitive market by combining a continuum of intermediate goods indexed by  $j \in [0, 1]$ , via the CES production function

$$y_t = \left( \int_0^1 y_{j,t}^{\frac{\theta_p-1}{\theta_p}} dj \right)^{\frac{\theta_p}{\theta_p-1}}, \quad (14)$$

where  $y_{j,t}$  denotes the overall demand addressed to the producer of intermediate good  $j$  and  $\theta_p$  is the elasticity of demand for a producer of intermediate good. The maximization of profits yields the typical demand function

$$y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_p} y_t, \quad (15)$$

where  $P_{j,t}$  denotes the price of intermediate good produced by firm  $j$ .

### 2.5 Intermediate Good Sector

**Production Function** Type- $j$  intermediate firms produce differentiated goods by assembling services of labor and capital, namely  $\ell_{j,t}$  and  $k_{j,t}$ , respectively. Capital services are rented from

the entrepreneur which owns the capital stock. Type- $j$  firm's total labor input,  $\ell_{j,t}$  is composed by household labor,  $\ell_{j,t}^h$ , and entrepreneurial labor,  $\ell_{j,t}^e$ , according to

$$\ell_{j,t} = [\ell_{j,t}^h]^\Omega [\ell_{j,t}^e]^{1-\Omega}.$$

Type- $j$  intermediate good is produced with the following constant return to scale technology

$$y_{j,t} = \ell_{j,t}^{1-\alpha} k_{j,t}^\alpha. \quad (16)$$

Each monopolistic firm chooses capital and labor services in order to minimize real cost subject to the production function (16), taking  $w_t$ ,  $w_t^e$  and  $z_t$  as given. Accordingly, labor and capital demands satisfy

$$w_t = s_t \Omega (1 - \alpha) \frac{y_t}{\ell_{j,t}}, \quad (17)$$

$$w_t^e = s_t (1 - \Omega) (1 - \alpha) \frac{y_t}{\ell_{j,t}^e}, \quad (18)$$

$$z_t = s_t \alpha \frac{y_t}{k_{j,t}}. \quad (19)$$

where  $s_t$  is the the real marginal cost associated to capital-labor inputs.

**Price Setting** In each period of time, type- $j$  monopolistic firm's price setting decision is modeled through the Calvo's (1983) staggering mechanism. In each period, a firm faces a constant probability,  $1 - \alpha_p$ , of being able to re-optimize its price. Firm  $j$  takes the demand function (15) into account when setting its price. Additionally, it takes into account the fact that this price rate will presumably hold for more than one period. If the firm cannot re-optimize its price, the latter is re-scaled is indexed to the most recently available inflation measure at a degree  $\gamma_p \in (0, 1)$ .

Let denote  $P_{j,t}^*$ , the nominal price chosen in time  $t$  and  $y_{j,t,T}^*$ , the demand for good  $j$  in period  $T$  if firm  $j$  last reoptimized its price in period  $t$ . Therefore, firm  $j$  selects  $P_{j,t}^*$  so as to maximize the present discounted sum of profit streams. The first order condition is given by

$$\mathbb{E}_t \sum_{T=t}^{\infty} (\beta \alpha_p)^{T-t} \lambda_T \frac{y_{j,t,T}^*}{P_{j,t}^*} \left\{ \frac{\delta_{t,T}^p P_{j,t}^*}{1 + \pi_{t,T}} - \mu_p s_t \right\} = 0, \quad (20)$$

where  $p_{j,t}^* \equiv P_{j,t}^*/P^*$ ,  $1 + \pi_{t,T} \equiv P_T/P_t$ ,  $\mu_p \equiv \theta_p/(\theta_p - 1)$  denotes the mark-up of the monopolistic firm, and  $\delta_{t,T}^p$  equals  $\prod_{j=t}^{T-1} (1 + \pi)^{1-\gamma_p} (1 + \pi_j)^{\gamma_p}$  when  $T > t$  and 1 otherwise.

## 2.6 Monetary Policy

The nominal interest rate follows a Taylor rule whenever such rule prescribes a non-negative level for the central bank's target interest rate. If this is not the case, then the central bank simply fixes

its target rate equal to zero. Following Reifschneider and Williams (2000) and Bodenstein *et al.* (2009), we introduce the concept of a notional nominal interest rate,  $R_t^{not}$  in gross terms, which is subject to the following rule

$$\frac{R_t^{not}}{\bar{R}} = \left( \frac{R_{t-1}^{not}}{\bar{R}} \right)^{\rho_R} \left[ \left( \frac{1 + \pi_t}{1 + \pi} \right)^{a_\pi} \left( \frac{y_t}{y_t^f} \right)^{a_y} \right]^{1 - \rho_R}, \quad (21)$$

where  $\rho_R \in (0, 1)$  denotes the interest rate smoothing parameter;  $a_\pi$  describes the elasticity of  $R_t^{not}$  to the deviation of inflation from its steady-state value,  $\pi$ ; finally,  $a_y$  is the elasticity of  $R_t^{not}$  to the output gap, with  $y_t^f$  denoting the output level that would prevail in the absence of nominal rigidities. Notice that  $\bar{R}$  denotes the steady-state level of the gross nominal interest rate, determined by  $(1 + \pi)\beta^{-1}$ .

Let  $R_t$  denote the actual short-term interest rate implemented by the central bank. Thus, the latter chooses its instrument according to

$$R_t = \max(1, R_t^{not}). \quad (22)$$

## 2.7 Fiscal Policy

The fiscal authority purchases consumption goods,  $g_t \equiv G_t/P_t$ , where  $G_t$  denotes government's consumption goods expressed in nominal terms. It also raises distortionary taxes ( $\tau_t$ ) and lump-sum taxes ( $\Upsilon_t$ ), and issues debt,  $D_{t+1}$ , consisting of one-period nominal discount bonds. The period budget constraint of the government reads as follows

$$\frac{D_{t+1}}{R_t} = D_t + P_t [g_t - \Upsilon_t - \tau_t y_t].$$

Letting  $\hat{d}_t \equiv (d_t - d)/y$ , the government's budget can be approximated as

$$\beta \hat{d}_{t+1} = \hat{d}_t + \gamma_g \hat{g}_t - \gamma_\Upsilon \hat{\Upsilon}_t - \tau(\hat{\tau}_t - \hat{y}_t),$$

where  $\gamma_g$ ,  $\gamma_\Upsilon$ , and  $\tau$  denote the steady state government-spending-to-output ratio, the lump-sum-taxes-to-output ratio, and the income tax rate, respectively. To select amongst the set of feasible time path, and to stabilize government debt, lump-sum taxes (or transfers) follows the rule

$$\hat{\Upsilon}_t = \tilde{a}_d \hat{d}_t + \tilde{a}_g \hat{g}_t, \quad (23)$$

where  $\tilde{a}_b$  and  $\tilde{a}_g$  are the elasticities of lump-sum taxes with respect to government debt and government spending, respectively. In addition, we assume that deviation of the income tax rate may

have three different patterns:

$$\tau\hat{\tau}_t = \begin{cases} 0 & \text{so } \tau_t = \tau \\ \chi_d\hat{d}_t + \chi_y\hat{y}_t & \text{Pro-cyclical taxation} \\ \rho_\tau\tau\hat{\tau}_{t-1} + \epsilon_{\tau,t} & \text{Fiscal stimulus} \end{cases} \quad (24)$$

where  $\chi_d > 0$ ,  $\chi_y > 0$  are the elasticities of income taxes with respect to government debt and output, respectively; while  $\rho_\tau$  defines a persistence parameter. The first specification simply defies that the income tax rate is constant. The second specification indicates that the income tax rate is pro-cyclical and that it is adjusted to public debt deviations. Finally, the third specification appeals to a fiscal policy aimed at affect economic activity. We will use this specification when we study a fiscal stimulus based on an income taxed cut. Let  $deficit_t$  denote the primary deficit of the government defined by  $deficit_t = \gamma_g\hat{g}_t - \gamma_Y\hat{Y}_t - \tau(\hat{\tau}_t - \hat{y}_t)$ . We assume that the steady-state of debt is zero ( $d = 0$ ) which implies that the primary budget is balanced at the steady-state. Finally, we assume that government expenditures evolve exogenously according to a ARMA( $p, q$ ) process

$$A(L)\hat{g}_t = B(L)\epsilon_{g,t}$$

where  $\epsilon_{g,t}$  is the government spending shock. The  $A(L)$  lag-polynomial allow us to model the percent deviations of government spending either as strict monotonic process (like an  $AR(1)$ ), or as a hump-shaped sequence (as an  $AR(2)$ ). The  $B(L)$  lag-polynomial allow to model implementation delays in government spending. We provide further details about the sequence of government spending in the following sections.

## 2.8 Resource Constraint and Equilibrium

The production of the final good is allocated to investment, total private consumption by households and entrepreneurs, public spending, and to monitoring costs paid by lenders

$$y_t = i_t + c_t + c_t^e + g_t + \mu G(\bar{\omega}_t)r_t^k q_{t-1}\tilde{k}_t.$$

In the symmetric equilibrium, all entrepreneurs, households and firms are identical and make the same decisions. In addition, equilibrium on the labor market yields  $\int_0^1 \ell_{j,t} dj = \ell_t^h$ . The symmetric equilibrium is characterized by an allocation  $\{y_t, c_t, i_t, \ell_t, k_t, \tilde{k}_t, n_t\}$  and a sequence of price and co-state variables  $\{\pi_t, r_t, r_t^k, q_t, \pi_t^w, z_t, \lambda_t\}$  that satisfy the optimally conditions in each sector, the monetary policy rule, and the stochastic shocks.

## 3 Methodology

### 3.1 Calibration

The model parameters are calibrated to fit the quarterly frequency. Table 1 describes the calibrated values for parameters related to households, firms, and economic authorities.

[ insert Table 1 here ]

The subjective discount factor,  $\beta$ , is set to 0.99, which entails a annual real interest rate of 4 per cent. The Frisch elasticity of labor supply,  $\varpi_w^{-1} \equiv \nabla_\ell / (\ell \nabla_{\ell\ell})$ , is set to unity. The degree of habit consumption,  $b$ , is set to 0.63, while the inverse of the inter-temporal elasticity of substitution,  $\sigma$ , is set to 0.2. All these values are set by Christiano *et al.* (2009b).

Regarding production, the capital share in the intermediate sector,  $\alpha$ , is set to 0.35; the depreciation capital rate,  $\delta$ , is equals 0.03, as in Christiano *et al.* (2009a); the investment adjustment cost,  $\varkappa \equiv \Phi''(1)$ , is calibrated to 5.86, following Smets and Wouters (2005); the elasticity of the utilization rate of capital,  $\vartheta_u \equiv u\delta''(u)/\delta'(u)$ , is calibrated to  $0.31^{-1}$ , similar to Queijo von Heideken (2008). Concerning price setting, we assume that the elasticity of substitution between intermediate goods,  $\theta_p$ , is set to 11, which implies a price mark-up of 10 per cent. Similarly, the elasticity of substitution between labor types,  $\theta_p$ , is set to 21, which translates into a wage mark-up of 5 per cent. The degrees of price and wage rigidities,  $\alpha_p$  and  $\alpha_w$ , are set equal to 0.67 and 0.68 respectively, implying that the average durations between price or wage re-optimization are half of a year. Price and wage indexation parameters,  $\gamma_p$  and  $\gamma_w$ , are set to 0.75 and 0.70, respectively. All these values have been estimated by Christiano *et al.* (2009a). The steady-state inflation,  $\pi$ , is equal to 0.

Table 2 shows calibrated values of parameters related to the financial sector, which are taken from Bernanke *et al.* (2000)'s results.

[ insert Table 2 here ]

The share of entrepreneurial wages in terms of income is set to 0.01, implying a value of  $\Omega = 0.9846$ . The steady-state share of capital investment that is financed by the entrepreneur's net worth,  $x = \tilde{k}/n$ , is calibrated to 2, meaning that the steady-state leverage ratio amounts to 50 per cent. The steady-state external finance premium,  $\tilde{r} = r^k/r$ , is set to  $1.02^{0.25}$ , corresponding to an annual risk spread of 200 basis points, equal to the sample average spread between the business prime lending rate and the three-month Treasury bill rate. Finally, the annual business failure rate,  $F(\bar{\omega})$ , is set to 3 per cent. It is assumed that the idiosyncratic productivity shock,  $\omega_t$ , has a log-normal distribution with positive support, and an unconditional expectation equal to 1. These

moments help to determine the steady-state survival probability of entrepreneurs,  $\gamma$ , which is set to 0.98, the monitoring costs to realized payoffs ratio,  $\mu$ , which amounts to 0.12, the steady-state variance of the entrepreneurs' idiosyncratic shock,  $\sigma_\omega$ , which is equal to 0.28, and the steady-state idiosyncratic threshold is set to 0.50.<sup>6</sup>

Table 3 shows calibrated values of parameters for monetary and fiscal policies.

[ insert Table 3 here ]

When it comes to the monetary policy, the interest rate smoothing parameter,  $\rho_R$ , is calibrated to 0.88; the elasticity of the notional interest rate with respect to inflation,  $a_\pi$ , is set to 1.85; and the elasticity of the interest rate with respect to the output gap,  $a_y$ , is set to 0.313/4. These values follow once more the estimations of Christiano *et al.* (2009a). The steady-state share of government purchases in total output is calibrated to 0.19, which corresponds to the last decade historical average. When it comes to the fiscal policy, the elasticity of lump-sum taxes with respect to debt and government spending are set to 0.33 and 0.10, respectively. These values corresponds to Galí *et al.*'s estimates (2007). The steady-state income tax rate is set at 0.30, as suggested by Bilbiie *et al.* (2008) and income taxes are set to be constant in the benchmark calibration ( $\psi_y = \psi_d = 0$ ).

### 3.2 Zero Lower Bound: Solution Strategy

The zero lower bound constraint, described in Equation (22), introduces an important non-linearity into the system. Had this constraint not appeared, we could proceed to analyze the dynamics of the economy using the linear rational expectations solution that can be derived from the above system. In fact, all the model equations can be linearized except for the nominal interest rate, which imposes different dynamics depending on whether the ZLB constraint is binding or not. We follow the piecewise-linear approach described in Bodenstein *et al.* (2009) to solve for the model dynamics, which is numerically equivalent to the method employed by Eggertson and Woodford (2003). In specific, we linearized all model equations around its non-stochastic steady-state, except for the monetary policy representation.<sup>7</sup> We then assume that an exogenous shock hits the economy and depress the nominal interest rate so that the zero lower bound is reached at period 1, and remains

<sup>6</sup>In technical terms,  $\bar{\omega}$ ,  $\sigma_\omega$ ,  $\gamma$  and  $\mu$  are chosen so as to satisfy the following system of steady-state equations:

$$\begin{aligned} F(\bar{\omega}) &= 0.03/4; \quad x = 1 + \Gamma_\omega(\bar{\omega})[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]/[1 - \Gamma(\bar{\omega})[\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})]], \\ (x-1)/x\bar{r} &= \Gamma(\bar{\omega}) - \mu G(\bar{\omega}); \quad n = \gamma[nr^k x[1 - \mu G(\bar{\omega})] - rn(x-1)] + (1-\tau)w^e. \end{aligned}$$

<sup>7</sup>The log-linear model is described in the appendix.

in place for  $T$  periods. The zero lower bound horizon  $T$  is determined by the time- $T$  value of the notional interest rate, which must satisfy the following condition

$$R_T^{not} < 1 \leq R_{T+1}^{not}.$$

In terms of percentage deviation from the steady-state, the latter becomes  $\hat{R}_T^{not} < -R \leq \hat{R}_{T+1}^{not}$ , where  $R = 1/\beta$ , which is the steady-state level of the gross nominal interest rate. According to the calibration of the preceding section, this equals 4 per cent in annual terms. The piecewise linear system is thus conformed with two different dynamic structures: First, the dynamics when the ZLB is binding and the interest rate is equal to  $-R$ ; and, second, the dynamics when the ZLB is not binding and the Taylor rule is operating. The second structure imposes the determinacy conditions of the whole system and is solved using the AIM implementation (see Anderson and Moore, 1985) of the Blanchard and Kahn (1980) method for linear rational expectations models. The dynamics with the ZLB are derived using backward-induction, and are deterministic, in the sense that people make their decisions knowing that in period  $T + 1$  the interest rate will follow the Taylor rule path. Bodenstein *et al.* (2009) present a detailed description of this solution algorithm.

## 4 Model's Dynamics under the Zero Lower Bound

The purpose of this section is to analyze the model's dynamics under a severe recession, in which the nominal interest rate is forced to stay at its zero bound for a sizable amount of time. The ingredients of the recession are a temporal 10 per cent decrease in the entrepreneurs' survival probability (such as  $\gamma_t$  falls from 0.98 to 0.88), and a negative preference shock,  $\varepsilon_t$ . We choose to combine these shocks for two reasons. First, the presence of the two disturbances increases the probability that the interest rate hit its zero bound. Schmitt-Grohe and Uribe (2007), among others, argue that in order to bind the zero lower bound constraint, single shocks would have to be quite big with respect to what is usually estimated. Thus, we can argue that, in reality, it is quite improbable that the nominal interest rate would be equal to zero due to the occurrence of a single shock. Second, a negative preference shock has an particular feature: it reflects an increase in the household's desire to save, which in turn increases the demand of bonds and puts downward pressure on consumption, output, and the nominal interest rate. Bodenstein *et al.* (2009), Christiano *et al.* (2009b), or Uhlig and Drautzburg (2010) use shocks with these characteristics to induce the zero lower bound in their analyses.<sup>8</sup> On the other hand, one may argue that a shift from household spending to saving

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<sup>8</sup>Uhlig and Drautzburg (2010) assume that there is an interest rate spread shock between the risk-free rate and the rate perceived by households. A shock of this kind has exactly the same effects than a preference shock. Not

typically happens when the consumer confidence index (or consumer sentiment) drops, which is likely to happen during a severe economic downturn.

Figure 1 compares the IRF of macroeconomic variables in response to combined negative shocks on the net worth and preferences, when the zero lower bound constraint is imposed (represented by the solid line) and when it is not (represented by the dash line). As suggested by Christiano *et al.* (2009), the autoregressive coefficient of the survival probability shock,  $\rho_\gamma$ , is calibrated to 0.561, and the autoregressive coefficient of the preference shock,  $\rho_\varepsilon$ , is calibrated to 0.903.

[ insert Figure 1 here ]

Without imposing the zero lower bound constraint, the model predicts that the nominal interest rate will be negative for a sizable amount of time. The decrease in the entrepreneurs' survival probability,  $\gamma_t$ , deteriorates the aggregate net worth. But the final effect on this variable can be explained by two factors. First, after the shock, the demand for capital is lower, which drives the price of capital down for a year, making the drop in the net worth more severe. And second, the decrease in inflation the real debt burden of entrepreneurs. Thus, entrepreneurs equity also decrease. The sharp drop in the net worth is then translated into an increase in the risk premium, which in turn depresses investment even more. The slowdown of the economy is also driven by a reduction in consumption, since households try to save more. Since the decrease in inflation is higher than the decrease in the nominal interest rate, the real interest rate rise at impact, which reinforces the drop in investment and consumption. The response of output features a convex hump-shaped pattern that peaks in the third quarter.

When the zero lower bound constraint is imposed, the economy experiences a stronger downturn. This is so because the central bank cannot further decrease the interest rate in order to stop inflation from falling. As it can be appreciated, the nominal interest rate binds its zero bound for 17 quarters, given that the central bank follows the monetary rule described in Equations (21) and (22). Although this might be seen excessive, it does not look unrealistic. Japan's call rate has been between 0 and 0.50 since October 1995. For the case of the U.S., Rudebusch (2009) computes a Taylor rule that fits very well the recent behavior of the fed funds rate. Such rule anticipates that the fed funds would remain for several years at its zero lower bound, and not only some months.

Since the nominal interest rate stays equal to zero, it cannot dampen as before the decrease in output, investment, and capital. As a consequence, these variables drop by more. This is due to

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only because both shocks appear in the Euler equation in a similar way, but because both of them create an increase in the desire to save inducing a drop in consumption.

the increase in the real interest rate, which is stronger than before. In turn, the cost of borrowing rises due to the increase in the external finance premium. The latter motivated by a sharp fall in net worth of entrepreneurs. The credit conditions in the zero lower bound scenario are even worse, providing poor incentives to invest. This explains the magnified increase in output. When it comes to inflation, it also displays more volatility when the zero lower bound is binding. Notice also that the nominal interest rate responds strongly after the zero lower bound period is over. This is so, because the interest rate rule tries to offset the strong increase in inflation from the recovery period.

## 5 Effects of a Fiscal Stimulus under a Financial Turmoil

The previous section shows that a recession that results from a strong drop in the net worth and a negative preference shock is deeply pronounced under the zero lower bound. We seek to highlight up to what extent can the fiscal stimulus help the economy to recover from a financial turmoil. A fiscal stimulus is illustrated by two types of policy: the government decides to either increase its public expenditures or to decrease the income tax rate. To pursue our analysis, we proceed in two steps.

First, we assess the model's dynamics resulting from an exogenous change in one of these two instruments, given that the financial recession drives the nominal interest rate to its zero-floor. The baseline scenario corresponds to the economy that suffers from a deep recession, as presented in the previous section. It is represented by the solid line in Figure 2. The dash line represents the case where government temporarily rises its purchases. The dotted line represents the case where the income tax rate temporarily decreases.

Second, for each fiscal shock, we quantitatively investigate the factors that modify their efficiency on output. Precisely, we determine how the fiscal multiplier is altered by the presence of credit market imperfections and a zero nominal interest rate. The net effect of fiscal policy is measured by subtracting the IRFs of the baseline scenario to the IRFs of the scenario with the fiscal stimulus. Let  $\hat{x}_t^0$  denote the response at time  $t$  of variable  $x$ , in terms of percent deviations from the steady-state, given the baseline recession scenario without the fiscal stimulus. Similarly, let  $\hat{x}_t^{fis}$  denote the response of the same variable with the fiscal stimulus. Here again, a stimulus is characterized either by an increase in government spending or an income tax cut. The net effect of fiscal expansion on  $x$  is given by  $\hat{x}_t^{net} \equiv \hat{x}_t^{fis} - \hat{x}_t^0$ . The quarter  $k$  net fiscal multipliers are thus given by

$$\frac{dy_{t+k}}{dg_t} \equiv \frac{1}{\gamma_g} \frac{\hat{y}_{t+k}^{net}}{\hat{g}_t^{net}} \quad \text{and} \quad \frac{dy_{t+k}}{d\tau_t y_t} \equiv -\frac{1}{\tau} \frac{\hat{y}_{t+k}^{net}}{(\hat{y}_t^{net} + \hat{\tau}_t^{net})}.$$

Figure 3 compares the value of the net fiscal multipliers under different configurations. The bench-

mark configuration corresponds to the economy under recession stimulated with an expansionary fiscal shock, in which the nominal interest rate binds its zero lower bound and financial frictions are present. The upper panel of the figure corresponds to the spending multiplier while the lower panel depicts the income tax multiplier.

## 5.1 Government Spending Expansion

We first assess the impact of an increase in government spending during a deep recession. We assume that government spending follows an AR(1) process, with a persistence coefficient equal to  $\rho_g = 0.945$ , as suggested by Christiano *et al.* (2009b). Since in the baseline scenario the recession is particularly deep, for the sake of simplicity we assume an important increase in government spending (with the later jumping from 19 per cent of GDP, its steady-state value, to 23 per cent of GDP at the impact period). Further, we assume that government spending increases at the same time that the recessionary shock hit the economy. In the next section, we relax this assumption to allow implementation delays in fiscal policy.

### 5.1.1 Model's dynamics

Figure 2 compares the IRFs of an economy that suffers from a deep recession with and without expansion in government spending.

[ insert Figure 2 here ]

It appears that a government spending expansion can potentially shorten the duration of the zero lower bound. In this example, government spending induces the nominal interest rate to become positive after 16 periods, instead of 17 quarters as in the baseline case. Regarding output, the fiscal stimulus effectively reduces the drop in production. However, as the effects of the stimulus fade out, output eventually converges towards its baseline path in the absence of the stimulus. The rise in debt-to-GDP ratio is financed by an increase in lump-sum taxes (equivalently a reduction in lump-sum transfers). The public deficit increases at impact, due to both the raise in government spending and the decrease in income tax revenues that follows the recession.

Perhaps the most emblematic effect of government spending in the model is its impact on the external finance premium and the rest of the financial sector. An increase in government spending expands aggregate demand, which translates into a lesser decrease of inflation with respect to the baseline scenario. As noticed earlier, a lower drop in inflation implies that entrepreneurs' real debt increases by less, due to the Fisher's debt-deflation channel. The latter has a positive effect on

the real capital gains of entrepreneurs, which collaborates to the lower decrease on the net worth. This reduces the moral hazard problem in the financial sector (since the collateral is greater in the fiscal stimulus scenario), which finally induces a smaller increase in the external finance premium. A similar explanation can be found in Fernández-Villaverde (2010). In addition, since the interest rate is constrained by its zero lower bound, investment is cheaper under the fiscal stimulus scenario than in the baseline one. Thus, investment does not fall as much when the fiscal stimulus is in place.

When it comes to consumption, Figure 2 does not show a sizable impact of government spending. On the one hand, the lesser decrease in output might also help consumption to fall by less. On the other hand, the economy is subject to the Ricardian equivalence, and thus an increase in government spending crowds-out consumption. These two effects offset each other in the final response of consumption, yielding the apparent lack of changes in this variable.

To conclude, the Fisher’s debt-deflation channel is a main component to explain the effects of government spending on output and investment. In addition, the crowding-out effect on investment following an positive government spending shock is lessened by the presence of the zero lower bound constraint on the nominal interest rate. To disentangle the role of these two factors, regarding the effect of a government spending expansion on output, we resort to the spending multiplier.

### 5.1.2 Determinants of the Spending Multiplier

The upper panel of Figure 3 displays the determinants of the government spending multiplier.

[ insert Figure 3 here ]

In the benchmark configuration, when the ZLB constraint and financial frictions are present, it appears that output increases by slightly more than a one-by-one basis to government spending. In fact, in the first three quarters the spending multiplier is larger than one, with a value of 1.14 at impact and a peak value of 1.16 in the second quarter. Two years after the stimulus, the fiscal multiplier reaches 0.69. The intuition behind this result is given by the effect of the fiscal stimulus on the external finance premium, and the fact that the nominal interest rate reacts only in period 16.

**Role of the Zero Lower Bound** One may ask if the benchmark spending multiplier is greater than one due to the presence of the zero lower bound. Figure 3 tackles this point: it shows that the government spending multiplier is lower than one when the nominal interest rate can move freely, even towards hypothetical negative levels. This is shown by the line *Fin. acc. with no ZLB*

in the figure. In such a case, the fiscal multiplier equals 0.87 at impact and 0.32 after two years. The lower-than-one value of the multiplier can be explained as follows. A fiscal stimulus, as any other positive shock to aggregate demand, tends to increase inflation. If nominal interest rate is allowed to be negative during the deep economic recession, it will be relatively less negative after the stimulus. Thus, the real interest would increase by more in the case where the nominal rate is not constraint by its zero-floor. Thus, investment would becomes expensier, strengthening the classic crowding-out effect. This illustrates the claim that the fiscal multiplier tends to be larger when the zero lower bound constraint is binding (Christiano *et al.*, 2009a).

**Role of Financial Frictions** What is the impact of credit market imperfections on the efficiency of government spending? Figure 3 shows that the government spending multiplier is reduced in comparison with the benchmark configuration when we consider an economy that does not feature financial imperfections (shown by the line *No-fin. acc. with ZLB*). Indeed, it is greater than one only at the very short-run, equaling 1.04 at impact and reaching 0.47 after two years. This result can be explained by the fact that in the no-financial accelerator model, entrepreneur’s decisions are not conditional on the external finance premium. Thus, the environment is equivalent to a standard New Keynesian model without credit markets imperfections. In this case, government spending has not a relevant impact on the net worth and on the risk premium. The Fisher debt-deflation channel is shut down, and thus the potency of government spending to reduce the risk premium is nil, which also foreclosures the additional incentives for investment coming from this mechanism. This result is consistent with Fernández-Villaverde (2010).

**Policy Mix Implementation** We also tackle the possibility that monetary and fiscal authorities undertake a policy mix corresponding to a zero interest rate commitment and a fiscal expansionary policy. To do so, we assume that economic authorities implement an increase in government purchase along with the announcement of keeping the future nominal interest rate equal to zero for a longer period than prescribed by the monetary policy rule (22).<sup>9</sup> We assume that the central bank commits to keep the nominal interest rate at zero for 27 periods, that is 10 periods more than

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<sup>9</sup>Eggertsson and Woodford (2003) discuss the implications for the economic dynamics of managing public expectations about the future path of the interest rate. The argument, accordingly, is that economic agents make their decisions taking into account their expectations about the future policy that the central bank is likely to implement. The bottom line of their discussion is that, according to their optimal path for monetary policy, the central bank should announce and maintain the nominal interest rate at very low levels for longer time than what would be normally prescribed by an strict inflation targeting rule. The latter is explained by the fact that, when agents expect a period of abundant liquidity, accompanied by a rise in inflation expectations, they will start to increase consumption and investment from today.

the recommendation of the Taylor rule. Figure 3 shows that the net value of the fiscal multiplier under this policy mix is greater (shown by the line *Policy mix, Fin. acc. with ZLB 27 periods*). In particular, the multiplier equals 1.23 at impact and 0.83 after two years, with a peak value of 1.31 in the second quarter. Since agents expect a low interest rate for a long period of time, they respond to the adverse economic conditions by smoothing their consumption and investment. This yields a lower drop in inflation, which in turn produce a lower increase in the real interest rate. On the one hand, a smaller increase in the real interest rate implies that the service of the debt that entrepreneurs have to pay is lower. On the other hand, a smaller decrease in inflation rises by less the real debt of entrepreneurs through the debt-deflation channel. These two effects lessen the crowding-out effect on investment and makes the recession milder. Therefore, this result tells us that a policy mix, characterized by a zero interest rate commitment and an increase in government spending, is the most effective measure to increase output.

## 5.2 Distortionary Tax Cut

We now turn to investigate the impact of a temporal reduction in the income tax rate. We assume in this case that percent deviation of the income tax rate,  $\hat{\tau}_t$ , follows an AR(1) process, with persistence parameter  $\rho_\tau$  equal to 0.80. The income tax shock corresponds to a temporary shift of the income tax rate from 0.3, its steady-state value, to 0.22.

### 5.2.1 Model's dynamics

Figure 2 compares the IRFs of an economy that suffers from a deep recession with and without the income tax cut. Our results are as follows.

A lower income tax rate does not affect the zero lower bound duration, since the latter still holds for 17 quarters. The recession is also dampened by the expansionary fiscal policy but by less than in the case of an increase in government spending. An exogenous reduction in the income tax rate strongly increases the debt ratio and the public deficit. This result can be easily explained by the size of the shock, which dramatically affects the debt ratio's dynamics. It is worth noticing that a negative income tax shock appears to slightly affect inflation, unlike to an aggregate demand shock. On the one hand, by increasing the returns of capital, the income tax cut is able to stimulate investment, which entails a positive impact on inflation. On the other hand, the income tax cut also decreases nominal wages, since households increase their labor supply. The latter diminish the labor costs that firms are facing, which eventually decreases inflation. At the end, the effect on inflation is ambiguous. Therefore, the Fisher's debt-deflation channel is weakened and do not

magnify the effect of the expansionary fiscal shock on investment due to an income tax cut.

### 5.2.2 Determinants of the Tax Multiplier

We now turn to discuss the determinants of the tax multiplier displayed in the lower panel of Figure 3. Interestingly, we show that the fiscal multiplier resulting from a tax cut is lower than one in all the configurations. In the benchmark case, the tax multiplier equals 0.07 at impact, it reaches a peak at the 10th quarter with a value of 0.27. A value of the tax multiplier lower than one confirms the intuition that a tax cut policy is less efficient than a government spending expansion. This can be explained by the fact that a reduction in the tax burden rises disposable incomes of households and entrepreneurs. However, even if this policy gives more incentives to consume and invest, it does not have the straightforward impact of an aggregated demand shock that directly fuels the economy.

**Role of the Zero Lower Bound** We now investigate the effect of the nominal interest rate zero-floor on the efficiency of a income tax cut. It appears from the line *Fin. acc. with no ZLB* in Figure 3 that the tax multiplier reaches higher values than in the benchmark configuration when the nominal interest rate can move towards negative values. It equals 0.08 at impact while its peak value is 0.30 at the 9th quarter. As suggested earlier, the decrease in the income tax rate rises the returns of capital of entrepreneurs, which have more incentive to invest. However, as described in Figure 2, the tax cut has a small effect on inflation. Consequently, in the absence of the zero lower bound constraint, when the nominal interest rate could drop by more, the apparent nil reaction of inflation would lead to a reduction in the real interest rate. This in turn would stimulate more investment and output. Therefore, the zero lower bound constraint limits the reach of an income tax cut policy.

**Role of Financial Frictions** When we drop financial frictions from the model, the value of the tax multiplier is even negative (displayed by the line *No-fin. acc. with ZLB*). Indeed, it equals  $-0.02$  at impact and peaks at 0.12 at the 12th quarter. This result highlights the crucial role of credit market imperfections in the efficiency of a tax cut policy. Without the financial accelerator mechanism, the increase in the returns of capital induced by the tax cut does not translates into a reduction in the risk premium. The latter means that the cost of credit, and thus investment, does not diminish as much when financial frictions are absent than when they are present. As a result, an income tax cut is more efficient when credit market imperfections are important.

**Policy Mix Implementation** We now turn to assess the effect of a policy mix that combines a zero interest rate commitment with an income tax cut. Interestingly, the line *Policy mix, Fin. acc. with ZLB 27 periods* in Figure 3 shows that the tax multiplier is negative at impact (equals to  $-0.03$ ) and it reaches 0.16 at the 13th quarter. This result suggests that keeping the nominal interest rate equal to zero for a longer time than the Taylor rule is far from being a positive determinant of an income tax policy. As we have shown above, the existence of the zero lower bound constraint reduces the value of the tax multiplier. Indeed, the real interest rate cannot decrease as much as if the nominal interest rate would be free to move towards negative levels. Therefore, binding the zero lower bound for a long time worsens the effect of cutting income taxes.

## 6 Government Spending Multiplier and Fiscal Strategies

Aggressive fiscal stimuli are usually followed by fiscal “exit strategies” in order to ensure public debt sustainability. For example, in March 2009, the Council of European Union recommended that a “fiscal consolidation in all EU Member States should start in 2011 at the latest” (Council conclusions on fiscal exit strategy, 2009). Another example would be the Obama’s administration that has constituted in February 2010 a commission aimed at bringing the federal budget deficit down to 3 per cent of the economy by 2015, and put the budget in balance except for payments on debt. In this section, we focus on the government spending multiplier. The goal is to investigate how different fiscal strategies affect its size in the presence of credit market imperfections.

### 6.1 Government Spending Multiplier with Different Distortionary Taxation

Financing a large increase in government spending can require to raise distortionary taxes that are known to reduce the effectiveness of fiscal policy, since the incentives for consumption and investment might be weakened. Erceg and Lindé (2010) show that the effects of an expansionary fiscal policy on output can be reversed if it is financed by labor income taxes. They emphasize that this effect is reinforced by the presence of a zero nominal interest rate.

In this paper, we compute the government spending multiplier for two types of distortionary taxes. The results are displayed in Figure 4, in which the left panel corresponds to a model with financial frictions and the right panel corresponds to a model without. The benchmark specification (shown by the line *Benchmark*) corresponds to a constant taxation rate on labor and capital incomes. In this case,  $\chi_y = \chi_g = 0$  implying that  $\tau = 0.30$  at each date. For the sake of comparison, we also assume that  $\tau$  can be equal to zero, implying that government spending is only financed by lump-sum taxes (represented by the line *No distortionary tax*). Finally, we assume that distortionary

taxes are chosen according to Rule (24) that specifies that incomes taxes are pro-cyclical ( $\chi_y > 0$ ). Precisely, we assume that a deviation of 3 per cent point of GDP from its steady-state value increases the income tax rate in 1 per cent point, from  $\tau = 0.30$  to  $\tau = 0.31$ . Thus,  $\chi_y$  is equal to  $1/3$ . We also assume that an increase of 5 percent in the debt ratio raises the income tax rate in 1 percent point, which corresponds to  $\chi_d = 1/5$ . This case is entitled *Time varying distortionary tax* in the figure.

[ insert Figure 4 here ]

Considering a model with credit market imperfection, we show that the value of the government spending multiplier is the highest when the government does not raise distortionary taxes. This is consistent with Erceg and Lindé (2010). The spending multiplier equals 1.17 at impact and peaks to 1.21 at quarter two. When distortionary income taxes are used to financed the deficit but they stay constant ( $\tau_t = 0.3 \forall t$ ), the government spending multiplier becomes smaller, reaching 1.14 at impact and peaking to 1.16 in quarter two. After two years, the value of the fiscal multiplier reaches 1.05 in the benchmark specification and 1.11 when deficit is fully financed by lump-sum taxes. The intuition behind this result is that the presence of distortionary taxes reduce households and entrepreneurs' disposable income. This automatically implies that output is less sensitive to government spending expansion. When it comes to the time-varying distortionary tax, the government spending multiplier is even lower. Precisely, it equals 1.12 at impact and 0.99 after two years. Indeed, the rise in debt implied by higher government spending is partially borne by an increase in income taxes that rise the tax burden for households and entrepreneurs.

We now turn to assess the role of distortionary taxes on the government spending when financial frictions are absent. Figure 4 shows that the fiscal multiplier is still lower when distortionary taxes are time-varying instead of being constant. Interestingly, we show that the value of the fiscal policy's efficiency is less altered by the fiscal rule when we drop the financial accelerator model. The explanation behind this result is straightforward. In the presence of credit market imperfections, a positive government spending shock increases time-varying income taxes that in turn rises the rental rate of capital and the external finance premium. Therefore, the efficiency of government spending policy is damped by the effect of distortionary taxes on investment decisions. In the absence of credit market imperfection, the effect of higher income taxes on the rental rate of capital is similar but the investment's dynamics is not magnified by an increase in the external finance premium.

## 6.2 Delays in the Implementation of Fiscal Policy

The timing in which a fiscal stimulus is implemented is a major concern when studying the final effects of fiscal policy on output. The consensus among economists is that the longer it takes to increase government spending, the less effective it will be to raise output. The intuition behind this statement is quite simple. If the strong pulse of the stimulus comes in a moment where the economy has already started to recover, the increase in government spending may raise faster the short- and long-run interest rates, eventually crowding-out investment. For instance, Erceg and Lindé (2010) show, using a model featuring no financial frictions and a liquidity trap, that the fiscal multiplier can be even negative when the government spending expansion is undertaken too late. This subsection aims at reviewing the effects of lagged implementation in government spending with and without financial frictions, while the zero lower bound constraint on the nominal interest rate is active.

For this purpose, we assume that government spending, as a percent deviation from its steady state-value, follows an ARMA(2,q) process

$$\hat{g}_t = \rho_{g1}\hat{g}_{t-1} + \rho_{g2}\hat{g}_{t-2} + b_0\epsilon_{g,t} + b_1\epsilon_{g,t-1} + \dots + b_q\epsilon_{g,t-q}.$$

The two autoregressive terms allow  $\hat{g}_t$  to display a (concave) hump-shaped pattern, which reflects the fact that any realistic fiscal stimulus would need some time to reach its peak. The moving average terms allow us to determine in which period the increase of government spending occurs. For instance, if we assume that  $b_4 > 0$  while all other  $b_{q'}$  are equal to zero, this means that government spending increases one year after its official announcement. The latter implies that, from period  $t = 0$ , all agents in the economy make their decisions knowing that in period  $t = 4$  several increases in government purchase would take place.<sup>10</sup>

Taking into account implementation lags in government spending deter us from using the impact fiscal multiplier introduced in section ??, since in most cases the percent deviation of government spending will be zero at the impact period. Thus, we follow Uhlig (2010) and Zubairy (2010) by introducing an alternative measure of efficiency of government spending, known as the present value multiplier  $k$ -periods ahead ( $PVM_k$ ) defined by

$$PVM_k = \frac{\mathbf{E}_t \sum_{j=0}^k \beta^j y_{t+k}^{net}}{\mathbf{E}_t \sum_{j=0}^k \beta^j g_{t+k}^{net}} = \frac{1}{\gamma_g} \frac{\mathbf{E}_t \sum_{j=0}^k \beta^j \hat{y}_{t+k}^{net}}{\mathbf{E}_t \sum_{j=0}^k \beta^j \hat{g}_{t+k}^{net}},$$

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<sup>10</sup>Since the model solution method implies a deterministic pattern for the economic dynamics, an ARMA(p,q) process for government spending allows us to introduce future shocks that are known by everybody from the initial period.

where  $PVM_k$  measures the total discounted net effect of government spending on output  $k$ -periods ahead in time, from the perspective of an agent in period  $t$ . We proceed as follows: we introduce the same negative net worth and preference shocks described in section ?? in order to generate a deep recession in models with and without financial frictions. Notice that the zero lower bound constraint is binding for 13 periods in the model without financial frictions, whereas it holds for 17 periods in the benchmark model (as shown in section ??). The persistence parameters of government spending are set to  $\rho_{g1} = 1.4$  and  $\rho_{g2} = -0.45$ .

Figure 5 displays the pattern of the different government spending shocks that we assume in the analysis.

[ insert Figure 5 here ]

All of these processes are rescaled such as the total amount of public expenditures for every different implementation is equal in present value. In order to maintain the duration of the zero lower bound constraint equal across all different fiscal policies, we consider a modest increase in government spending. This assumption allow us to focus only on the effect of the delays themselves.

We first focus on the long-run effect of government spending expansion on output with different delays on its implementation. Table 4 shows the value of  $PVM_k$  for  $k = 50$ , for different lag structures in government spending and model environments. The table considers only the implementation delays that are inside the duration of the zero lower bound constraint for each model.

[ insert Table 4 here ]

As it can be seen, a fiscal stimulus is always more efficient on the long-run when we consider financial frictions instead of omitting them. Furthermore, the long-run spending multiplier,  $PVM_{50}$ , is lower than one in a model without credit market imperfections, whatever the lag implementation structure is. On the contrary, the  $PVM_{50}$  for the 0, 4, and 8 lags cases are greater than one in the model with financial frictions. Following the same logic than in the preceding sections, this is a signal than the net effect of government spending on investment is positive, which is mainly due to the debt-deflation channel and the zero lower bound constraint. Consequently, Table 4 confirms the standard view that, at least in the long-run, implementation delays decrease the efficiency of fiscal policy, as the government spending multiplier becomes smaller the longer are the delays.

We now turn to assess the short-run impact of the fiscal stimuli implemented with various lags. Figure 6 displays the value of  $PVM_k$  for different horizons for models with and without financial frictions.

[ insert Figure 6 here ]

First, consider the model without financial frictions. In that case, Figure 6 again shows that implementation delays decrease the value of the multiplier, with even negative values when government spending is increased just one period before the end of the duration of the zero lower bound constraint (12 lags case).

Interestingly, when financial frictions are present, this result is reversed. Indeed, a small delay in the fiscal stimulus implementation increases the value of the multiplier in the very short-run. This apparent counter-intuitive result can be explained as follows. When government spending is announced to increase in, say, 1 year from now, agents adjust their inflation expectations upwards from the present period. The latter activates the debt-deflation channel by increasing net worth and decreasing the risk premium, since inflation starts rising from today. Thus, investment and output decrease by less even before the fiscal stimulus physically takes place. In one year from now, when government spending eventually rises, output has already accumulated some net effect from the announced stimulus, which makes the size of the present value multiplier much greater than one. Eventually, the value of the multiplier decreases in order to establish the ordering found in the long-run, as summarized by Table 4. However, it is worth noticing that even if small delays seem not to be as bad under financial frictions, longer delays are definitely worse. Considering the case that the fiscal stimulus is pre-announced 15 quarters in advance, that is, just 2 periods before interest rates becomes positive again, it turns out that the present value multiplier is negative. The latter is in line with the results of Erceg and Lindé (2010).

### **6.3 Spending Reversal Effect**

## **7 Conclusion**

This paper investigates the qualitatively impact a deep recession, driven by negative financial and preference shocks, that make the zero lower bound on the nominal interest rate binding. The shock on the credit market we consider are a decrease in the survival probability of entrepreneurs (or a negative net worth shock). We describe how the economy is subject to bigger fluctuations when the central bank cannot reduce further the nominal interest rate.

We explore economic policies that search to steer the economy back to the long run equilibrium. The economic measures we consider are a fiscal stimulus, in terms of an increase in government purchases and a income tax cut. We also investigate the efficiency of these expansionary policies for different fiscal strategies.

Several results emerge. When the nominal interest rate hit its zero bound, the monetary authorities lost its policy instrument, resulting in a stronger volatility real and nominal variables. In addition,

an increase in government purchases leads to a spending multiplier greater than one for at least one year while the tax multiplier is lower than one. We also emphasize the importance of financial frictions when a government spending expansion is financed by distortionary taxes and when the government spending expansion is implemented with a lag.

The importance of the banking intermediation sector has been spotlighted during the recent financial turmoil. In our model, we have focused on the role of entrepreneurs' balance sheet in transmission mechanisms of shocks, assuming that the financial intermediary is perfectly competitive. The next step of this paper would be to assess how imperfect competitive banks modify the transmission mechanisms of financial shocks that make the zero lower bound on the nominal interest rate binding. We leave this topic for future research.

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## 8 Appendix

The log-linear model can be sum up as following:

### 8.0.1 Household's Preferences

#### Euler equation

$$(1 - \beta b) \sigma \hat{\lambda}_t = \beta b \mathbf{E}_t \{ \hat{c}_{t+1} \} - (1 + \beta b^2) \hat{c}_t + b \hat{c}_{t-1} + \sigma \hat{\varepsilon}_t - \beta b \sigma \mathbf{E}_t \{ \hat{\varepsilon}_{t+1} \}, \quad (25)$$

where  $\sigma^{-1} = -\mathbb{U}_{cc}c/\mathbb{U}_c$ .

### Risk free bond equation

$$\hat{\lambda}_t - \hat{R}_t = \mathbb{E}_t \left\{ \hat{\lambda}_{t+1} - \hat{\pi}_{t+1} \right\} \quad \text{and} \quad \hat{r}_t = \hat{R}_t - \mathbb{E}_t \{ \hat{\pi}_{t+1} \}. \quad (26)$$

### Wage Phillips curve

$$\hat{\pi}_t^w - \gamma_w \hat{\pi}_{t-1} = \frac{(1 - \alpha_w)(1 - \beta \alpha_w)}{\alpha_w(1 + \omega_w \theta_w)} \left[ \omega_w \hat{\ell}_t^h - \hat{\lambda}_t - \hat{w}_t + \hat{\tau}_t \right] + \beta \mathbb{E}_t \{ \hat{\pi}_{t+1}^w - \gamma_w \hat{\pi}_t \}, \quad (27)$$

where  $\omega_w = \ell^h \nabla_{\ell\ell} / \nabla_{\ell}$ .

### Wage inflation definition

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t. \quad (28)$$

## 8.0.2 Intermediate Good Sector

### Production function

$$\hat{y}_t = (1 - \alpha) \hat{\ell}_t + \alpha \hat{k}_t \quad \text{and} \quad \hat{\ell}_t = \Omega \hat{\ell}_t^h. \quad (29)$$

### Cost minimization

$$\hat{w}_t = \hat{s}_t + \hat{y}_t - \hat{\ell}_t^h, \quad \hat{w}_t^e = \hat{s}_t + \hat{y}_t, \quad \text{and} \quad \hat{z}_t = \hat{s}_t + \hat{y}_t - \hat{k}_t \quad (30)$$

### Phillips curve

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \frac{(1 - \alpha_p)(1 - \beta \alpha_p)}{\alpha_p} \hat{s}_t + \beta \mathbb{E}_t \{ \hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t \}, \quad (31)$$

## 8.0.3 Entrepreneur

$$\hat{x}_t = \hat{q}_{t-1} + \hat{k}_t - \hat{n}_t, \quad \text{and} \quad \hat{r}_t = \hat{r}_t^k - \hat{r}_{t-1} \quad (32)$$

### Optimal contract equations

$$\mathbb{E}_t \{ \hat{r}_{t+1} \} = \mathbb{E}_t \{ \hat{\Lambda}_{t+1} \} [1 - \check{r} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]], \quad (33)$$

$$\mathbb{E}_t \{ \hat{\Lambda}_{t+1} \} = \mathbb{E}_t \{ \hat{\bar{\omega}}_{t+1} \} \bar{\omega} \left[ \frac{\Gamma_{\omega\omega}(\bar{\omega})}{\Gamma_{\omega}(\bar{\omega})} - \frac{\Gamma_{\omega\omega}(\bar{\omega}) - \mu G_{\omega\omega}(\bar{\omega})}{\Gamma_{\omega}(\bar{\omega}) - \mu G_{\omega}(\bar{\omega})} \right], \quad (34)$$

$$\hat{x}_{t+1} = \mathbb{E}_t \{ \hat{r}_{t+1} \} [x - 1] + \mathbb{E}_t \{ \hat{\bar{\omega}}_{t+1} \} \bar{\omega} \check{r} x [\Gamma_{\omega}(\bar{\omega}) - \mu G_{\omega}(\bar{\omega})]. \quad (35)$$

where

$$\Gamma(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega) d\omega + \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega, \quad \text{and}$$

$$\mu G(\bar{\omega}_{t+1}) = \mu_t \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega.$$

## Return of capital

$$\mathbb{E}_t\{\hat{r}_{t+x1}^k\} = \mathbb{E}_t\{\hat{z}_{t+1} - \hat{\tau}_t\} \frac{z(1-\tau)}{r^k} + \mathbb{E}_t\{\hat{q}_{t+1}\} \frac{(1-\delta)}{r^k} - \hat{q}_t. \quad (36)$$

## Net worth

$$\hat{n}_{t+1} \frac{1}{x r^k} = (\hat{\gamma}_t + \hat{v}_t) \gamma [1 - \Gamma(\bar{\omega})] + (\hat{w}_t^e - \hat{\tau}_t) \left[ \frac{1}{r^k x} - \gamma [1 - \Gamma(\bar{\omega})] \right], \quad (37)$$

$$\hat{v}_t [1 - \Gamma(\bar{\omega})] = \hat{r}_t^k [1 - \mu G(\bar{\omega})] + \hat{n}_t [1 - \Gamma(\bar{\omega})] + \hat{x}_t \left[ 1 - \frac{1}{\tilde{r}} - \mu G(\bar{\omega}) \right] - \hat{r}_{t-1} \frac{1}{\tilde{r}} \left[ 1 - \frac{1}{x} \right] - \hat{\omega}_t \bar{\omega} \mu G_\omega(\bar{\omega}), \quad (38)$$

$$\hat{c}_t \frac{c^e}{k r^k} = (\hat{v}_t (1 - \gamma) - \hat{\gamma}_t \gamma) \varrho [1 - \Gamma(\bar{\omega})]. \quad (39)$$

### 8.0.4 Capital Producer

#### Law of motion of capital

$$\hat{k}_{t+1} \frac{1}{\delta} = \hat{u}_t + \hat{k}_t \left[ \frac{1}{\delta} - 1 \right] - \hat{u}_t \frac{z}{\delta}. \quad (40)$$

#### Price of capital

$$\hat{q}_t \frac{1}{\varkappa} = \hat{u}_t [1 + \beta] - \hat{u}_{t-1} - \beta \mathbb{E}_t\{\hat{u}_{t+1}\}, \quad (41)$$

where  $\varkappa = \Phi''(1)$ .

#### Capital utilization rate

$$\hat{k}_t = \hat{u}_t + \hat{k}_t \quad \text{and} \quad \hat{z}_t = \vartheta_u \hat{u}_t + \hat{q}_t, \quad \text{where} \quad \vartheta_u = u \delta''(u) / \delta'(u). \quad (42)$$

### 8.0.5 Resource Constraint

$$\hat{y}_t = \hat{c}_t \frac{c}{y} + \hat{i}_t \frac{i}{y} + \hat{c}_t^e \frac{c^e}{y} + \hat{g}_t \frac{g}{y} + \left[ \hat{r}_t^k + \hat{q}_{t-1} + \hat{k}_t \right] \left[ \mu G(\bar{\omega}) r^k \frac{\tilde{k}}{y} \right] + \hat{\omega}_t \bar{\omega} \mu G_\omega(\bar{\omega}) r^k \frac{\tilde{k}}{y}. \quad (43)$$

### 8.0.6 Non-Constraint Monetary Policy

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) a_\pi \hat{\pi}_t + (1 - \rho_R) a_y \hat{y}_t \quad (44)$$

### 8.0.7 Fiscal Policy

$$\beta \hat{d}_{t+1} = \hat{d}_t + \gamma_g \hat{g}_t - \gamma_Y \hat{Y}_t - \tau(\hat{\tau}_t - \hat{y}_t), \quad (45)$$

$$\hat{Y}_t = a_d \hat{d}_t + a_g \hat{g}_t, \quad (46)$$

$$\tau \hat{\tau}_t = \chi_d \hat{d}_t + \chi_y \hat{y}_t, \quad (47)$$

$$deficit_t = \gamma_g \hat{g}_t - \gamma_Y \hat{Y}_t - \tau(\hat{\tau}_t - \hat{y}_t). \quad (48)$$

Table 1. Calibrated Parameters

<b>Preferences and Technology</b>		Value
$\beta$	Discount factor	0.99
$b$	Degree of habit consumption	0.63
$\sigma$	Inv. of the elasticity of intertemp. substitution	0.20
$\omega_w$	Elasticity of labor disutility	1.00
$\phi$	Elasticity of value added wrt capital	0.35
$\delta$	Capital depreciation rate	0.03
$\varkappa$	Investment adjustment cost	5.86
$\vartheta_u$	Utilization rate of capital parameter	1/0.31
<b>Nominal Rigidities</b>		
$\theta_p$	Elasticity of substitution of goods	11.00
$\alpha_p$	Degree of price stickiness	0.67
$\gamma_p$	Degree of price indexation	0.75
$\theta_w$	Elasticity of substitution of labor	21.00
$\alpha_w$	Degree of wage stickiness	0.69
$\gamma_w$	Degree of wage indexation	0.70

Table 2. Calibrated Parameters

<b>Financial Accelerator Mechanism</b>		Value
$\Omega$	Share of household labor in aggr. labor	0.9846
$x$	Steady-state ratio of capital to net worth	2.00
$\check{r}$	Steady-state risk spread	$1.02^{0.25}$
$\gamma$	Survival rate of entrepreneurs	0.9785
$\bar{\omega}$	Threshold value of idiosyncratic shock	0.4982
$\sigma_{\omega}$	Standard error of idiosyncratic shock	0.2764
$\mu$	Monitoring cost	0.1175

Table 3. Calibrated Parameters

<b>Monetary Policy</b>		Value
$\rho_R$	Interest rate smoothing	0.88
$a_\pi$	Elasticity of the interest rate wrt inflation	1.85
$a_y$	Elasticity of the interest rate wrt output gap	0.08
<b>Fiscal Policy</b>		Value
$g/y$	Share of government expenditure in output	0.19
$\phi_b$	Elasticities of lump-sum taxes wrt debt	0.33
$\phi_g$	Elasticities of lump-sum taxes wrt government spending	0.10
$\psi_y$	Elasticities of distortionary taxes wrt output	[0; 0.05]
$\psi_d$	Elasticities of distortionary taxes wrt debt	[0; 0.05]
$\tau$	Income tax rate	[0; 0.30]

Table 4. Value of  $PVM_k$ , with  $k = 50$ 

	No lags	4 lags	8 lags	10 lags	12 lags	15 lags
With financial frictions	1.27	1.25	1.09	0.92	0.70	0.37
Without financial frictions	0.54	0.51	0.50	0.48	0.48	-

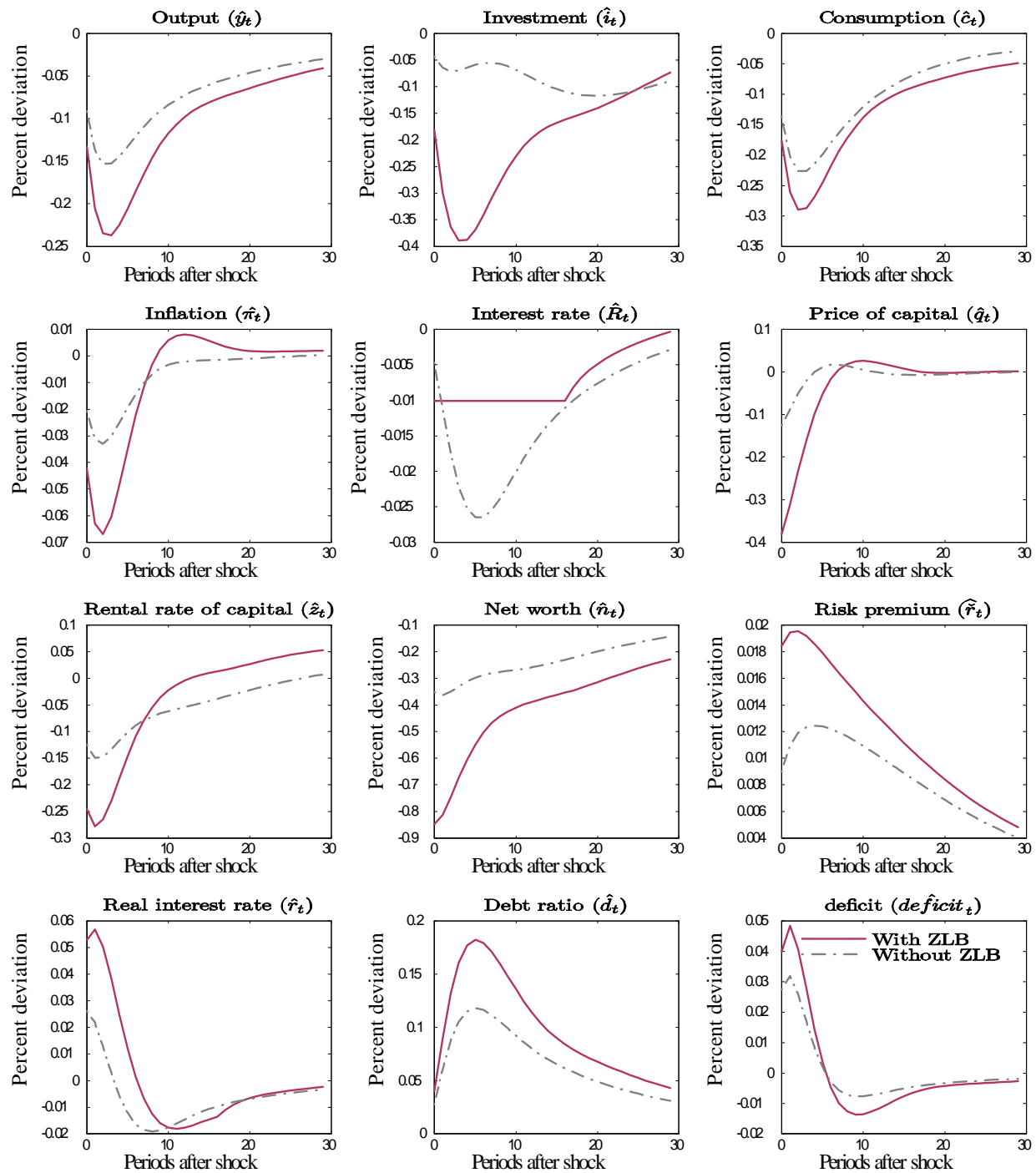


Figure 1: Impulse Response Functions to a negative financial shock (decrease in  $\gamma_t$ ) and a negative preference shock (decrease in  $\varepsilon_t$ ). In the specification represented by solid lines, the lower bound constraint is imposed. In the specification represented by dash lines, the lower bound constraint is omitted.

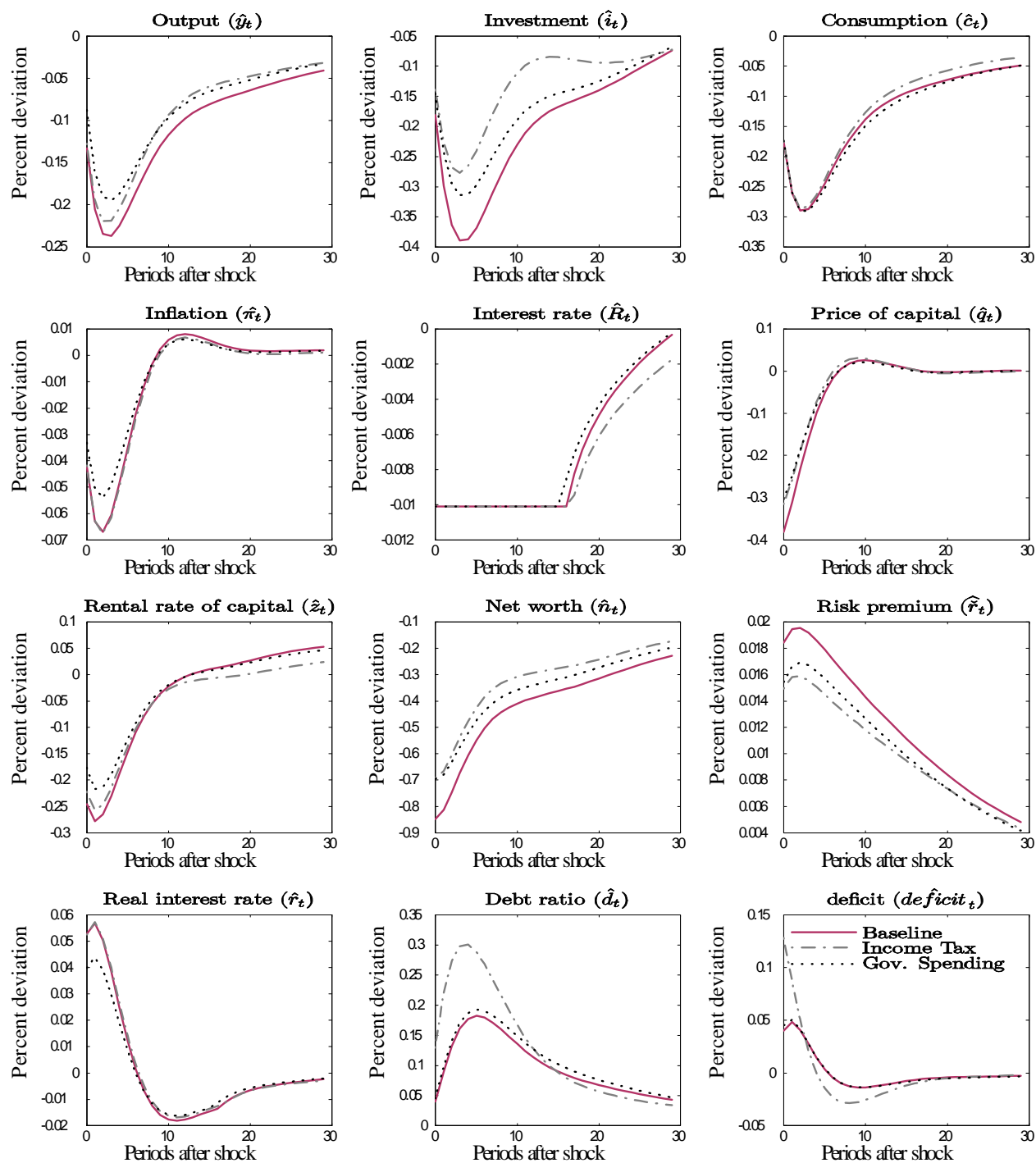


Figure 2: Impulse Response Functions to a negative financial shock (decrease in  $\gamma_t$ ) and a negative preference shock (decrease in  $\varepsilon_t$ ). The solid lines correspond to baseline model's dynamics under the deep recession and without fiscal expansion. The dash lines represent the IRF to an increase in distortionary taxes under the deep recession. The dotted lines represent the IRF to a positive fiscal government shock under the deep recession.

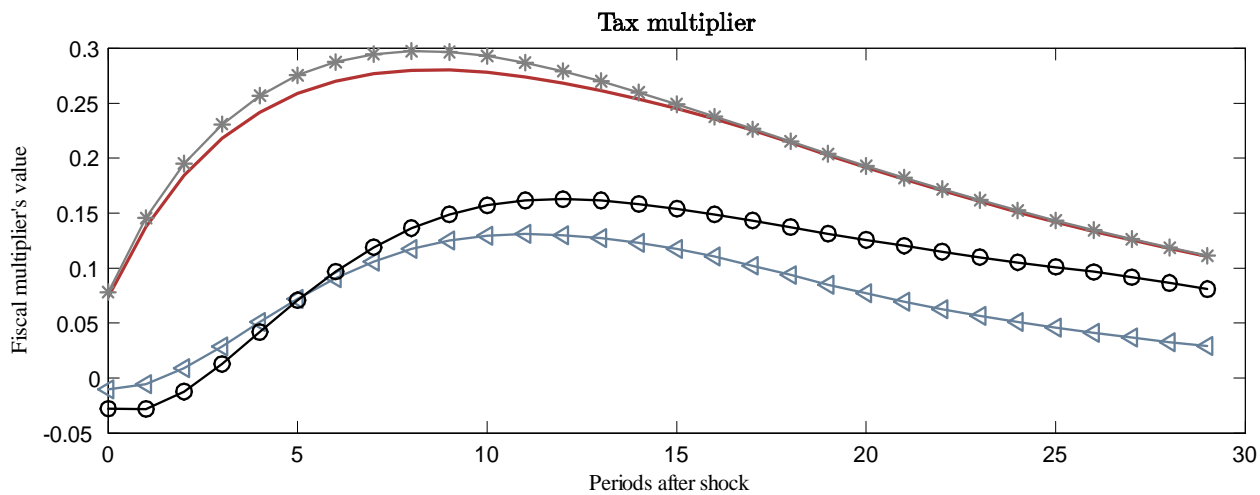
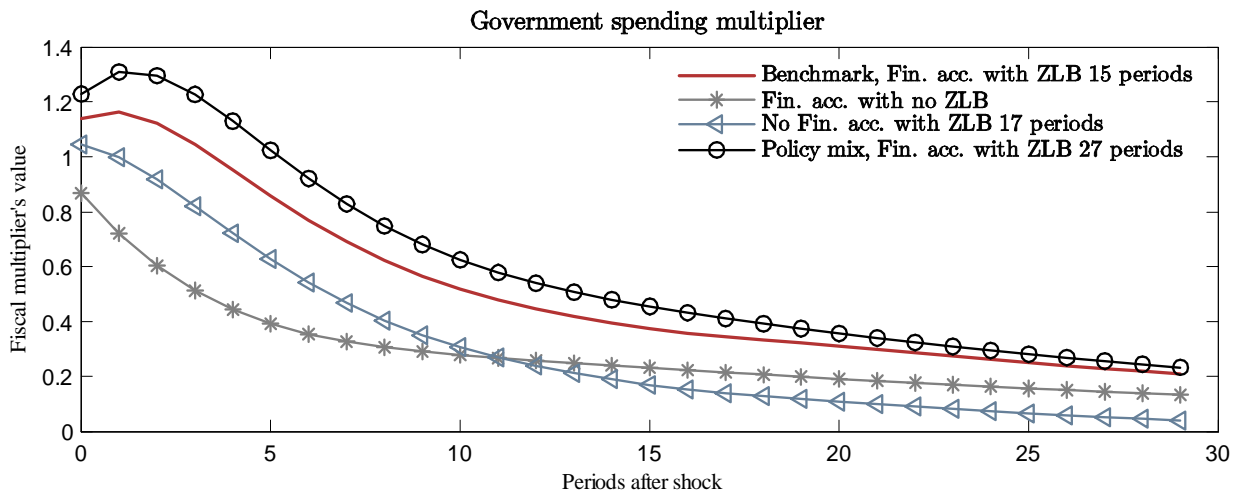


Figure 3: Fiscal multiplier for various model's specification. The solid line corresponds to the benchmark case. The line with cross corresponds to a model without ZLB constraint. The line with triangle corresponds to a model without financial frictions. The line with dot corresponds to a model without ZLB and financial frictions. The line with circle corresponds to a benchmark case with a zero interest rate period of 27 quarters.

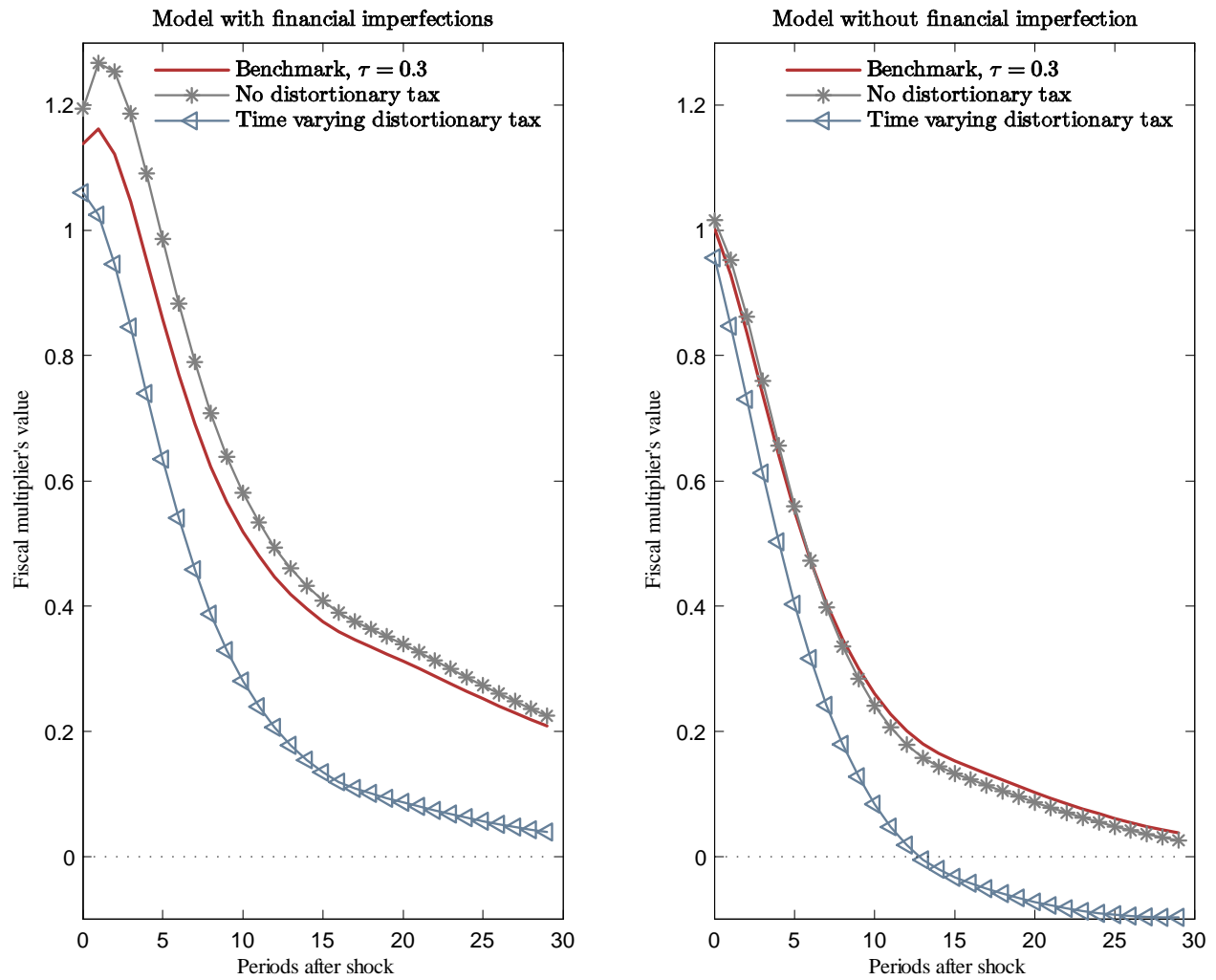


Figure 4: Fiscal multiplier for various taxation policies. The solid line corresponds to the benchmark case. The line with cross corresponds to a model without distortionary income tax. The line with triangle corresponds to a model with income taxes that varies with output and debt. The left panel corresponds to the benchmark model with the right panel corresponds to a model without financial frictions.

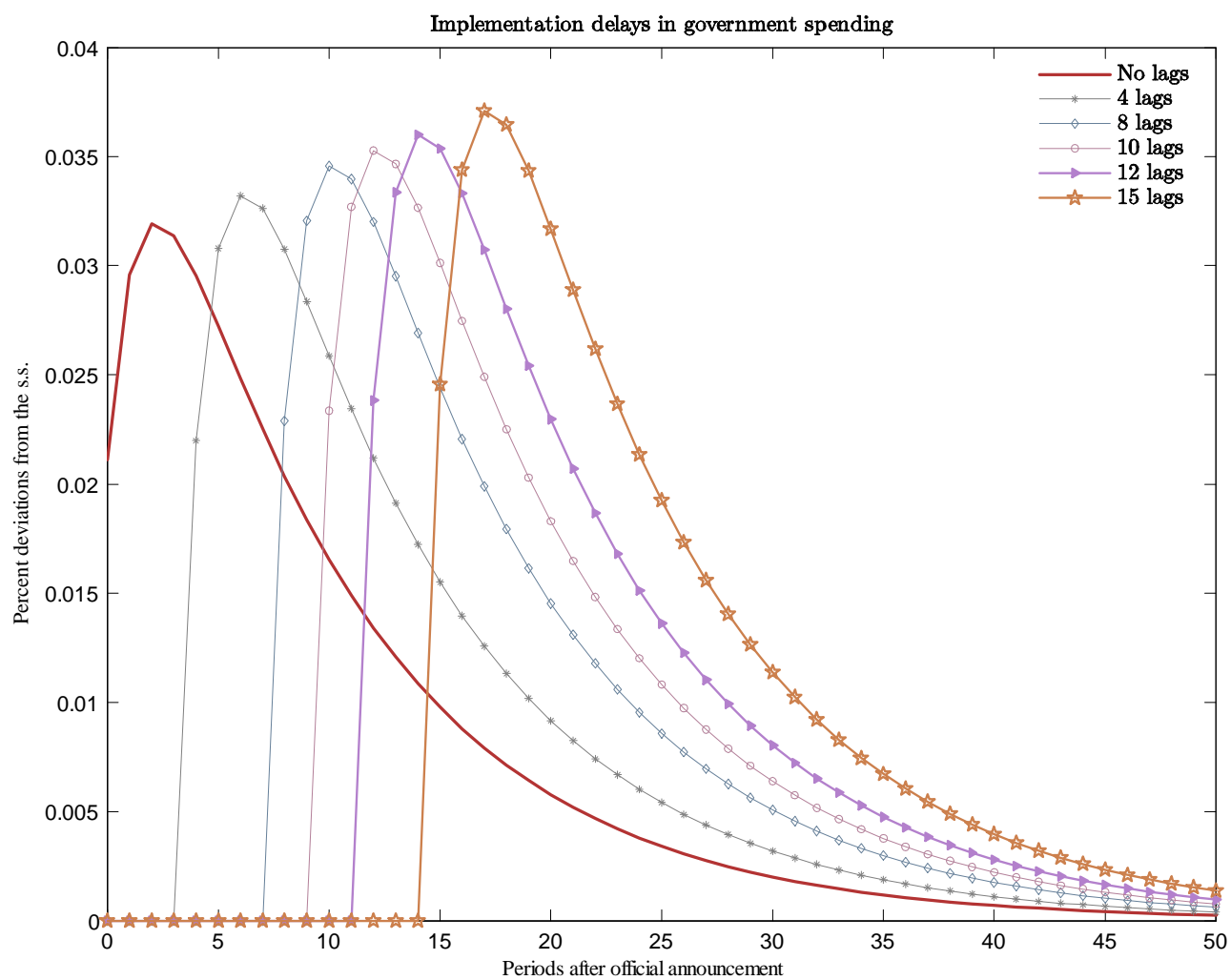


Figure 5: Different delays in the implementation of government spending.

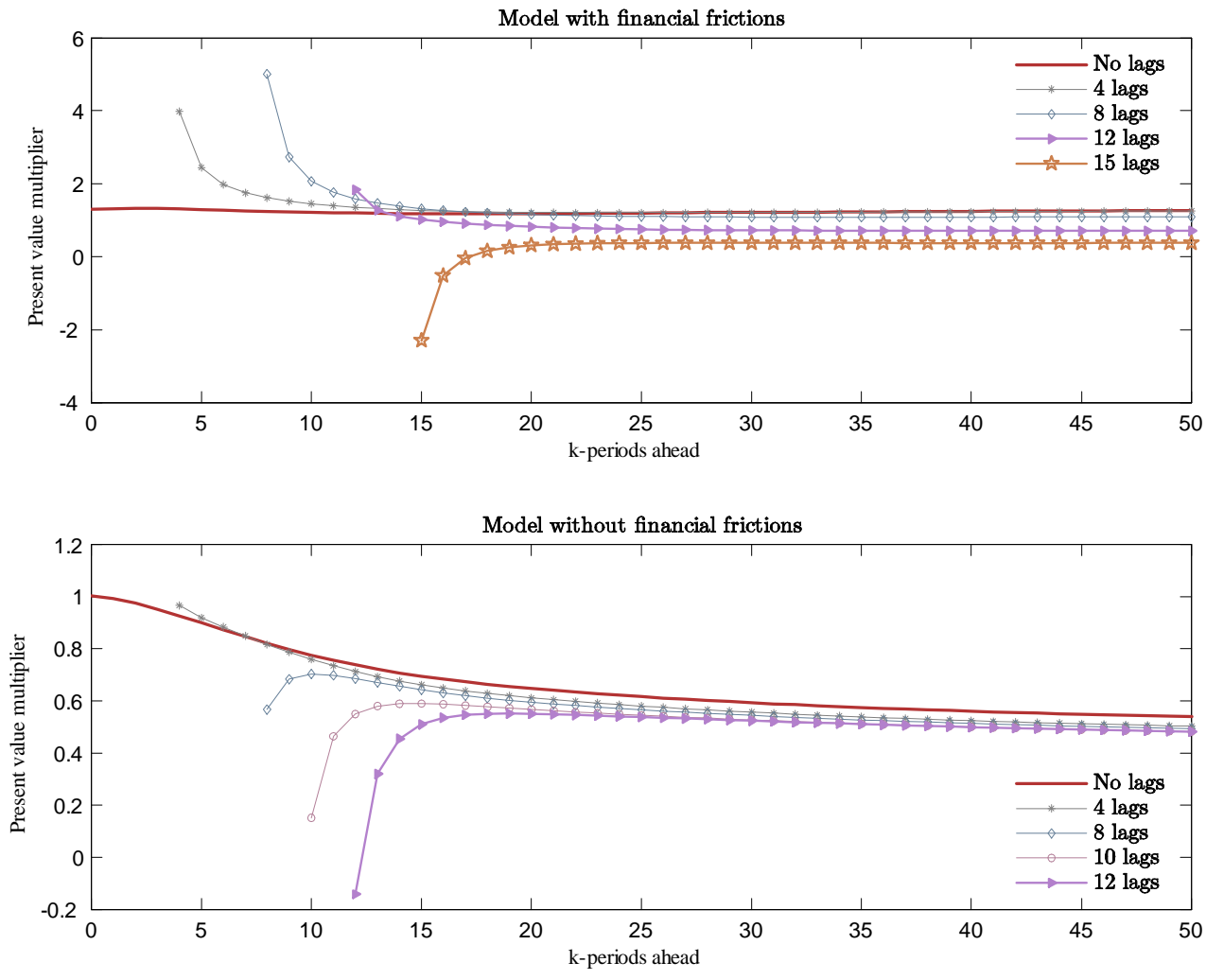


Figure 6: Present value multiplier k-periods ahead for government spending.