Abstract

I study whether US Tax Policies affected economic volatility during the post World War II period. I employ a Real Business Cycle model with distorting taxation on household income and tax rules, and assume that taxes respond to the cyclical conditions of the economy. I estimate the deep parameters of the model using Bayesian techniques. My findings are; (a) tax policies display a strong countercyclical behavior, (b) help to reduce the cyclical and raw volatility of GDP, consumption, investment when the government can issue debt, and (c) unexpected changes in tax policies do not affect the volatility of the macroeconomic variables.

Keywords: Fiscal Policy and Business Cycles, Bayesian Methods.


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1 Introduction

Can fiscal adjustment change the features of the Business Cycle? Is it true that a countercyclical fiscal policy helps to smooth fluctuations? Which fluctuations, raw and/or cyclical? These topics have been at the center of public discussion in the past two decades. In Europe, because of the creation of a single currency area and the relinquishment of national monetary policy, the debate has focused on the role of national fiscal policy and the nature of the Stability and Growth Pact. In this respect, Gali and Perotti (2004) ask whether after the Maastricht Treaty national fiscal policies have become less countercyclical, and they find no evidence to support this view. In the US, few years ago a Treasury Department study concluded that the absence of automatic stabilizers at the peak of the US recession in 2001 would have added an additional 1.5 million people to the ranks of the unemployed. Moreover, a consideration clearly influencing recent policy decisions as of the beginning of 2009, if not in earlier years, is the zero-nominal-interest rate bound facing monetary policy. Thus, a renovated activism on fiscal policy to fight the recent crises brought governments to rethink the role of fiscal policy (see Auerbach (2009)).

At the core of this debate there is the Keynesian prescription that a countercyclical fiscal policy has stabilizing effects that work through both automatic stabilizers and occasionally discretionary actions; many economists share this view\(^1\). At the opposite end of the debate, some recent studies question the stabilizing role of fiscal policies. Jones (2002) shows empirically that post war fiscal policy did not help the US economy to smooth Business Cycle fluctuations, for example. From a theoretical standpoint, Gordon and Leeper (2005) highlight that countercyclical fiscal policy might amplify recessions through the policy expectations channel.

In this paper, I revisit the issue of whether fiscal policy matters for business cycle fluctuations and ask whether US tax policy has been an important source for economic volatility, and (if so) at which frequencies is the tax instrument more important. I assume that taxes respond to cyclical conditions of the economy by reacting to economic fluctuations. I estimate the deep parameters of the model from a vector of time series using Bayesian techniques. The main findings read as follows. First, consistent with the less structural analysis of Romer and Romer (2007) and Cohen and Follette (2000), US tax policies display a strong procyclical reaction to GDP or employment in the period

considered. Second, while with a balanced budget assumption the procyclical tax behavior has little room to stabilize the economy, with a government budget constrain with debt it acts as an important stabilizing device. Indeed, when government can issue debt the automatic response of the labor and capital tax to cyclical conditions reduces the volatility of consumption, GDP, investment. This is true regardless of the horizon that we look at, and in particular counterfactuals with raw and cyclical simulated data provide the similar conclusions. Unexpected changes in taxes generate very little economic volatility especially at business cycles frequencies.

For the purpose of this paper, one of the most delicate issues is the definition of a policy rule that summarizes the evolution of tax over time. As explained in Gali and Perotti (2004), the fiscal response function can be seen as the combination of a cyclical component and a structural or discretionary component; the first part comprises all the variations outside the direct control of the fiscal authority (like changes in the tax base). The second part should be interpreted as the part which is intentionally chosen by the policymaker, as in Fatás and Mihov (2001). Within this discretionary part, there is an endogenous or systematic response by which the policymaker automatically responds to cyclical economic conditions and there is a non-systematic or exogenous component. The former component arises from spending programs and tax cut that adjust systematically with economic conditions. The latter captures all the changes that do not correspond to systematic variations to cyclical conditions; we can interpret these exogenous changes as actions that are meant to sustain or fasten long run growth (Romer and Romer (2007)), or changes in the political process (Gali and Perotti (2004)). This paper attempts to estimate these two components from the data. There are different fiscal instruments that could be taken into consideration. For instance, Fatas and Mihov (2006) consider government spending. Gali and Perotti (2004) or Auerbach (2003) instead consider the primary deficit. Jones (2002) considers average tax rates and government spending. Here I focus on tax policies and government debt and I deliberately ignore government spending as a fiscal instrument. The reason for that is that government expenditure is rather inflexible and while it can be easily increased it is very difficult to decrease it. This cast doubts on its validity as a stabilizing tool.

An interesting extension to this work is to introduce sticky prices and a monetary rule and to study the interactions between fiscal and monetary policy\(^2\). However, the main

\(^2\)I thank an anonymous referee to point this out.
focus of this paper is to show that the results of Jones (2002) are driven by the balanced budget assumption, and postulating a nominal side would make the comparison less clear. Moreover, parameters identification is problematic in DSGE model estimation (see Canova and Sala (2009)). For a given sample size I am aware that the larger the model the more difficult it becomes to identify parameters, making our inference weaker.

Several empirical studies have used VAR techniques to address the issue of interest in this paper; Mountford and Uhlig (forthcoming) study the transmission mechanism of fiscal shocks. Canova and Pappa (2007) and Perotti (2002) shows that government spending has a significant output multiplier. There are endogenous feedbacks between economic activity and tax policies; on the one hand, taxes directly affect household consumption and labor decisions, and therefore economic activity. On the other hand, the fiscal authority sets tax policies by looking at the economic activity. Given the strong endogenous relations between household decisions and tax policy, I choose to employ a general equilibrium framework. There are several papers that look at fiscal policies in a general equilibrium framework. McGrattan (1994) estimates the fiscal response from a vector of autoregression considering a broad general class of fiscal responses; she concludes that a relevant portion of business cycles fluctuations is due to fiscal instruments. Braun (1994) estimates in a GE model the reaction function of fiscal instruments, taxes and government spending. Jones (2002) estimates various fiscal policy rules form the US postwar data. I extend their analysis by introducing debt in the model and as a measurable variable. Furthermore, I consider proxies for the marginal tax rates using Mendoza, Razin and Tesar (1994) methodology, where marginal tax rate are are constructed using national accounts and revenue statistics3.

Following a standard practice in the literature of DSGE models I estimate structural parameters using a Bayesian approach. Bayesian techniques have gained a predominant role for the DSGE models estimation, representing ideally the toolkit of every applied researcher, see An and Schorfheide (2007). There are several reasons for that. First, the Bayesian paradigm provides a coherent framework to treat model uncertainty and to take decisions based on risk. Second, while not treated as the 'true' data

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3The method allows to compute time series of effective tax rates on consumption, capital income, and labor income using information publicly available from the national accounts. The three rates are measured as ad-valorem estimates by classifying virtually all forms of tax revenue at the general government level into one of the three taxes. Each measure of tax revenue is then expressed as a fraction of a precise estimate of the corresponding tax base. These ad-valorem tax rates reflect specific (or per-unit) tax rates faced by a representative agent in a general equilibrium framework.
generating process, DSGE models are just considered as an approximation of the law of motion for the data. Third, Bayesian estimators have desirable properties in small samples. Fourth, priors allow to incorporate external information to the model and to consistently combine pre-sample information with the observed data.

The paper is organized as follows: Section 2 presents the model. The estimation procedure is described in Section 3. Section 4 discusses different specifications and Section 5 presents the results and the model fit. Section 6 draws policy implications and Section 7 concludes.

2 Model

I employ a prototype RBC model with no frictions and with a time varying ‘wedge’ on labor and capital accumulation, and an efficiency ‘wedge’ on the production side. Chari, Kehoe and McGrattan (2007) have shown that this prototype economy is equivalent to a large class of models with various types of frictions and can reasonably well account for the U.S. postwar Business Cycle fluctuations.

The model consists of a single representative firm, a representative household, and a government. The firm and the household behave in the standard fashion; the firm maximizes profit, and the household maximizes its discounted lifetime utility. The government, on the other hand, does not have an objective function and it has to finance an exogenous expenditure process using distortionary taxation and issuing real debt.

The supply side of the economy is very stylized. The numeraire is final output $Y_t$, which is produced by a representative price-taking firm. The firm faces a Cobb-Douglas production function:

$$Y_t = A_t K_{t-1}^\alpha N_t^{1-\alpha},$$

(1)

where $K_{t-1}$ and $N_t$ denote the capital and labor available at time $t$, respectively. $A_t$ is the exogenous stochastic technology process. I assume

$$a_t = \rho a_{t-1} + \epsilon^a_t,$$

where $a_t = \ln A_t$, and $\epsilon^a_t$ is an i.i.d. shock with zero mean and variance $\sigma^2_a$. By perfect competition assumption, the cost of renting capital, $r_t$, and real wages, $w_t$, are equal to their marginal products, i.e. $w_t = (1 - \alpha) \frac{Y_t}{N_t}$ and $r_t = \alpha \frac{Y_t}{K_{t-1}}$.

On the demand side, the economy is populated by a single representative household
who maximizes an infinite stream of discounted utility,

$$\max_{\{C_t, N_t, I_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t - 1}{1 - \eta} - X_t N_t \right]$$

(2)

where $\beta$ is the time discount factor and $1/\eta$ is the intertemporal elasticity of substitution; $C_t, N_t$ are respectively consumption and hours worked at time $t$. All the variables are expressed in per capita terms. $X_t$ is an exogenous preference shock which evolves according to

$$\chi_t = \rho \chi_{t-1} + \epsilon_t^\chi$$

where $\chi_t = \ln X_t$ and $\epsilon_t^\chi$ is an i.i.d. shock with zero mean and variance $\sigma_\chi^2$. The representative household faces a budget constraint,

$$I_t + C_t + B_t = (1 - \tau_w^t) w_t N_t + (1 - \tau_k^t) r_t K_{t-1} + (1 + r_b^t) B_{t-1}$$

(3)

I indicate with $\tau_w^t$ and $\tau_k^t$ the taxes on labor and capital income, respectively; $B_{t-1}$ is the real debt issued by the government at time $t - 1$ which gives a net interest rate of $r_b^t$. I assume also that the law of motion of capital is

$$V_t I_t = K_t - (1 - \delta) K_{t-1}$$

(4)

where $\delta$ is the rate of depreciation of capital and $V_t$ is an investment specific shock which follows an AR(1) process, i.e.

$$v_t = \rho v_{t-1} + \epsilon_t^v$$

where $v_t = \ln V_t$, and $\epsilon_t^v$ is an i.i.d. shock with zero mean and variance $\sigma_v^2$. The representative household problem is to maximize (2) subject to the budget constraint (3) and (4); the first order conditions for the household problem are

$$C_t^\eta = (1 - \tau_w^t)(1 - \alpha) \frac{Y_t}{X_t N_t},$$

(5)

$$1 = \beta E_t \{ V_t (\frac{C_t}{C_{t+1}})^\eta R_{t+1} \}$$

(6)

$$R_{t+1} = (1 - \tau_k^{t+1}) \alpha \frac{Y_{t+1}}{K_t} + \frac{1 - \delta}{V_{t+1}}.$$  

(7)

$$1 = \beta E_t \{ (\frac{C_t}{C_{t+1}})^\eta (1 + r_b^{t+1}) \}$$

(8)

Equation (5) is the intra-temporal optimality condition between consumption and leisure; equation (6) is the usual Euler equation and $R_t$ is net depreciation after tax.
interest rate. Equation (8) is the intertemporal optimality condition for debt demand.

In equilibrium, it must be the case that the no arbitrage condition between the after tax interest rate and the bond interest rate holds:

\[ V_{t-1}[(1 - r^k_t) r_t + \frac{1 - \delta}{V_t}] = 1 + r^b_t \]  

(9)

The government satisfies a period by period budget constraint,

\[ G_t + (1 + r^b_t) B_{t-1} = \tau^w w_t N_t + r^k_t r_t K_{t-1} + B_t \]  

(10)

where \( G_t \) is government spending. The literature considers several specifications for the fiscal policy instruments. Some authors consider government spending as an instrument that stimulates private consumption (see Galí, López-Salido and Vallés (2007)). My goal here is to study the ability of taxes to affect the Business Cycle. Moreover, government expenditure is rather inflexible and while it can be easily increased it is very difficult to decrease it. Therefore, I postulate that the government spending evolves as AR(1) process,

\[ g_t = \rho g_{t-1} + \epsilon^g_t \]

where \( g_t = \ln G_t \) and \( \epsilon^g_t \) is an i.i.d. shock with zero mean and variance \( \sigma^2_g \). As mentioned, the fiscal literature defines the fiscal rule as a combination of two main elements; an endogenous automatic response to the economic conditions, through which the policymaker reacts automatically to cyclical conditions and an exogenous component meant to be an unexpected reply to economic cycles. In line with this literature, I assume that the tax deviation from its steady state responds to the GDP log deviations from its steady state and to the debt-GDP deviation; the latter variable is included in order to avoid explosive paths of government debt. It is also common to include lagged value of the taxes to account for sluggish reaction of the fiscal instrument. Rules governing tax policies\(^4\) take the following form:

\[ \tilde{\tau}_t^w = \varphi^y \tilde{y}_t + \varphi^w \tilde{\tau}_{t-1}^w + \varphi^y y_t + \epsilon^w_t \]  

(11)

\[ \tilde{\tau}_t^k = \psi^y \tilde{y}_t + \psi^k \tilde{\tau}_{t-1}^k + \psi^y y_t + \epsilon^k_t \]  

(12)

where \( \tilde{\tau}_t^j \) is the tax \( j \) in deviation from its steady state, i.e. \( \tilde{\tau}_t^j = \tau_t^j - \tau^j \), for \( j = w, k \), \( \tilde{y}_t \) is the deviation of the debt-GDP ratio from its steady state, i.e. \( \tilde{y}_t = B_t/Y_t - B/Y \), and \( y_t \) is the log deviation of the GDP from its steady state, i.e. \( y_t = \ln \frac{Y_t}{Y} \). \( \epsilon^j_t \) are i.i.d.

\(^4\)Other specifications for the FP rule are considered in the following sections.
policy shocks with zero mean and variance $\sigma^2_j$, with $j = w, k$.

Finally, total output is absorbed by private and public consumption and investments, i.e. $Y_t = I_t + C_t + G_t$.

3 Estimation strategy

The DSGE model presented is log linearized around a non stochastic steady state and solved, where variables in deviation from their steady states are interpreted as a relevant measure of cycles. The solution of the linearized model takes the from,

$$y^1_{t+1} = \rho_y(\theta)y^1_t + \rho_i(\theta)u_{t+1}$$

where $\rho_y(\theta) \text{ and } \rho_i(\theta)$ are matrixes which are function of the structural parameters of the DSGE model, $\theta = [\alpha, \eta, \delta, K, \tau^w, \tau^k, \varphi^v, \varphi^y, \varphi^w, \varphi^by, \varphi^y, \psi^y, \rho_a, \rho_g, \rho_y, \rho_k, \rho_w, \rho_y, \rho_k, \rho_w]$. $u_{t+1}$ is the vector of structural innovations whose variance covariance matrix is diagonal and has on its main diagonal $\sigma^2 = [\sigma^2_a, \sigma^2_g, \sigma^2\chi, \sigma^2_k, \sigma^2w, \sigma^2w, \sigma^2ym, \sigma^2km]$. $\sigma_{yw}$ and $\sigma_{wk}$ are the standard deviations of measurement shocks on the marginal tax rates.$^5$

The model describes the behavior of several variables and it is assumed that eight variables are observed: real GDP, consumption, investment, hour worked, labor and capital taxes, government debt and government spending from 1966q3 to 2007q2. Details on data construction are reported in the appendix. In the literature on DSGE models estimation$^6$, it is standard practice to assume a unit root behavior for the technology process to account for the long run movements of the data. This assumption implies that real variables grow at the same rate, the technology rate, and real data in first difference has a direct counterparts with real model-based variables in first difference (plus a noise). Usually, only real and nominal variable are considered, and 'hybrid' quantities such as debt-GDP ratio, marginal tax rates and government spending are not included in the set of observable variable. In undertaking this task many problems arise. First, each series displays different upward trends and low frequencies movements. Looking at the data plotted in the first two rows of Figure 1, 'eyeballs econometric' revels that each series displays very idiosyncratic pattern both at low and medium frequencies. Standard model based filtering is not implementable since the

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$^5$Since I describe the evolution of eight observable variables, I need to introduce measurement errors to avoid stochastic singularity. Since marginal tax rate are the variables that are more ‘latent’ and less directly observable, I decided to attach a measurement error on them.

likelihood is ill behaved using the raw series of debt-GDP ratio, marginal tax rates and government spending. While a possible option is to filter the data, how to extract cyclical information from the raw series is not a trivial question. Indeed, the third and fourth lines of Figure 1 display the linear detrended, HP filtered and first difference data. Clearly, cycles are statistically different across different filters. For the sake of the argument, consider the debt-GDP ratio filtered with a linear detrending and an HP filter: an HP filter on debt-GDP ratio (fourth row seventh column) produces cycles with an average periodicity of 10 quarters from peak to peak, whereas linear detrended debt-GDP ratio (second row seventh column) displays cycles with a periodicity of 100 quarters! Moreover different filters produce different posterior distributions of the parameters and thus different conclusions for the model (see Canova and Ferroni (2008) or Castelnuovo (2009)).

To the best of my knowledge, there are two different approaches that deal with this problem, namely a multiple filtering device proposed by Canova and Ferroni (2008) and a trend agnostic method proposed by Ferroni (2009). The former procedure exploits data-rich environments and treats data filtered with alternative procedures as contaminated proxy of the relevant model-based quantities. Structural and nonstruc-
tural parameters are jointly estimated using an unobservable component structure. The latter approach proposes a one step procedure, where DSGE structural parameters are jointly estimated with trends parameters. This allows to test hypothesis about the most likely trend specification for individual series and/or use the resulting information to construct robust estimates by Bayesian averaging. Both procedures display nice properties with simulated data. For the aim of this paper, Ferroni (2009) setup is preferable. While with the multiple filtering device we can only make inference on the cyclical behavior of the observed variables, with the one step approach we can make inference about standard deviation of both cyclical and raw variables.

The estimation strategy assumes that data, $y_t$, is made up of a non-stationary trend component, $y^\tau_t$, and a cyclical component, $y^c_t$, so that

$$ y_t = y^\tau_t + y^c_t $$

(14)

where $y_t = [\ln C_t, \ln GDP_t, \ln N_t, \ln I_t, \tau^w_t, \tau^k_t, B_t Y_t, \ln G_t]^T$.

It is assumed that the log linearized model provides a good approximation of the cyclical component of the data,

$$ y^c_t = S y^\dagger_t $$

(15)

$$ y^\dagger_{t+1} = \rho_y(\theta) y^\dagger_t + \rho_1(\theta) n_{t+1} $$

(16)

$$ y^\tau_t = \mathcal{F}(y_t) $$

(17)

where $S$ is a selection matrix that picks the model variables that are observables, and $\mathcal{F}(y_t)$ is the filter that extracts the trend $y^\tau_t$ from the data. Let

$$ \mathcal{F}_{lt} : y^\tau_t = A + B * t + \eta_t $$

(18)

$$ \mathcal{F}_{rd} : y^\tau_t = \gamma + y_{t-1} + \eta_t $$

(19)

$$ \mathcal{F}_{hp} : y^\tau_{t+1} = y^\tau_t + \mu_t $$

(20)

$$ \mu_{t+1} = \mu_t + \zeta_{t+1} $$

where equation (18) postulates a linear trend, equation (19) a unit root, and equation (20) a smooth integrated of order II random walk. $\eta_t$ and $\zeta_t$ are independent zero mean shocks with diagonal covariance matrices, $\Sigma_\eta$ and $\Sigma_\zeta$ respectively. For each trend specification, there is a set of non-structural parameters to be estimated: for the linear

\footnote{Since marginal tax rates and debt-GDP ratio are variables bounded to be between 0 and 1, taking logs make little sense. Also in the log linearization of the model these variables are rewritten in terms of deviation from the steady state.}
trend $\vartheta_t = [A, B, \Sigma_\eta]$, for the unit root $\vartheta_{fd} = [\gamma, \Sigma_\eta]$, and for the smooth integrated of order II random walk $\vartheta_{hp} = \Sigma_\zeta$. Harvey and Jaeger (1993) show that the random walk of order II (equations (20)) is equivalent to a Hodrey Prescott trend where the smoothing parameter is the ratio between the variance of cycles and the variance of the second difference of the trend.

It is easy to show that the system of of equations (14)-(17) can be cast into a linear state space, whose likelihood can be computed using the Kalman filter.

Parameters, $\nu = [\theta, \vartheta]$, are estimated using Bayesian methods. Given the DSGE model, $M$, and a trend specification, $F$, posterior distributions of the parameters is proportional to the product of prior and sample information, so that

$$p(\nu | y, M, F) \propto p(\nu) L(y | \nu, M, F)$$

where $p(\nu)$ is the prior distribution of the parameters, and $L(y | \nu, M, F)$ is the likelihood of raw data computed with the Kalman filter. Given the large number of parameter involved, there is no analytical solution and simulation methods are needed. In particular, I used Monte Carlo Markov Chain (MCMC) methods. The main idea of MCMC simulators is to define a transition distribution for the parameters that induce an ergodic Markov chain. After a large number of iterations, draws obtained from the chain are draws from the limiting target distribution (see Schorfheide (2000), DeJong, Ingram and Whiteman (2000) or Canova (2007) Ch 9). I run 600,000 draws for each specification and I tune up the RWM variance in order to achieve a 30%-40% acceptance rate. Convergence for all the parameters is achieved after 300,000 draws.

I briefly discuss the prior selection. I fix the depreciation rate to 0.025, which implies an annual depreciation rate of 10%, and estimate the remaining parameters. In Table 4, I report the parameters description and the priors assumptions. Following standard practice, I choose Beta distributions for those parameters that must lie within the unit interval, like capital share or steady state taxes. The persistence parameters of the AR(1) processes are assumed to follow a Beta distribution as well, with mean 0.7 and standard deviation 0.1. The Beta distribution covers the range between 0 and 1, but a small standard error was used to have a clearer separation between stationary and non stationary shocks, as in Smets and Wouters (2003). For the coefficient of

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8To save space, convergence diagnostics are not presented here, but they are available upon request.
relative risk aversion, $\eta$, I pick a Gamma distribution with mean 2.5, which implies an intertemporal elasticity of substitution, $1/\eta$, of 0.4, close to the RBC literature values. Standard deviations are assumed to be distributed as Uniform with 0-1 support. For the fiscal policy parameters I choose Normal distributions centered at positive values, 0.2, with a large standard deviation, 0.5. This implies that a priori fiscal policies are countercyclical on average but there is a positive probability that the coefficients are negative. About the trend parameters, I use uniform priors for standard deviations and normal with large variance for drifts or slopes.

4 Trend and Fiscal Policy specifications

As mentioned, fiscal policy rules are linear combinations of automatic stabilizers and unexpected changes. This broad definition leaves space for arbitrariness in writing down the exact fiscal policy function. For instance, Jones (2002) defines the (log deviation from the steady state) labor and capital tax as a linear combination of present and lagged values of GDP, hour worked and lagged values of taxes and government consumption. Davig and Leeper (2007) let the (deviation from the steady state) tax depend on the past level of debt-GDP ratio, current GDP, government consumption. In Gali and Perotti (2004), the primary deficit responds to the expected value of output gap, to debt, to the past level of the deficit and an orthogonal shock. To account for this variety, I experiment with different possibilities. The specifications that I consider are

- Taxes respond to GDP, $S_1$:

$$\tilde{\tau}_t^{w} = \varphi_w \tilde{\tau}_{t-1}^{w} + \varphi_y y_t + \varphi_by \tilde{y}_t + \xi_t^{w}$$
$$\tilde{\tau}_t^{k} = \psi_k \tilde{\tau}_{t-1}^{k} + \psi_y y_t + \psi_by \tilde{y}_t + \epsilon_t^{k}$$

- Taxes respond to employment, $S_2$:

$$\tilde{\tau}_t^{w} = \varphi_w \tilde{\tau}_{t-1}^{w} + \varphi_n n_t + \varphi_by \tilde{y}_t + \xi_t^{w}$$
$$\tilde{\tau}_t^{k} = \psi_k \tilde{\tau}_{t-1}^{k} + \psi_n n_t + \psi_by \tilde{y}_t + \epsilon_t^{k}$$

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• Taxes respond to lagged GDP, $S_3$:

\[
\tilde{\tau}^j_{t} = \varphi_w \tilde{\tau}^j_{t-1} + \varphi_y y_{t-1} + \varphi_b \tilde{b}_{y t} + \epsilon^w_t
\]

\[
\tilde{\tau}^k_{t} = \psi_k \tilde{\tau}^k_{t-1} + \psi_y y_{t-1} + \psi_b \tilde{b}_{y t} + \epsilon^k_t
\]

• Taxes respond to expected GDP as in Gali and Perotti (2004), $S_4$:

\[
\tilde{\tau}^w_{t} = \varphi_w \tilde{\tau}^w_{t-1} + \varphi_y E_{t+1} y_t + \varphi_b \tilde{b}_{y t} + \epsilon^w_t
\]

\[
\tilde{\tau}^k_{t} = \psi_k \tilde{\tau}^k_{t-1} + \psi_y E_{t+1} y_t + \psi_b \tilde{b}_{y t} + \epsilon^k_t
\]

The rationale behind these specifications is twofold: (a) test which is the most likely cyclical indicator for fiscal policy rule among GDP or employment, (b) search for the correct timing of the fiscal policy reaction to changes in the cyclical conditions, i.e. past, current or expected cyclical changes affect today tax.

To test among different specifications I use Posterior Odd ratios. The Posterior Odds ratio is constructed by comparing the Bayes Factor, which is the ratio of the predictive densities of the data conditional on different models, and prior odds, which is the ratio of prior probabilities associated to each model. Given a prior $p(\nu)$, the predictive density of the data, $y$, conditional on a fiscal policy specification, $S$, and on a trend specification, $F$, is

\[
p(y|S, F) = \int \mathcal{L}(y|\nu; S, F)p(\nu)d\nu
\]

Therefore, for the given trend specification -say- $F_{lt}$, the Posterior Odds ratios between $S_1$ and $S_2$ is

\[
PO_{S_1, F_{lt}|S_2, F_{lt}} = \frac{p(S_1, F_{lt})}{p(S_2, F_{lt})} \times \frac{p(y|S_1, F_{lt})}{p(y|S_2, F_{lt})}
\]

where $p(S_1, F_{lt})$ and $p(S_2, F_{lt})$ are prior probabilities on the trend and fiscal policy specifications.
Table 1 reports the Posterior Odds for each specification and for each trend specification. First thing that stands out is the poor relative fit of the linear trend specification compared to other trend specifications, independently on the fiscal policy. While HP or first difference filters are relatively comparable, a linear detrended filter has posterior density of the data much lower than the other filters. This is true for each fiscal policy specification. This is not surprising by looking at cycles extracted by a linear trend. As mentioned, linear detrended debt-GDP ratio displays cycles with large periodicity (second row seventh column in Figure 1), of roughly 100 quarters, and the remaining linear detrended variables do not display similar periodicity. This clearly unbalances the likelihood since the filtered data has different times series properties.

Second thing to notice is that no matter what filter we use, the specification $S_1$ is always preferred over the remaining specifications; so, if we use a linear trend we would choose $S_1$, same for a unit root or a II order random walk. This is an important result because it means that the choice of the fiscal policy specification is independent on the way in which we extract the cyclical information from the data.

Therefore, concerning the two questions of interest (namely, the choice of the cyclical indicator and timing of the reaction), with a 0-1 loss function the PO ratios reveals that the set up with $S_1$ and fd is the preferred one. Therefore, the fiscal policy and trend

| Specification | Acc | Prior | $p(y|S_k, F_m)$ | $\ln(PO_{k,m|S_1,tt})$ |
|---------------|-----|-------|----------------|----------------------|
| $S_1$ & lt    | 25.95 | 1/12  | 2367.2         | 0.0                  |
| $S_1$ & hp    | 24.82 | 1/12  | 4018.3         | 1651.1               |
| $S_1$ & fd    | 44.66 | 1/12  | 4061.6         | 1694.4               |
| $S_2$ & lt    | 27.36 | 1/12  | 2347.1         | -20.1                |
| $S_2$ & hp    | 18.43 | 1/12  | 3946.7         | 1579.5               |
| $S_2$ & fd    | 25.49 | 1/12  | 3373.9         | 1006.8               |
| $S_3$ & lt    | 26.77 | 1/12  | 2344.5         | -22.6                |
| $S_3$ & hp    | 27.35 | 1/12  | 3942.9         | 1575.7               |
| $S_3$ & fd    | 32.26 | 1/12  | 3633.3         | 1266.2               |
| $S_4$ & lt    | 26.17 | 1/12  | 2356.6         | -10.6                |
| $S_4$ & hp    | 27.66 | 1/12  | 3963.2         | 1596.0               |
| $S_4$ & fd    | 34.22 | 1/12  | 3880.4         | 1513.2               |
specification I discuss and adopt for policy experiments is the unit root specification for raw data with a fiscal policy rule that responds to contemporaneous variations.\footnote{I also tried with a nested version of the fiscal policy rule, but I decided to not to present it. First, because the marginal increase in the posterior density of the data is small. Second, more importantly parameters estimation becomes problematic: parameter convergence is harder, and some parameters of the fiscal policy rule is not identifiable.}

\section{Parameters Estimates and Moments}

In this section I discuss the posterior estimates of the parameters and the model fit. Table 5 reports the mean, median, standard deviation, and probability sets of the structural and non structural parameters estimates obtained with the RWM algorithm. Figures 9 and 10 summarize this information visually by plotting the prior distribution (dashed line) and the posterior one (solid line). Figures 7 and 8 show the Cumulative Sum Statistics for all parameters, and indicate that convergence for all parameters is achieved after 300,000 draws. Overall, most of the parameters are estimated to be significantly different from zero.

Analyzing, first, the estimated stochastic processes, it appears that the variance of the preference shock, $\sigma_\chi$, is larger than the technology shock, $\sigma_a$, which is similar to what it is found by Smets and Wouters (2003, 2007). The standard deviations of the tax shocks are significantly different from zero. Notably, the standard deviations of the measurement errors are generally larger than structural shocks ones, implying that they are capturing a lot of variation not explained by the model. Turning to behavioral parameters, the overall picture is pretty much in line with what is available in the RBC literature.

Moving now to the fiscal policy parameters, $\theta_f = [\varphi_w, \varphi_{by}, \varphi_k, \varphi_{by}, \psi_y]$, we can notice that generally posterior standard deviations are smaller than the prior ones. The parameters controlling government debt, $\varphi_{by}$ and $\psi_{by}$, are greater than zero as one would expect; since they are meant to avoid the explosiveness of public debt, the tax reaction should be positively correlated to debt changes. The two taxes are weakly correlated. For our purpose the most interesting parameters estimates are $\varphi_y$ and $\psi_y$, which summarize the automatic response of the fiscal policy to cyclical conditions. The automatic response of the labor tax, $\varphi_y$, is strictly positive, and we can rule out the possibility that these coefficients are zero or negative. This is clear when looking at plots of prior and posterior distributions (Figure 9 eight from top left); indeed, the
left tail of the posterior assigns (almost) zero probability to the event $\varphi_y \leq 0$. This fact implies that the labor tax function is procyclical, corroborating the idea that fiscal policy has been countercyclical along the period considered. Figure 10 indicates that in most of the cases the posteriors of the fiscal parameters do not overlap with the priors, meaning that the fiscal policy parameters are identifiable.

The main focus of this paper is assess the ability of the tax policy to affect the volatility of the macro variables. Therefore, the first check to do is to see how the model is able to replicate standard deviations. Table 6 displays the data standard deviation and the model standard deviations. The first two rows show standard deviation computed on raw data, the last two row standard deviation computed on cyclical data. In general, model statistics are constructed by simulating the full model using one every 1,000 draws of MCMC chain after discarding the first 300,000. Overall, the model does a good job in replicating standard deviations. The only exception is the volatility of hours worked, where the model over-estimates the standard deviation both at cyclical and non cyclical frequencies. Figure 2 plots the autocorrelations function for the observables. The model is able to replicate the ACF for raw data with the only exception being $\tau_k$. Moreover, the ACF of the model cyclical component seizes pretty well the autocorrelation function of the filtered data, see Figure 3. The only exception is debt-GDP ratio, whose cyclical persistence is not captured by the model. Overall, the model has a good fit and provides an adequate framework for policy analysis.

### 6 Tax Policies and Stabilization

The question addressed in this paper is whether US tax policies helped to reduce fluctuations. As mentioned, there are two channels through which the fiscal authority can adjust tax rates: there is an endogenous or 'systematic' channel, by which the
policymakers automatically respond to cyclical fluctuation, and I identified it with the
two coefficients in the fiscal policy rule $\varphi_y$, $\psi_y$. The second channel is through the
orthogonal part of the fiscal policies rule. The estimated tax responses are

$$\tilde{\tau}_w^t = 0.39\tilde{\tau}_w^{t-1} + 0.47y_t + 0.86\tilde{y}_t + \epsilon_t^w$$

$$\tilde{\tau}_k^t = 0.38\tilde{\tau}_k^{t-1} + 0.54y_t + 0.24\tilde{y}_t + \epsilon_t^k$$

The task here is to see whether they are important in influencing the amplitude of
fluctuations both at cyclical and non cyclical frequencies, and if shutting them would
affect the volatility of the macro variables. So, the question is: what happens to the
standard deviations of GDP, consumption, investment and hours worked if I set these
coefficients to zero, i.e. $\varphi_y = \psi_y = 0$. Unfortunately, standard counterfactuals can not
be performed. With the mean, the median or the mode of the posterior distributions
of the parameters and setting $\varphi_y = \psi_y = 0$, the model can not be solved. Thus, data
can not be simulated or theoretical moments can not be computed. To overcome this
problem, I experiment two roads: (a) compute the standard deviations for $\varphi_y \rightarrow 0$ and
$\psi_y \rightarrow 0$, (b) perform counterfactuals a l`a Canova and Gambetti (2009), where structural
parameters are re-estimated with the constraints. Figure 4 plots the changes in
standard deviations of GDP, consumption, investment and hours worked if we move
Figure 3: ACF for cyclical variables in the data and in the model (solid line mean estimates, dashed line 90% probability sets).

away from the median estimates of the FP parameters. With respect to the vertical red line, towards left I reduce the fiscal policy parameters, viceversa toward right (notice that the scales of the left and the right part are different). Each point in the blue line represent the standard deviation associated to a policy change. The standard deviation is an average over 100 simulated data sets. For all the policy changes, I use the same set of draws from the white noise innovations so that the differences among policies do not arise from different stochastic realizations.

Clearly, the negative slopes indicates that the more we approach zero the bigger the standard deviations become. From the graph, it stands out that GDP is reacting less relative to the other variable to change in policy. The remaining variables display a significant increase in standard deviation; the percentage increase is 26% for consumption, 93% for hours worked, 82% for investment. Similar results apply at cyclical frequencies (see Figure 5) even thought the results are less significant. The increase in standard deviation is 2% for GDP, 3% for consumption, 8% for hours worked, 27% for investment. However, these are standard deviation computed for positive values of $\varphi_y$ and $\psi_y$, that are relatively far from zero; precisely, we can not compute standard deviations for values of fiscal policy parameters smaller than 0.17. Overall, we can be relatively confident that the relation between volatility of macro variables and values of $\varphi_y$ and $\psi_y$ is negative.
Figure 4: Blue line standard deviation of raw variables with a fiscal policy change. The intersection between the red and the blue line shows the standard deviation of the model using the median estimates. Moving toward left we reduce the values of $\varphi_y$ and $\psi_y$, vice versa toward right. Notice that right and left scale are different.

To explore to what extent the absence of countercyclical fiscal policy is important to smooth fluctuations we need to re-estimate the model. I try to answer to four questions: (1) does the volatility increase if $\varphi_y = \psi_y = 0$ ? (2) with a balanced budget assumption as in Jones (2002) is it still true that the volatility increase if $\varphi_y = \psi_y = 0$ ? (3) Which of the two tax instrument, labor or capital tax, is more important to reduce fluctuations ? (4) Do fiscal policy shocks matter ? Table 3 collects the answers to the previous questions. The first half of the table presents the standard deviations computed at business cycles frequencies, and the second half reports the standard deviations computed on raw data. The first row reports the data standard deviations, the second row (fm) the standard deviation of the full model, and the third row (r1) the standard deviations with $\varphi_y = \psi_y = 0$. The fourth row (r2) report the standard deviation assuming a Balanced Budget assumption. To replicate a balanced budget assumption I assume that tax besides reacting to GDP is very sensitive to debt fluctuations. Setting the coefficients of debt-GDP ratio, $\varphi_y$ and $\psi_y$, to 5, we are imposing a strong reaction of the tax to debt variations. This mimics a Balance Budget assumption where debt does not fluctuate along the cycles and the government budget constraint is adjusted by tax variations. Figure 11 plot the implied path of debt where the FP parameters are set to large values. Clearly, the implied path of debt-GDP is much less
Figure 5: Blue line standard deviation of cyclical variables with a fiscal policy change. The intersection between the red an the blue line shows the standard deviation of the model using the median estimates. Moving toward left we reduce the values of $\varphi_y$ and $\psi_y$, vice versa toward right. Notice that right and left scale are different.

Contrasting (fm) and (r1), we can notice that the standard deviations of GDP, consumption and investment increase considerably if omit the cyclical reaction of the tax. This is somehow expectable given the previous analysis. Hour worked is quite insensitive to changes in policies. Indeed, along all the specifications hours worked standard deviation ranges between 1.01 to 1.11, so it is unlikely that fiscal policy affects the volatility of hours worked. It is interesting to disentangle the impact of the two taxes on the volatility of GDP, consumption and investment. Confronting (r4) and (r5), it seems that the labor tax cyclical reaction influence volatilities more than the capital tax. Concerning the exogenous channel, tax shocks virtually have no impact on standard deviations. The latter finding is consistent with what is found in the narrative approach of Romer and Romer (2007), where they show that fiscal shocks explain 9% of GDP growth rate volatility. Finally, Balance Budget rule offsets completely the
Table 3: Standard deviations with different restrictions.

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>y</th>
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<tr>
<td>Cyclical Data</td>
<td>0.65</td>
<td>0.82</td>
<td>0.86</td>
<td>2.24</td>
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<tr>
<td>fm: Full model</td>
<td>0.69</td>
<td>0.73</td>
<td>1.11</td>
<td>2.48</td>
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<tr>
<td>r1: $\varphi_y = \psi_y = 0$</td>
<td>2.28</td>
<td>1.33</td>
<td>1.01</td>
<td>3.45</td>
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<tr>
<td>r2: $\varphi_{by} = \psi_{by} = 5$</td>
<td>0.77</td>
<td>0.73</td>
<td>1.14</td>
<td>2.65</td>
</tr>
<tr>
<td>r3: $\varphi_{by} = \psi_{by} = 5$ and $\varphi_y = \psi_y = 0$</td>
<td>0.83</td>
<td>0.74</td>
<td>1.13</td>
<td>3.02</td>
</tr>
<tr>
<td>r4: $\varphi_y = 0$</td>
<td>2.08</td>
<td>1.38</td>
<td>1.05</td>
<td>3.70</td>
</tr>
<tr>
<td>r5: $\psi_y = 0$</td>
<td>0.68</td>
<td>0.72</td>
<td>1.07</td>
<td>2.63</td>
</tr>
<tr>
<td>ns: $\sigma_w = \sigma_k = 0$</td>
<td>0.69</td>
<td>0.73</td>
<td>1.11</td>
<td>2.47</td>
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<tr>
<td>Raw Data</td>
<td>26.5</td>
<td>23.6</td>
<td>7.3</td>
<td>39.4</td>
</tr>
<tr>
<td>fm: Full model</td>
<td>25.4</td>
<td>19.2</td>
<td>18.2</td>
<td>44.1</td>
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<tr>
<td>r1: $\varphi_y = \psi_y = 0$</td>
<td>71.7</td>
<td>40.4</td>
<td>12.9</td>
<td>46.2</td>
</tr>
<tr>
<td>r2: $\varphi_{by} = \psi_{by} = 5$</td>
<td>30.4</td>
<td>25.3</td>
<td>19.5</td>
<td>51.5</td>
</tr>
<tr>
<td>r3: $\varphi_{by} = \psi_{by} = 5$ and $\varphi_y = \psi_y = 0$</td>
<td>32.5</td>
<td>16.0</td>
<td>55.6</td>
<td>75.3</td>
</tr>
<tr>
<td>r4: $\varphi_y = 0$</td>
<td>62.9</td>
<td>37.8</td>
<td>11.4</td>
<td>55.7</td>
</tr>
<tr>
<td>r5: $\psi_y = 0$</td>
<td>45.8</td>
<td>33.7</td>
<td>19.7</td>
<td>87.7</td>
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<tr>
<td>ns: $\sigma_w = \sigma_k = 0$</td>
<td>21.7</td>
<td>18.3</td>
<td>16.9</td>
<td>37.7</td>
</tr>
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stabilizing role of fiscal policy. In fact, contrasting (r2) and (r3) there are no clear differences in terms of standard deviations with or without countercyclical fiscal policy. So, by omitting debt we are mismeasuring the impact of tax policy on volatilities. The second part of the table shows the changes of standard deviations with raw data, and results are very similar.

Concluding, this analysis suggests that the countercyclical reaction of fiscal policy is important for smoothing fluctuations both at cyclical and non cyclical frequencies.

6.1 Welfare Cost

What are the welfare cost for the representative consumer associated to fiscal policy changes? Very small.

In this part of the paper I compute the welfare cost associated to fiscal policy changes. It is useful to introduce some notation. Let $A$ be the benchmark economy where tax policy reacts as in the full estimated model, and let $\{C^A_t, N^A_t\}_{t=0}^\infty$ be the relative optimal consumption-working plan. For a given path of structural shocks, I denote the lifetime utility, $V$, of $\{C^A_t, N^A_t\}_{t=0}^\infty$ as

$$V(\{C^A_t, N^A_t\}_{t=0}^\infty) = \sum_{t=0}^\infty \beta^t \left[ \frac{(C^A_t)^{1-\eta} - 1}{1 - \eta} - X_t N^A_t \right]$$
Let $B$ be the economy where fiscal policy parameters have changed, and let $\{C_t^B, N_t^B\}_{t=0}^\infty$ be the relative optimal consumption-working plan. The utilitarian welfare gain of policy change, $\omega$, is defined by

$$E_0V((1 + \omega)C^A_t, N^A_t)_{t=0}^\infty = E_0V(C_t^B, N_t^B)_{t=0}^\infty$$

The premium, $\omega$, can be thought of as a percentage of lifetime consumption that agent in economy A is prepared to give up to get the policy change. As mentioned, with

![Figure 6: Percentage of lifetime consumption that agent in benchmark economy (A) is prepared to give up to be indifferent with the policy change (B).](image)

the mean or the median of the posterior distributions of the parameters and setting $\varphi_y = \psi_y = 0$, the model can not be solved. Thus, data can not be simulated and utilitarian welfare gain can not be computed for all combination of $\varphi_y$ and $\psi_y$. As before, what I can do is to approach zero as much as possible. Figure 6 reports the changes of the premium $\omega$ as $\varphi_y \rightarrow 0$ and $\psi_y \rightarrow 0$. The policy change towards a less countercyclical tax policy is clearly not preferred. The scale on the $y$ axis reveals that the change is small, and the representative consumer to be indifferent with the policy change needs to reduce his lifetime consumption by 0.08%. However, a more interesting welfare analysis of the fiscal policy would require to move to a heterogenous agents setup, where the cost of deeper fluctuations is perceived differently by household with distinct income distributions. I leave this interesting topic for future research.
7 Conclusions

In this paper, I study whether US tax policies affected the volatility of the macro variables both at cyclical and non cyclical frequencies. There are endogenous feedbacks between economic activity and tax policies; on the one hand, the latter directly affects household decisions influencing consumption and labor choices, and therefore economic activity. On the other hand, the fiscal authority sets the tax policy by responding to cyclical conditions of the economy. The task of this work was to estimate from the data the feedbacks between economic activity and tax policies, and in particular to test whether tax policies are useful in reducing economic volatility. To answer the question of interest, I chose to employ a General Equilibrium model that provides a theoretical framework to identify endogenous interactions. I found that tax policies helped to reduce economic volatility when the government has no balanced budget constraint. In particular, the automatic response to cyclical conditions has been very important in shaping macroeconomic stability; indeed, if we assume that the labor and capital taxes do not respond to GDP variations, the volatility of the main macro variables would increase. I also found that unexpected changes in the tax policy do not affect much the economic volatility.
References


A Steady State Analysis

I shall indicate the variable without time subscript as the variable at the steady state.

From the Euler equation, (6) and (7), we get that

$$1 = \beta R$$

$$R = (1 - \tau^k)\alpha \frac{Y}{K} + 1 - \delta$$

Therefore, $1/\beta = (1 - \tau^k)\alpha \frac{K}{Y}^{-1} + 1 - \delta$. From the production function, equation (1), we get

$$\frac{N}{Y} = \frac{K}{Y} \frac{a}{1 - \alpha}$$

Moreover, from the intertemporal optimality condition, equation (5), we get that

$$C^\eta = (1 - \tau^w)(1 - \alpha)\frac{Y}{N}$$

At the non stochastic steady state, the exogenous process are identical to 1, $X = G = V = A = 1$. Thus the law of motion for capital, equation (4), becomes

$$I = \delta K$$

and the feasibility constraint,

$$\frac{1}{Y} = 1 - \delta \frac{K}{Y} - C\frac{1}{Y}$$

which can be rewritten as

$$Y = \frac{1 + C}{1 - \delta \frac{K}{Y}}$$

Finally, we can obtain the debt-gdp ratio form the government budget constraint, equation (10),

$$\frac{1}{Y} + \frac{B}{Y}(R - 1) = (1 - \alpha)\tau^w + \alpha \tau^k$$

Therefore,

$$\frac{B}{Y} = \frac{(1 - \alpha)\tau^w + \alpha \tau^k - \frac{1}{Y}}{(1 - \tau^k)\alpha \frac{K}{Y}^{-1} - \delta}$$

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B Log-linearization of the equilibrium conditions.

In this part, I develop the log linearization of the equations that characterize the economy. Except for some cases (taxes and interest rate), I denote the log deviation of a variable $X_t$ from its steady state path, $X$ (without time subscript), with small letter, i.e.

$$x_t = \ln(X_t/X).$$

The production function is given by

$$Y_t = A_tK_t^{\alpha}N_t^{1-\alpha}$$

At the non stochastic steady state we have that the variables are constant and the shocks are zero; the log linear version is obtained by dividing the equation by its value at steady state and by taking logarithm; i.e.

$$y_t = a_t + \alpha k_{t-1} + (1 - \alpha) n_t$$

The log linearized version of equation (5) is as follow

$$\eta C_t = -(1 - \tau^w) \left( 1 - \frac{\tau^w}{\tau^w} \right) \eta C_t \eta + (1 - \tau^w) (1 - \alpha) \frac{Y}{X} y_t -$$

$$- (1 - \tau^w) (1 - \alpha) \frac{Y}{X} n_t - (1 - \tau^w) (1 - \alpha) \frac{Y}{X} \chi_t$$

where I denote with $\tilde{\tau}^w$ the log deviation of the tax on labor income from its steady state level, $\tau^w$, i.e. $\tilde{\tau}^w = \ln \frac{\tau^w}{\tau^w}$. The latter equation can be rewritten as

$$\eta C_t = -(1 - \tau^w) (1 - \alpha) \frac{Y}{X} \tilde{\tau}^w + (1 - \tau^w) (1 - \alpha) \frac{Y}{X} y_t -$$

$$- (1 - \tau^w) (1 - \alpha) \frac{Y}{X} n_t - (1 - \tau^w) (1 - \alpha) \frac{Y}{X} \chi_t$$

using (??), the expression simplifies to

$$\eta C_t = - \frac{1}{1 - \tau^w} \tilde{\tau}^w + y_t - n_t - \chi_t$$

where we used the fact that $\tilde{\tau}^w = \ln \frac{\tau^w}{\tau^w} \cong \frac{\tau^w - \tau^w}{\tau^w} = \tau^w$ for $j = w, k$. The log linearized version of the Euler equation can be derived by applying a first order approximation to equation (6) and (7); we get

$$0 = \beta E_t \left\{ \frac{C_t}{C_t} R v_t + \eta \frac{C_{t+1}^{-1}}{C_t} RC_c t - \eta \frac{C_t}{C_t+1} RC_c t + \frac{C_t}{C_t} R \tilde{\tau}^w \right\}$$

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where $\hat{r}_t = \ln \frac{R_t}{R}$. Using the fact that $1 = \beta R$ the previous equation can be simplified to

$$0 = E_t\{v_t + \eta(c_t - c_{t+1}) + \hat{r}_{t+1}\} \quad (21)$$

The interest rate equation, (7)

$$R\hat{r}_t = -\alpha \frac{Y}{K}(1 - \tau^k_t) \frac{\tau^k_t}{1 - \tau^k_t} \hat{\tau}_w^t + (1 - \tau^k_t)\alpha \frac{1}{K} Y y_t - (1 - \tau^k_t)\alpha \frac{Y}{K^2} Y k_{t-1} - (1 - \delta) v_{t+1}$$

$$R\hat{\gamma}_t = \alpha (1 - \tau^k_t) \frac{Y}{K}(y_t - k_{t-1} - \frac{\tau^k_t}{1 - \tau^k_t} \hat{\tau}_w^t) - (1 - \delta) v_t$$

Using the steady state equation for the interest rate, i.e. $\mu = \alpha (1 - \tau^k_t) \frac{Y}{K} = R - 1 + \delta$, we get

$$\hat{r}_t = \frac{\mu}{\mu + 1 - \delta} (y_t - k_{t-1} - \frac{\tau^k_t}{1 - \tau^k_t} \hat{\tau}_w^t) - \frac{1 - \delta}{\mu + 1 - \delta} v_t \quad (22)$$

and combining the two equations, i.e. (22) and (21), we get

$$0 = E_t\{v_t + \eta(c_t - c_{t+1}) + \frac{\mu}{\mu + 1 - \delta} (y_{t+1} - k_t - \frac{1}{1 - \tau^k_t} \hat{\tau}_w^t) - \frac{1 - \delta}{\mu + 1 - \delta} v_{t+1}\}$$

Recall the government budget constraint, i.e.

$$G_t + (1 + r^b_t) B_{t-1} = \tau^w_t w_t N_t + \tau^k_t r_t K_{t-1} + B_t$$

$$G_t + (1 + r^b_t) B_{t-1} = [(1 - \alpha) \tau^w_t + \alpha \tau^k_t] Y_t + B_t \quad (23)$$

where the latter equality is obtained by substituting the optimality condition of the firm. Rewriting it in terms of the debt-GDP ratio $B_t/Y_t$

$$\frac{G_t}{Y_t} + (1 + r^b_t) \frac{B_{t-1}}{Y_{t-1}} \frac{Y_{t-1}}{Y_t} = (1 - \alpha) \tau^w_t + \alpha \tau^k_t + \frac{B_t}{Y_t} \quad (24)$$

I shall indicate with $by_t$ the logarithm deviation of debt-GDP ratio from its steady state at time $t$, i.e.

$$by_t = \ln \frac{B_t}{Y_t} \quad (25)$$

The log linear version is derived as follows,

$$\frac{G}{Y} y_t - \frac{G}{Y} y_{t-1} - (1 + r^b) \frac{B}{Y} y_{t-1} - (1 + r^b) \frac{B}{Y} y_t + (1 + r^b) \frac{B}{Y} by_{t-1} + \frac{B}{Y} r^b by_t =$$

$$= (1 - \alpha) \tau^w_t \hat{\tau}_w^t + \alpha \tau^k_t \hat{\tau}_k^t + \frac{B}{Y} by_t$$

where $\hat{r}_t^b = \ln(r_t^b)$. Moreover, by the no arbitrage condition we know that

$$1 + r^b_t = V_{t-1} R_t$$
The log linearized version of the no-arbitrage condition gives the following
\[ r^b_t = v_{t-1} + R^b_t = v_{t-1} + \mu(y_t - k_{t-1} - \frac{\tau^k}{1 - \tau^k} \tilde{r}^k_t) - (1 - \delta)v_t \] (25)
where the last equality follows from equation (22). We can get rid of the bond interest rate and express the government budget constraint in terms of GDP, debt-GDP, taxes and capital. The budget constraint becomes
\[ \frac{G}{Y} y_t - \frac{G}{Y} y_t + (\mu + 1 - \delta) \frac{B}{Y} y_{t-1} - (\mu + 1 - \delta) \frac{B}{Y} b_{yt-1} + \frac{B}{Y} \mu(y_t - k_{t-1} - \frac{\tau^k}{1 - \tau^k} \tilde{r}^k_t) = (1 - \alpha) \tau^w w_t + \alpha \tau^{k \times k} w_t + \frac{B}{Y} b_{yt} + \frac{B}{Y} [(1 - \delta)v_t - v_{t-1}] \]
Simplifying and rearranging the terms we obtain
\[ \frac{G}{Y} y_t + \lambda_1 y_{t-1} + \lambda_2 \tilde{b}_{yt-1} = \tilde{b}_y_t + (1 - \alpha) \tau^w w_t + \lambda_1 \tilde{r}^k w_t + \lambda_2 y_{t-1} + \lambda_3 k_{t-1} + \frac{B}{Y} [(1 - \delta)v_t - v_{t-1}] \]
where
\[ \lambda_1 = \alpha + \frac{B}{Y} \mu \frac{1}{1 - \tau^k} \]
\[ \lambda_2 = \frac{G}{Y} + \frac{B}{Y} (1 - \delta) \]
\[ \lambda_3 = \frac{B}{Y} \mu \]
\[ \lambda_4 = \mu + 1 - \delta \]
where I am using the fact that \( b_{yt} = \ln \frac{B_t}{Y_t} \approx \frac{B_t / Y_t - B_t / Y}{B_t / Y} = \frac{\tilde{b}_{yt}}{B_t / Y} \). Finally, the log linear version of the feasibility constrain is
\[ Y y_t = I i_t + C c_t + G g_t \]
\[ y_t = \frac{I}{Y} i_t + \frac{C}{Y} c_t + \frac{G}{Y} g_t. \]
and the log linear version of the law of motion of capital is
\[ IV i_t + IV v_t = K k_t - (1 - \delta) K k_{t-1}. \] (26)
where \( V = 1 \) at the non stochastic steady state.
C Tax Series construction.

I use quarterly values for real series of gdp, consumption, hours worked, investment, government debt and government spending. Except taxes, all the times series are taken from the FRED database (Federal Reserve Bank of St. Louis http://research.stlouisfed.org/fred2/); average tax rate are constructed from the times series of the Bureau of Economic Analysis (www.bea.gov). The hours worked are constructed as follows: I took the average weekly hours (Average Weekly Hours of Production Worker) and I normalized to 1 unit measure. Then, I multiply the series for the level of employment (All Employees), and divide by the population (Total Population). The series of investment is the sum of Fixed Investment plus Durable Consumption. Government spending is the real Government Consumption series.

To calculate the average tax rates, I follow closely Mendoza et al. (1994). All these items are indexed by table and line number. I start with finding $\tau^p$, the average personal income tax rate:

$$
\tau^p = \frac{FIT + SIT}{W + PRI/2 + CI} \frac{PRI/2 + RI + CP + NI}{CI = PRI/2 + RI + CP + NI}
$$

where

$FIT =$ Federal Income taxes (3.2: line 3);

$SIT =$ State and local income taxes (3.3: line 3);

$W =$ Wages and salaries (1.14: line 5);

$CI =$ Capital income;

$PRI =$ Proprietor’s income (1.14: line 13);

$RI =$ Rental income (1.14: line 17);

$CP =$ Corporate profits (1.14: line 11);

$NI =$ Net interest (1.14: line 25).

As discussed by Joines (1981), the division of proprietor’s income into capital and income is somewhat arbitrary. Joines analyzes both cases, I follow Jones (2002), who takes ’a middle ground’ and splits proprietor’s income evenly between capital and labor income. The labor tax rate, $tax^w$, is then calculated as

$$
tax^w = \frac{\tau^p(W + PRI/2) + CSI}{PRI/2 + EC}
$$

where

$CSI =$ Total contributions to social insurance (3.1: line 7);
EC = Total employee compensation (1.14: line 4).

In addition to wages and salaries, employee compensation includes contribution to social insurance and untaxed benefits. Tax capital rate is calculated as

$$\text{tax}^k = \frac{\tau p CI + CT + PT}{PT + CI}$$

where

- $CT = \text{Corporate taxes (3.1: line 5)}$;
- $PT = \text{Property taxes (3.3: line 9)}$. 
<table>
<thead>
<tr>
<th>ν</th>
<th>Parameters Description</th>
<th>F</th>
<th>μ</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>capital share</td>
<td>B(18, 20)</td>
<td>0.5</td>
<td>0.1</td>
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<tr>
<td>η</td>
<td>inverse of intert elasticity</td>
<td>Γ(2, 1.25)</td>
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<td>1.8</td>
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<tr>
<td>K</td>
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<td>N(2.5, 0.1)</td>
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<td>0.1</td>
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<td>τw</td>
<td>steady state</td>
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<td>0.1</td>
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<tr>
<td>τk</td>
<td>steady state</td>
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<td>0.1</td>
</tr>
<tr>
<td>ϕw</td>
<td>labor tax autoreg</td>
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<td>0.5</td>
</tr>
<tr>
<td>ϕby</td>
<td>labor tax response to debt-GDP</td>
<td>N(0.2, 0.5)</td>
<td>0.2</td>
<td>0.5</td>
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<tr>
<td>ϕy</td>
<td>labor tax response to debt-GDP</td>
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<td>0.5</td>
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<tr>
<td>ψk</td>
<td>capital tax autoreg</td>
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<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>ψby</td>
<td>capital tax response to debt-GDP</td>
<td>N(0.2, 0.5)</td>
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</tr>
<tr>
<td>ρa</td>
<td>AR technology</td>
<td>B(18, 8)</td>
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<td>0.1</td>
</tr>
<tr>
<td>ρg</td>
<td>AR government</td>
<td>B(18, 8)</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>ρχ</td>
<td>AR preference</td>
<td>B(18, 8)</td>
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<td>0.1</td>
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<tr>
<td>ρv</td>
<td>AR investment</td>
<td>B(18, 8)</td>
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<td>0.1</td>
</tr>
<tr>
<td>σa</td>
<td>SD technology</td>
<td>U(0, 1)</td>
<td>0.5</td>
<td>0.3</td>
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<tr>
<td>σχ</td>
<td>SD preference</td>
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<td>SD investment</td>
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<td>0.3</td>
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<td>SD labor tax</td>
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<td>SD capital tax</td>
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<tr>
<td>σg</td>
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<td>SD meas τw</td>
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<td>0.3</td>
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<tr>
<td>σmk</td>
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<td>0.3</td>
</tr>
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<td>SD trend</td>
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<td>0.3</td>
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<tr>
<td>ϑfd</td>
<td>Drifts</td>
<td>N(0, 1)</td>
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<td>1.0</td>
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<tr>
<td>ση</td>
<td>SD trends</td>
<td>U(0, 1)</td>
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<td>0.3</td>
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<tr>
<td>Bj</td>
<td>Slopes</td>
<td>N(0, 1)</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>σj</td>
<td>SD trends</td>
<td>U(0, 1)</td>
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<td>0.3</td>
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Table 4: Prior Distribution for the parameter ν
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<tr>
<th>( \nu )</th>
<th>mean</th>
<th>median</th>
<th>sd</th>
<th>95%</th>
<th>5%</th>
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<tbody>
<tr>
<td>( \alpha )</td>
<td>0.461</td>
<td>0.458</td>
<td>0.010</td>
<td>0.479</td>
<td>0.449</td>
</tr>
<tr>
<td>( \eta )</td>
<td>1.266</td>
<td>1.272</td>
<td>0.037</td>
<td>1.313</td>
<td>1.192</td>
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<tr>
<td>( k/y )</td>
<td>3.368</td>
<td>3.400</td>
<td>0.087</td>
<td>3.477</td>
<td>3.244</td>
</tr>
<tr>
<td>( \tau_w )</td>
<td>0.458</td>
<td>0.460</td>
<td>0.028</td>
<td>0.497</td>
<td>0.414</td>
</tr>
<tr>
<td>( \tau_k )</td>
<td>0.273</td>
<td>0.273</td>
<td>0.023</td>
<td>0.310</td>
<td>0.236</td>
</tr>
<tr>
<td>( \phi_w )</td>
<td>0.395</td>
<td>0.377</td>
<td>0.067</td>
<td>0.520</td>
<td>0.302</td>
</tr>
<tr>
<td>( \phi_{by} )</td>
<td>0.866</td>
<td>0.865</td>
<td>0.038</td>
<td>0.926</td>
<td>0.803</td>
</tr>
<tr>
<td>( \phi_y )</td>
<td>0.470</td>
<td>0.467</td>
<td>0.037</td>
<td>0.526</td>
<td>0.410</td>
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<tr>
<td>( \psi_k )</td>
<td>0.381</td>
<td>0.383</td>
<td>0.029</td>
<td>0.426</td>
<td>0.333</td>
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<tr>
<td>( \psi_{by} )</td>
<td>0.243</td>
<td>0.261</td>
<td>0.059</td>
<td>0.306</td>
<td>0.118</td>
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<tr>
<td>( \psi_y )</td>
<td>0.545</td>
<td>0.554</td>
<td>0.049</td>
<td>0.605</td>
<td>0.440</td>
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<tr>
<td>( \rho_s )</td>
<td>0.719</td>
<td>0.724</td>
<td>0.069</td>
<td>0.805</td>
<td>0.609</td>
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<tr>
<td>( \rho_g )</td>
<td>0.586</td>
<td>0.597</td>
<td>0.048</td>
<td>0.648</td>
<td>0.478</td>
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<tr>
<td>( \rho_\chi )</td>
<td>0.708</td>
<td>0.715</td>
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</tr>
<tr>
<td>( \rho_v )</td>
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<td>0.571</td>
<td>0.067</td>
<td>0.674</td>
<td>0.452</td>
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<tr>
<td>( \sigma_\alpha )</td>
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<td>0.045</td>
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<td>( \sigma_\chi )</td>
<td>0.089</td>
<td>0.088</td>
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<td>( \sigma_\psi )</td>
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<td>0.489</td>
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<td>0.551</td>
<td>0.437</td>
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<td>( \sigma_\sigma )</td>
<td>0.783</td>
<td>0.780</td>
<td>0.055</td>
<td>0.883</td>
<td>0.691</td>
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<td>( \sigma_{\psi_w} )</td>
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<td>0.150</td>
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<td>( \sigma_{mw} )</td>
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<td>0.904</td>
<td>0.090</td>
<td>1.007</td>
<td>0.751</td>
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<tr>
<td>( \sigma_{mk} )</td>
<td>1.099</td>
<td>1.126</td>
<td>0.153</td>
<td>1.302</td>
<td>0.806</td>
</tr>
<tr>
<td>( \sigma_\sigma )</td>
<td>0.364</td>
<td>0.364</td>
<td>0.036</td>
<td>0.425</td>
<td>0.306</td>
</tr>
<tr>
<td>( \sigma_\sigma )</td>
<td>0.610</td>
<td>0.610</td>
<td>0.034</td>
<td>0.668</td>
<td>0.557</td>
</tr>
<tr>
<td>( \sigma_\sigma )</td>
<td>0.831</td>
<td>0.828</td>
<td>0.047</td>
<td>0.904</td>
<td>0.755</td>
</tr>
<tr>
<td>( \sigma_\sigma )</td>
<td>0.336</td>
<td>0.346</td>
<td>0.181</td>
<td>0.625</td>
<td>0.055</td>
</tr>
<tr>
<td>( \sigma_\sigma )</td>
<td>0.229</td>
<td>0.184</td>
<td>0.175</td>
<td>0.564</td>
<td>0.019</td>
</tr>
<tr>
<td>( \sigma_\sigma )</td>
<td>0.359</td>
<td>0.267</td>
<td>0.287</td>
<td>0.883</td>
<td>0.016</td>
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<tr>
<td>( \sigma_\sigma )</td>
<td>0.127</td>
<td>0.112</td>
<td>0.095</td>
<td>0.299</td>
<td>0.011</td>
</tr>
<tr>
<td>( \sigma_\sigma )</td>
<td>0.908</td>
<td>0.906</td>
<td>0.053</td>
<td>0.992</td>
<td>0.826</td>
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<tr>
<td>( \gamma_\sigma )</td>
<td>0.468</td>
<td>0.475</td>
<td>0.185</td>
<td>0.749</td>
<td>0.129</td>
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<tr>
<td>( \gamma_\sigma )</td>
<td>0.420</td>
<td>0.421</td>
<td>0.107</td>
<td>0.584</td>
<td>0.229</td>
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<tr>
<td>( \gamma_\sigma )</td>
<td>0.227</td>
<td>0.214</td>
<td>0.184</td>
<td>0.564</td>
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</tr>
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<td>( \gamma_\sigma )</td>
<td>0.597</td>
<td>0.604</td>
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<td>( \gamma_\sigma )</td>
<td>0.065</td>
<td>0.067</td>
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<td>( \gamma_\sigma )</td>
<td>-0.001</td>
<td>0.011</td>
<td>0.133</td>
<td>0.189</td>
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<tr>
<td>( \gamma_\sigma )</td>
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<td>0.091</td>
<td>0.322</td>
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<tr>
<td>( \gamma_\sigma )</td>
<td>0.173</td>
<td>0.169</td>
<td>0.076</td>
<td>0.307</td>
<td>0.048</td>
</tr>
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</table>

Table 5: Parameters Estimates with Specification \( S_1 \) with filter \( fd \). Standard deviations are expressed in percentage terms.
Table 6: Data standard deviation and Model standard deviations. Standard deviations are in % terms in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>$\tau_k$</th>
<th>$\tau_k$</th>
<th>$by$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw data</td>
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<td>2.81</td>
<td>13.11</td>
<td>10.93</td>
</tr>
<tr>
<td>Full model raw data</td>
<td>7.82(3.66)</td>
<td>9.81(5.34)</td>
<td>10.67(5.37)</td>
<td>8.97(4.66)</td>
</tr>
<tr>
<td>Cyclical Data</td>
<td>0.78</td>
<td>1.18</td>
<td>0.69</td>
<td>0.94</td>
</tr>
<tr>
<td>Full model cyclical data</td>
<td>1.08(0.08)</td>
<td>1.25(0.10)</td>
<td>0.76(0.06)</td>
<td>0.91(0.07)</td>
</tr>
</tbody>
</table>
Figure 7: Convergence Statistics for structural parameters: cumulative sum of draws.

Figure 8: Convergence Statistics for non structural parameters: cumulative sum of draws.
Figure 9: Prior and Posterior distributions for structural parameters. The solid lines represent the posterior and the dotted lines the prior.

Figure 10: Prior and Posterior distributions for non structural parameters. The solid lines represent the posterior and the dotted lines the prior.
Figure 11: Blue line represent the debt-DGP ratio implied by the full model (fm), the red line the debt-DGP ratio implied by a model with $\varphi_{by} = \psi_{by} = 5$ ($r2$). The debt-GDP ratio path is the average among 100 simulations using the median values of posterior estimates.