New Perspectives on Depreciation Shocks as a Source of Business Cycle Fluctuations*

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Abstract

In this paper we study the transmission for capital depreciation shocks. The existing literature in the Real Business Cycle tradition has concluded that these shocks are irrelevant for business cycle fluctuations. We show that these shocks are a potentially important drivers of aggregate fluctuations in a New Keynesian model. Nominal rigidities and some persistence in the shock process are the key ingredients to generate co-movement across real variables.

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1 Introduction

In a recent paper, Liu, Waggoner and Zha (2010) estimate a series of Dynamic Stochastic General Equilibrium models with regime switches in shock variances and in the inflation target. Their estimated models suggest that shocks to the rate of capital depreciation are important sources of business cycle fluctuations and of the shift in the characteristics of fluctuations represented by the Great Moderation. Depreciation shocks have not been much researched, but the result is surprising given the available literature. In particular, a study by Ambler and Paquet (1994) suggests that capital depreciation shocks are largely irrelevant for fluctuations in output and other key macroeconomic variables. In their calibrated Real Business Cycle model, capital depreciation shocks are important only because they interact with total factor productivity shocks to improve the model's implications for the correlation of hours worked and labour productivity.

The objective of this paper is twofold. First, the paper seeks to reconcile these two very different sets of results by analysing the propagation mechanism of capital depreciation shocks with emphasis on their potential as important driving forces of business cycle fluctuations. Of particular interest are the conditions under which the shocks generate co-movement of key macroeconomic variables as co-movement is an important feature of empirically recognisable business cycles. Second, the paper compares the transmission of capital depreciation shocks to quality of capital shocks and investment-specific adjustment costs. These alternative shocks to the capital accumulation process have recently received a lot of attention in the literature. Gertler and Karadi (2010) generate co-movement of hours, consumption, investment and output following a capital quality shock, which plays an important role in their model's ability to simulate the recent Great Recession. Similarly, investment-specific technology shocks have been found to be important drivers of the business cycle although they have some difficulties in generating co-movement in particular of consumption, cf. Justiniano, Primiceri and Tambalotti (2009, 2010), Jaimovich and

Rebelo (2009) and Furlanetto and Seneca (2010).

To achieve these objectives, we build a New Keynesian DSGE model similar to the model by Liu, Waggoner and Zha (2010). This model nests the RBC model considered by Ambler and Paquet (1994) as a special case without the nominal and real rigidities that are central to the New Keynesian tradition.

In a nutshell, our analysis shows that nominal rigidities and persistence in the depreciation shock modify the transmission mechanism highlighted in the RBC literature. These features are shown to be crucial for achieving co-movement of key macroeconomic variables following a shock to the rate of capital depreciation. Thus, they can be considered to be important parts of the reason why depreciation shocks may become important drivers of aggregate fluctuations in DSGE models.

Turning to our second objective, we find that capital depreciation shocks differ from the shocks to the quality of capital considered by Gertler and Karadi (2010) only through an initial timing effect. Hence, co-movement conditional on these shocks equally relies on persistence and nominal rigidity in our model. In comparision to the investment-specific technology shocks, we find that co-movement is easier to achieve following depreciation shocks. In particular, co-movement conditional on depreciation shocks is not dependent on preferences with a high complementarity between hours and consumption as is the case following shocks to investment-specific technology.

The paper is organised as follows. Section 2 presents the model. Section 3 discusses how depreciation shocks can become potentially important sources of business cycle fluctuations. Section 4 compares the dynamics generated by capital depreciation shocks to the ones generated by shocks to the quality of capital and to investment-specific technology. Finally, section 5 concludes.

2 The model

The model is a standard New Keynesian dynamic stochastic general equilibrium model extended with endogenous capital accumulation, variable capital utilisation and investment-adjustment costs. The economy consists of a continuum of firms, a continuum of households, and an inflation-targeting central bank. There is monopolistic competition in goods and labour markets, and perfect competition in capital rental markets.

Using Cobb-Douglas technology, each firm combines rented capital with an aggregate of the differentiated labour services supplied by individual households to produce a differentiated intermediate good. It sets the price of its good according to a Calvo price-setting mechanism and stands ready to satisfy demand at the chosen price. Given this demand, and given wages and rental rates, the firm chooses the relative factor inputs to production to minimise its costs.

Each household consumes a bundle of the intermediate goods produced by individual firms. Each period, it chooses how much to consume of this final good (in addition to its composition) and how much to invest in state-contingent one-period bonds. As in Christiano, Eichenbaum and Evans (2005), it also chooses how much to invest in new capital subject to investment adjustment costs, and it chooses the utilisation rate of its current capital stock subject to utilisation costs. Finally, the household chooses the hourly wage rate for its labour service, and it stands ready to meet demand at the chosen wage.

We consider two specifications of the household felicity function. The first is a standard specification with constant elasticities of intertemporal subsitution, while the second, due to Greenwood, Hercowitz and Huffman (1988), is one that eliminates wealth effects on household labour supply decisions. We allow for habit persistence in consumption in both specifications.

Each period begins by the realisation of shocks to the economy. We concentrate on the shocks to the rate of capital depreciation that Liu, Waggoner and Zha (2010) found to be important drivers of the business cycle. But we also consider Gertler and Karadi's (2010) shocks to the quality of capital and the investment-specific technology shocks of Justiniano, Primiceri and Tambalotti (2010). We abstract from other shocks that may affect the economy.

2.1 Monopolistic competition

The labour used in production in each firm $i \in [0,1]$, denoted by $N_t(i)$, is a Dixit-Stiglitz aggregate of the differentiated labour services supplied by households

$$N_t(i) = \left(\int_0^1 N_t(i,j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \, dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{1}$$

where ε_w is the elasticity of substitution between labour services, and $N_t(i,j)$ represents the hours worked by household $j \in [0,1]$ in the production process of firm i. Denoting the wage rate demanded by household j by $W_t(j)$, cost minimisation by the firm (for a given level of total labour input) leads to a downward-sloping demand schedule for the labour service offered by this particular households. Aggregating over firms gives the economy-wide demand for the work hours offered by household j

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t \tag{2}$$

where ε_w represents the elasticity of demand, and $N_t = \int_0^1 N_t(i) \, di$ represents total hours worked in firms across the economy. W_t is the wage index defined as

$$W_t = \left(\int_0^1 W_t(j)^{1-\varepsilon_w} \, dj \right)^{\frac{1}{1-\varepsilon_w}} \tag{3}$$

This wage index has the property that the minimum cost of employing workers for N_t hours is given by $W_t N_t$.

Similarly, the final consumption good that enters household j's utility function is a Dixit-Stiglitz aggregate of the differentiated intermediate goods supplied by firms

$$C_t(j) \equiv \left(\int_0^1 C_t(i,j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{4}$$

where ε_p is the elasticity of substitution between product varieties, and $C_t(i,j)$ represents the consumption by household j of the good produced by firm i. Denoting the price demanded by firm i by $P_t(i)$, expenditure minimisation by the household (for a given level of final goods consumption) leads to a downward-sloping demand schedule for the intermediate good produced by this particular firm. Aggregating over households gives the economy-wide consumption demand for good i

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} C_t \tag{5}$$

where ε_p represents the elasticity of demand, and $C_t = \int_0^1 C_t(j)\, dj$ is aggregate consumption. P_t is the price index defined as

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon_p} di \right)^{\frac{1}{1-\varepsilon_p}} \tag{6}$$

This price index has the property that the minimum expenditure required to purchase C_t units of the composite good is given by $P_t C_t$.

Assuming that the elasticity of substitution between varieties of goods is the same when purchased for investment and for maintenance of machinery as when consumed, aggregate demand for an intermediate good i is given by

$$Y_t^d(i) \equiv C_t(i) + I_t(i) + M_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} (C_t + I_t + M_t) \tag{7}$$

where $I_t(i)$ represents goods produced by firm i that households devote to capital accumulation, while $M_t(i)$ denotes those devoted to covering capital utilisation costs, which we may think of as maintenance of the existing capital stock. Omission of firm indices indicate corresponding economy-wide variables (in per capita terms).

Aggregate output is defined as

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{8}$$

where $Y_t(i)$ is the output of firm i. Market clearing requires that $Y_t^d(i) = Y_t(i)$. The aggregate resource constraint in the economy is therefore

$$Y_t = C_t + I_t + M_t \tag{9}$$

2.2 Households

Each household $j \in [0, 1]$ maximises its expected discounted utility given by

$$E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}(j), N_{t+k}(j)\right) \tag{10}$$

where β is the subjective discount factor.

We consider two specifications of the instantaneous utility function. The first is standard in the New Keynesian literature (see for instance Galí, 2008)

$$U\left(C_t(j), N_t(j)\right) = \frac{\left(C_t(j) - hC_{t-1}\right)^{1-\sigma}}{1-\sigma} - \chi \frac{N_t(j)^{1+\eta}}{1+\eta} \tag{11}$$

The second follows Greenwood, Hercowitz and Huffman (1988)

$$U\left(C_t, N_t\right) = \frac{1}{1-\sigma} \left(C_t(j) - hC_{t-1} - \chi \frac{N_t(j)^{1+\eta}}{1+\eta} \right)^{1-\sigma} \tag{12}$$

We refer to the class of preferences represented by this utility function as GHH preferences. With both specifications we allow for habit formation in consumption, where $h \geq 0$ is the degree of habit persistence (there is no habit in consumption when $h = 0$). The habit formation is external to the household in the sense that the household ignores the effect of its current consumption choice on habit formation;

it is lagged *aggregate* consumption that enters the felicity function next period.

With standard preferences, the marginal utilities of consumption and labour are

$$MU_{C,t}^{STD}(j) = (C_t(j) - hC_{t-1})^{-\sigma} \tag{13}$$

and

$$MU_{N,t}^{STD} = -\chi N_t(j)^{\eta} \tag{14}$$

respectively. With GHH preferences, we get

$$MU_{C,t}^{GHH}(j) = \left(C_t(j) - hC_{t-1} - \chi \frac{N_t(j)^{1+\eta}}{1+\eta} \right)^{-\sigma} \tag{15}$$

and

$$MU_{N,t}^{GHH}(j) = -\chi \left(C_t(j) - hC_{t-1} - \chi \frac{N_t(j)^{1+\eta}}{1+\eta} \right)^{-\sigma} N_t(j)^{\eta} \tag{16}$$

The two specifications therefore result in different marginal rates of substitution between consumption and labour effort. With standard preferences, we get

$$MRS_t^{STD} = -\frac{MU_{N,t}^{S}(j)}{MU_{C,t}^{S}(j)} = \chi N_t(j)^{\eta} (C_t(j) - hC_{t-1})^{\sigma} \tag{17}$$

while the marginal rate of substitution with GHH preferences

$$MRS_t^{GHH} = -\frac{MU_{N,t}^{G}(j)}{MU_{C,t}^{G}(j)} = \chi N_t(j)^{\eta} \tag{18}$$

is independent of consumption. Hence, the supply of labour is determined independently of the intertemporal consumption allocation.

Households own the capital stock and let this capital to firms in a perfectly competitive rental market at the real rental rate R_t^K. The accumulated capital stock, $K_t(j)$, is subject to a quality shock, $Z_{\xi,t}$. Each household chooses the rate at which its capital is utilised, $U_t(j)$, which transforms the quality-adjusted accumulated capital stock, $Z_{\xi,t}K_t(j)$, into effective capital in period t, $\tilde{K}_t(j)$, according to

$$\tilde{K}_t(j) = U_t Z_{\xi,t} K_t(j) \tag{19}$$

The quality of capital shock evolves according to the autoregressive process

$$\log Z_{\xi,t} = \rho_\xi \log Z_{\xi,t-1} + \epsilon_{\xi,t} \tag{20}$$

where $0 < \rho_\xi < 1$, and $\epsilon_{\xi,t}$ is white noise.

Following Christiano, Eichenbaum and Evans (2005), the cost of capital utilisation is determined by the increasing and convex function $a(U_t(j))$ so that $M_t(j) = a(U_t(j))Z_{\xi,t}K_t(j)$. Steady-state utilisation is normalised to $U = 1$, and we assume $a(1) = 0$ and $a'(.), a''(.) > 0$.

The capital accumulation equation is given by

$$K_{t+1}(j) = (1-\delta_t)Z_{\xi,t}K_t(j) + Z_{I,t}\left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right)I_t(j) \tag{21}$$

where $I_t(j)$ is the amount of the final good acquired by the household for investment purposes, and $S(.)$ is a function representing investment-adjustment costs. We assume that $S(1) = S'(1) = 0$ and $S''(1) > 0$.

The rate of depreciation is given by $\delta_t = \delta Z_{\delta.t}$ where $0 \leq \delta \leq 1$ is the steady-state rate of depreciation and $Z_{\delta.t}$ is a shock to the rate of depreciation. The depreciation shock evolves according to the autoregressive process

$$\log Z_{\delta.t} = \rho_\delta \log Z_{\delta.t} + \epsilon_{\delta,t} \tag{22}$$

where $0 < \rho_\delta < 1$, and $\epsilon_{I,t}$ is white noise.

$Z_{I,t}$ is the investment-specific technology shock, which affects the extent to which resources allocated to investment (net of investment-adjustment costs) increase the capital stock available for use in production next period. It is therefore a shock to the marginal efficiency of investment. The shock evolves according to the autoregressive process

$$\log Z_{I,t} = \rho_I \log Z_{I,t-1} + \epsilon_{I,t} \tag{23}$$

where $0 < \rho_I < 1$, and $\epsilon_{I,t}$ is white noise.

Household maximisation is subject to a sequence of budget constraints taking the following form

$$\begin{aligned} & P_t C_t(j) + I_t(j) + M_t(j) + E_t(\Lambda_{t,t+1} B_{t+1}(j)) \\ \leq \; & B_t(j) + W_t(j) N_t(j) + T_t(j) + P_t R_t^K K_t(j) \end{aligned} \tag{24}$$

The left-hand side gives the allocation of resources to consumption, investment, capital adjustment costs, and to a portfolio of bonds, $E_t(\Lambda_{t,t+1} B_{t+1}(j))$, where $\Lambda_{t,t+1}$ is the stochastic discount factor and $B_{t+1}(j)$ represents contingent claims.[1] Hence, the risk-free (gross) nominal interest rate is defined by $R_t = (E_t \Lambda_{t,t+1})^{-1}$. The right-hand side gives available resources as the sum of bond holdings, labour income, dividends from firms, denoted by T_t, and rental income from capital.

First-order conditions with respect to consumption and bond holdings gives rise to an Euler equation summarising the intertemporal consumption allocation choice of households. It takes the standard form

$$1 = R_t E_t \Lambda_{t,t+1}. \tag{25}$$

where the stochastic discount factor is given as

$$\Lambda_{t,t+1} = \beta \frac{MU^l_{C,t+1}}{MU^l_{C,t}} \frac{P_t}{P_{t+1}}$$

$l \in \{STD, GHH\}$ is an index for the type of preferences assumed so that $MU^l_{C,t}$ is

[1] The stochastic discount factor $\Lambda_{t,t+1}$ is defined as the period-t price of a claim to one unit of currency in a particular state in period $t+1$, divided by the period-t probability of that state occuring.

the marginal utility of consumption as specified above. The assumption of complete markets allows us to drop household indices in this expression (and in many of those that follow). First-order conditions imply that risk-sharing is complete in consumption and investment under the complete market assumption as long as initial endowments are identical. That is, $C_t(j) = C_t$, $I_t(j) = I_t$, $\bar{K}_t(j) = \bar{K}_t$ and $U_t(j) = U_t$ for all $j \in [0,1]$.

First-order conditions with respect to investment and capital equates marginal cost and benefits of additional investment and capital

$$\begin{aligned} 1 &= Q_t Z_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] \\ &\quad + E_t\left[\Lambda_{t,t+1}\frac{P_{t+1}}{P_t}Q_{t+1}Z_{I,t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{aligned} \tag{26}$$

$$Q_t = \beta E_t\left\{\Lambda_{t,t+1}\frac{P_{t+1}}{P_t}Z_{\xi,t+1}\left[R^K_{t+1}U_{t+1} - \frac{M_{t+1}}{Z_{\xi,t+1}K_{t+1}} + Q_{t+1}\left(1-\delta_{t+1}\right)\right]\right\} \tag{27}$$

The variable Q_t, representing Tobin's q, is equal to the ratio of the Lagrange multipliers attached to the capital accumulation equation and the budget constraint, respectively.

Similarly, the first-order condition with respect to capital utilisation equates the marginal benefit of raising capital utilisation with the marginal cost of doing so. This first-order condition becomes

$$R^K_t = a'\left(U_t\right) \tag{28}$$

Households set wages following a Calvo mechanism. Each period a measure $(1-\theta_w)$ of randomly selected households get to set a new wage rate, while remaining households must keep theirs constant. A household allowed to reoptimise at time t sets $W_t(j) = W^*_t$ to maximise its expected life-time utility, (10), subject to its budget constraint, (24), the demand for its labour service, (2), and the restriction from the Calvo mechanism that

$$W_{t+k+1}(j) = \begin{cases} W^*_{t+k+1} & \text{w.p.}\,(1-\theta_w) \\ W_{t+k}(j) & \text{w.p.}\theta_w \end{cases} \tag{29}$$

The first-order condition is given by

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k}(j) \left[\frac{W^*_t}{P_{t+k}} MU^l_{C,t} + \mu_w MU^l_{N,t+k}(j) \right] \right\} = 0 \tag{30}$$

where $\mu_w \equiv \varepsilon_w (\varepsilon_w - 1)^{-1}$ is the household's desired mark-up of the real wage over the marginal rate of substitution. Again, $l \in \{STD, GHH\}$ denotes the class of preferences.

2.3 Firms

Each firm $i \in [0,1]$ produces a differentiated good, $Y_t(i)$, according to

$$Y_t(i) = \tilde{K}_t(i)^{\alpha} N_t(i)^{1-\alpha} \tag{31}$$

where $\tilde{K}_t(i)$ denotes the period-t effective capital stock rented by firm i, and $N_t(i)$ is the number of hours worked in the production process of firm i. We abstract from shocks to total factor productivity.

Firm i's marginal cost can be found as the Lagrange multiplier from the firm's cost minimisation problem

$$MC_t(i) = \frac{W_t/P_t}{(1-\alpha)\left(\tilde{K}_t(i)/N_t(i)\right)^{\alpha}} = \frac{R^K_t}{\alpha\left(N_t(i)/\tilde{K}_t(i)\right)^{1-\alpha}} \tag{32}$$

where R^K_t denotes the real rental rate of capital. Conditional factor demand schedules imply that firm i will choose factor inputs such that

$$\frac{\tilde{K}_t(i)}{N_t(i)} = \frac{\alpha}{1-\alpha}\frac{W_t/P_t}{R_t^K} \tag{33}$$

This equation implies that, on the margin, the cost of increasing capital in production equals the cost of increasing labour. Since all firms have to pay the same wage for the labour they employ, and the same rental rate for the capital they rent, it follows that marginal costs (of increasing output) are equalised across firms regardless of any heterogeneity in output induced by differences in prices. Hence, $MC_t(i) = MC_t\ \forall i$ where

$$MC_t = \frac{1}{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-a}\left(\frac{W_t}{P_t}\right)^{1-\alpha}\left(R_t^K\right)^{\alpha} \tag{34}$$

follows from combining (32) and (33).

Consequently, the marginal product of labour

$$MPL_t(i) = (1-\alpha)\,Y_t(i)\,/N_t(i) = \frac{W_t/P_t}{MC_t(i)} \tag{35}$$

is also equalised across firms so that $MPL_t(i) = MPL_t\ \forall i$.

Firms follow a Calve price-setting mechanism when setting prices. Each period, a measure $(1-\theta_p)$ of randomly selected firms get to post new prices, while remaining firms must keep their prices constant. A firm allowed to choose a new price at time t sets $P_t(i) = P_t^*$ to maximise the value of the firm to its owners, the households. At time t, this value is given by

$$\sum_{k=0}^{\infty} E_t\left\{\Lambda_{t,t+k}\left[P_{t+k}(i)\,Y_{t+k}(i) - \Psi\left(Y_{t+k}(i)\right)\right]\right\} \tag{36}$$

where $\Lambda_{t,t+k}$ is the stochastic discount factor, and $\Psi(.)$ is the cost function (i.e. the value function from the cost minimisation problem described above). Optimisation is subject to the demand for the firm's product, (7), its production technology, (31), and the restriction from the Calvo mechanism that

$$P_{t+k+1}(i) = \begin{cases} P^*_{t+k+1} & \text{w.p.}\,(1-\theta_p) \\ P_{t+k}(i) & \text{w.p.}\theta_p \end{cases} \tag{37}$$

The first-order condition is given by

$$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ \Lambda_{t,t+1} Y_{t+k}(i) \left[P_t^* - \mu P_{t+k} MC_{t+k} \right] \right\} = 0 \tag{38}$$

where $\mu_p \equiv \varepsilon_p (\varepsilon_p - 1)^{-1}$ is the desired mark-up of price over nominal marginal cost. This condition reflects the forward-looking nature of price-setting; firms take not only current but also future expected marginal costs into account when setting prices.

2.4 Monetary policy

We assume that the central bank reacts to inflation $\Pi_{P,t} = (P_t - P_{t-1})/P_{t-1}$ according to a simple Taylor rule

$$\frac{R_t}{R} = \left(\frac{\Pi_{P,t}}{\Pi_P} \right)^{\phi_\pi} \tag{39}$$

where the omission of time subscripts indicate steady-state values and $\phi_\pi > 1$.

2.5 Calibration

We calibrate the model's parameter values and solve it numerically after log-linearising the equilibrium conditions. The steady state around which we log-linearise is characterised in appendix A, and the log-linear relations are summarised in appendix B.

We consider the length of a period to be one quarter, and we let $\beta = 0.99$ implying that the annual interest rate is about 4 per cent in steady state. We set the steady-state depreciation rate to $\delta = 0.025$ and the capital share to $\alpha = 0.33$.

We assume that utility is logarithmic by setting $\sigma = 1$, and we set the inverse of the labour supply elasticity to $\eta = 1$. Desired mark-ups in both labour and goods markets are assumed to be 20 per cent, which we achieve by setting $\varepsilon_p = \varepsilon_w = 6$. We use χ to pin down hours in steady state to $N = 1/3$ of available time. These are values in line with those commonly found in the New Keynesian literature, see, e.g., Christiano, Eichenbaum and Evans (2005), Galí (2008), Golosov and Lucas (2007) and Smets and Wouters (2007). Monetary policy is such that $\tau_\pi = 1.5$ as originally suggested by Taylor (1993).

We consider versions of the model with and without investment adjustment costs. When we assume that investment adjustment is costly, we set the inverse of the second derivative of the investment adjustment cost function to $\lambda_s = 0.2$, slightly larger than the 0.17 estimated by Smets and Wouters (2007), but smaller than the 0.4 estimated by Christiano, Eichenbaum and Evans (2005), and the 0.34 found by Justiniano, Primiceri and Tambalotti (2010). In the log-linear model, this is the only characteristic of the investment adjustment function with implications for the model's propagation mechanism. By reducing the convexity of the adjustment cost function, an increase in λ_s leads to a smaller investment adjustment cost for a given change in investment. Hence, the sensitivity of households' investment decisions to changes in the current value of installed capital (Tobin's q) will increase as λ_s increases. Without investment adjustment costs, we let $\lambda_s \to \infty$.

We consider versions of the model with and without habit persistence. When allowing for habit persistence, we set $h = 0.9$. Similarly, we consider the model both with and without variable capacity utilisation. When allowing for variable capacity utilisation, we set $\lambda_a = 2.3$. These are the values estimated by Liu, Waggoner and Zha (2010).

Throughout we consider both the case with flexible wages and prices, i.e. $\theta_w = \theta_p = 0$, and the case with nominal wage and price rigidity with $\theta_w = \theta_p = 0.7$. This choice strikes a balance between the microdata evidence provided by Bils and Klenow (2004) and Nakamura and Steinsson (2008) for prices, and the slightly larger values usually considered for wages, while keeping the degrees of nominal rigidity in labour and goods markets equal for convenience.

3 How capital depreciation shocks may become important

The analysis in Amber and Paquet (1994) does not suggest that capital depreciation shocks should be important for business cycle fluctuations. In contrast, Liu, Waggoner and Zha (2010) find that capital depreciation shocks act as important drivers of the business cycle. In this section we try reconcile these opposing conclusions. The aim is to provide an understanding of the model features needed for the shocks to play an important role in business cycle fluctuations.

3.1 The RBC model with white noise shocks

We first simulate a version of the model resembling the RBC model by Ambler and Paquet (1994), i.e. a version of the model in the previous section without any of the real and nominal frictions that are central to DSGE models in the New Keynesian tradition. Thus, we first consider a model with flexible wages and prices, fixed utilisation of capital, free adjustment of investment, and without habit persistence in consumption. Throughout this section, preferences take the standard form in (11). Also, we let the depreciation shocks be white noise as assumed by Ambler and Paquet (1994).

Figure 1 shows the impulse responses to a white-noise shock to the rate of capital depreciation in the RBC model. The shock essentially works to destroy a part of the existing capital stock. As the shock is purely temporary, this is a one-off event, and the economy's adjustment therefore follows the transitional dynamics known from the neoclassical growth model, see e.g. King and Rebelo (1999).

The shock reduces the capital stock below its steady-state level, and this drop in the economy's productive capacity results in lower output. Agents seek to rebuild the capital stock by increasing investment and postponing consumption, which is facilitated by an increase in the real return. At the same time, a negative wealth effect makes it optimal for households to reduce consumption and leisure, and to increase working hours. Hence, output and consumption fall, while hours and investment rise. As the capital stock is rebuilt, the economy gradually reverts to it's steady state.

To see the interest rate effect, note that the real return in the log-linearised RBC version of the model (abstracting from quality and investment shocks) is simply

$$\begin{aligned} r_t - E_t \pi_{P,t+1} &= (1 - \beta(1-\delta)) E_t r^K_{t+1} - \beta\delta E_t z_{\delta,t+1} \\ &= (1 - \beta(1-\delta)) E_t r^K_{t+1} - \beta\delta\rho_\delta z_{\delta,t} \end{aligned} \tag{40}$$

where the rental rate, r_t^K, is equal to the marginal product of capital. This follows from the first-order condition for capital. Thus, when the shock is white noise so that $\rho_\delta = 0$, the real return is simply given as the expected marginal product of capital in the next period. Therefore, the real rate increases on impact of the depreciation rate shock as effective capital falls in the next period. Without habit persistence, the Euler equation is

$$E_t c_{t+1} - c_t = r_t - E_t \pi_{P,t+1} \tag{41}$$

Hence, an increase in the real rate makes it optimal for households to postpone consumption over time by chosing an increasing consumption path. The wealth effect of the shock makes sure that initial consumption falls to allow an increase in consumption back to the steady-state. By the aggregate resourse constraint, the consumption response is mirrored through intertemporal substitution by an initial increase in investment and a declining investment path.

There is one caveat, however. The timing assumption of the depreciation shock means that the impact response deviates from the transitional dynamics just described. The shock leaves the initial effective capital unchanged, affecting only the capital available for production in the next period. This means that agents get a chance to work hard to counter the effect on the capital stock before the marginal product of their labour falls as a consequence of the shock. The shock therefore leads to an initial expansionary effect. When investment is free to adjust without cost, the response is strong enough to increase output on impact of the shock as households increase the supply of labour. But in the next period, when the shock affects effective capital, the standard transitional dynamics take over.

The labour market equilibrium condition provides a different perspective on the propagation mechanism. This relation equates the marginal product of labour (labour demand) to the marginal rate of substitution (labour supply) up to a proportionality factor, μ_t, given as the product of the desired mark-ups in goods and labour markets. That is

$$(1-\alpha)\,\tilde{K}_t^{\alpha} N_t^{-\alpha} = \mu_t \chi {N_t}^{\eta} {C_t}^{\sigma} \tag{42}$$

With flexible wages and prices, $\mu_t = \mu$ is constant. Since capital is a predetermined variable, this equilibrium condition implies that consumption and hours must move in opposite directions on impact of the shock to the rate of depreciation. When preferences are standard and capital utilisation is fixed, only shocks to the marginal product of labour (e.g. neutral technology shocks) or shocks to the marginal rate of substitution (e.g. labour supply shocks) could induce co-movement of consumption and hours, cf. Barro and King (1984). As the depreciation shock causes intertemporal substitution from consumption to investment as well as a reduction in consumption through a wealth effect, hours worked are driven up through a shift to the right of the labour supply curve.

In sum, following an initial impact response driven by the timing assumption, the RBC model features negative co-movement of investment and hours (both increase) on the one hand, and output and consumption (both decline) on the other. Given that empirical business cycles are characterised by significant positive co-movement of all these four variables, it is not surprising that Ambler and Paquet (1994) find a negligible role for depreciation shocks in driving the business cycle in their RBC model. As emphasised by King and Rebelo (1999), the capital accumulation process is central to business cycles in the RBC framework not as a source of shocks, but as a key propagation mechanism for total factor productivity shocks.

3.2 The RBC model with persistent shocks

We now consider the implications of increasing the persistence of the depreciation shock to the $\rho_\delta = 0.93$ estimated by Liu, Waggoner and Zha (2010), while keeping the simple frictionless RBC framework. In contrast to total factor productivity shocks, increasing the persistence of depreciation rate shocks has non-trivial implications for the propagation mechanism beyond simply drawing out responses over time to compensate for the weak internal propagation of the RBC model.

Responses to a persistent depreciation shock are shown in figure 2. In contrast to the case with white noise shocks, investment falls along with output on impact of the shock. The reason for this is a strong interest rate effect that induces intertemporal substitution *away* from investment.

From (40), the real return is affected by the expected future depreciation rate as well as the marginal product of labour. Hence, when ρ_δ is large enough, i.e. when the shock is sufficiently persistent, the real return may fall despite an expected increase in the marginal product of capital. By (41), the optimal consumption path is then declining. Consequently, agents may find it optimal not to reconstruct the capital destroyed, but rather to substitute away from investment into consumption. After all, the resources they devote to investment will slowly but surely depreciate, while they gain instant utility from consumption. Only as the capital depreciation shock has abated will investment eventually pick up to reconstruct the lost capital and bring it back to its steady-state level. The persistence of the shock effectively delays the transitional dynamics of the neoclassical growth model; the higher the persistence of the shock, the sharper the contraction in investment in the initial periods, and the farther into the future the recovery in the capital stock.

Without costs to adjusting investment and consumption, the substitution is strong enough to actually increase consumption in the initial periods. This is despite the negative wealth effect caused by the substantial destruction of the economy's productive capacity implied by the persistence in the depreciation shock. With flexible wage and prices, hours and consumption move in opposite directions as before. Specifically, agents find it optimal to increase also the consumption of leisure in the initial periods by increasing labour supply.

Hence, while both investment and hours now fall in line with output on impact of a depreciation shock, it comes at the cost of a countercyclical impact response of consumption. Therefore, an increase in the persistence of depreciation shocks alone, while fundamentally changing the responses of key variables, is unable to deliver the co-movement needed to generate empirical recognisable business cycles in the RBC model.

3.3 The New Keynesian model

We now consider responses to a persistent shock in the New Keynesian model with nominal and real frictions. Specifically, we consider a basic New Keynesian model with nominal rigidities and investment adjustment costs ($\theta_w = \theta_p = 0.7$ and $\lambda_s = 0.2$), and the complete New Keynesian model described in section 2, which is similar to the model in Liu, Waggoner and Zha (2010), also including habit persistence in consumption ($h = 0.9$) and variable capacity utilisation ($\lambda_a = 2.3$).

When wages and prices are sticky, mark-ups in goods and labour markets generally deviate from their desired levels, and they vary over time as the economy is hit by shocks. Hence, $\mu_t \neq \mu$ in (42). Specifically, nominal rigidities prevent firms from reducing prices and households from reducing wages when faced with a contraction in demand. The nominal rigidities therefore imply countercyclical mark-ups that will shift labour demand down following a positive depreciation shock. This both allows consumption and hours to move in the same direction on impact of the shock, cf. (42), and it works as an additional contractionary force in the model. The negative wealth effect on consumption is therefore more likely to dominate the interest rate effect.[2] Consequently, consumption may fall along with output, hours and investment in response to a persistent depreciation shock.

Indeed, this is what we find for both the basic and the complete New Keynesian model, represented in figure 3 by solid and dashed lines, respectively. The recession following a positive depreciation shock is slightly smaller in the full model as the effect of the shock is partially off-set by an increase in capacity utilisation. Also, habit persistence in consumption slows the decline in consumption down. But in both models, output, consumption, investment and hours all fall following a postive depreciation shock. Hence, the combination of persistence in the shock process (reducing the real return to investment) and nominal rigidities (shifting labour demand to the left) changes the propagation mechanism of the RBC model fundamentally. In particular, it induces the co-movement across real variables needed for the depreciation shocks to be potentially important drivers of the business cycle as found by Lie, Waggoner and Zha (2009).

3.4 Discussion

The discussion so far leaves open the question about the empirical relevance of the two features needed to generate co-movement. Nominal rigidities are commonly assumed, but remain a controversial ingredient in modern business cycle models, while capital depreciation shocks have not received a lot of attention in the literature.

The debate on the plausibility of nominal rigidities is beyond the scope of this

[2] If the interest rate effect on consumption is very strong, e.g. if investment falls a lot because of very small investment adjustment costs, consumption may still increase on impact of very persistent shocks. But with nominal rigidities, the investment response is unreasonably large without some cost to the adjustment of investment.

paper. We simply note that any mechanism that would generate countercyclical mark-ups have the potential to stand in for nominal rigidities in this analysis. In contrast, a discussion of the plausibility of persistant capital depreciation shocks is warranted.

According to Ambler and Paquet (1994), as far as we know the first to study capital depreciation shocks explicitly in a business cycle context, these shocks could capture three factors that are not specified in standard models. First, they could represent natural disasters, or more generally weather conditions, with impact on the capital stock. Second, they could represent heterogeneity of capital across sectors so that shifts in the relative importance of sectors could be reflected in shocks to the aggregate depreciation rate. And third, they could capture the obsolescence of capital used to produce certain products that become outdated.

It is difficult to maintain the interpretation that these shocks represent natural disasters when the shocks become persistent and when they appear to be one of the most important drivers of the US business cycle. More plausible are the interpretations that the shocks reflect persistent shift in the relative importance of heterogeneous sectors or extended periods in which part of the capital stock becomes economically obsolete, which is the interpretation emphasised by both Liu, Waggoner and Zha (2010) and Gertler and Karadi (2010). But we believe capital depreciation shocks can be interpreted more broadly. The capital accumulation equation can be seen as a production function for capital goods so that capital is produced by combining undepreciated capital with investment. A shock to the depreciation rate may then more generally reflect a shock to the process by which investment goods are transformed into productive capital.

With such an interpretation, the assumption of persistence in the shock process becomes more appealing, and it seems more plausible that the shocks could be important drivers of the business cycle. A positive shock to the depreciation rate could, for instance, reflect a negative technology shock in the production of capital goods. Alternatively, a shock to the capital accumulation process may reflect disturbances to the intermediation ability of the financial system to finance investment projects as suggested by Justiniano, Primiceri and Tambalotti (2009, 2010). A persistent positive depreciation shock may then reflect a lengthy restructuring process in the financial sector, while a negative shock may reflect a period with easy access to external financing. In any event, we would be sceptical that persistent depreciation shocks of the size needed for them to be able to generate empirically recognisable business cycles in DSGE models should be given a strictly structural interpretation.

4 The depreciation rate shock and its rivals

Shocks to the capital accumulation equation take different forms in the literature. The shock considered by Justiniano, Primiteri and Tambalotti (2010) is an investment-specific technology shock, equivalent to $Z_{I,t}$, rather than a depreciation shock, $Z_{\delta,t}$. They find that this shock is the most important driver of the US business cycle, even if it fails to generate co-movement of consumption. In a recent paper, Gertler and Karadi (2010) study a capital accumulation shock, which they call a shock to the quality of capital, in a model differentiating capital producing firms from intermediate good producing firms. In our model, this shock is equivalent to $Z_{\xi,t}$. Gertler and Karadi (2010) find that this shock generates co-movement across real variables, and importantly, they show how a financial friction may amplify the effects as well as the co-movement in a way that enables them to similate a crisis of the character and magnitude experienced in the past couple of years.

In this section we compare the shock to the rate of capital depreciation, $Z_{\delta,t}$, first with the shock to the quality of capital, $Z_{\xi,t}$, considered by Gertler and Karadi (2010), then with the investment-specific technology shock, $Z_{I,t}$, considered by Justiniano, Primiteri and Tambalotti (2010).

4.1 Quality of capital

Figure 4 compares the responses to a white-noise shock to the rate of capital depreciation with a white-noise shock to the quality of capital in the basic RBC model. The shocks are normalised to give the same impact on effective capital. A one unit standard-deviation increase in the depreciation rate increases depreciation from 2.5 to 5 per cent of the capital stock. This compares to a 0.025 unit standard deviation decline in the quality of capital, reducing effective capital by 2.5 per cent. Hence, we are comparing a positive depreciation rate shock to a negative shock to the quality of capital.

As the figure shows, the shocks induce essentially identical dynamics in all periods except the first one. In contrast to the shock to the rate of capital depreciation, the shock to the quality of capital destroys a part of the capital stock immediately on impact of the shock. This means that effective capital is immediately affected, and the economy's response is reflected entirely by the traditional transition dynamics from a capital stock that is initially below its steady-state value. The timing assumption implicit in the quality shock is the more reasonable one, if we think it unlikely that agents receive warning about future exogenous developments in the effective capital stock. On the other hand, the initial response to the depreciation rate shock may be indicative of the responses following news announcements concerning capital production.

Figure 5 compares responses to the two shock with a high level of persistence ($\rho_\delta = \rho_\xi = 0.93$) in the basic New Keynesian model with nominal rigidities and investment adjustment costs. When the shocks are persistent, the responses (solid and dashed lines, respectively) are almost identical, as the initial anticipation response to the shock to the depreciation rate is dominated by similar responses in subsequent periods (this holds also without nominal rigidity). Hence, the shocks differ only in terms of initial timing effects that become unimportant as persistence increases. When shocks are persistence, the choise between quality shocks and depreciation shocks is therefore essentially inconsequential.

Consequently, the reasoning in the previous section also applies to the capital quality shock used by Gertler and Karadi (2010). Indeed, they assume four quarters of price rigidity and consider a persistent shock with an autoregressive coefficient of 0.66. Our analysis suggests that nominal rigidity and persistence in the shock process are important assumptions needed to generate co-movement in their model.[3]

As a final note, the analysis also suggest an alternative amplification mechanism of a quality shock to the financial frictions considered by Gertler and Karadi (2010). The dotted lines in figure 5 show responses to a persistent quality shock when the standard utility specification in (11) is replaced by the GHH specification in (12) eliminating the wealth effect on labour supply. This specification introduces a complementarity between consumption and hours that tend to further reduce hours and consumption. This amplifies the contraction in aggregate output. In contrast to the financial friction, the amplification through a weak wealth effect on labour supply mainly works through consumption instead of investment.

4.2 Investment-specific technology

Figure 6 compares the responses to a white-noise shock to the rate of capital depreciation with a white-noise shock to investment specific technology in the basic RBC model. The shocks are normalised to give similar impacts on effective capital. Hence, we consider a negative shock to investment-specific technology of a size that reduces effective capital in the next period by 2.5 per cent when compared to a one unit standard deviation depreciation shock. As the change in effective capital is brought about indirectly through the investment response, the reponses of flow variables will be much stronger for a given change in effective capital. In other words,

[3] When we simulate our model using parameter values chosen by Gertler and Karadi (2009), we find impulse responses that are very similar to the ones they report for the case without financial frictions (their figure 2, dashed lines). In this case, the initial drop in investment is modest. According to our analysis, this is because they assume flexible wages and a moderately persistent shock with a persistence parameter at 0.66.

investment shock generate fluctuations of a given size for much smaller movements in the capital stock than depreciation shocks.

From the figure, we see that output, investment and hours fall on impact of a negative shock to investment-specific technology, while consumption increases. The drop in investment brings the capital stock below its steady-state value in the next period, and since the shock is white noise, the economy reverts to the standard transition dynamics. But the initial responses set the investment-specific technology shocks apart from depreciation shocks despite the fact that both are shocks to capital accumulation.

To understand this difference, note that the investment shock is inversely related to Tobin's q. Without costs to investment adjustment we have $q_t = -z_{I,t}$. Thus, subject to investment shocks, the real return becomes

$$\begin{aligned} r_t - E_t \pi_{P,t+1} &= (1-\beta(1-\delta)) E_t r^K_{t+1} - \beta(1-\delta) E_t z_{I,t+1} + z_{I,t} \\ &= (1-\beta(1-\delta)) E_t r^K_{t+1} + (1-\beta(1-\delta)\rho_I) z_{I,t} \end{aligned} \tag{43}$$

By increasing the cost of investment, an adverse investment-specific technology shock works to reduce the real return. In contrast to depreciation shocks, the real return will fall regardless of the persistence of the shock. But the negative effect on the real return is stronger, the lower the persistence of the shock, as persistence reduces the loss of value on the newly acquired capital stock. A fall in the rate of return induces intertemporal substitution from investment to consumption. Therefore, investment falls and consumption increases on impact of the shock. By the labour market equilibrium condition, hours fall.

Figure 7 compares the two shocks for a degree of persistence equal to 0.93. In this case, the depreciation shock also reduces the real return on impact of the shock. Therefore, responses are similar when the shocks are highly persistent. However, the wealth effect allows for a larger increase in consumption on impact of the investment shock. By implication, the negative wealth effect is less likely to dominate the interest rate effect when we introduce countercyclical mark-ups.

Figure 8 compares responses of the four key macroeconomic variables in the basic New Keynesian model for different levels of persistence. We see that co-movement is achieved for sufficiently persistent depreciation shocks, while the investment-specific technology shocks fail to generate comovement of consumption. Only for highly temporary investment shocks will the consumption response be flat initially.

Figure 9 reproduces figure 8 for the alternative GHH preferences. In this case, we may generate moderate co-movement for low levels of persistence for the investment-specific shocks. In contrast, the co-movement result for depreciation shock is not sensitive to the preference specification. GHH preferences amplifies the effects to a capital depreciation shock, but it is the combination of nominal rigidity and persistence that allows us to generate the co-movement characteristic of empirically recognisable business cycles.

5 Concluding remarks

We have studied the propagation of shocks to the rate of capital depreciation in a DSGE model allowing for nominal rigidities. We have found that it is the combination of nominal rigidities and persistence in the shock process that induces the co-movement across real variables needed for depreciation shocks to be potentially important drivers of business cycles. The shocks differ from quality shocks of capital only by an implicit timing assumption that becomes unimportant for persistent shocks. Compared to investment-specific technology shocks, depreciation shocks generate co-movement under more general assumptions about the structure of the economy.

In the existing literature, depreciation shocks have been found to be important drivers of the business cycle, and they have been used to explain both the beginning and the end of the Great Moderation. Interpreting depreciation shocks broadly as shocks to the capital accumulation process, possibly reflecting the intermediation ability of the financial system, we believe the further study of depreciation shocks as potentially important drivers of the business cycle holds promise for improving our understanding of macroeconomic fluctuations.

A The steady state

Steady-state variables are indicated by omission of time subscripts. In steady state we have $U = (P^*/P) = 1$ and $\Pi^p = \Pi^W = 0$ where Π^W represents steady-state wage inflation. Hence from (19) $\bar{K} = K$. From (21) we get $I = \delta K$ and from (25) $R = \beta^{-1}$. From (26) we get $Q = 1$ and so from (27) $R^K = \left(\beta^{-1} - 1 + \delta\right)$. (28) now gives a restriction on $a'(1) = R^K$. (38) implies $MC = \mu^{-1}$.

Combining (31) and (32) then gives the restriction

$$\gamma_k \equiv \frac{K}{Y} = \frac{\alpha MC}{R^K} \tag{44}$$

so that

$$\gamma_i \equiv \frac{I}{Y} = \frac{\delta\alpha}{\mu\left(\beta^{-1} - 1 + \delta\right)} \tag{45}$$

Then, from (9) we get

$$\gamma_c \equiv \frac{C}{Y} = 1 - \gamma_i \tag{46}$$

Combining (31) and (21) gives

$$Y = N\left(\gamma_i \delta^{-1}\right)^{\frac{\alpha}{1-\alpha}} \tag{47}$$

and consequently

$$C = \gamma_c Y \tag{48}$$

while (33) now gives

$$\frac{W}{P} = (1 - \alpha)\, MC \frac{Y}{N} \tag{49}$$

Taking N as given, a restriction on χ follows (or, alternatively, given χ we can find N) from (30). With standard preferences, this restriction is

$$\chi = \frac{W/P}{\mu_w N^\eta \left(C - hC\right)^\sigma} \tag{50}$$

and with GHH preferences

$$\chi = \frac{W/P}{\mu_w N^\eta} \tag{51}$$

This completes the solution of the model in steady state.

For future reference, we define

$$\sigma_r \equiv \sigma\,(1-h)\left(1-h-\frac{\chi}{1+\eta}\frac{N^{1+\eta}}{C}\right)^{-1} \tag{52}$$

B Log-linearisation

We log-linearise the equilibrium dynamics outlined in section 2 around the steady state described in appendix A. Lower case letters denote the log-deviation of a variable from its steady state value.

The relation between the stock of capital and effective capital, (19) becomes

$$\tilde{k}_t = u_t + z_{\xi,t} + k_t \tag{53}$$

while the capital accumulation equation (21) in log-linear form is given by

$$k_{t+1} = (1-\delta)\,(k_t + z_{\xi,t}) + \delta\,(i_t + z_{I,t}) - d\delta_t \tag{54}$$

where $d\delta_t = \delta z_{\delta,t}$ with maintenance costs and $d\delta_t = \delta z_{\delta,t} + \left(\beta^{-1} - 1 + \delta\right) u_t$ with user costs.

With standard preferences, the consumption Euler equation (25) takes the form

$$c_t = \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} - \frac{1}{\sigma}\frac{1-h}{1+h}\left(r_t - E_t\pi^p_{t+1}\right) \tag{55}$$

With GHH preferences, it becomes

$$\begin{aligned} c_t &= \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} - \frac{1}{\sigma_r}\frac{1-h}{1+h}\left(r_t - E_t \pi^p_{t+1}\right) \\ & -\frac{1-h}{1+h}\left(1+\eta\right)\left(1-\sigma\sigma_r^{-1}\right)\left(E_t n_{t+1} - n_t\right) \end{aligned} \tag{56}$$

where σ_r is defined in (52).

The linearised first-order conditions with respect to investment and capital read

$$i_t = \frac{1}{1+\beta}\left(\beta E_t i_{t+1} + i_{t-1} + \lambda_s \left(q_t + z_{I,t}\right)\right) \tag{57}$$

$$\begin{aligned} q_t &= -\left(r_t - E_t \pi_{t+1}\right) + \left(1 - \beta\left(1-\delta\right)\right) E_t r^k_{t+1} \\ & +\beta\left(1-\delta\right) E_t q_{t+1} + E_t z_{\xi,t+1} - \beta\delta E_t z_{\delta,t+1} \end{aligned} \tag{58}$$

where the value of $\lambda_s^{-1} \equiv S''\left(1\right) > 0$ governs investment-adjustment costs.

With the Christiano, Eichenbaum and Evans (2005) specification of capital utilisation costs, the first-order condition with respect to capital utilisation (28) becomes

$$r^k_t = \lambda_a u_t \tag{59}$$

in its log-linear form where

$$\lambda_a \equiv \frac{a''\left(U\right)U}{a'\left(U\right)} = \frac{a''\left(1\right)}{a'\left(1\right)} \tag{60}$$

is the elasticity of the marginal costs of capital utilisation.

By combining (30) with the law of motion of the wage index, a standard New Keynesian Phillips curve for wage inflation, π^W_t, is derived

$$\pi_{W,t} = \beta E_t \pi_{W,t+1} + \kappa_w \left(mrs^l_t - \left(w_t - p_t\right)\right) \tag{61}$$

for $l \in \{STD, GHH\}$ where $mrs_t^{std} = \sigma (1-h)^{-1} (c_t - hc_{t-1}) + \eta n_t$ is the economy's average marginal rate of substitution under standard preferences, and $mrs_t^{ghh} = \eta n_t$ is the same average under GHH preferences. The slope is given by

$$\kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w (1+\eta\varepsilon_w)}$$

Up to a first-order approximation, aggregate production is given by

$$y_t = \alpha \tilde{k}_t + (1-\alpha) n_t \tag{62}$$

By combining (38) with the law of motion of the price index, the standard New Keynesian Phillips curve is derived

$$\pi_{P,t} = \beta E_t \pi_{P,t+1} + \kappa_p mc_t \tag{63}$$

where $\kappa_p = (1-\beta\theta_p)(1-\theta_p)\theta_p^{-1}$ and

$$mc_t = (1-\alpha)(w_t - p_t) + \alpha r_t^k \tag{64}$$

The factor input relation (33) becomes

$$r_t^k = (w_t - p_t) + n_t - \tilde{k}_t \tag{65}$$

The aggregate resource contraint (9) in log-linear from is given as

$$y_t = \gamma_c c_t + \gamma_i i_t + \gamma_k \left(\beta^{-1} - 1 + \delta\right) u_t \tag{66}$$

where the last term in u_t drops out with user cost of capital. The monetary policy rule, (39), is

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\phi_\pi \pi_{P,t} + (1-\rho_r)\phi_y (y_t - y_{t-1}) \tag{67}$$

while the exogenous driving forces are specified as

$$z_{\xi,t} = \rho_\xi z_{\xi,t-1} + \epsilon_{\xi,t} \tag{68}$$

$$z_{I,t} = \rho_I z_{I,t-1} + \epsilon_{I,t} \tag{69}$$

and

$$z_{\delta,t} = \rho_\delta z_{\delta,t-1} + \epsilon_{\delta,t} \tag{70}$$

where $\epsilon_{\xi,t} \overset{iid}{\sim} \left(0, \sigma_\xi^2\right)$, $\epsilon_{I,t} \overset{iid}{\sim} (0, \sigma_I^2)$ and $\epsilon_{\delta,t} \overset{iid}{\sim} (0, \sigma_\delta^2)$.

Finally, the model in log-linear form is closed by adding the identity

$$\pi_{W,t} - \pi_{P,t} = (w_t - p_t) - (w_{t-1} - p_{t-1}) \tag{71}$$

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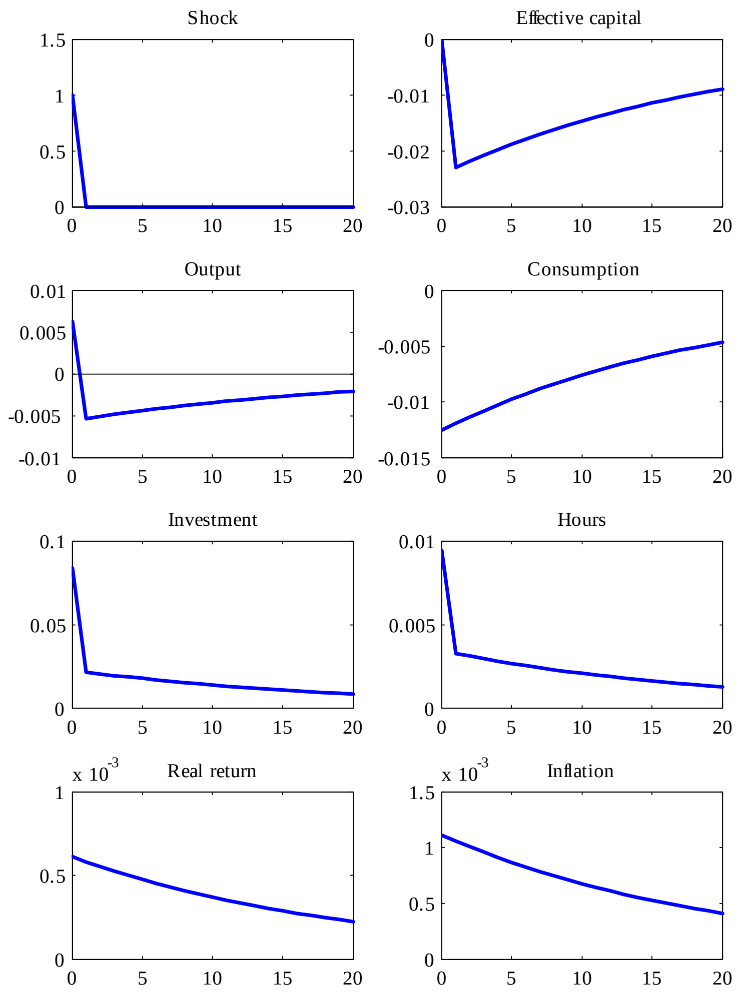

Figure 1: Responses to a white noise shock to the rate of capital depreciation ($\rho_\delta = 0$) in the basic RBC model.

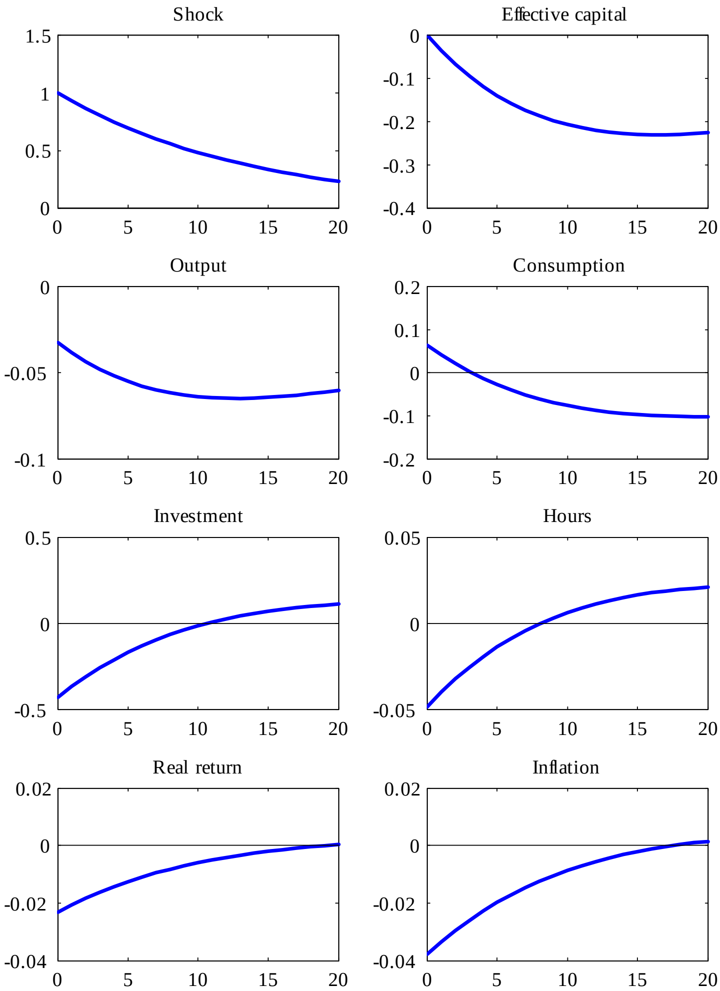

Figure 2: Responses to a persistent shock to the rate of capital depreciation ($\rho_\delta = 0.93$) in the basic RBC model.

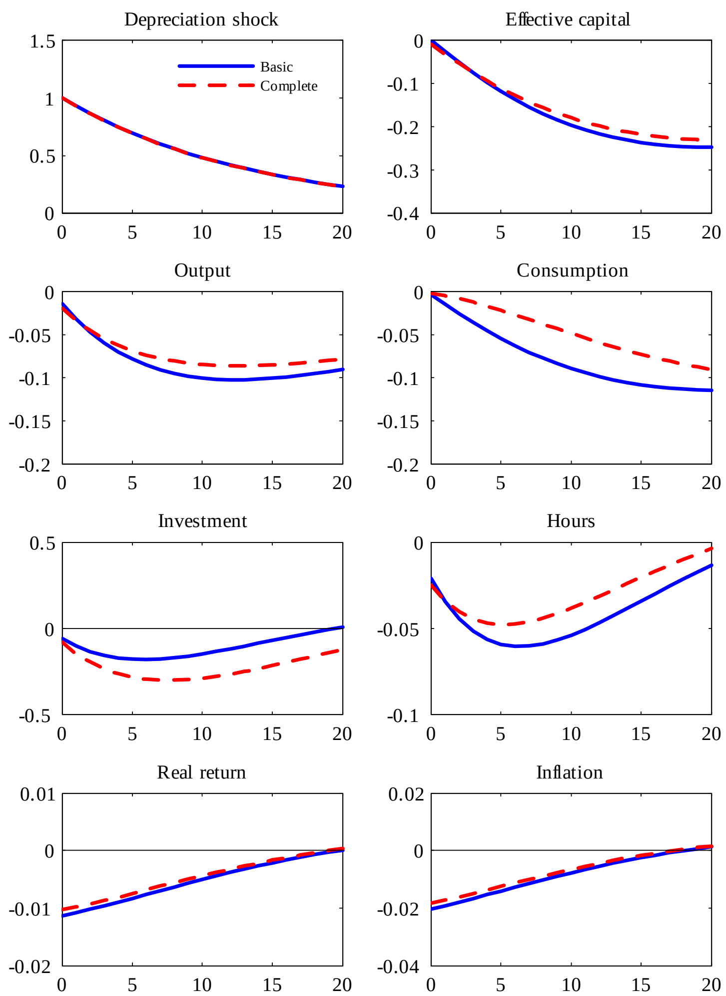

Figure 3: Responses to a persistent shock to the rate of capital depreciation ($\rho_\delta = 0.93$) in the New Keynesian model.

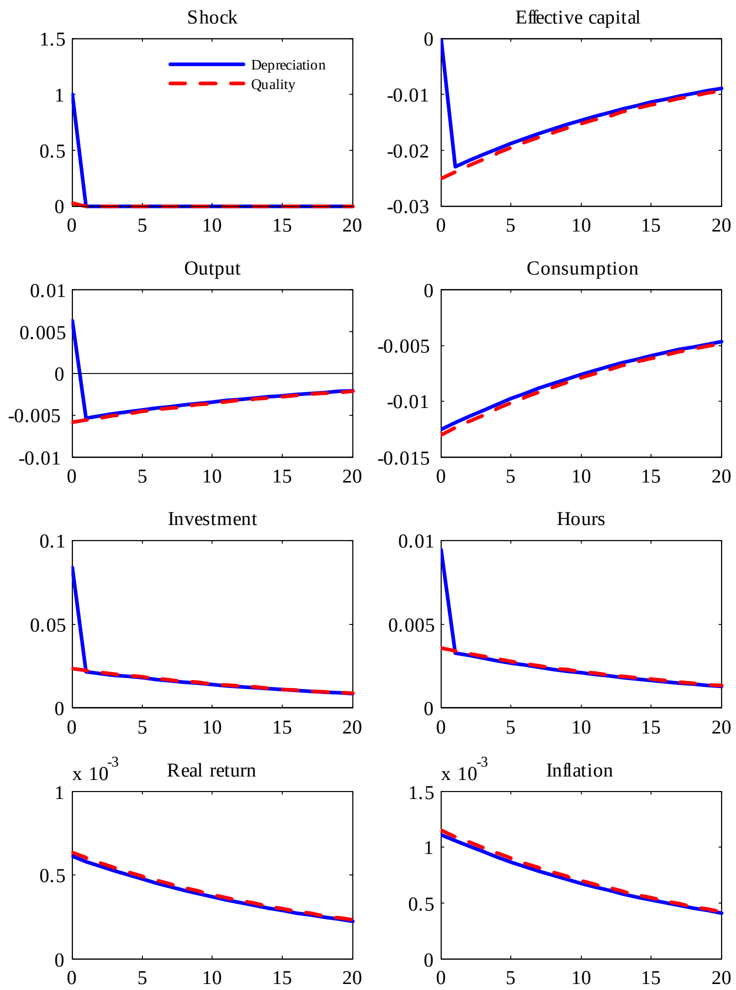

Figure 4: Responses to a white noise shock to the rate of capital depreciation and to the quality of capital ($\rho_\delta = \rho_\xi = 0$) in the basic RBC model.

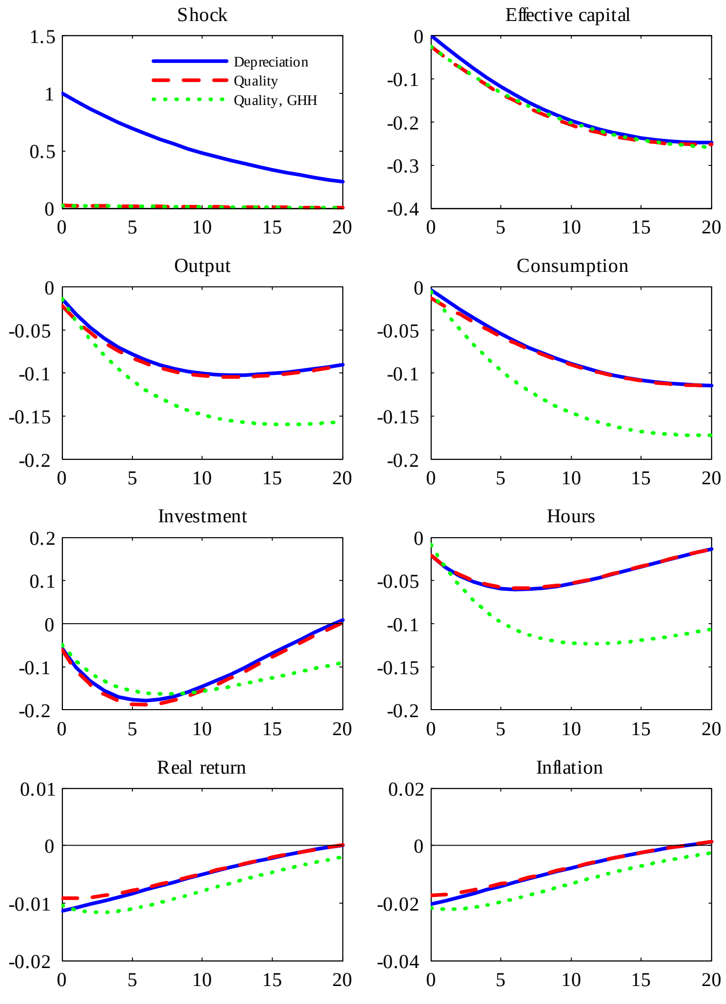

Figure 5: Responses to a persistent shock ($\rho_\delta = \rho_\xi = 0.93$) to the rate of capital depreciation (solid lines) and the quality of capital (dashed lines) in the basic New Keynesian model, plus responses to a persistent shock to the quality of capital in the New Keynesian model with GHH preferences (dotted lines).

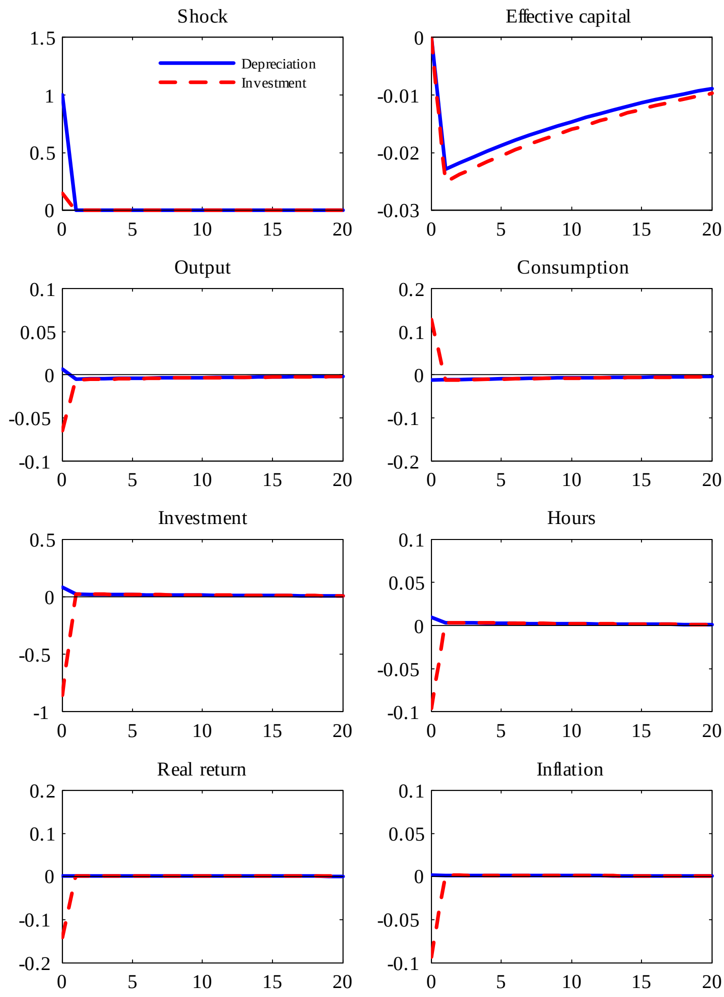

Figure 6: Responses to a white noise shock to the rate of capital depreciation and to investment-specific technology ($\rho_\delta = \rho_I = 0$) in the basic RBC model.

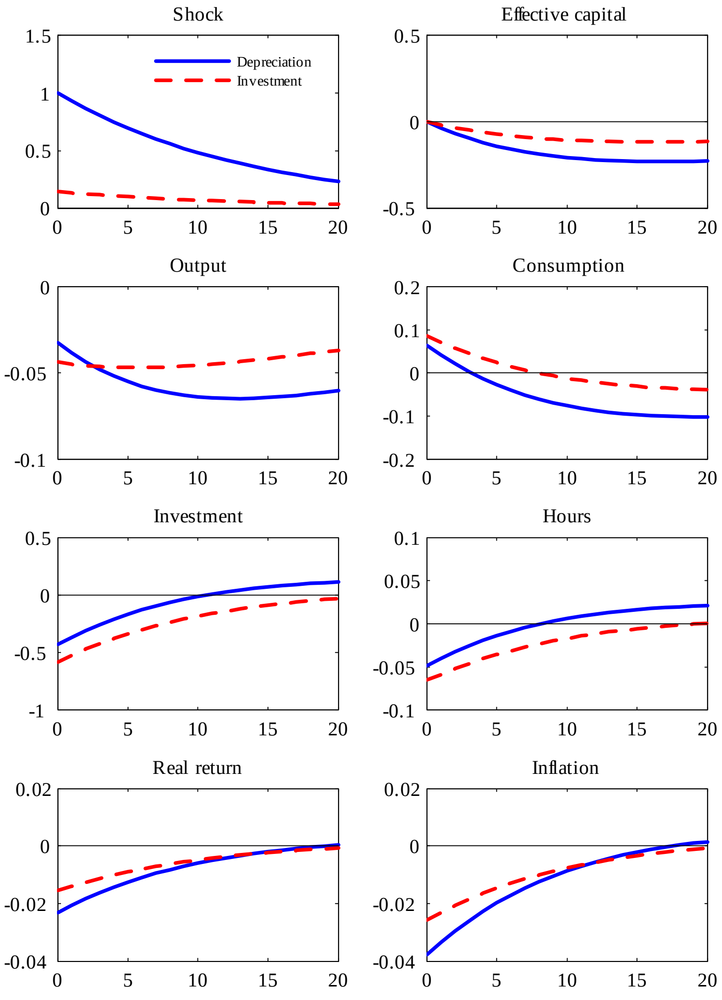

Figure 7: Responses to a persistent shock to the rate of capital depreciation and to investment-specific technology ($\rho_\delta = \rho_I = 0.93$) in the basic RBC model.

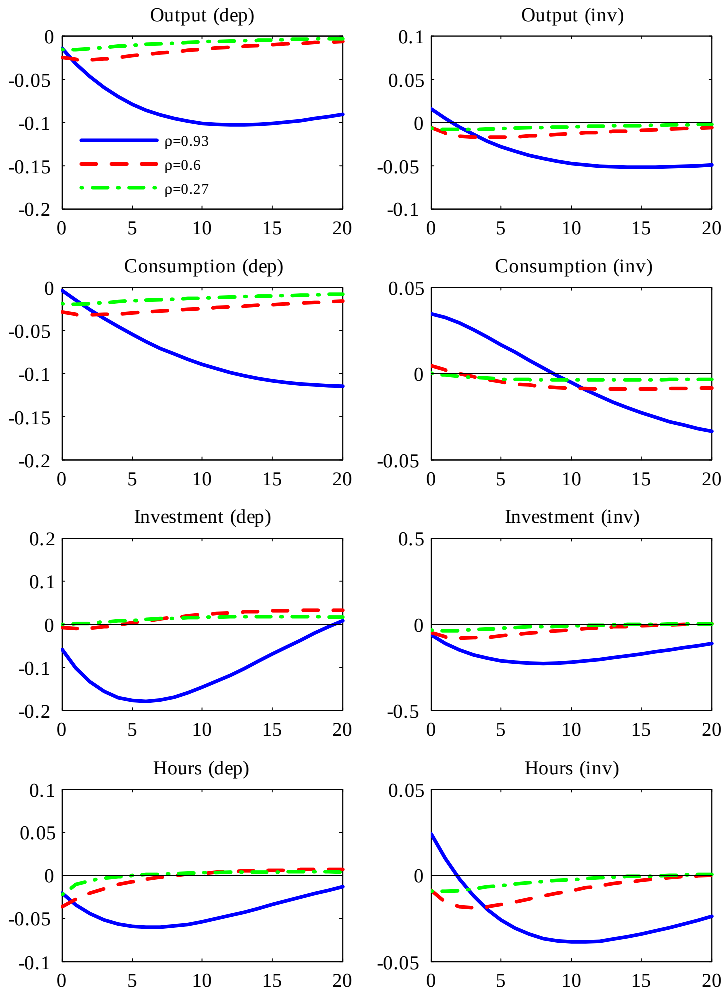

Figure 8: Responses to shocks to the rate of depreciation (left panels) and to investment-specific technology (right panels) for different levels of persistence in the basic New Keynesian model with standard preferences.

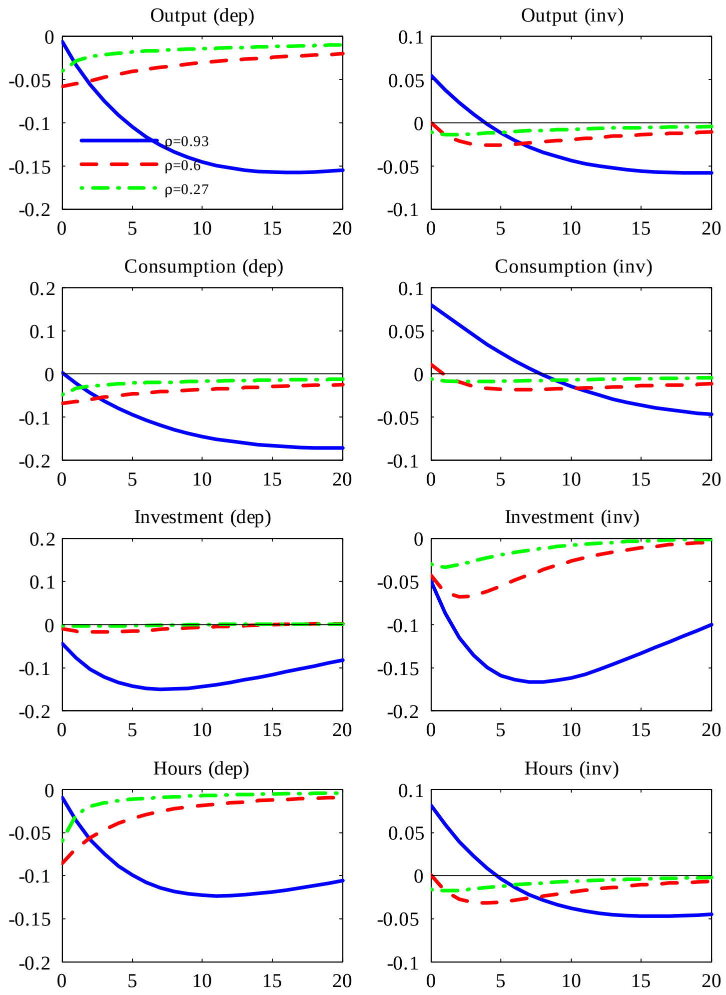

Figure 9: Responses to shocks to the rate of depreciation (left panels) and to investment-specific technology (right panels) for different levels of persistence in the basic New Keynesian model with GHH preferences.