

Progressive Taxation of Labor Income, Taylor Principle and Monetary Policy

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Preliminary Version

June 17, 2010

Abstract

Progressive labor income taxation in an otherwise standard NK model: (i) introduces a trade-off between output and inflation stabilization; (ii) enlarges the determinacy region in the parameter space, substantially altering the so-called Taylor principle; (iii) has non-linear dynamic effects and changes the responses of the economy to a technology and to a government spending shock; (iv) sensibly alters the prescription for the optimal discretionary interest rate rule; (v) the welfare gains from commitment are a decreasing function of the degree of the progressiveness of the labor income tax.

The key point is that, whatever the set up, the literature on monetary policy cannot disregard the progressive taxation on labor income which characterized the most of OECD countries.

JEL CODES: E50, E52, E58

1 Introduction

Since its inception, the New Keynesian research program has aimed at constructing microfounded models that could be immune from the Lucas critique

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and so represent a useful tool for policy analysis. Whether or not this research agenda has achieved its goals is still the subject of heated debate¹; there is no doubt, however, that New Keynesian DSGE models have become increasingly successful in replicating the behavior of macroeconomic data² and represent today a standard point of reference for the implementation of monetary policy by major central banks.

In its attempt to provide an increasingly realistic interpretation of the dynamics of actual economies, however, the New Keynesian literature has not paid enough attention to the structure of taxation. This is hardly surprising, given the emphasis of this literature on monetary policy but, as we argue in this paper, the absence of an adequate description of the working of modern tax systems may lead to misleading results and so reduce the empirical performance of the model. The way in which taxes are designed in actual economies, in fact, severely distort the behavior of economic agents and may significantly affect the response of time series to the relevant shocks. Large part of the NK literature, such as, among others, Schmitt-Grohe and Uribe (2005 and 2010) and Benigno (2006), has concentrated on the issue of optimal fiscal and monetary policy, employing distorsive fiscal rules with flat taxes and on the issues concerning to the financing of government debt, but do not consider the possibility to tax income progressively.

In this paper, we concentrate on the macroeconomic consequences of progressive labor income taxation. As we can see from table 1, almost all governments tax labor income progressively although the degree of progressiveness shows huge variability across countries. If we restrict the attention to OECD countries, we go from countries like the Czech Republic where wages are subject to an approximately 20% tax rate independently of the tax base, to countries like Sweden, where labor income tax rates are 23.4% for workers earning 67% of the average wage, more than double for workers that earn 137% of the average wage and reach 56.4% for workers earning 167% of the average wage. Therefore, a model that ignores the fact that labor income can be taxed progressively may run the risk of ignoring one of the main channels through which the effects of technology or demand shocks are propagated through the economy. For instance, shouldn't we expect significant differences in the dynamics of the Czech and Swedish economies? Shouldn't monetary policy respond differently in a country like Sweden where wage increases above average are so penalized by the tax system than in countries like the Czech Republic where labor income taxes are basically flat?

¹See for example Chari et al (2009)

²See for example Christiano Eichenbaum and Evans (2005) and Smets and Wouters (2003).

These questions resemble those concerning the macroeconomic consequences of wage rigidity. While the rigidity of wages however, cannot be taken as given, but must be derived as an equilibrium outcome, the degree of progressiveness can be regarded as a structural characteristic of an economy. The aim of progressive taxation is to achieve a more egalitarian distribution of income and therefore is crucially linked to the preferences of society or to the social contract and therefore, in analyzing short run stabilization policy, can be safely taken as parametric. In order to introduce progressive taxation in an otherwise standard New Keynesian model we follow the approach of Guo (1999) and Guo and Lansing (1998) that analyze this issue in a Real Business Cycle (RBC) framework and suggest a convenient and tractable way to model progressive taxation in representative agents economies.

The consequences of progressive taxation for the NK model are quite staggering. First, we find that in economies characterized by progressive labor income taxation, policy makers face a trade-off between inflation stabilization and output stabilization. This is quite interesting since it is well known that the NK model, in its standard version, does not imply any policy trade-offs³, while these trade-offs are usually perceived by central banks as a major challenge in formulating monetary policy.

Second, not only we find that in a model with progressive taxation the New Keynesian Phillips curve (NKPC) is significantly affected by productivity and government spending shocks, but also we find that the response of inflation to movements in the output gap increases as the labor income tax becomes more progressive, i.e., the Phillips curve becomes steeper. Economies characterized by a more progressive tax structure, therefore, will typically face a larger trade-off between inflation stabilization and output stabilization. The reason is quite intuitive. Following a productivity shock, output increases, the labor demand schedule shift outwards and real wages must increase. When taxes are progressive, as the real wage increases labor income taxes increase more than proportionally. The supply of labor therefore increases less than in the basic NK model. In order to produce the same

³See for example Galì (2008) ch. 4. In the literature this problem has been dealt with by amending, in some ad hoc way, the NK model. In their "Science of Monetary Policy" Clarida Galì and Gertler (1999) amended the standard NK Phillips curve by adding an exogenous cost-push shock. In a further paper, Clarida Galì and Gertler (2002) proposed a NK model with variable markups. Woodford (2003) discusses a source of monetary policy trade-offs different from cost-push shocks created by the presence of transactions friction. More recently, Blanchard-Gali have shown that economies characterized by real wage rigidities experience a policy trade-off. While these authors simply assume that the current real wage is a function of the past real wage. Mattesini and Rossi (2009) show that a significant policy trade-off arises also in a two sector economy were one of the two sectors is unionized.

amount of output, firms must offer an higher real wage at the cost of setting higher prices and therefore higher inflation. As a consequence, the NKPC becomes steeper.

Third, by approximating the model up to a first order, we find that a progressive labor income tax has non-linear dynamic effects and changes the responses of the economy to a technology and to a government spending shock and acts as an automatic stabilizer.

A fourth interesting result of our paper is that progressive taxation enlarges the determinacy region. The more progressive is the labor income tax, the larger is the number of Taylor rules that are able to guarantee a unique rational expectations equilibrium. Progressive taxation, therefore, acts as an automatic stabilizer, in the sense that it reduces the possibility that inflationary bursts (like the Great Inflation of the 1970s) are caused by self-fulfilling expectations.

Finally, we derive a linear-quadratic (LQ) approximation of the households' utility function around a distorted steady state and we show that progressive labor income taxation alters the prescriptions for optimal discretionary monetary policy. In particular, we find that a central bank that operates in an economy characterized by a more progressive tax structure should pursue more aggressive monetary policies than a central bank operating in an economy where the degree of progressiveness is lower. Finally, when studying the welfare gains from commitment we find that their are a decreasing function of the degree of the progressiveness of the labor income tax.

The paper is organized as follows. Section 2 describes the structure of the model. Section 3 studies the model dynamics. Section 4 derives analytically the Central Bank welfare function as a linear quadratic approximation of the households' utility function and study the optimal discretionary monetary policy. Section 5 concludes.

2 The model

2.1 Consumer Optimization

We consider an economy populated by many identical, infinitely lived worker-households, each of measure zero. Households demand a Dixit Stiglitz composite consumption bundle C_t produced by a continuum of monopolistically competitive firms. The life-time expected utility function of the representa-

tive household is given by:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^{\tau-t} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right] \quad \sigma, \phi > 0 \quad (1)$$

where $0 < \beta < 1$ is the subjective discount rate, and $N_t \in (0, 1)$ are is the supply of labor hours. Aggregate consumption C_t is defined as a Dixit-Stiglitz consumption basket. Labor income is taxed at the rate τ_t . The individual flow budget constraint is:

$$P_t C_t + R_t^{-1} B_t \leq (1 - \tau_t) W_t N_t + B_{t-1} + \Pi_t(j) - P_t T_t^l \quad (2)$$

where P_t is the price level, B_t is the stock of risk-free nominal bonds purchased at the beginning of period t and maturing at the end of the period. R_t is the gross nominal interest rate. W_t is the nominal wage and Π_t is the profit income. Households pay a lump sum tax T_t^l and taxes on labor income τ_t . Following Guo (1999) and Guo and Lansing (1998) we postulate that τ_t take the form:

$$\tau_t = 1 - \eta \left(\frac{Y_n}{Y_{n,t}} \right)^{\phi_n}, \quad \eta \in (0, 1], \quad \phi_n \in [0, 1] \quad (3)$$

where $Y_n = WN/P$ represents a base level of income, taken as given by the household. We set this level to the steady state level of per capita income. When the actual income of the household $Y_{n,t} = W_t N_t / P_t$ is above Y_n then the tax rate is higher than when the taxable income is below Y_n . The parameters η and ϕ_n govern the level and the slope of the tax schedule, respectively. When $\phi_n > 0$ the tax rate increases in the household's taxable income. We impose restrictions on these parameters to ensure that $0 \leq \tau_t < 1$, and households have an incentive to supply labor to firms.

In order to understand the progressiveness of the taxation scheme it is useful to distinguish between the average tax rate which is given by (3) and the marginal tax rate which is given by

$$\tau_t^m = \frac{\partial(\tau_t Y_{n,t})}{\partial Y_{n,t}} = 1 - \eta(1 - \phi_n) \left(\frac{Y_n}{Y_{n,t}} \right)^{\phi_n}. \quad (4)$$

Here we consider an environment where τ_t is strictly less than 100% and where also τ_t^m is strictly less than 100% so that households have an incentive to supply labor to firms. Since $\tau_t^m = \tau_t + \eta\phi_n \left(\frac{Y_n}{Y_{n,t}} \right)^{\phi_n}$ the marginal tax rate is above the average tax rate when $\phi_n > 0$. In this case the tax schedule is

said to be “progressive”. When $\phi_n = 0$, the average and marginal tax rates of labor income are both equal to $1 - \eta$, and the labor tax schedule is said to be “flat”.

Households maximize (1) subject to (2). Therefore, the optimal labor supply and the consumption-saving decision are given by:

$$C_t^\sigma N_t^\phi = \frac{W_t}{P_t} (1 - \tau_t^m) \quad (5)$$

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] R_t \quad (6)$$

Equation (5) states that the marginal rate of substitution between leisure and consumption equals the real wage net of taxes. Notice, that the presence of τ_t^m in equation (5) is due to the fact that households internalize the effects of the marginal tax rate, when they choose their supply of labor hours. Equation (6) is the standard Euler equation. Note that substituting (3) in the households labor supply (5) we get:

$$C_t^\sigma N_t^\phi = \eta \left(\frac{Y_n}{Y_{n,t}} \right)^{\phi_n} \frac{W_t}{P_t} = \eta Y_n^{\phi_n} \left(\frac{W_t}{P_t} \right)^{(1-\phi_n)} N_t^{-\phi_n} \quad (7)$$

It is useful to rewrite equations (7) and (6) as log-deviations from their steady state values. In particular, from equation (7) we get:

$$\sigma \hat{c}_t + \phi \hat{n}_t = (1 - \phi_n) \hat{\omega}_t - \phi_n \hat{n}_t, \quad (8)$$

where $\hat{\omega}_t$ is the log-deviation of the real wage. When $\phi_n = 0$ (the labor income tax is flat) then we get the standard labor supply equation.

From the log-linearization of the Euler equation (6) we finally get:

$$\hat{c}_t = E_t \{ \hat{c}_{t+1} \} - \frac{1}{\sigma} (\hat{r}_t - E_t \{ \hat{\pi}_{t+1} \}) \quad (9)$$

where $\hat{r}_t = r_t - \sigma \log \beta$ is the log-deviation of the nominal interest rate from its steady state value.

Notice that, the log deviation of τ_t in equation (3) from its steady state, which is given by:

$$\hat{\tau}_t = \frac{\eta \phi_n}{\tau} \hat{y}_{n,t}. \quad (10)$$

Since in the steady state $\tau = 1 - \eta$ and the log-linearization of $Y_{n,t} = W_t N_t / P_t$ yields $\hat{y}_{n,t} = (\hat{\omega}_t + \hat{n}_t)$ we can rewrite (10) as:

$$\hat{\tau}_t = \frac{\eta \phi_n}{1 - \eta} (\hat{\omega}_t + \hat{n}_t). \quad (11)$$

2.2 The Role of the Government

The government always runs a balanced budget. Therefore, in each period the following Government budget constraint holds:

$$G_t = \tau_t \frac{W_t}{P_t} N_t + T_t^l \quad (12)$$

We assume that public consumption evolves exogenously, so that:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t} \quad (13)$$

where $\hat{g}_t = \ln(G_t/G)$ is an exogenous AR(1) process.⁴

2.3 Firms

2.3.1 The Final Goods-Producing Sector

A perfectly competitive final-good-producing firm employs $Y_t(u)$ units of each intermediate good $u \in [0, 1]$ at the nominal price $P_t(u)$ to produce Y_t units of the final good, using the following constant return to scale technology:

$$Y_t = \left[\int_0^1 Y_t(u)^{\frac{\theta-1}{\theta}} du \right]^{\frac{\theta}{\theta-1}} \quad (14)$$

where $Y_t(u)$ is the quantity of intermediate good u used as input.

The final good is allocated to consumers and to the Government. Profit maximization yields the following demand for intermediate goods

$$Y_t(u) = \left(\frac{P_t(u)}{P_t} \right)^{-\theta} Y_t \quad (15)$$

where $Y_t(u) = C_t(u) + G_t(u)$. From the zero profit condition, instead, we have

$$P_t = \left[\int_0^1 P_t(u)^{1-\theta} du \right]^{\frac{1}{1-\theta}}. \quad (16)$$

The aggregate resource constraint of the economy is:

$$Y_t = C_t + G_t. \quad (17)$$

Log-linearizing we get:

$$\hat{y}_t = \gamma_c \hat{c}_t + (1 - \gamma_c) \hat{g}_t \quad (18)$$

where $\gamma_c = \frac{C}{Y}$.

⁴Notice that the presence of the lump-sum transfer is need to balance the Government budget in every period. This assumption does not alter the result on the optimal monetary policy in response to a technology shocks and give us the advantage to study the effect of an exogenous government spending shock

2.3.2 Intermediate Goods Producing Firms

Intermediate goods producing firms produce a differentiated good with a linear technology represented by the following constant return to scale production function:

$$Y_t(u) = A_t N_t(u) \quad (19)$$

where $u \in (0, 1)$ is a firm specific index. A_t is a technology shock and $a_t = \ln(A_t/a)$ follows an AR(1) process, i.e.,

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad (20)$$

where $\rho_a < 1$ and ε_t^a is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_a .

Given the constant return to scale technology and the aggregate nature of shocks, real marginal costs are the same across the intermediate good producing firms. Solving the cost-minimization problem of the representative firm and imposing the symmetric equilibrium we obtain the following aggregate labor demand:

$$\frac{W_t}{P_t} = MC_t A_t. \quad (21)$$

It is useful to rewrite equation (21) in log-deviations:

$$\widehat{mc}_t = \hat{\omega}_t - a_t \quad (22)$$

The aggregate production in log-deviations is instead:

$$\hat{y}_t = a_t + \hat{n}_t \quad (23)$$

2.3.3 Staggered Price Setting

Firms choose $P_t(u)$ in a staggered price setting à la Calvo (1983). Solving the Calvo problem and log-linearizing we find a typical forward-looking Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \widehat{mc}_t \quad (24)$$

where $\lambda = \frac{(1-\varphi)(1-\beta\varphi)}{\varphi}$ and φ is the probability that prices are reset.

2.3.4 Real Marginal Costs and the flexible price equilibrium output

We now want to find an expression for the aggregate real marginal costs and, after imposing that prices are flexible, derive the flexible price equilibrium

output. Given the flexible price equilibrium output (or natural output) we will then derive the output gap, which is defined as the difference between the actual and the flexible price equilibrium output. Let us first consider equilibrium in the labor market which is obtained equating the aggregate demand for labor (21) and the aggregate labor supply (8). This allows us to find the equation for the aggregate real marginal costs, that in log-deviations is given by:

$$\widehat{mc}_t = \frac{(\phi + \phi_n)}{(1 - \phi_n)} \hat{n}_t + \frac{\sigma}{1 - \phi_n} \hat{c}_t - a_t. \quad (25)$$

Using equations (18), (8) and (23) we can rewrite equation (25) as:

$$\widehat{mc}_t = \frac{\gamma_c (\phi + \phi_n) + \sigma}{\gamma_c (1 - \phi_n)} \hat{y}_t - \frac{(1 + \phi)}{(1 - \phi_n)} a_t - \frac{\sigma (1 - \gamma_c)}{\gamma_c (1 - \phi_n)} \hat{g}_t. \quad (26)$$

We know that under the flexible price equilibrium the log of real marginal costs equals the log of its steady state value, i.e. $mc_t = \log\left(\frac{\theta-1}{\theta}\right)$, then $\widehat{mc}_t = 0$. Therefore, imposing this last condition (which holds only when prices are flexible) and solving for \hat{y}_t , we find the flexible price equilibrium output which is given by:

$$\hat{y}_t^n = \frac{\gamma_c (1 + \phi)}{(\phi + \phi_n) \gamma_c + \sigma} a_t + \frac{\sigma (1 - \gamma_c)}{(\phi + \phi_n) \gamma_c + \sigma} \hat{g}_t \quad (27)$$

As we said before, when $\phi_n = 0$, the average and marginal tax rates of labor income are both equal to $1 - \eta$, and therefore the labor income tax becomes a flat tax. In this case, the flexible price equilibrium output is:

$$\hat{y}_t^{f,flat} = \frac{(1 + \phi) \gamma_c}{\sigma + \phi \gamma_c} a_t + \frac{\sigma (1 - \gamma_c)}{\sigma + \phi \gamma_c} \hat{g}_t \quad (28)$$

Note that the difference between (27) and (28) is:

$$\hat{y}_t^n - \hat{y}_t^{f,flat} = -\frac{(1 + \phi) \phi_n \gamma_c^2}{((\phi + \phi_n) \gamma_c + \sigma) (\sigma + \phi \gamma_c)} a_t - \frac{\sigma (1 - \gamma_c) \phi_n \gamma_c}{((\phi + \phi_n) \gamma_c + \sigma) (\sigma + \phi \gamma_c)} \hat{g}_t \quad (29)$$

In an economy characterized by progressive taxation of labor income, the flexible price equilibrium output is always lower than the one that would arise in an economy where the labor income tax is flat.

2.4 The Phillips Curve

Note that from (26) and (27) we are able to rewrite real marginal costs in terms of output gap, which is defined as the difference between the actual

and the flexible price equilibrium output, $y_t - y_t^f$:

$$\widehat{mc}_t = \frac{\sigma + \gamma_c(\phi + \phi_n)}{\gamma_c(1 - \phi_n)} (y_t - y_t^f). \quad (30)$$

Using (30) the NKPC can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \varsigma_x (\hat{y}_t - \hat{y}_t^n) \quad (31)$$

where $\varsigma_x = \frac{\sigma + \gamma_c(\phi + \phi_n)}{\gamma_c(1 - \phi_n)}$. We following Benigno and Woodford (2005) to rewrite the NKPC in terms of the welfare relevant output as,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{x}_t + \kappa u_t \quad (32)$$

where $\kappa = \lambda \varsigma_x$, $u_t = (\hat{y}_t^* - \hat{y}_t^n)$ and $\hat{x}_t = (\hat{y}_t - \hat{y}_t^*)$ is the welfare relevant output gap.

In Appendix (??) we show that u_t is completely exogenous, indeed it is a function of technology and government spending shocks. Therefore, unlike what happens in the standard NK model, the difference between welfare relevant output and flexible price equilibrium output is not constant, but is a function of the relevant shocks that hit the economy. What Blanchard and Galì (2007) define as the "divine coincidence" does not hold since a policy that brings the economy to its natural level is not necessarily optimal.

We are therefore able to state the following

Proposition 1 Endogenous trade-off. *With progressive taxation on labor income an endogenous trade-off between stabilizing inflation and the output gap emerges. The New Keynesian Phillips Curve is affected by technology and government spending shocks.*

The endogenous trade-off between inflation stabilization and output stabilization is a consequence of the progressiveness of the tax rate. Suppose a positive productivity shock hits the economy. Efficient output and natural output both increase, but natural output increases less. This means that the welfare relevant output increases more than the natural output.⁵ The reason is the following. Because of the productivity shock the demand

⁵As shown by Benigno and Woodford (2005) and Gnocchi (2009) among other, when the model economy is log-linearized around a distorted steady state, the welfare relevant output is a function of the efficient and of the natural output.

for labor increases and the real wage increases in order to restore the labor market equilibrium. If taxes were flat the efficient and the flexible price equilibrium output would be identical but for a constant, which disappears when we write the two outputs in log-deviation from the steady state. When taxes are progressive, instead, as the real wage increases, labor income tax increases more than proportionally. The supply of labor therefore increases less than in the efficient economy and the increase in the natural output is smaller. Since natural output increases less than the welfare relevant output following a productivity shock, to reduce the welfare relevant output gap the central bank has to accept a higher rate of inflation.

It is important to notice that, in this model, the endogenous trade-off between inflation stabilization and output stabilization arises without any assumption on real wage rigidity like in Blanchard-Gali (2007), or on the structure of labor contracts like in Mattesini and Rossi (2009). Rather, it is a simple consequence of the structure of taxation that, in most countries, shows some degree of progressiveness. Notice also that the effect of progressive taxation is the opposite of the effect of real wage rigidity. When real wages are rigid, following a productivity shock, natural output tends to increase more than the welfare relevant output, while in the case of progressive taxation natural output tends to increase less than the welfare relevant output. Therefore, progressive labor income taxation acts as an automatic stabilizer. The stabilizing effect of ϕ_n will be shown in detail in the next section, in which we study the dynamics of our model.

Differentiating ς_x with respect to ϕ_n we obtain:

$$\frac{d\varsigma_x}{d\phi_n} = \frac{1}{\gamma_c(\phi_n - 1)^2} (\sigma + \gamma_c + \phi\gamma_c) > 0 \quad (33)$$

hence:

Corollary 1. *the higher the degree of the progressiveness of the labor income tax, ϕ_n , the steeper becomes the NKPC.*

Notice that for $\phi_n = 0$, the Phillips curve collapses to the standard NK forward looking Phillips curve. By closing the gap \hat{x}_t , the Central Bank is able to obtain an inflation rate equal to zero.

The economic intuition of Corollary 1 is the following. *Ceteris paribus*, in order to produce more output firms need more labor and real wage must increase. If taxes on labor income are progressive, the increase in real wage

is followed by an increase in the marginal tax rate τ_t^m and the supply of labor increases less than in the basic NK model. Therefore, with respect to the standard NK model, to produce the same amount of output, firms must pay higher real wages at the cost of setting higher prices and therefore higher inflation. This is the reason why the NKPC becomes steeper as ϕ_n increases.

2.5 The IS curve

The reduced form solution of the model is given by the IS curve and the NKPC. In order to find the IS curve, we combine together the Euler equation (9), the aggregate resource constraint (18), the aggregate production function (23) and the aggregate labor supply (8). After some algebra we get:

$$\hat{y}_t = E_t \hat{y}_{t+1} - (1 - \gamma_c) E_t \Delta \hat{g}_{t+1} - \frac{\gamma_c}{\sigma} (\hat{r}_t - E_t \{\hat{\pi}_{t+1}\}) \quad (34)$$

we can rewrite the IS curve in terms of the welfare relevant output gap as follows:

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{\gamma_c}{\sigma} (\hat{r}_t - E_t \{\hat{\pi}_{t+1}\} - \hat{r}_t^*). \quad (35)$$

where \hat{r}_t^* is the interest rate that characterizes an efficient-frictionless economy.

3 The model Dynamics

In this section we study how the progressiveness of the labor income tax affects the dynamics of model. In particular, we look at how ϕ_n affects: (i) the responses of output and inflation to a technology and to a government spending shock; (ii) the conditions under which the rational expectation equilibrium is determinate.

3.1 Impulse Response Functions

The aim of this paragraph is to analyze how the dynamics of the log-linearized model depends on the parameter ϕ_n of labor income taxation. In particular, we look at the implied dynamics of the main economic variables in response to a positive productivity shock and to a positive government spending shock. To close the model we need to specify an equation for the monetary authority. We assume that the Central Bank sets the short run nominal interest rate according to the following standard Taylor-type rule:

$$\hat{r}_t = \alpha_\pi \hat{\pi}_t + \alpha_Y \hat{x}_t. \quad (36)$$

We calibrate the model using the following parameters specification: $\sigma = 1$, $\phi = 1$, $\beta = 0.99$, $\varepsilon = 6$, $\theta = 0.75$, $\lambda = \frac{(\varepsilon-1)\theta}{(1-\theta)(1-\beta\theta)}$.⁶ From the steady state we find that $\gamma_c = \frac{1+\eta(\varepsilon-1)}{\varepsilon}$. The persistence of technology and of Government spending shock are respectively: $\rho_a = 0.9$ and $\rho_g = 0.7$. Both shocks are calibrated to have 1% standard deviation. Since the monetary authority implements the standard Taylor (1993) rule, we set $\alpha_\pi = 1.5$ and $\alpha_y = 0.5/4$. None of the qualitative results depends on the calibration values chosen.

Figures 1 and 2 show the responses of inflation, nominal interest rate, output, labor hours, real wages and the marginal income tax to a positive technology shock and to a positive Government spending shock for different values of ϕ_n .

- Figures 1 and 2 about here -

Figure 1 shows that, in response to a positive technology, output and real wages increase on impact while inflation and labor hours decrease; they return to their initial level after almost forty periods. Such a response of employment is consistent with much of the recent empirical evidence on the effects of technology shocks. Notice that, given the Taylor rule (36), the improvement in technology is partly accommodated by the Central Bank, which lowers nominal and real rates. However, the Central Bank is not able to close a negative output gap, which is responsible for the decline in inflation. Under the baseline calibration output increases, even if less than the welfare relevant output and employment declines. Notice that the higher is ϕ_n , i.e. the higher is the progressiveness of the labor income tax, the lower is the effect of the technology shock on output, output gap and inflation and, consequently, the higher the effect on labor hours. In order to understand why this happens remember that both the technology and the government spending shocks enter the NKPC and that the NKPC becomes steeper as ϕ_n increases. Indeed, as real wages increase, the marginal income tax increases and the labor supply is lower with respect to the standard NK model. This implies, that with a progressive labor income taxation actual output increases even less and employment decreases even more than under the standard NK model.

We find similar results for the government spending shock. As shown in Figure 2, the higher ϕ_n the lower is the increase in inflation and the increase in output and in labor hours following a positive government spending shock. Also in this case a progressive labor income tax acts as an automatic

⁶The parameters θ is used only for the staggered price case.

stabilizer. Therefore, our model confirms the well know stylized fact that progressive tax acts as an automatic stabilizers (see for example Vanhala 2006, Heer and Maussner 2006). The advantage of this instrument is of course that the Government can stabilize the economy without changing the government side (i.e. without moving the government expenditure). Necessarily the presence of a progressive taxation originates a further trade-off between stabilization and efficiency of the equilibrium.

3.2 Determinacy and the Taylor principle

To assess the determinacy of the rational expectations equilibrium (REE henceforth), we first substitute the Taylor rule (36) into the IS curve and then we write the structural equations in the following matrix format

$$X_t = \mathbf{B}E_t X_{t+1} + \mathbf{B}a_t, \quad (37)$$

where vector X_t includes the endogenous variables of the model while a_t is the technology shock. Determinacy of REE is obtained if the standard Blanchard and Kahn (1980) conditions are satisfied. Our model is isomorphic to the standard NK model, and matrix B is exactly the same which characterizes the determinacy properties of the standard model. The only difference is due to the value of the progressiveness of the labor income tax which affects the slope of the NKPC. Therefore, we can state the following:

Proposition 2. Necessary and sufficient conditions for determinacy of REE. *Let $\alpha_\pi \in [0, \infty)$, $\alpha_y \in [0, \infty)$ and at least one different from zero. Determinacy of REE under progressive labor income taxation obtains if and only if*

$$1 < \alpha_\pi + \frac{(1 - \beta)}{\kappa} \alpha_y \quad (38)$$

where $\kappa = \lambda \frac{\sigma + \gamma_c(\phi + \phi_n)}{\gamma_c(1 - \phi_n)}$ is the long-run elasticity of output to inflation (see Appendix (A.1)).

Note that, as stressed by Woodford (2001, 2003) among others, condition (38) is a generalization of the standard Taylor principle: to ensure determinacy of REE the nominal interest rate should rise by more than the increase of inflation in the long run. Indeed, the coefficient $\frac{(1 - \beta)}{\kappa}$ represents the long run multiplier of inflation on output in a standard NKPC log-linearized

around the zero-inflation steady state (see Appendix (??)). In other words, the Taylor principle has to be intended as,

$$\left. \frac{\partial \hat{i}}{\partial \hat{\pi}} \right|_{LR} = \alpha_\pi + \left. \frac{\partial \hat{y}}{\partial \hat{\pi}} \right|_{LR} \alpha_y > 1. \quad (39)$$

where $\left. \frac{\partial \hat{y}}{\partial \hat{\pi}} \right|_{LR} = \frac{(1-\beta)(\sigma+\gamma_c(\phi+\phi_n))}{\lambda\gamma_c(1-\phi_n)}$.

We now look at the effects of progressive taxation on the determinacy region. In Appendix (A.1) we show the following.

Proposition 3. The effects of progressive taxation on the determinacy region. *Let $\alpha_\pi \in [0, \infty)$, $\alpha_y \in [0, \infty)$ and at least one different from zero. Then*

$$\frac{d \left(\frac{(1-\beta)(\sigma+\gamma_c(\phi+\phi_n))}{\lambda\gamma_c(1-\phi_n)} \right)}{d\phi_n} = \frac{1}{\lambda\gamma_c} \frac{1-\beta}{(\phi_n-1)^2} (\sigma + \gamma_c + \phi\gamma_c) > 0 \quad (40)$$

which is always positive.

Corollary 2. *Let $\alpha_\pi \in [0, \infty)$, $\alpha_y \in [0, \infty)$ and at least one different from zero. Then, as the degree of the progressive taxation increases the determinacy region enlarges in the parameter space (α_π, α_y) .*

According to corollary 2 progressive taxation on labor income enlarges the determinacy region. This means that for a given α_y the condition (??) is satisfied for lower values of α_π .

Figure 3 shows the effect of an increase in the degree of progressiveness of the labor income tax. In Figure 3a we have the usual graph of the Taylor principle in the space (α_π, α_y) for the case $\phi_n = 0$, which is identical to the one we get in Proposition 1. In fact, condition (39) implies $\alpha_y > (1 - \alpha_\pi) / \left. \frac{\partial \hat{y}}{\partial \hat{\pi}} \right|_{LR}$, where $\left. \frac{\partial \hat{y}}{\partial \hat{\pi}} \right|_{LR, \bar{\pi}=1} = \frac{1-\beta}{\kappa} = \frac{(1-\beta)(\sigma+\gamma_c\phi)}{\lambda\gamma_c}$. As the parameter ϕ_n increases, Propositions 2 shows that $\left. \frac{\partial \hat{y}}{\partial \hat{\pi}} \right|_{LR}$ increases, and the line rotates anti-clockwise (see figure 3b) enlarging the determinacy region.

- Figures 3 about here -

In order to get some intuition about this result, Suppose that, in the absence of any shock to fundamentals that could justify it, there was an increase in the level of economic activity. The increase in output would be associated with increases in hours and in real wages, lower markups (because of sticky prices), and persistently high inflation. In the basic NK model the central bank increases the real interest, so that consumption and the aggregate demand decrease. By reducing the aggregate demand, the central bank is able to avoid self-fulfilling inflation. In economy with progressive taxation, when real wage increases the marginal tax increases as well, so that the net wage received by households and consequently the supply of labor hours are lower than in a basic NK model. This effect will push down output reinforcing the initial reduction in the aggregate demand. Hence, the monetary authority is able to avoid self-fulfilling inflation using a looser interest rate rule.

Since progressive taxation increases the determinacy region and the likelihood of multiple equilibria based on self-fulfilling expectations becomes smaller, we can regard progressive taxation as some kind of automatic stabilizer: the more progressive is taxation the smaller is the need that the Central Bank reacts to inflation with a strong increase in the interest rate to achieve a unique equilibrium. This is quite interesting in light of current controversies on monetary policy. Consider for example the debate on the Great Inflation of the 1970s. Some authors such as Clarida, Galì and Gertler (2000) and Lubik and Schorfede (2004) have suggested that, indeed, the Great Inflation of the 1970s was the result of the unwillingness of the Fed to fight inflation aggressively. This "bad policy" was inconsistent with equilibrium determinacy and self-fulfilling expectations were at the root of the inflationary tensions that characterized that decade. Our results suggest that, in order to fully analyze this issue, it is necessary to consider a much more complex model of the economy. Other characteristics of the economy, such as progressive taxation, might have been at work in that period, thus reducing the probability that indeterminacy and self-fulfilling expectations were the real causes of the Great Inflation.

4 Optimal Monetary Policy

We follow Benigno and Woodford (2005) to derive a second order approximation of the household utility function under a distorted steady state.⁷ The welfare-loss function obtained can be write as follows

⁷In Appendix (A.1) we show how to derive the loss-function.

$$\min_{\{x_t, \pi_t\}} -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [q_x \hat{x}_t^2 + q_\pi \pi_t^2] + T_{t_0} + t.i.p. + O(\|\xi\|^3) \quad (41)$$

s.t.

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + \kappa u_t \quad (42)$$

where q_x and q_π are complicated convolution parameters depending on the structural parameters of the model reported in appendix 3.⁸ Note that the linear term \hat{x}_t captures the fact that any increase in output positively affects welfare. Notice that by following the methodology developed by Benigno and Woodford (2005), the Central Bank's problem has the convenient linear-quadratic form even with a distorted steady state.⁹

4.1 Discretion

If the Central Bank cannot credibly commit in advance to a future policy action or to a sequence of future policy actions, then the optimal monetary policy is discretionary, in the sense that the policy makers choose in each period the value to assign to the policy instrument i_t . The Central Bank minimizes the welfare-based loss function, subject to the Phillips curve, taking all expectations as given. Therefore:

$$\min_{\{x_t, \pi_t\}} -\frac{1}{2} [\pi_t^2 + \alpha_x \hat{x}_t^2] + F_t$$

s.t.

$$\pi_t = \kappa x_t + f_t$$

where $f_t = \beta E_t \pi_{t+1} + \kappa u_t$, $\alpha_x = \frac{q_x}{q_\pi}$ and $F_t = E_0 \sum_{i=1}^{\infty} \beta^i [\alpha_x x_{t+i}^2 + \pi_{t+i}^2]$.

Solving the problem we find that optimality requires the following targeting rule:

$$\hat{x}_t = -\frac{\kappa(\phi_n)}{\alpha_x(\phi_n)} \pi_t \quad (43)$$

⁸As shown by Benigno and Woodford (2005), the term T_{t_0} is a transitory component, defined in the appendix, which is predetermined at the time of the policy choice.

⁹As shown by Benigno and Woodford (2005) in the face of large distortions the presence of a linear term in (41) would require the use of a second-order approximation to the equilibrium condition connecting output gap and inflation. The technical Appendix (A.1) shows in detail how we get equation (41).

Proposition 4. *A unit increase in inflation requires a decrease in the output gap which is not independent from the degree of the progressiveness of the labor income tax.*

To show our result in a more readable and tractable way, we now consider the effect of a technology shock. In order to do so we assume that public expenditure is always at its steady state level, this implies that $\hat{g}_t = 0$ for each t .¹⁰ In this case

$$\frac{\kappa}{\alpha_x} = \frac{(1 + \phi + \Phi\phi_n)\varepsilon}{((1 + \phi)((1 - \phi_n) + \phi_n\Phi) + \phi_n\Phi)} \quad (44)$$

where $\Phi = \frac{\varepsilon - \eta(\varepsilon - 1)}{\varepsilon} < 1$ is the steady state distortion. Taking the derivative of $\frac{\kappa}{\alpha_x}$ with respect to ϕ_n we get:

$$\frac{d\left(\frac{\kappa}{\alpha_x}\right)}{d\phi_n} = \varepsilon(1 - \Phi) \frac{(\phi + 1)^2}{(\phi - \phi_n + 2\phi_n\Phi + \phi_n\phi(\Phi - 1) + 1)^2} > 0$$

Corollary *A unit increase in inflation requires a decrease in the output gap. Such decrease is larger the higher is the degree of progressiveness of the tax system.*

Substituting (43) in the IS curve we finally find the optimal interest rate, which is given by:

$$\hat{i}_t^* = \hat{i}_t^{Eff} + \left(1 + \frac{\kappa(1 - \rho_a)}{\alpha_x}\right) E_t\pi_{t+1} \quad (45)$$

then:

Proposition 5. *With progressive taxation the Central Bank implements the Taylor principle, i.e. the monetary authority increases the nominal interest rate more than proportionally with respect to the increase in the inflation rate. Since the source of inflation is the technology shock, this means that the monetary authority responds procyclically to a positive technology shock.¹¹*

¹⁰We obtain more tractable analytical results we also assume that $\sigma = 1$. Results are not affected by this simplifying hypothesis.

¹¹Symmetrically, when the shock is a government spending shock or a cost-push shock, which rise inflation on impact, the monetary authority responds by increasing the nominal interest rate.

In the technical appendix we show that

$$\frac{d\left(1 + \frac{\kappa(1-\rho_a)}{\alpha_x}\right)}{d\phi_n} > 0 \quad (46)$$

Corollary 3. *The higher the degree of the progressiveness of the labor income tax the more aggressive is the optimal policy.*

Finally, to evaluate welfare we can compute the unconditional welfare-loss. Combining the NKPC with (43) and iterating forward we get:

$$\pi_t = \alpha_x \varsigma_a \Psi a_t \quad (47)$$

where $\Psi = \frac{1}{\kappa^2 + \alpha_x(1-\beta\rho_a)}$ and $\varsigma_a = \lambda \frac{(1+\phi)(1-\Phi)}{(1-\phi_n)q_y}$
Combining (47) with (43) we obtain:

$$\hat{x}_t = -\kappa \varsigma_a \Psi a_t \quad (48)$$

we can substitute (47) and (48) in the loss-function to get the unconditional welfare-loss under discretion,

$$L_D = \frac{1}{2} [Var(\pi_t) + \alpha_x Var(\hat{x}_t)] = \frac{1}{2} [(\alpha_x \varsigma_a \Psi)^2 Var(a_t) + \alpha_x (\kappa \varsigma_a \Psi)^2 Var(a_t)] \quad (49)$$

4.2 Gains from Commitment

We now assess the welfare gains that the Central Bank may obtain by committing to a state-contingent rule of the kind studied by Clarida et al (1999):

$$\hat{x}_t^c = -\omega a_t \quad (50)$$

Under the state-contingent rule (50) inflation evolves according to:

$$\pi_t^c = \frac{\varsigma_a - \kappa\omega}{1 - \beta\rho_a} a_t \quad (51)$$

and the NKPC implies

$$\pi_t^c = \frac{\kappa}{1 - \beta\rho_a} \hat{x}_t^c + \frac{\varsigma_a}{1 - \beta\rho_a} a_t \quad (52)$$

this means that under constrained commitment to the rule (50) a 1% contraction in the output gap \hat{x}_t^c reduces inflation π_t^c by the factor $\frac{\kappa}{1-\beta\rho_a}$. Under

discretion the same reduction in the output gap produces a fall in π_t only equal to $\kappa < \frac{\kappa}{1-\beta\rho_a}$. This means that the the gains from reducing inflation under constrained commitment are higher than the ones under discretion. As in Clarida et al (1999), the problem of the Central Bank is to choose the optimal value of the feedback parameter ω by minimizing the welfare-loss subject to equation (52). The first order conditions imply:

$$\hat{x}_t^c = -\frac{\kappa}{\alpha_x^c} \pi_t^c \quad (53)$$

where $\alpha_x^c = \alpha_x(1 - \beta\rho_a) < \alpha_x$. The equilibrium solution for \hat{x}_t^c and π_t^c are obtained by combining (53) and (52) and imply

$$\hat{x}_t^c = -\kappa\zeta_a\Psi^c a_t \quad (54)$$

$$\pi_t^c = \alpha_x^c\zeta_a\Psi^c a_t \quad (55)$$

where $\Psi^c = \frac{1}{\kappa^2 + \alpha_x^c(1-\beta\rho_a)}$. Finally, to evaluate welfare we can compute the unconditional welfare-loss, which is given by

$$L_C = \frac{1}{2} [Var(\pi_t) + \alpha_x Var(\hat{x}_t)] = \frac{1}{2} [(\alpha_x^c\zeta_a\Psi^c)^2 Var(a_t) + \alpha_x (\kappa\zeta_a\Psi^c)^2 Var(a_t)] \quad (56)$$

As measure of the welfare gains implied by commitment we take the ratio between the unconditional loss under discretion L_D and the unconditional loss under commitment L_C . This measure the welfare loss in welfare of moving from the constrained commitment to the discretionary equilibrium.

$$W_G = \frac{L_D}{L_C} = \frac{(\kappa^2 + \alpha_x^c(1 - \beta\rho_a))^2 (\alpha_x^2 + \alpha_x (\kappa\zeta_a\Psi)^2)}{(\kappa^2 + \alpha_x(1 - \beta\rho_a))^2 (\alpha_x^{c2} + \alpha_x (\kappa\zeta_a\Psi^c)^2)} \quad (57)$$

Notice that with $\rho_a = 0$ the welfare gains are nil, in other words $W_G = 1$. The intuition is rather simple. If the shock does not have any persistence then the trade-off between inflation and output will last for just one period. In this case, the central bank is not able to improve the performance of monetary policy by driven future expectation, since the effect of the shock last for just one period. This means the more persistent the shocks the higher are the welfare gains from commitment. Figure 4 shows what happens to the welfare gains when the parameter of the degree of the progressiveness of the labor income tax increases.

- Figure 4 about here -

Figure 4 shows that the higher is ϕ_n the lower are the welfare gains from commitment. In other words, the welfare gains from committing to a state-contingent rule like (50) are a decreasing function of the degree of the progressiveness of the labor income tax. To understand why it happens, remember that by committing to a state contingent rule as in (50) the monetary authority yields a more stable inflation. As we show in section 3, however, a progressive labor income tax has a stabilizing effect. Therefore, the stabilization gains from commitment becomes lower as ϕ_n increases.

5 Conclusions

We introduce progressive taxation on labor income in a New Keynesian model and we study the dynamics of the model and the optimal discretionary monetary policy. We find some interesting results. First of all, we show that progressive taxation on labor income introduces a trade-off between output and inflation stabilization. Moreover, the NKPC becomes steeper the higher is the degree of the progressiveness of the labor income tax.

Second, a progressive labor income tax sensibly alters the so called Taylor-principle. Indeed we find that it enlarges the determinacy region. The higher is the degree of the progressiveness of the labor income tax the larger is the number of Taylor rules which can guarantee the determinacy of the equilibrium.

Third, by approximating the model up to a first order we find that a progressive labor income tax has non-linear dynamic effects and changes the responses of the economy to a technology and to a government spending shock.

We also show that a progressive labor income taxation affects the prescriptions for the optimal discretionary monetary policy. Finally we find that, the welfare gains from commitment are a decreasing function of the degree of the progressiveness of the labor income tax.

Since progressive taxation is common to almost all OECD countries the paper suggests that the literature on monetary policy should carefully consider such important institutional aspect of advanced economies in order to provide a satisfactory interpretation of the dynamics of modern economies.

References

- Benigno, P. and M. Woodford (2006). Optimal taxation in an RBC model: A linear quadratic approach. *Journal of Economic dynamics and Control* 30, 1445–1489.
- Blanchard, O. and J. Galí (2007). Real wage rigidities and the new keynesian model. *Journal of Money, Credit and Banking* 39, 35–65.
- Blanchard, O. J. and C. M. Kahn (1980). The solution of linear difference models under rational expectations. *Econometrica* 48, 1305–1311.
- Calvo, G. A. (1983). Staggered prices in a utility-maximising framework. *Journal of Monetary Economics* 12, 383–398.
- Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2009). New keynesian models: Not yet useful for policy analysis. *American Economic Journal: Macroeconomics* 1, 242–266.
- Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113(1), 1–45.
- Clarida, R., J. Galí, and M. Gertler (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature* 37, 1661–1707.
- Galí, J. (2008). *Monetary Policy, Inflation and the Business Cycle* (1 ed.). Princeton and oxford: princeton university press.
- Guo, J. T. (1999). Multiple equilibria and progressive taxation of labor income. *Economics Letters* 65, 97–103.
- Guo, J.-T. and K. J. Lansing (1998). Indeterminacy and stabilization policy. *Journal of Economic Theory* 82, pages 481–490,.
- Mattesini, F. and L. Rossi (2009). Optimal monetary policy in economies with dual labor markets. *Journal of Economic Dynamics and control* 33, 1469–1489.
- Schmitt-Grohé, S. and M. Uribe (2005). Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomics Annual*, pp. 383–425. Cambridge, MA: MIT Press.
- Schmitt-Grohé, S. and M. Uribe (forthcoming). Optimal, simple, and implementable monetary and fiscal rules. *Journal of Monetary Economics*.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association* 1, 1123–1175.

Woodford, M. (2001). The Taylor rule and optimal monetary policy. *American Economic Review Papers and Proceedings* 91, 232–237.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

A Technical Appendix

A.1 Derivation of the Central Bank Welfare Function with a Distorted Steady State

A.1.1 The steady state distortion: the case of small distortions

As shown in appendix A1, if the steady state is efficient labor market equilibrium implies

$$C^\sigma N^\phi = A = \frac{Y}{N}$$

We now check whether the steady state of our model is efficient or not.

In our model labor supply is:

$$C_t^\sigma N_t^\phi = \frac{W_t}{P_t} (1 - \tau_t) \quad (58)$$

which in steady state becomes

$$C^\sigma N^\phi = \frac{W}{P} (1 - \tau) \quad (59)$$

given that in SS $\tau = \eta$, then the steady state labor supply (59) can be rewritten as:

$$C^\sigma N^\phi = \frac{W}{P} \eta. \quad (60)$$

Firms' labor demand is

$$\frac{W_t}{P_t} = MC_t \frac{Y_t}{N_t}$$

which in steady state implies:

$$\frac{W}{P} = MC \frac{Y}{N} = \frac{\varepsilon - 1}{\varepsilon} \frac{Y}{N} \quad (61)$$

then, by equating (60) and (61) we get steady state labor market equilibrium is

$$C^\sigma N^\phi = MC\eta \frac{Y}{N} = \frac{\varepsilon - 1}{\varepsilon} \eta \frac{Y}{N}. \quad (62)$$

In order to derive the second order approximation of the Central Bank welfare function it is useful to rewrite the previous equation as follows:

$$\frac{V_N(N)}{U_C(C)} = MC\eta \frac{Y}{N} \quad (63)$$

This means that (63) can be rewritten as:

$$\begin{aligned} \frac{V_N(N)}{U_C(C)} &= MC\eta \frac{Y}{N} = \frac{\eta(\varepsilon - 1)}{\varepsilon} \\ &= 1 + \frac{\eta(\varepsilon - 1)}{\varepsilon} - 1 = 1 - \frac{\varepsilon - \eta(\varepsilon - 1)}{\varepsilon} \\ &= 1 - \Phi \end{aligned} \quad (64)$$

where $\Phi < 1$ is the steady state distortion, i.e., the steady-state wedge between the marginal rate of S substitution between consumption and leisure and the marginal product of labor, and hence the inefficiency of the steady-state output level. Remember in Benigno and Woodford (2005) the steady state distortion is

$$\Phi = 1 - \frac{\varepsilon - 1}{\varepsilon} (1 - \tau)$$

in our model η correspond to $(1 - \tau)$ of Benigno and Woodford (2005).

This means that:

$$V(N)N = U_C(C)Y(1 - \Phi)$$

A.1.2 Derivation of the CB Welfare Based Loss function under a distorted steady state

We derive the second order approximation of the household utility function step by step. first of all, given that the utility function is separable in consumption and leisure we approximate the two part separately. Therefore, given that $U(C_t) = U(Y_t - G_t)$ we have that up to a second order:

$$\begin{aligned} U(Y_t - G_t) &\approx U(Y - G) + U_C \frac{dC}{dY} (Y_t - Y) + \frac{1}{2} U_{CC} \frac{dC}{dY} (Y_t - Y)^2 + \\ &+ U_{gy} \frac{dC}{dY dG} GY (Y_t - Y) (G_t - G) + U_C \frac{dC}{dG} (G_t - G) + \frac{1}{2} U_{CC} \frac{dC}{dG} (G_t - G)^2 + O(\alpha^3) \end{aligned}$$

where $\frac{dC}{dY} = 1 = -\frac{dC}{dG}$. Up to a second order: $Y_t - Y = Y\hat{y}_t + \frac{1}{2}Y\hat{y}_t^2$ and $(Y_t - Y)^2 = Y^2\hat{y}_t^2 + O(\alpha^3)$. Analogously: $G_t - G = G\hat{g}_t + \frac{1}{2}G\hat{g}_t^2$ and $(G_t - G)^2 = G^2\hat{g}_t^2 + O(\alpha^3)$. Then, the previous equation becomes:

$$\begin{aligned} U(Y_t - G_t) &\approx U(Y - G) + U_C Y \left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2 \right) + \frac{1}{2}U_{CC}Y\hat{y}_t^2 + U_{gy}GY\hat{g}_t\hat{y}_t \\ &\quad + U_C G \left(\hat{g}_t + \frac{1}{2}\hat{g}_t^2 \right) + \frac{1}{2}U_{CC}G^2\hat{g}_t^2 + O(\alpha^3) \end{aligned} \quad (65)$$

Given that: $\frac{U_{CC}C}{U_C} = -\sigma$, we rewrite equation as:

$$\begin{aligned} U(Y_t - G_t) &\approx U(Y - G) + U_C Y \left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2 + \frac{1}{2}\frac{U_{CC}C}{U_C}\frac{Y}{C}\hat{y}_t^2 - \frac{U_{CC}C}{U_C}\frac{G}{C}\frac{Y}{Y} \right) \\ &\quad - U_C Y \left(\hat{g}_t + \frac{1}{2}\hat{g}_t^2 - \frac{1}{2}\frac{U_{CC}C}{U_C}\left(\frac{G}{Y}\right)^2\frac{Y}{C}\hat{g}_t^2 \right) + O(\alpha^3) \end{aligned} \quad (66)$$

or

$$\begin{aligned} U(Y_t - G_t) &\approx U(Y - G) + U_C Y \left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2 - \frac{1}{2}\frac{\sigma}{\gamma_c}\hat{y}_t^2 - \frac{\sigma(1-\gamma_c)}{\gamma_c}\hat{g}_t\hat{y}_t \right) \\ &\quad + U_C G \left(\hat{g}_t + \frac{1}{2}\hat{g}_t^2 - \frac{1}{2}\frac{\sigma(1-\gamma_c)^2}{\gamma_c}\hat{g}_t^2 \right) + O(\alpha^3) \end{aligned} \quad (67)$$

the collecting terms and recalling $U(Y_t - G_t) - U(Y - G) = \tilde{U}(Y_t - G_t)$:

$$\begin{aligned} \tilde{U}(Y_t - G_t) &\approx U_C Y \left(\begin{aligned} &\hat{y}_t + \frac{1}{2}\hat{y}_t^2 - \frac{\sigma}{\gamma_c}\hat{y}_t^2 - \frac{\sigma(1-\gamma_c)}{\gamma_c}\hat{g}_t\hat{y}_t \\ &+ \hat{g}_t + \frac{1}{2}\hat{g}_t^2 - \frac{1}{2}\frac{\sigma(1-\gamma_c)^2}{\gamma_c}\hat{g}_t^2 \end{aligned} \right) + O(\alpha^3) \end{aligned} \quad (68)$$

Now consider the second order approximation of the utility of leisure $V(N_t)$:

$$\begin{aligned} V(N_t) &\approx V(N) + V_N(N_t - N) + \frac{1}{2}V_{NN}(N_t - N)^2 + O(\alpha^3) \\ &\approx V(N) + V_N N \left(\hat{n}_t + \frac{1}{2}\hat{n}_t \right) + \frac{1}{2}V_{NN}N^2\hat{n}_t^2 + O(\alpha^3) \end{aligned} \quad (69)$$

collecting terms

$$V(N_t) \approx V_N N + V_N N \left(\hat{n}_t + \frac{1}{2}\hat{n}_t + \frac{1}{2}\frac{V_{NN}N}{V_N}\hat{n}_t^2 \right) + O(\alpha^3) \quad (70)$$

Given that $\frac{V_{NN}N}{V_N} = \phi$ and recalling $V(N_t) - V_N N = \tilde{V}(N_t)$, we can rewrite (70) as follows:

$$\tilde{V}(N_t) \approx V_N N \left(\hat{n}_t + \frac{1}{2}\hat{n}_t + \frac{\phi}{2}\hat{n}_t^2 \right) + O(\alpha^3) \quad (71)$$

subtracting (71) from (68) we get:

$$\begin{aligned} \tilde{W}_t = & U_C Y \left(\begin{array}{c} \hat{y}_t + \frac{1}{2} \hat{y}_t^2 - \frac{\sigma}{\gamma_c} \hat{y}_t^2 - \frac{\sigma(1-\gamma_c)}{\gamma_c} \hat{g}_t \hat{y}_t \\ + \hat{g}_t + \frac{1}{2} \hat{g}_t^2 - \frac{1}{2} \frac{\sigma(1-\gamma_c)^2}{\gamma_c} \hat{g}_t^2 \end{array} \right) + \\ & - V_N N \left(\hat{n}_t + \frac{1}{2} \hat{n}_t + \frac{\phi}{2} \hat{n}_t^2 \right) + O(\alpha^3). \end{aligned} \quad (72)$$

We now that in the steady state $V(N)N = U_C(C)Y(1-\Phi)$, therefore we can rewrite (72) as follows:

$$\begin{aligned} \tilde{W}_t = & U_C Y \left(\begin{array}{c} \hat{y}_t + \frac{1}{2} \hat{y}_t^2 - \frac{\sigma}{\gamma_c} \hat{y}_t^2 - \frac{\sigma(1-\gamma_c)}{\gamma_c} \hat{g}_t \hat{y}_t \\ + \hat{g}_t + \frac{1}{2} \hat{g}_t^2 - \frac{1}{2} \frac{\sigma(1-\gamma_c)^2}{\gamma_c} \hat{g}_t^2 \end{array} \right) + \\ & - U_C(C)Y(1-\Phi) \left(\hat{n}_t + \frac{1+\phi}{2} \hat{n}_t^2 \right) + O(\alpha^3). \end{aligned} \quad (73)$$

From the economy production function we know that $\hat{n}_t = \hat{y}_t - a_t + d_t$ and $\hat{n}_t^2 = (\hat{y}_t - a_t)^2$ and then:

$$\tilde{W}_t = U_C Y \left(\begin{array}{c} \hat{y}_t + \frac{1-\sigma}{\gamma_c} \hat{y}_t^2 + \frac{\sigma(1-\gamma_c)}{\gamma_c} \hat{g}_t \hat{y}_t \\ + \hat{g}_t + \frac{1}{2} \hat{g}_t^2 - \frac{1}{2} \frac{\sigma(1-\gamma_c)^2}{\gamma_c} \hat{g}_t^2 \end{array} \right) - U_C(C)Y(1-\Phi) \left(\begin{array}{c} \hat{y}_t - a_t + d_t + \\ + \frac{1+\phi}{2} (\hat{y}_t - a_t)^2 \end{array} \right) + O(\alpha^3). \quad (74)$$

Collecting terms:

$$\tilde{W}_t = U_C Y \left[\begin{array}{c} \Phi \hat{y}_t + \frac{1}{2} \left(\frac{1-\sigma}{\gamma_c} - (1-\Phi)(1+\phi) \right) \hat{y}_t^2 - (1-\Phi) d_t \\ + \frac{\sigma(1-\gamma_c)}{\gamma_c} \hat{g}_t \hat{y}_t + (1-\Phi)(1+\phi) \hat{y}_t a_t \end{array} \right] + t.i.p. + O(\alpha^3). \quad (75)$$

Remembering lemma 1 and lemma 2 of Woodford (2003):

Lemma 1

$$d_t = \frac{\varepsilon}{2} \text{var}_i \{p_{i,t}\} \quad (76)$$

Lemma 2

$$\sum_{t=0}^{\infty} \beta^t \text{var}_i \{p_{i,t}\} = \sum_{t=0}^{\infty} \beta^t \lambda^{-1} \text{var}_i \{p_{i,t}\} \quad (77)$$

Using Lemma 1 and Lemma 2 we finally get the following intertemporal Welfare Based Loss function:

$$W_0 = -\frac{1}{2} U_C Y E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} -\Phi \hat{y}_t + \frac{\gamma_c(1-\Phi)(1+\phi)-(1-\sigma)}{\gamma_c} \hat{y}_t^2 \\ - \frac{\sigma(1-\gamma_c)}{\gamma_c} \hat{g}_t \hat{y}_t - (1-\Phi)(1+\phi) \hat{y}_t a_t + (1-\Phi) \frac{\varepsilon}{\lambda} \pi_t^2 \end{array} \right] + t.i.p. \quad (78)$$

Notice that (78) when $\Phi > 0$ there is a non-zero linear term in (78) which means that we cannot evaluate this equation to second order using a log-linear approximation for the path of aggregate output. Thus we cannot study optimal policy using the log-linear approximation of the competitive equilibrium economy. Rotemberg and Woodford (1997) avoid this problem by introducing subsidies to ensure that $\Phi = 0$. We relax this assumption and in solving the optimal problem we follow Benigno and Woodford (2005) who show that the first order term in equation (78) can be eliminated by taking a second-order approximation of the aggregate supply relation, that is by taking the second order approximation of the following equation:

$$\begin{cases} \frac{1-\varphi\Pi_t^{\varepsilon-1}}{1-\varphi} = \left(\frac{K_t}{F_t}\right)^{1-\varepsilon} \\ K_t = MC_t Y_t + \varphi\beta E_t \{ \Pi_{t+1}^{\varepsilon} K_{t+1} \} \\ F_t = Y_t + \varphi\beta E_t \{ \Pi_{t+1}^{\varepsilon-1} F_{t+1} \} \end{cases} \quad (79)$$

where (79) is the standard first order condition we get when solving the Calvo price-setting problem. A second order approximation of the aggregate supply (79) yields

$$V_0 = \lambda \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} \mu_y \hat{y}_t + \mu_y^2 \hat{y}_t^2 + (1 + \mu_\pi) \frac{\varepsilon}{\lambda} \pi_t^2 + \\ -\mu_y \left(\frac{1+\phi}{1-\phi_n} \right) \hat{y}_t a_t - \mu_y \left(\frac{\sigma(1-\gamma_c)}{\gamma_c(1-\phi_n)} \right) \hat{y}_t g_t \end{array} \right] + t.i.p. \quad (80)$$

where $\lambda = \frac{(1-\varphi)(1-\varphi\beta)}{\varphi}$, $\mu_y = \frac{\sigma+\gamma_c(\phi+\phi_n)}{(1-\phi_n)\gamma_c}$, and $\mu_\pi = \frac{\phi+\phi_n}{(1-\phi_n)}$. Subtracting $\mu_y^{-1} U_c Y \frac{\Phi}{\lambda} V_0$ from W_0 we get

$$W'_0 = -\frac{1}{2} U_c Y E_0 \sum_{t=0}^{\infty} \beta^t [q_y \hat{y}_t^2 + q_\pi \pi_t^2 - q_g \hat{g}_t \hat{y}_t - q_a \hat{y}_t a_t] + t.i.p. \quad (81)$$

where $q_y = \left(\frac{\gamma_c(1-\Phi)(1+\phi)-(1-\sigma)}{\gamma_c} + \mu_y \Phi \right)$, $q_\pi = \left((1-\Phi) + \frac{(1+\mu_\pi)\Phi}{\mu_y} \right) \frac{\varepsilon}{\lambda}$, $q_a = (1-\Phi)(1+\phi) + \left(\frac{1+\phi}{1-\phi_n} \right) \Phi$ and $q_g = \frac{\sigma(1-\gamma_c)}{\gamma_c} + \left(\frac{\sigma(1-\gamma_c)}{\gamma_c(1-\phi_n)} \right) \Phi$

To simplify the analysis we now consider the case in which $\sigma = 1$ and $\gamma_c = 1$. The coefficient of equation (81) becomes

$$\begin{cases} q_y = \left((1-\Phi)(1+\phi) + \frac{1+\phi+\phi_n}{1-\phi_n} \Phi \right) \\ q_\pi = \left((1-\Phi) + \frac{1+\phi}{1+\phi+\phi_n} \Phi \right) \frac{\varepsilon}{\lambda} = \frac{1+\phi+\Phi\phi_n}{1+\phi+\phi_n} \frac{\varepsilon}{\lambda} \\ q_a = (1-\Phi)(1+\phi) + \left(\frac{1+\phi}{1-\phi_n} \right) \Phi \end{cases} \quad (82)$$

which are the coefficient of the welfare-loss (41) in the main text. From (81) it is now easy to define the welfare relevant output gap as a weighted average of

the natural and the efficient equilibrium output:

$$\hat{y}_t^* = q_y^{-1} \left((1 + \phi) (1 - \Phi) \hat{y}_t^{Eff} + \left(\frac{1 + \phi + \phi_n}{1 - \phi_n} \right) \Phi \hat{y}_t^n \right) \quad (83)$$

where $\hat{y}_t^{Eff} = a_t$ and $\hat{y}_t^n = \frac{1+\phi}{1+\phi+\phi_n} a_t$, so that \hat{y}_t^* can be rewritten in terms of the shock.

$$\hat{y}_t^* = (1 + \phi) (1 - \Phi) a_t + \left(\frac{1 + \phi}{1 - \phi_n} \right) \Phi a_t \quad (84)$$

Notice that the weights in (83) depends on the steady state distortion Φ . The welfare-loss can be now rewritten in terms of output gap from the welfare relevant output as in the main text (equation (41)).

The NKPC is

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (\hat{y}_t - \hat{y}_t^n) \quad (85)$$

where $\kappa = \lambda \frac{1+\phi+\phi_n}{1-\phi_n}$. Adding and subtracting \hat{y}_t^*

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - \hat{y}_t^*) + \kappa (\hat{y}_t^* - \hat{y}_t^n) \quad (86)$$

In the main text we call $u_t = (\hat{y}_t^* - \hat{y}_t^n)$.

Notice that given the definition of q_y , then \hat{y}_t^* can be written as:

$$\hat{y}_t^* = \alpha_1 \hat{y}_t^{Eff} + (1 - \alpha_1) \hat{y}_t^n \quad (87)$$

where $\alpha_1 = \frac{(1+\phi)(1-\Phi)}{q_y}$ so that $\frac{\left(\frac{1+\phi+\phi_n}{1-\phi_n}\right)\Phi}{q_y} = 1 - \alpha_1$ and (86) can be rewritten as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \frac{1 + \phi + \phi_n}{1 - \phi_n} (y_t - \hat{y}_t^*) + \kappa \alpha_1 (\hat{y}_t^{Eff} - \hat{y}_t^n) \quad (88)$$

or

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \frac{1 + \phi + \phi_n}{1 - \phi_n} (y_t - \hat{y}_t^*) + \lambda \frac{(1 + \phi) (1 - \Phi) \phi_n}{(1 - \phi_n) q_y} a_t \quad (89)$$

We can finally rewrite the NKPC used to study optimal monetary policy, both under discretion and commitment, as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - \hat{y}_t^*) + \zeta_a a_t \quad (90)$$

where $\zeta_a = \lambda \frac{(1+\phi)(1-\Phi)\phi_n}{(1-\phi_n)q_y}$

Finally, notice that for $\phi_n = 0$, then we get the same equation as Benigno and Woodford (2005) and Gnocchi (2009) among others. Indeed,

$$\begin{aligned} q_y &= (1 + \phi) \\ q_\pi &= \frac{\varepsilon}{\lambda} \\ q_a &= (1 + \phi) \end{aligned}$$

and the welfare loss becomes

$$W'_0 = -\frac{1}{2}U_c Y E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \frac{\lambda}{\varepsilon} (1 + \phi) x_t^2 \right] + t.i.p. \quad (91)$$

notice also that in the case in which $\phi_n = 0$, the coefficient $\zeta_a = 0$ and the NKPC (90) becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - \hat{y}_t^*) \quad (92)$$

This means that the monetary authority does not face any trade-off in stabilizing inflation and output gap. Therefore, by stabilizing inflation, i.e. with $\pi_t = 0$, the output gap becomes also equal to zero and the welfare-loss is nil.

Country	% AW			
	67%	100%	133%	167%
Australia	35,5%	31,5%	41,5%	41,5%
Austria	26,8%	30,4%	30,4%	37,5%
Belgium	40,8%	40,8%	45,3%	46,7%
Canada	19,6%	30,1%	31,1%	33,0%
Czech Republic	20,2%	20,3%	20,2%	20,3%
Denmark	34,6%	41,4%	55,0%	55,0%
Finland	36,5%	41,6%	41,6%	41,6%
France	18,0%	18,0%	30,1%	30,1%
Germany	30,6%	37,3%	42,9%	44,3%
Greece	22,7%	22,7%	22,7%	31,1%
Hungary	27,0%	45,0%	36,0%	36,0%
Iceland	34,3%	34,3%	34,3%	34,3%
Ireland	20,0%	20,0%	41,0%	41,0%
Italy	29,2%	29,2%	39,1%	38,7%
Japan	8,7%	13,6%	23,3%	25,6%
Luxembourg	21,9%	34,7%	34,7%	34,7%
Mexico	10,8%	10,8%	17,8%	19,9%
Netherlands	11,1%	40,5%	52,0%	52,0%
New Zealand	21,0%	33,0%	36,0%	39,0%
Norway	28,0%	37,0%	37,0%	40,0%
Poland	9,7%	9,7%	9,7%	19,2%
Portugal	23,5%	23,5%	34,0%	34,0%
Slovak Republic	16,5%	16,5%	16,7%	16,7%
Spain	22,5%	26,2%	26,2%	37,0%
Sweden	23,4%	44,4%	51,4%	56,4%
Switzerland	16,0%	17,7%	22,7%	26,8%
Turkey	17,6%	17,6%	23,5%	23,5%
United Kingdom	20,0%	20,0%	40,0%	40,0%
United States	21,7%	21,7%	31,7%	31,7%

Table1. Marginal Personal Income Tax Rate, (year: 2008). Source: OECD

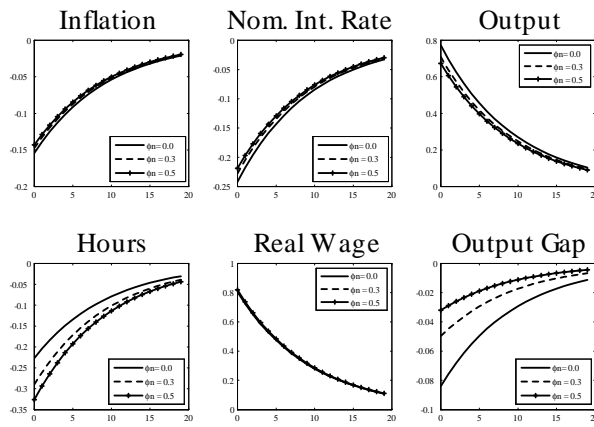


Fig. 1: IRFs to a 1 sd positive technology shock under different degree of the labor income tax ϕ_n .

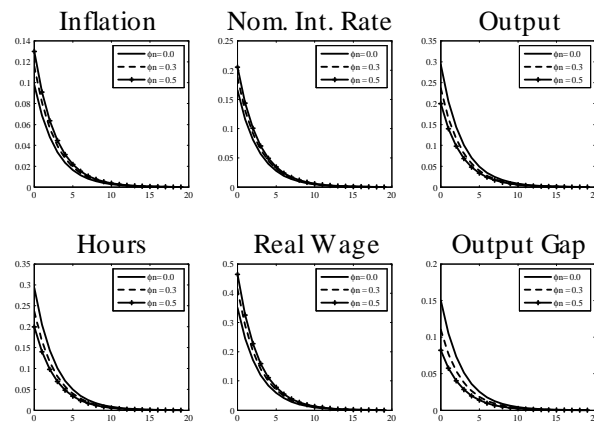


Fig. 2: IRFs to a 1 sd positive government spending shock under different degree of the labor income tax ϕ_n .

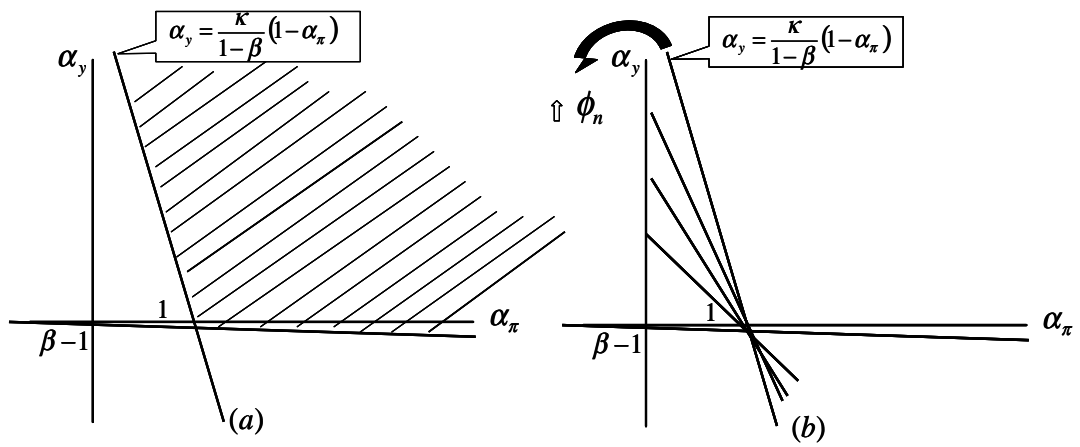


Fig. 3: Determinacy regions for different degrees of progressiveness of the labor income tax.

Effect on Welfare of Varying ρ_a and ϕ_n on the welfare gains from commitment

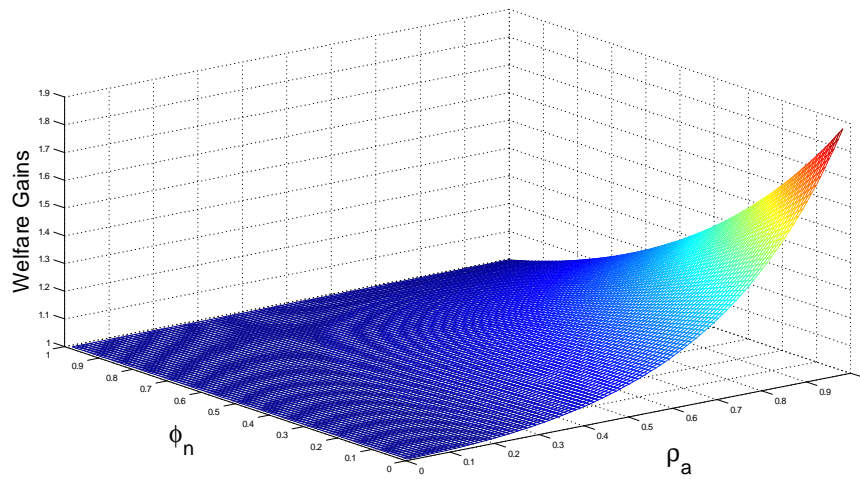


Fig. 4 Welfare Gains from Commitment