

Simple Monetary and Fiscal Rules in a non Ricardian Economy with Capital: Leeper revisited

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Abstract

In this paper, we study the interaction between monetary and fiscal policies within the framework of a model with non-Ricardian consumer and capital accumulation. Taking into account the presence of wealth effects and the respect of a lower bound on the nominal interest rate makes the global dynamics of the economy very complex, despite the simplicity of the monetary and fiscal policies chosen. We can notably observe the coexistence of the four types of equilibria described by Leeper (1991) in one of the founder articles of the "Fiscal Theory of the Price Level", but for a unique set of the economic policy parameters. We show in particular that a liquidity trap equilibrium, which is also characterized by a high real interest rate and a high public debt-to-GDP ratio, owns the usually required properties for determination, as well as the more traditional equilibrium targeted by the monetary and fiscal authorities. The model is calibrated based on annual data and assesses the implications of a fiscal shock according to the long-term equilibrium reached.

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1 Introduction

The study of the interaction between monetary and fiscal policies has recently been the object of a renewed and vigorous interest.

The development of the stochastic general equilibrium models integrating simultaneously nominal (and real) rigidities and fiscal distortions allow to review, with new theoretical tools, an old and a major subject for macroeconomists. The contributions of Benigno and Woodford (2006), Schmitt-Grohé and Uribe (2006) and Leith and Wren-Lewis (2006) provide a good starting point for addressing this rich and complex literature.

Alongside this work, a current research continues to explore the same question within the framework of intertemporal monetary models with flexible prices, in the line of the works of Sargent and Wallace (1981), Ayagari and Gertler (1985) and, recently, Leeper (1991), Sims (1994) and Woodford (1994) around the particularly provocative subject of the "Fiscal Theory of the Price Level" (FTPL).

According to this theory, to the traditional framework allocating to a monetary authority the care of stabilizing the price level and to a fiscal authority the care of guaranteeing the balanced government budget, can be substituted a very different vision of the respective roles of these two institutions and of the economy equilibrium which results from it. There is in particular an equilibrium situation, meeting in all respects to the requirements of a rational expectations equilibrium, characterized by a fiscal authority "dominant" within the meaning of Sargent and Wallace (1981), and "passive" monetary authority, allowing the stabilization of the public debt.

In the simplest form of the thesis, which is presented in the pioneer article of Leeper (1991), the monetary authority conserve the control of long run inflation rate but loose the control of the short run price level. As it has been shown by Sims (1994) and Woodford (1994), the price level established is then the only one that verifies the intertemporal government budget constraint which becomes an equilibrium condition rather than a traditional constraint.

The thesis is provocative and has received a mixed response from the profession. Buiter (1999), Cushing (1999) and McCallum (2001) believe (perhaps rightly) that it is risky to suggest that the fiscal authorities do not have to worry about their solvency. But these criticisms are essentially external and do not affect the convictions of the supporters of the theory, which are based on the logical analysis of rigorous and orthodox models.

It is also possible to understand the Fiscal Theory of the Price Level by reviewing the validity conditions of the Ricardian equivalence setting highlighted by Barro (1994). The latter only applies when the fiscal authority ensures the government solvency by respecting its intertemporal budget for

any sequence of the price level. In the terms of Woodford (1995), the fiscal authority adopts then a Ricardian policy; in the terms of Leeper (1991), the fiscal authority is "passive" and leaves the monetary authority pursues an "active" or "dominant" policy allowing to control inflation.

If, on the contrary, the fiscal authority adopts a non Ricardian policy, the intertemporal government budget constraint holds only in equilibrium. The present and future taxes (and seigniorage) have then a direct effect on the price level and the latter is determined, even though monetary authority is content to set the nominal interest rate at a constant level, contradicting thus the results of Sargent and Wallace (1975). In this case, the Ricardian equivalence¹ does not hold because notably of the effect of the price changes on the amount of the inflationary tax borne by private agents.

In two independent contributions, Cushing (1999) and Bénassy (2000) study the consequences of pegging the nominal interest rate in the presence of another source of failure of the Ricardian equivalence linked to a particular form of heterogeneity among households. The heterogeneity comes from the regular arrival of new families (or "Dynasties") over time. As Weil (1987, 1991) has demonstrated, the arrival of these new agents, gives to the studied model some of the characteristics of an overlapping generations model and explains notably the existence of wealth effects, which are absent in the model with a representative agent.

But, while Cushing (1999) tries to show that the price level is always indeterminate in the presence of these effects, even locally, Bénassy (2000) shows, more clearly according to us, that the "nominal" indeterminacy described by Sargent and Wallace (1975) disappears around the steady state which is locally determined. There is nevertheless, as a general rule, another stationary equilibrium locally indeterminate towards which converge the multiple trajectories emphasized by Cushing (1999).

The link between these results and those of the "fiscal theory of the price level" is not immediate. The presence of wealth effects does not any more allow to consider a simple fiscal rule as satisfying or not satisfying the criteria of a Ricardian policy. The difference between the conclusions of Cushing (1999) and Bénassy (2000) actually results from the little operational character of this concept in a non Ricardian world.

The model developed in this paper proposes a generalization of Cushing (1999) and Bénassy (2000) in the case of more complex monetary policies evaluating a nominal interest rule more or less active, but always respecting a zero lower bound. We study more exactly the interactions between monetary and fiscal policies within the framework of a non Ricardian model with

¹Or, more precisely, his extended version to the presence of money.

capital accumulation. Actually, we pursue the works of Annicchiarico (2007), Annicchiarico, Giammarioli and Piergallini (2006) and Leith and von Thadden (2007) who share this concerns but do not take into account the zero lower bound on the nominal interest rate. Taking into account the wealth effects and the respect of the zero lower bound on the nominal interest rate make the global dynamics of this economy strongly non linear and complex and this, in spite of the simplicity of the adopted monetary and fiscal policy rules. We can notably see coexisting four types of equilibria described by Leeper (1991) in the founder article of the "Fiscal Theory of the Price Level", but for one set of economic parameters. We notably show that a liquidity trap, also characterized by a higher real interest rate and a higher level of real debt, possesses the usually required properties of determinacy, like the more traditional equilibrium targeted by the monetary and fiscal authorities. The model is calibrated on annual data and allows to estimate the implications of a fiscal shock according to the reached long run equilibrium.

The article is organized in the following way. In section 2, we build the model of a non Ricardian economy with money and capital. The section 3 studies the steady state equilibria. The section 4 is dedicated to the study of the local properties of the equilibria and proposes a discussion about the global dynamics. We make, in section 5, some stochastic simulations of fiscal shocks according to the reached equilibrium but also to the exactly adopted monetary policy.

2 The model

We use an expanded version of Weil's (1987, 1991) overlapping-generations structure. The economy consists of many infinitely-lived families of agents. Each period new and identical infinitely-lived families appear in the economy without initial wealth. The economy also consists of identical infinitely-lived firms using capital and labor to produce a unique good, of the fiscal authority (the Government) and of the monetary authority (the Central Bank). We use a stochastic framework and we assume that markets are complete.

2.1 Households

In period t , the economy is populated by a large number N_t of agents. Each period a new dynasty appears consisting of $(N_t - N_{t-1}) = nN_{t-1}$ agents where $n \geq 0$ represents at the same time the population growth rate and the birth rate.

Each household belonging to the dynasty $j \leq t$ has preferences defined over consumption and real money balances described by the utility function:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U \left(c_{j,s}, \frac{M_{j,s}}{P_s} \right) \quad (1)$$

where E_t denotes the mathematical expectations operator conditional on information available at time t , $\beta \in [0, 1]$ represents a subjective discount factor, and $U(\cdot, \cdot)$ is a period utility index assumed to be strictly increasing in its two arguments and strictly concave. The variables, $c_{j,t}$, P_t and $M_{j,t}$, represent respectively, the consumption of the household j in period $t \geq j$, the price of consumption good, and the nominal money balances held by household j in period $t \geq j$.

At the beginning of the period t , the household $j < t$ holds the initial nominal wealth, $V_{j,t}$, defined by:

$$V_{j,t} = M_{j,t-1} + (1 - \delta + \kappa_t) P_t k_{j,t} + D_{j,t} \quad (2)$$

where $(1 - \delta + \kappa_t) P_t k_{j,t}$ is the nominal value of the capital stock, including the capital incomes net of depreciation, and $D_{j,t}$ is the beginning-of-period state-contingent value of all other financial assets, whether privately issued or claims on the Government.

In each period, agents supply an inelastic and constant amount of labour and receive a real wage, $w_{j,t}$. Each agent uses his total financial wealth augmented by the wage incomes net of taxes, $P_t \tau_{j,t}$, to consume and to reconstitute his financial holdings. We can write the household's flow budget constraint as follows:

$$P_t c_{j,t} + M_{j,t} + E_t Q_{t,t+1} D_{j,t+1} + P_t k_{j,t+1} \leq V_{j,t} + P_t (w_{j,t} - \tau_{j,t}) \quad (3)$$

where $Q_{t,t+1}$, is the stochastic discount factor².

Markets are supposed to be complete. This assumption implies the existence of the risk-free one-period nominal interest rate defined by:

$$1 + i_t = [E_t Q_{t,t+1}]^{-1} \quad (4)$$

Finally, the household is subject to an appropriate set of borrowing limits which prevents "Ponzi Games". In the absence of financial market frictions,

²To be more precise, $Q_{t,t+1}$ is the asset price in period t , that gives one unit of money in a given state of the world in period $t + 1$, weighted by the probability (or density function) of such state. $E_t Q_{t,t+1} D_{j,t+1}$ can be rewritten as $\sum q_{t,t+1} D_{j,t+1}$ (where $q_{t,t+1}$ is an asset price) and represents the state-contingent assets portfolio. We have more generally: $Q_{t,T} = Q_{t,t+1} \times Q_{t+1,t+2} \times \dots \times Q_{T-1,T}$ and $Q_{t,t} = 1$.

the borrowing constraint takes the form:

$$V_{j,t+1} \geq - \sum_{k=1}^{\infty} E_{t+1} [Q_{t+1,k} P_k (w_{j,k} - \tau_{j,k})] \quad \forall j, \forall t. \quad (5)$$

This constraint implies that the household cannot borrow in period t an amount greater than the present discounted value of future labor incomes net of taxes in each state of the world that may be realized at date $t + 1$.

The representative household of generation j maximizes his intertemporal utility (1) subject to the budget constraint (3) and the borrowing constraint (5), where $V_{j,t}$ is defined by equation (2).

Denoting $U_{x_i}(t) = \partial U(x_{1,t}, x_{2,t}) / \partial x_{i,t}$, $i = 1, 2$, the first-order conditions for this maximizing problem can be written as follows:

$$\beta \frac{U_{c_j}(t+1)}{U_{c_j}(t)} = Q_{t,t+1} \frac{P_{t+1}}{P_t} \quad (6)$$

$$\frac{U_{m_j}(t)}{U_{c_j}(t)} = \frac{i_t}{1 + i_t} \quad (7)$$

$$\left(E_t Q_{t,t+1} \frac{P_{t+1}}{P_t} \right)^{-1} = 1 + \kappa_{t+1} - \delta \equiv R_t \quad (8)$$

$$E_t Q_{t,t+1} V_{j,t+1} + P_t c_{j,t} + \frac{i_t}{1 + i_t} M_{j,t} = V_{j,t} + P_t (w_{j,t} - \tau_{j,t}) \quad (9)$$

$$\lim_{T \rightarrow +\infty} E_t Q_{t,T} V_{j,T} = 0 \quad (10)$$

Equation (6) is a stochastic Euler equation summarizing the intertemporal arbitrage between present and future consumptions in each possible state of the world. Equation (7) represents an arbitrage condition between real money balances and present consumption. Equation (8) is a no-arbitrage condition relative to the saving choice in terms of capital accumulation or in terms of nominal state-contingent assets. Note that the net return on capital, $\kappa_{t+1} - \delta$, is not associated to an expectation operator because we assume a risk-free production. Thus, R_t represents the real risk-free gross interest rate, and it is known in period t . Equation (9) is the household j 's balanced budget constraint obtained by combining equations (3), (2) and (8). Finally, Equation (10) corresponds to the transversality condition and states that the discounted value of the financial wealth (or debt) tends to zero when time goes to infinity.

Iterating Equation (3) forward, with the use of (10), leads to the following household j 's intertemporal budget constraint:

$$V_{j,t} = E_t \sum_{s=t}^{+\infty} Q_{t,s} \left[P_s c_{j,s} + \frac{i_s}{1+i_s} M_{j,s} - P_s (w_{j,s} - \tau_{j,s}) \right] \quad (11)$$

In order to obtain an explicit outcome for individual consumption, one specifies the utility function as follows:

$$U \left(c_{j,t}, \frac{M_{j,t}}{P_t} \right) = \xi \ln c_{j,t} + (1 - \xi) \ln \frac{M_{j,t}}{P_t}$$

Equations (6) and (7) can be rewritten as:

$$P_t c_{j,t} = \beta^{-1} Q_{t,t+1} P_{t+1} c_{j,t+1} \quad (12)$$

and

$$P_t c_{j,t} = \xi \left[P_t c_{j,t} + \frac{i_t}{1+i_t} M_{j,t} \right]$$

Introducing these results into equation (11), one can easily show that the optimal consumption of agent j is a constant fraction of his consolidated wealth (financial wealth + human wealth).

$$P_t c_{j,t} = \xi (1 - \beta) (V_{j,t} + P_t h_{j,t}) \quad (13)$$

where

$$h_{j,t} = \frac{1}{P_t} E_t \sum_{s=t}^{\infty} Q_{t,s} [P_s (w_{j,s} - \tau_{j,s})]$$

is the household j 's human capital and corresponds to the discounted value of future labor incomes net of taxes.

2.2 Aggregation

Noting that the generation j is composed of $N_j - N_{j-1}$ agents, the following aggregation rule is applied to get *per capita* aggregate variables:

$$x_t = \sum_{j \leq t} \frac{(N_j - N_{j-1})}{N_t} x_{j,t} \quad (14)$$

for $x_{j,t} = c_{j,t}$, $V_{j,t}$ as well as $M_{j,t}$, $D_{j,t}$ and $k_{j,t}$.

We assume that the agent's inelastic supply of labor corresponds to one unit of labor, whatever the age of the agent, and we assume that taxes are independent of the age. Therefore, $w_{j,t} = w_t$ and $\tau_{j,t} = \tau_t$, $\forall j$ and so that $h_{j,t} = h_t \quad \forall j$.

Finally, notice that applying the aggregate rule (14) in period t to the variable $V_{j,t+1}$, we get:

$$\begin{aligned} \sum_{j \leq t} \frac{(N_j - N_{j-1})}{N_t} V_{j,t+1} &= \frac{N_{t+1}}{N_t} \sum_{j \leq t} \frac{(N_j - N_{j-1})}{N_{t+1}} V_{j,t+1} \\ &= (1+n) \left[\sum_{j \leq t+1} \frac{(N_j - N_{j-1})}{N_{t+1}} V_{j,t+1} - \frac{n}{1+n} V_{t+1,t+1} \right] \\ &= (1+n) V_{t+1} \end{aligned}$$

since $V_{t+1,t+1} = 0$, the dynasty $j = t + 1$ having no financial wealth in period $t + 1$.

Using this result and applying the aggregate rule (14) to equation (12) where we replace $P_{t+1}c_{j,t+1}$ by its expression given by equation (13) expressed in $t + 1$, we obtain:

$$P_t c_t = \xi (1 - \beta) \beta^{-1} Q_{t,t+1} [(1+n) V_{t+1} + P_{t+1} h_{t+1}]$$

Finally, by incorporating (13) expressed in $t + 1$ in the previous equation, it can be rewritten:

$$P_t c_t = \beta^{-1} Q_{t,t+1} P_{t+1} c_{t+1} + n \xi (\beta^{-1} - 1) Q_{t,t+1} V_{t+1} \quad (15)$$

This equation is the aggregate stochastic Euler equation which differs from the individual Euler condition (12) as long as the population growth rate is different from zero³.

Defining

$$R_{t,t+1} = \left(Q_{t,t+1} \frac{P_{t+1}}{P_t} \right)^{-1} \quad (16)$$

as the stochastic gross real interest rate corresponding to real return of the state-contingent nominal asset⁴, we rewrite equation (15) as:

$$c_t = \beta^{-1} \frac{c_{t+1}}{R_{t,t+1}} + \Psi \frac{V_{t+1}/P_{t+1}}{R_{t,t+1}} \quad (17)$$

where $\Psi = n \xi (\beta^{-1} - 1) \geq 0$ if $n \geq 0$.

The aggregate Euler equation includes a real wealth effect which is characteristic of a non Ricardian economy. The growth rate of individual consumption is greater than the aggregate growth rate, reflecting the heterogeneity of

³Recall that in Weil's model the population growth rate couldn't be negative since the absence of death.

⁴Note that according to (8), we have: $R_t = \left[E_t \left(1/\tilde{R}_{t,t+1} \right) \right]^{-1}$.

individual wealth. An increase in the expected beginning-of-period financial wealth in $t + 1$ benefits only to currently alive consumers in period t and thus it can't be proportionally distributed amongst present and future aggregate consumptions.

2.3 Firms

It is assumed that there exists a larger number of competitive firms with access to a standard neoclassical technology:

$$Y_t = F(K_t, L_t) \tag{18}$$

where Y_t , K_t and L_t denote the aggregate levels of production, physical capital and labour demand, respectively. The production function is homogeneous of degree one, concave, twice continuously differentiable and satisfies the Inada conditions⁵.

Firms are price takers in input and output markets. In a competitive equilibrium, labour market clearing requires $L_t = N_t$. Let $k_t = K_t/N_t$ and $y_t = Y_t/N_t$ denote the *per capita* capital stock and the *per capita* output, we have:

$$y_t = F(k_t, 1) \equiv f(k_t) \tag{19}$$

Competitive profit-maximizing firms leads to the standard conditions that factor prices equal their respective marginal products:

$$\kappa_t = f_k(k_t) \tag{20}$$

$$w_t = f(k_t) - f_k(k_t) k_t \tag{21}$$

Given the constant return to scale, factor payments exhaust firm revenues.

2.4 Monetary and Fiscal Authorities

The Government collects lump-sum taxes in the amount of $P_t T_t$, spends $P_t G_t$, prints money M_t and issues one-period nominally risk-free bonds B_t at the nominal price of $(1 + i_t)^{-1}$.

Denoting:

$$\Omega_t = M_{t-1} + B_{t-1} \tag{22}$$

the total beginning-of-period t government debt, including money balances,

⁵Production function satisfies: $F(0, l) = 0$, $F_k(k, l) \geq 0$, $F_{kk}(k, l) \leq 0$, $\lim_{k \rightarrow 0} F_k(k, 1) = \infty$, and $\lim_{k \rightarrow 0} F_k(k, 1) = 0$.

the Government flow budget constraint can be written:

$$\frac{\Omega_{t+1}}{(1+i_t)} + \frac{i_t}{1+i_t}M_t + P_tT_t = \Omega_t + P_tG_t \quad (23)$$

2.4.1 Fiscal Rule

We assume that in order to determine the amount of the lump-sum taxes, the fiscal authority applies the following simple rule:

$$T_t = z_tY_t + \theta\frac{\Omega_t}{P_t} - \frac{i_t}{1+i_t}\frac{M_t}{P_t} \quad (24)$$

The first term on the right-hand side of equation (24), z_tY_t , represents the part of taxes proportional to the output. z_t is a choice variable of fiscal authority, but it's perceived by agents as stochastic. The second component reflects the fact that the government debt is partially backed by direct taxes. It generalizes the rule proposed by Leeper (1991) to the total government debt, Ω_t , instead of B_{t-1} . The parameter θ verifies : $0 \leq \theta \leq 1$. Finally, the Government transfers all its seigniorage revenues, $\frac{i_t}{1+i_t}M_t/P_t$, to agents. The last two assumptions will considerably simplify the model by neutralizing the effects of seigniorage on the total government debt dynamic.

The government expenditures are assumed to be proportional to the output:

$$G_t = g_tY_t \quad (25)$$

where g_t is determined by fiscal authority but it's also perceived as stochastic by the agents.

Inserting (24) and (25) into the budget constraint (23) and using the definition of the nominal gross interest rate (4) and the definition of the stochastic real gross interest rate (16), we obtain the equation:

$$E_t \left(\frac{\omega_{t+1}}{R_{t,t+1}} \right) = \frac{1}{1+n} [(1-\theta)\omega_t + (g_t - z_t)y_t] \quad (26)$$

which describes the dynamic of the total *per capita* government debt in real terms: $\omega_t = \Omega_t/P_tN_t$.

To simplify the analysis, we will assume that in long run the fiscal authority imposes the condition: $g = z$, in order to guaranty that the primary deficit can equal zero when the debt is entirely paid back.

2.4.2 Monetary Rule

Taking up the assumption introduced by Leeper (1991) and then generalized and popularized by Taylor (1993, 1999) we assume that monetary authority has, in the short-run, leverage over the nominal interest rate that responds to the deviation (or the ratio) of inflation from its long-run target, $\bar{\Pi}$.

In order to take into account a lower bound constraint on the nominal interest rate⁶, we specify the following class of non linear monetary rules:

$$1 + i_t = \Phi(\bar{R}_t, \Pi_t; \bar{\Pi}) \quad (27)$$

where \bar{R}_t is a gross real interest rate target and the function $\Phi(\cdot)$ is assumed to have the following properties:

$$\begin{aligned} \Phi(R, \bar{\Pi}; \bar{\Pi}) &= R\bar{\Pi} \quad \forall \bar{\Pi} \text{ such as } R\bar{\Pi} > 1, \\ \Phi(\cdot) &> 1 \quad \forall \bar{R}_t, \forall \Pi_t, \\ \Phi_{\Pi}(\cdot) &\geq 0, \quad \Phi_R(\cdot) \geq 0. \end{aligned} \quad (\text{H1})$$

The first condition helps to guaranty that the inflation target $\bar{\Pi}$ can be reached at stationary state when the real interest rate target, \bar{R} , equal the long run value of the real interest rate, R , as long as the resulting value of the nominal interest rate is strictly positive⁷. The second condition generalizes the zero lower bound on the nominal interest rate constraint to all possible values of the gross real interest rate target and the gross inflation rate. The two last conditions help to preclude atypical rules. We can verify that the following rule respects all previous conditions:

$$\Phi(\bar{R}_t, \Pi_t; \bar{\Pi}) = \max \left\{ \bar{R}_t \bar{\Pi} \left[a \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi} + 1 - a \right]; 1 + \underline{i} \right\} \quad (28)$$

where $0 \leq a \leq 1$, $\underline{i} > 0$ and $\phi \geq 0$.

The case where \bar{R}_t represents a constant target, *i.e.* $\bar{R}_t = \bar{R} \forall t$, is often used in the literature, notably by Taylor (1993). Nevertheless we will analyze the case where \bar{R}_t is equal to the real gross interest rate, *i.e.* $\bar{R}_t = R_t$, that could be wise to stabilize inflation around its target when the stationary level of capital is not yet reached.

⁶This point was analysed particularly by Benhabib, Schmitt-Grohé and Uribe (2001).

⁷Taking into account the logarithmic form of the utility function, the zero bound on nominal interest rate can never be reached. A positive lower bound, $\underline{i} > 0$, has to be defined (see Alstadheim and Henderson [2006]). The limit case $\underline{i} = 0$ can be considered in a cashless economy, when $\xi = 1$.

2.5 Market Clearing

In equilibrium, the surplus of state-contingent assets supplied by agents equals zero thus their financial holdings are composed of government bonds, money and capital:

$$\frac{V_{t+1}}{P_{t+1}} = \frac{M_t + B_t}{P_{t+1}} + R_t k_{t+1} = \omega_{t+1} + R_t k_{t+1}$$

It follows that the stochastic aggregate Euler equation (17) takes the form:

$$c_t = \beta^{-1} \frac{c_{t+1}}{R_{t,t+1}} + \Psi \frac{\omega_{t+1} + R_t k_{t+1}}{R_{t,t+1}}$$

Using (8) and (20), we define the function $\tilde{R}(k_{t+1})$ that determines the value of the gross real interest rate according to the capital accumulated in t :

$$R_t = 1 - \delta + f_k(k_{t+1}) \equiv \tilde{R}(k_{t+1}) \quad (29)$$

Equilibrium is then described by the following set of equations:

$$c_t = \beta^{-1} \frac{c_{t+1}}{R_{t,t+1}} + \Psi \left[\frac{\omega_{t+1}}{R_{t,t+1}} + \frac{\tilde{R}(k_{t+1})}{R_{t,t+1}} k_{t+1} \right] \quad (30)$$

$$k_{t+1} = \frac{1}{1+n} [(1-\delta)k_t + (1-g_t) \cdot f(k_t) - c_t] \quad (31)$$

$$E_t \left(\frac{\omega_{t+1}}{R_{t,t+1}} \right) = \frac{1}{1+n} [(1-\theta)\omega_t + (g_t - z_t) f(k_t)] \quad (32)$$

$$E_t \left(\frac{1}{R_{t,t+1}} \right) = \frac{1}{\tilde{R}(k_{t+1})} \quad (33)$$

$$E_t \left(\frac{1}{R_{t,t+1} \Pi_{t+1}} \right) = \frac{1}{1+i_t} \quad (34)$$

$$1 + i_t = \Phi(\bar{R}_t, \Pi_t; \bar{\Pi}) \quad (35)$$

If the period $t + 1$ is characterized by S_{t+1} possible states of the world then the later system of equations is composed by $5 + S_{t+1}$ equations allowing to find the values of c_t , k_{t+1} , ω_t , Π_t , i_t and the S_{t+1} values of $R_{t,t+1}$, subject to equilibrium existence and uniqueness. Notice that it is possible, in theory at least, to eliminate the variables, i_t and $R_{t,t+1}$, both non-predetermined and non-dynamic, in order to reduce the size of the system. So we can consider a representation⁸ composed of four dynamic equations where two

⁸Appendix 4 gives details of such a representation.

variables, c_t and Π_t , are non-predetermined and two variables, k_t and x_t , are predetermined, with $x_t = \Pi_t \omega_t = (M_{t-1} + B_{t-1}) / N_t P_{t-1}$. This choice would theoretically permit to solve the problem posed by the dynamic status of $\omega_t = (M_{t-1} + B_{t-1}) / N_t P_t$ and $\Pi_t = P_t / P_{t-1}$, whose values can jump but not independently of each other. More satisfactory from a conceptual point of view, such a representation is not sufficiently malleable on a technical level. We will use later in this paper sneaky ways to analyze the previous model.

3 Steady State

A deterministic steady state equilibrium is a vector (c, k, ω, Π) verifying a four equations system which is obtained by deleting the indications of time and uncertainty in equations (30) to (35), by using $g = z$, and by replacing i in (34) by its value given by (35). One obtains:

$$\left[\tilde{R}(k) - \beta^{-1} \right] c = \Psi \left[\tilde{R}(k) k + \omega \right] \quad (36)$$

$$c = (1 - g) f(k) - (n + \delta) k \quad (37)$$

$$\left[\frac{1 + n}{\tilde{R}(k)} - (1 - \theta) \right] \omega = 0 \quad (38)$$

and:

$$\begin{aligned} \tilde{R}(k) \Pi &= \Phi(\bar{R}, \Pi) \\ & (= 1 + i) \end{aligned} \quad (39)$$

The first three equations are independent of Π . The system is then dichotomous and allows to find (c, k, ω) independently of the monetary policy. For a given value of k , equation (39) allows to find the equilibrium value(s) of Π according to the target, \bar{R} which can (or cannot) be chosen to be equal to the actual steady state value of $\tilde{R}(k)$.

Notice that this long run dichotomy is not a fundamental characteristic of such a model. This it is one of the consequences of the assumptions that we had adopted about the use of a simple monetary and fiscal rules, on the one hand, and the use of the variable ω , the beginning-of-period real debt instead of $x = \Pi \omega$, the end-of-period debt, on the other hand.

3.1 Equilibrium Inflation

We begin this subsection by analyzing the equation (39). According to assumption (28), and when the real interest rate target coincide with the long

run real interest rate : $\bar{R} = \tilde{R}(k)$, (39) has at least one solution corresponding to the inflation target, $\bar{\Pi}$.

Benhabib, Schmitt-Grohé and Uribe (2001) show that the possibility of the existence of a second steady state equilibrium is one of the unexpected consequences of the zero lower bound on the nominal interest rate. It is notably the case when the rule is active ⁹ in the sense of Leeper (1991), that is when the elasticity of the function $\Phi(\cdot)$ with respect to Π_t is greater than 1, when $\Pi_t = \bar{\Pi}$. A second equilibrium appears, corresponding to a lower inflation rate, potentially negative and reminding the Keynesian liquidity trap. We illustrate this case by the following figures where we assume that $\bar{R} = \tilde{R}(k)$:

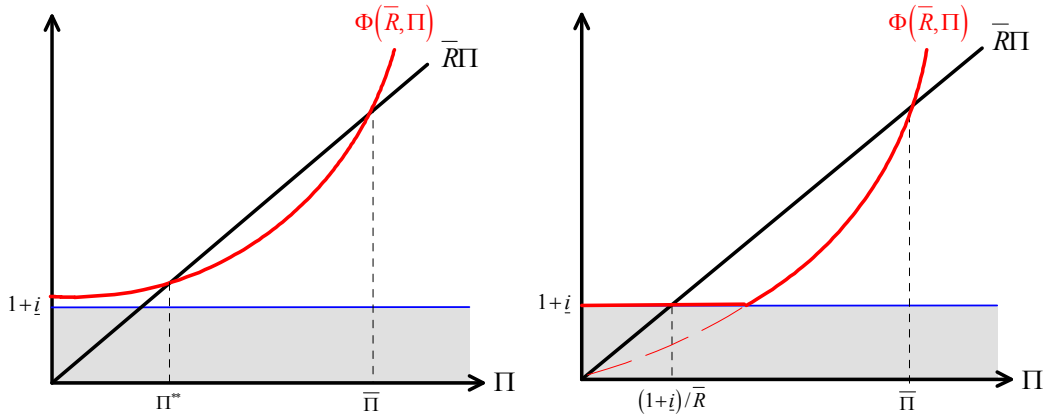


Figure 1

Figure 2

Figures 1 and 2 represent the function $\Phi(\cdot)$ given in the example (28), when $\phi > 1$. The figure 1 corresponds to the case: $(1 - a) \bar{R} \bar{\Pi} > 1 + \underline{i}$ and the figure 2 corresponds to the case $a = 1$. In the later case, the function $\Phi(\cdot)$ crosses the horizontal axis defined by $1 + \underline{i}$ for a value of Π greater than $(1 + \underline{i}) / \bar{R}$, which determines the lower equilibrium value in $\Pi^{**} = (1 + \underline{i}) / \bar{R}$. The associated nominal interest, i , is in its minimal value, \underline{i} , and then the liquidity trap is reached.

⁹One can also say that such a rule respects, locally, the "Taylor principle". The initial condition retained by Leeper (1991) is slightly different. Leeper formalizes a linear rule without referring to the interest rate target before linearizing the other equations of the model.

3.2 Debt, Capital and Interest

We rewrite the steady state equilibrium condition (38) in a slightly different form:

$$\left[\frac{1+n}{(1-\theta)} - \tilde{R}(k) \right] \omega = 0 \quad (40)$$

and the condition (36) in the two alternative forms:

$$\tilde{R}(k) = \frac{\beta^{-1} + \Psi(\omega/c)}{1 - \Psi(k/c)} \quad (41)$$

$$\iff \omega = \Psi^{-1} [R(k) - \beta^{-1}] c - \tilde{R}(k) k \quad (42)$$

Equation (40) admits two evident solutions, $\omega^* = 0$ and $\tilde{R}(k^{**}) = \frac{1+n}{(1-\theta)}$, corresponding to two stationary equilibrium vectors of the variables c , k and ω . In what follows, we will study the properties of these vectors.

3.2.1 “Autarkic Equilibria”

First, we study the solution corresponding to a zero public debt in steady state. The equation (41) together with equation (37), allows to obtain the value of the capital stock *per capita* and the consumption *per capita* in an implicit form. We get:

$$\omega^* = 0 \quad (43)$$

$$R^* = \tilde{R}(k^*) = \frac{\beta^{-1}}{1 - \Psi k^*/c^*} \quad (44)$$

$$c^* = (1-g) f(k^*) - (n+\delta) k^* \quad (45)$$

The second equation allows to verify that the equilibrium gross interest rate, R^* , verifies:

$$R^* > \beta^{-1}$$

where β^{-1} is the gross interest rate in the Ricardian economy, obtained by assuming that $n = 0$.

Because the parameter Ψ given by $n\xi(\beta^{-1} - 1)$ is weak, the gap between R^* and β^{-1} is likely to be low. Besides, the value of the equilibrium debt equals zero, as in the Ricardian case¹⁰. With reference to the standard OLG model, where this kind of equilibria corresponds to the absence of exchange among generations and Government intervention, we call this first kind of equilibria: "Autarkic Equilibria".

¹⁰Since the equation (40) has to be verified when $\Psi = 0$, the stationary debt level is: $\omega^R = 0$ in the ricardian case.

3.2.2 “Debt Equilibria”

The second solution of equation (40) allows to compute the equilibrium value of the real public debt according to the equation (42). One obtains:

$$\omega^{**} = \Psi^{-1} \left(\frac{1+n}{1-\theta} - \beta^{-1} \right) c^{**} - \frac{1+n}{1-\theta} k^{**} \quad (46)$$

$$R^{**} = \tilde{R}(k^{**}) = \frac{1+n}{1-\theta} \quad (47)$$

$$c^{**} = (1-g) f(k^{**}) - (n+\delta) k^{**} \quad (48)$$

where the values of k^{**} and c^{**} in (46) are given by (47) and (48), respectively.

Comparing the Autarkic and Debt Equilibria, we can express the following proposition, the demonstration of which is given in appendix 1:

Proposition 1 *The real value of the per capita public debt is positive in a Debt Equilibrium if and only if the associated real interest rate is greater than the Autarkic real interest rate, i.e.:*

$$R^{**} = \frac{1+n}{1-\theta} \geq R^* \iff \omega^{**} \geq 0.$$

Afterward, we will assume that the condition:

$$\theta \geq 1 - \frac{1+n}{R^*} \quad (\text{H2})$$

is satisfied which guarantees the positivity of ω^{**} .

An example

Assuming that the production function is of the kind: $f(k) = Ak^\alpha$, the equation (29) allows to rewrite the real gross interest rate according to the average productivity of capital:

$$\tilde{R}(k) = 1 - \delta + \alpha f(k)/k = R.$$

Using this result and denoting the real debt-to-GDP ratio by $\tilde{\omega} = \omega/f(k)$, and combining the equations (36) and (37), one obtains :

$$\tilde{\omega} = \frac{R - \beta^{-1}}{\Psi} \left((1-g) - \frac{\alpha(n+\delta)}{R-1+\delta} \right) - \frac{\alpha R}{R-1+\delta}$$

which is monotone and increasing for $R > \beta^{-1}$ and that can be evaluated in $R = \frac{1+n}{1-\theta}$ and in $\tilde{\omega} = 0$.

We adopt the following annual calibration for the model's parameters :

Table 1:

<i>Definition</i>	<i>Parameter</i>	<i>Value</i>
Discount factor:	β	0,96
Consumption weight in utility function:	ξ	0,95
Capital share of output:	α	0,3
Depreciation rate of capital:	δ	0,1
Population growth rate:	n	0,02
Public expenditure-to-GDP ratio:	g	0,2
GDP parameter in the fiscal rule:	z	0,2
Debt parameter in the fiscal rule:	θ	0,025

implying that $\Psi = n\xi(\beta^{-1} - 1) \simeq 0.00079$. We easily obtain the following values of the real gross interest rate in the Ricardian, Autarkic and Debt Equilibria, respectively :

Table 2:

<i>Real Interest Rate</i>	
Ricardian Equilibria Interest Rate:	$\beta^{-1} - 1 = 4,17\%$
Autarkic Equilibria Interest Rate:	$R^* - 1 = 4,48\%$
Debt Equilibria Interest Rate:	$R^{**} - 1 = 4,62\%$

The real debt-to-GDP ratio corresponding to a Debt Equilibrium is: $\tilde{\omega}^{**} = 0.99$, approximately 100%.

3.3 Multiple Equilibria

The presence of wealth effects generates, in the case of simple fiscal rule as used in our model, two possible configurations in the long run for the vector of real variables (c, k, ω) . The first configuration is very close to the one (unique) described in the Ricardian economy. Consider a naive version of the dynamic equation of the real debt (32), with $g_t = z_t$, supposing that the gross interest rate equals its long run value R^* and neglecting the uncertainty:

$$\frac{\omega_{t+1}}{R^*} = \frac{1 - \theta}{1 + n} \omega_t$$

We observe that the real debt convergence to its stationary value, $\omega^* = 0$, is insured by the assumption (H2): $\theta \geq 1 - (1 + n)/R^*$. Using the Leeper's terminology, the envisaged fiscal rule is said to be (locally) passive around this equilibrium. Such a condition is not insured around a Debt Equilibrium,

when $R^{**} = (1 + n)/(1 - \theta)$. The endogenous real interest rate does not allow our simple fiscal rule to offset the increasing level of the debt burden associated to a higher debt. In Leeper's terms, the fiscal rule is said to be (locally) active in R^{**} . It is this mechanism which is responsible for the emergence of a second steady state configuration.

We remind that in section 3.1 we have found that, for a given level of the real interest rate, two values of the steady state inflation rate existed, when one of the two was associated to an active monetary policy. Each of the two gross interest rates R^* and R^{**} could be associated to two possible inflation rates, so our economy potentially admit four equilibria. These equilibria are represented on the figures 3 and 4, each representing a particular version of the monetary rule:

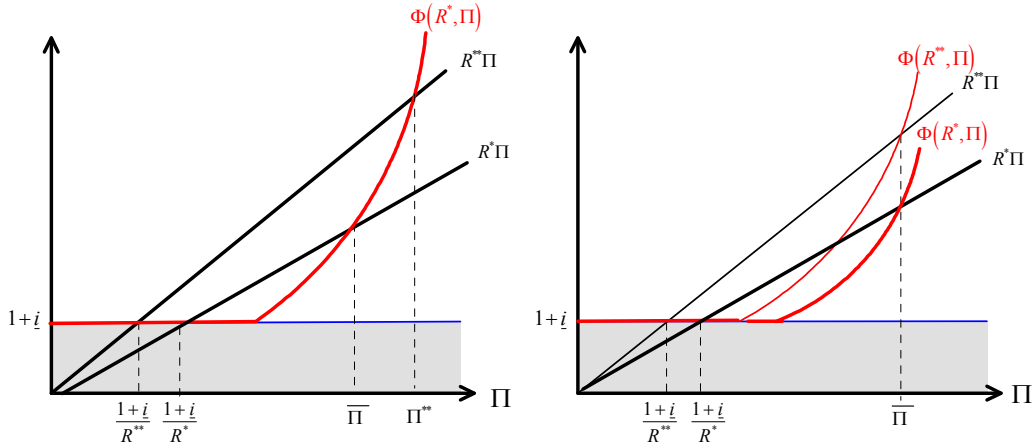


Figure 3

Figure 4

In the first case, represented on figure 3, the monetary rule depends on a constant real interest rate target¹¹, corresponding to the Autarkic Equilibrium : $\bar{R}_t = R^*$. If the actual real gross interest rate is R^{**} then the target $\bar{\Pi}$ does not be reached and an inflationary bias appears. In the case of the rule (28), with $a = 1$ (and always $\phi > 1$), this inflationary bias is given by : $\Pi^{**}/\bar{\Pi} = (R^{**}/R^*)^{\frac{1}{\phi-1}}$.

In the second case, represented on figure 4, the monetary rule depends on the current real interest rate: $\bar{R}_t = R_t$. As we can note, this rule presents the advantage of not making the long term inflation rate depending on the equilibrium value of the real interest rate, except in the liquidity trap. So, the inflation target $\bar{\Pi}$ can be reached in R^* and in R^{**} . On the other hand,

¹¹The rule used in this case is similar to the Leeper-Taylor's rule.

the assumption adopted about the representation of a liquidity trap does not allow us to obtain the uniqueness of the lower inflation rate¹².

To resume, our four equilibria correspond to the four configurations described by Leeper (1991), but unlike the case of the Ricardian economy (and without liquidity trap) considered by Leeper, these four equilibria can exist for a unique configuration of the fundamental parameters. In the following section, we will verify if these four equilibria have, locally, the same dynamics properties as those analyzed by Leeper.

4 Dynamics and Stability

In order to study the dynamics of the economy, we start by analyzing the local stability around the stationary equilibria. Then we discuss these results from a more global point of view.

4.1 Local dynamics

As we have already noted, the most relevant linearized model would be the one constituted of the predetermined variables k_t and $x_t = (M_{t-1} + B_{t-1})/N_t P_{t-1}$ and the non predetermined variables c_t and Π_t . Then the Blanchard and Kahn (1981) conditions would theoretically allow to characterize the local dynamics of the four stationary equilibria. We shall adopt this procedure, in the last section, in order to simulate numerically the model, but the dimension of the system does not allow us to characterize analytically the equilibria. On the other hand, the system composed of the variables c_t , k_t , ω_t and Π_t offers some interesting possibilities that we are going to investigate.

Because one of the variables, ω_t , could equal zero in the long run, we linearize the equations (30) to (35) around any stationary equilibrium, by defining each variable in difference : $\hat{u}_t = u_t - u$, where u represents the variable u_t evaluated in one of the stationary equilibria. We obtain :

$$\hat{c}_t = \frac{\beta^{-1}}{R} E_t \hat{c}_{t+1} + \frac{\Psi}{R} E_t \hat{\omega}_{t+1} + \left(\Psi - \frac{\beta^{-1}c + \Psi\omega}{R^2} f_{kk} \right) E_t \hat{k}_{t+1} \quad (49)$$

$$\hat{k}_{t+1} = \frac{1}{1+n} \left([R - g f_k] \hat{k}_t - \hat{c}_t - f \cdot \hat{g}_t \right) \quad (50)$$

$$E_t \hat{\omega}_{t+1} = \frac{R}{R^{**}} \hat{\omega}_t + \frac{\omega f_{kk}}{R^{**}} \hat{k}_{t+1} + \frac{R}{1+n} f \cdot (\hat{g}_t - \hat{z}_t) \quad (51)$$

$$E_t \hat{\pi}_{t+1} = \phi_\pi \hat{\pi}_t + (\phi_R - 1) \frac{\Pi}{R} f_{kk} \hat{k}_{t+1} \quad (52)$$

¹²It would not be the case if the represented function $\Phi(\cdot)$ corresponded to the case $(1-a)R^*\bar{\Pi} > 1 + \dot{i}$.

where $\phi_\pi = \Pi\Phi_\Pi/\Phi$ and $\phi_R = R\Phi_R/\Phi$ are the elasticity of the function $\Phi(\cdot)$ and where we have used $R^{**} = \frac{1+n}{1-\theta}$.

Denoting $\hat{Y}_t = [\hat{k}_t \ \hat{\omega}_t \ \hat{c}_t \ \hat{\pi}_t]'$, the vector of the endogenous variables, using the long run equations (36) to (39) and neglecting the shocks \hat{g}_t and \hat{z}_t , the equations (49) to (52) could be combined in order to get the following state-space form :

$$E_t \hat{Y}_{t+1} = J_4(k, \omega, c, \pi) \cdot \hat{Y}_t \quad (53)$$

where the Jacobian matrix $J_4(k, \omega, c, \pi)$ is given by:

$$J_4(\cdot) = \begin{pmatrix} \frac{R-gf_k}{1+n} & 0 & -\frac{1}{1+n} & 0 \\ \frac{R-gf_k}{1+n} f_{kk} \frac{\omega}{R^{**}} & \frac{R}{R^{**}} & -\frac{1}{1+n} f_{kk} \frac{\omega}{R^{**}} & 0 \\ \frac{R-gf_k}{1+n} (c \frac{f_{kk}}{R} - \Psi\beta R) & -\Psi\beta \frac{R}{R^{**}} & \beta R - \frac{c \frac{f_{kk}}{R} - \Psi\beta R}{1+n} & 0 \\ \frac{R-gf_k}{1+n} (\phi_R - 1) \frac{\Pi}{R} f_{kk} & 0 & -(\phi_R - 1) \frac{\Pi}{R} \frac{f_{kk}}{1+n} & \phi_\pi \end{pmatrix} \quad (54)$$

The vector \hat{Y}_t is composed of a predetermined variable, \hat{k}_t , a non predetermined variable, \hat{c}_t , and the two variables $\hat{\omega}_t$ and $\hat{\pi}_t$, both potentially non predetermined but linked to one another by the relation:

$$\hat{\omega}_t = \frac{1}{\Pi} \hat{x}_t - \frac{\omega}{\Pi} \hat{\pi}_t$$

where \hat{x}_t is predetermined. It is therefore necessary, in order to apply the Blanchard and Kahn conditions, to consider one of the two variables ($\hat{\pi}_t$ or $\hat{\omega}_t$) as predetermined and the other one ($\hat{\omega}_t$ or $\hat{\pi}_t$) as non predetermined. The matrix $J_4(k, \omega, c, \pi)$, evaluated in one of the stationary states, has to possess two eigenvalues inside the unit circle and two eigenvalues outside in order to let the associated equilibrium locally determined.

The interest of the matrix $J_4(k, \omega, c, \pi)$, with regard to the Jacobian matrix $\mathcal{J}_4(k, x, c, \pi)$ which would be associated to the vector variables $\hat{Y}_t' = [\hat{k}_t \ \hat{x}_t \ \hat{c}_t \ \hat{\pi}_t]$, lies in its decomposability property. The three first lines of the last column are composed of zero, what means that we can study the properties (the eigenvalues) of the sub-systems $J_3(k, \omega, c)$ and $J_1(\pi)$ independently of each other, with :

$$J_3(k, \omega, c) = \begin{pmatrix} \frac{R-gf_k}{1+n} & 0 & -\frac{1}{1+n} \\ \frac{R-gf_k}{1+n} f_{kk} \frac{\omega}{R} & \frac{R}{R^{**}} & -\frac{1}{1+n} f_{kk} \frac{\omega}{R} \\ \frac{R-gf_k}{1+n} ([c+d] \frac{f_{kk}}{R} - \Psi\beta R) & -\Psi\beta \frac{R}{R^{**}} & \beta R - \frac{[c+d] \frac{f_{kk}}{R} - \Psi\beta R}{1+n} \end{pmatrix} \quad (55)$$

and :

$$J_1(\pi) = \phi_\pi$$

The eigenvalue associated to $J_1(\pi)$ is its unique component, ϕ_π . If the function $\Phi(\cdot)$ is of the form used in the figures 3 and 4, we have $\phi_\pi > 1$ in $\bar{\Pi}$, as well as in Π^{**} , and $\phi_\pi = 0$, around the liquidity trap equilibria in $(1 + \underline{i})/R^*$ and in $(1 + \underline{i})/R^{**}$.

The sub-system $J_3(k, \omega, c)$ is easier to study when the type of the considered steady state is specified.

4.1.1 Autarkic Equilibria

In the autarkic steady state equilibrium, the real debt equals zero, which allows to simplify the matrix $J(k, \omega, c)$:

$$J_3(k^*, \omega^*, c^*) = \begin{pmatrix} \frac{R^* - g f_k^*}{1+n} & 0 & -\frac{1}{1+n} \\ 0 & \frac{R^*}{R^{**}} & 0 \\ \frac{R^* - g f_k^*}{1+n} \left(c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^* \right) & -\Psi \beta \frac{R^*}{R^{**}} & \beta R^* - \frac{c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^*}{1+n} \end{pmatrix}$$

Rearranging the variables, it is once again possible to decompose this matrix into two sub-systems $J_2(k^*, c^*)$ and $J_{1'}(\omega^*)$, with :

$$J_2(k^*, c^*) = \begin{pmatrix} \frac{R^* - g f_k^*}{1+n} & -\frac{1}{1+n} \\ \frac{R^* - g f_k^*}{1+n} \left(c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^* \right) & \beta R^* - \frac{c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^*}{1+n} \end{pmatrix}$$

and:

$$J_{1'}(\omega^*) = R^*/R^{**}$$

Under the assumption (H2), the eigenvalue R^*/R^{**} is strictly less than 1 and we show, in the appendix 2, that the condition :

$$R^* < \frac{(1+n) - (1-\delta)g}{1-g} \quad (\text{H3})$$

is sufficient for the matrix $J_2(k^*, c^*)$ to admit one and only one eigenvalue less than unity in absolute value. These results are summarized by the following proposition¹³:

¹³See appendix 2 for the démonstration. We verify that the condition (H3) is implied by (H2) for the calibration that we held in the example in the section 3.2. In general, the assumptions (H2) and (H3) can be summarized by: $R^* < \min\left(\frac{1+n}{1-\theta}; \frac{(1+n) - (1-\delta)g}{1-g}\right)$.

Proposition 2 *Under assumptions (H2) and (H3), the autarkic equilibrium associated to the inflation target, $\bar{\Pi}$, is locally determined when $\phi_\pi > 1$. The liquidity trap equilibrium is locally indeterminate.*

Equivalent results are obtained in the case of Ricardian economy by putting $n = 0$ and by replacing R^* with β^{-1} .

4.1.2 Debt Equilibria

The matrix $J_3^{**} = J_3(k^{**}, \omega^{**}, c^{**})$ corresponding to the Debt Equilibria is obtained by putting $R = R^{**}$ in (55). One obtains :

$$J_3^{**} = \begin{pmatrix} \frac{R^{**} - g f_k^{**}}{1+n} & 0 & -\frac{1}{1+n} \\ \frac{R^{**} - g f_k^{**}}{1+n} f_{kk}^{**} \frac{\omega^{**}}{R^{**}} & 1 & -\frac{1}{1+n} f_{kk}^{**} \frac{\omega^{**}}{R^{**}} \\ \frac{R^{**} - g f_k^{**}}{1+n} \left(c^{**} \frac{f_{kk}^{**}}{R^{**}} - \Psi \beta R^{**} \right) & -\Psi \beta & \beta R^{**} - \frac{c^{**} \frac{f_{kk}^{**}}{R^{**}} - \Psi \beta R^{**}}{1+n} \end{pmatrix}$$

In appendix 3, we analyze the characteristic polynomial $\mathcal{P}^{**}(\lambda)$ associated to this matrix which allows us to show that the matrix J_3^{**} admit one eigenvalue in absolute value less than unity and two eigenvalues, greater than unity. One can deduce the following proposition¹⁴:

Proposition 3 *Under the assumption (H2) and (H3), the "debt" equilibrium associated to the higher inflation rate, $\bar{\Pi}$ or Π^{**} , is locally overdetermined when $\phi_\pi > 1$. The trap equilibrium is locally determined.*

4.2 Global dynamics: Discussion

Based on propositions 2 and 3, it can be verified that the four potential stationary equilibria of our economy have locally, the properties of the four possible configurations of fiscal and monetary policies identified by Leeper (1991).

Within the framework considered by Leeper, monetary and fiscal policies simultaneously passive lead to indeterminacy and active policies¹⁵ to overdetermination (instability). Only the configurations where one of the two policies is active and the other passive provide the determination, *i.e.* the local uniqueness, of the equilibrium.

¹⁴The proof is in the appendix 3

¹⁵Recall that for Leeper, a fiscal policy is called active when the fiscal authority pays no attention to the debt stabilization objective. The rule is then not very reactive to the level of debt.

Our results can be interpreted by mobilizing the concepts of active Vs passive policies of Leeper, but with a difference. In our case, these four configurations exist for or a unique set of the economic policies parameters. The characterization of a policy as passive or active can no longer be global.

The difficulty with the definition of an interest rate policy corresponding to a globally active monetary rule was noted by Benhabib, Schmitt-Grohé and Uribe (2001).

The zero bound on the nominal interest rate (the liquidity trap) fails to ensure the application of an interest rate rule sufficiently reactive to inflation (active) when the rates are low. We have seen that the required non-linearity of the Monetary rule doubled the number of stationary equilibria and no longer ensured the determination of the Autarkic Equilibrium when the fiscal policy was locally passive (reactive to the level of debt).

The second source of difficulty arises from the accumulation of debt. The exchange economy considered by Leeper permits to characterize a simple fiscal rule whose properties do not depend on the level of the initial public debt¹⁶

The mere presence of production and capital accumulation is not sufficient to modify this result. The Ricardian equivalence insulates the real interest rate from the real debt level. However, the presence of wealth effects, characteristics of a non-Ricardian economy, result in the dependence of the real interest rate level of the public debt. In this case, a simple fiscal policy no longer compensate for the increasing burden associated with a high real debt, even if it is sufficiently responsive (passive in terms of Leeper) for a low level of debt (zero in our case).

Paradoxically, these are opposing characteristics of the two rules that we used that explain the multiplicity of equilibria. The monetary rule would not be a problem if it was linear¹⁷, and a non-linear fiscal rule, becoming more responsive to the level of debt as the real interest rate rises, would easily allow to ensure the convergence of debt to zero.

From another point of view, the double multiplicity of equilibrium results from a double non-linearity: that of a interest rate rule which respects a lower bound and that of a simple rule of public debt accumulation in the presence of wealth effects. The most original result of our model lies probably in the coexistence of two steady state equilibria locally determined, that is associated with saddle trajectories¹⁸ locally unique. The complexity of a

¹⁶The term "initial" can be misleading, insofar as the general level of prices can jump so that the real Government debt is just covered by expected income, as in the highly controversial "Fiscal Theory of the Price Level".

¹⁷This would require the possibility of negative nominal interest rates...

¹⁸It is in fact about a stable variety of dimension 2, that is of " saddle plans ".

universe of dimension 4 makes difficult a global analysis of stability. Naturally, our results are enough to determine the absence of a global uniqueness (or determination) of the equilibrium...

The most interesting question cannot have of definitive answer: the economy by being situated around an equilibrium without debt (autarkic) can it jump, in the possible favour of an important shock (fiscal?), on a trajectory leading it towards the debt equilibrium with liquidity trap? We emit the guess of a positive answer. In such a case, our model could offer an alternative explanation to the more traditional explanations brought by Krugman (1998), Svensson (2001) and Eggertson and Woodford (2003) of the Japanese recession of the 90s. For these authors, Japanese liquidity trap is the consequence of a very negative shock on the natural interest rate in a context of the inflation stabilization around a too weak target. The hypothesis of a the liquidity trap of Benhabib, Schmitt-Grohé and Uribe (2001) explains the weakness of the nominal interest rates and the incapacity of the monetary authority to stabilize the economy in such a context, but does not allow to explain the entrance in recession and the persistence of this one¹⁹. Our trap and debt equilibrium does not have this defect. The higher level of the real interest rate provokes an eviction of the private investment and reduces well the level of the production. A last point deserves our attention. the trap and debt equilibrium possesses the local properties of an equilibrium of the kind " fiscal theory of the price level " that Leeper (1991), Sims (1994) and Woodford (1994) presented and analyzed.

5 Simulation

In this section, we report the response of the economy to a temporary fiscal expansion ($\hat{z}_t < 0$). The fiscal change analyzed is assumed to correspond to a decrease in taxes to 1 point of the autarkic GDP and the considered monetary policy is a modified non linear Taylor Rule: $1 + i_t = \Phi(R_t, \Pi_t; \bar{\Pi})$. We use the same calibration than in section 3.2.

The response of the economy to this kind of fiscal change has different characteristics depending on whether the economy is Ricardian or not and, when not, on the considered (Debt or Autarkic) Equilibrium.

When the economy is Ricardian a taxes decrease is represented by an equivalent increase of the future public debt. In this economy the Ricardian

¹⁹We could even expect a more important level of activity in a model where the weakness of the nominal interest rates reduces the level of the monetary distortions and increases for the labor supply...

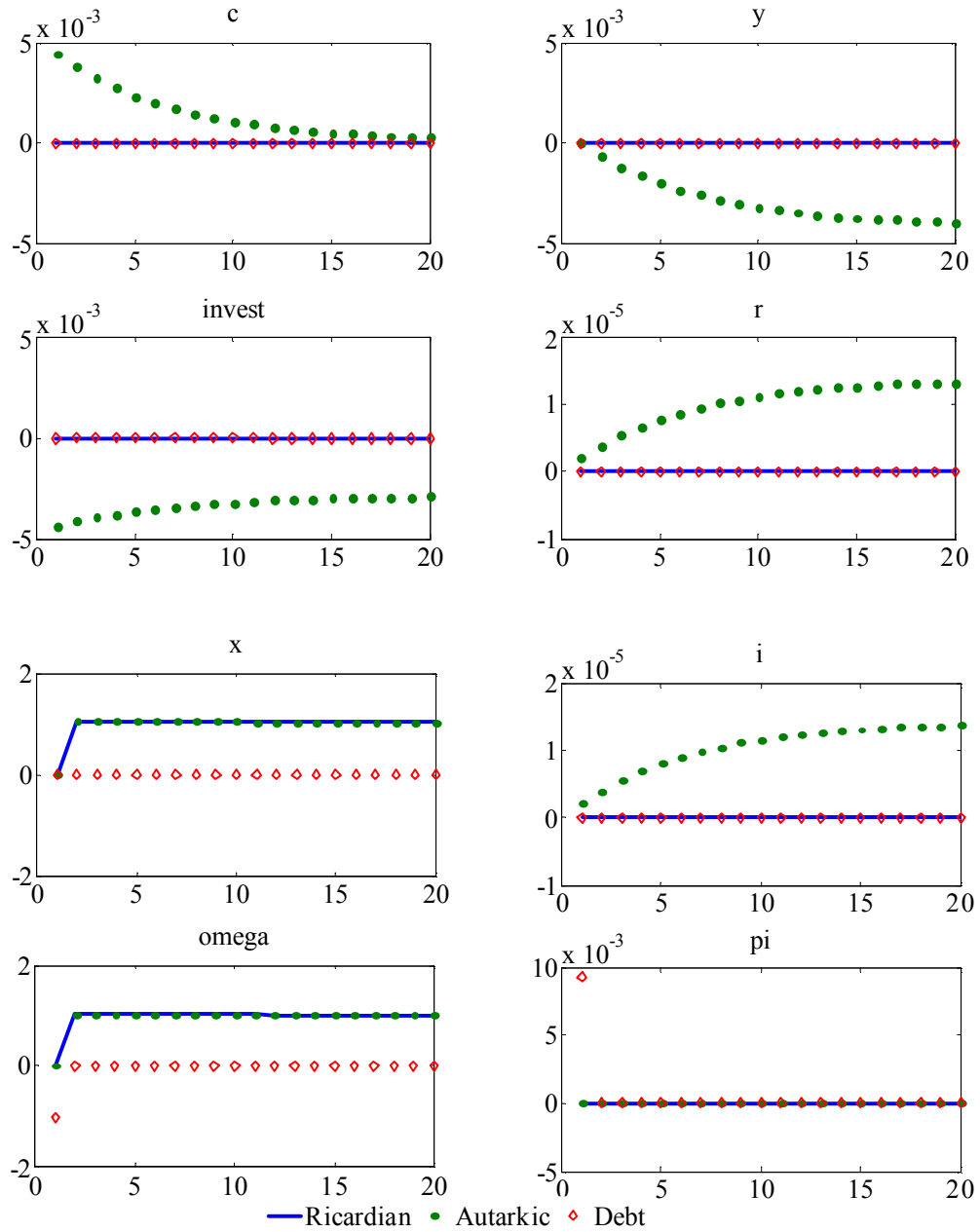
equivalence is satisfied what means that the higher public debt is not net wealth for the agents. The taxes decrease has no effect on the economy.

In the Autarkic Equilibrium, a taxes decrease augments the public debt, the real and nominal interest rates and the consumption and it depresses the investment and the output. These results are easily explained. When the Ricardian equivalence is not satisfied, a taxes decrease augments the human capital of the current generations which fully benefit from the fiscal expansion and share the future taxes increase with the future - not yet born - generations. To increase their consumption the agents reduce their savings which implies a higher real interest and as a consequence a decrease of the investment. The inflation does not react to the taxes decrease because the considered monetary rule completely stabilizes the inflation.

In the Debt Equilibrium, the economy is behaving in a very different way and, actually, rather approximates the Ricardian economy. Around this equilibrium, the real variables do not respond to the taxes decrease. In addition, the real public debt decrease in the first period. In order to understand this result, we should note that this equilibrium have the characteristics of an FTPL²⁰ equilibrium. A decrease of the fiscal incomes must be offset by an equivalent decrease in the real public debt to ensure a balanced intertemporal budget constraint of the government. This is allowed by a jump of the price

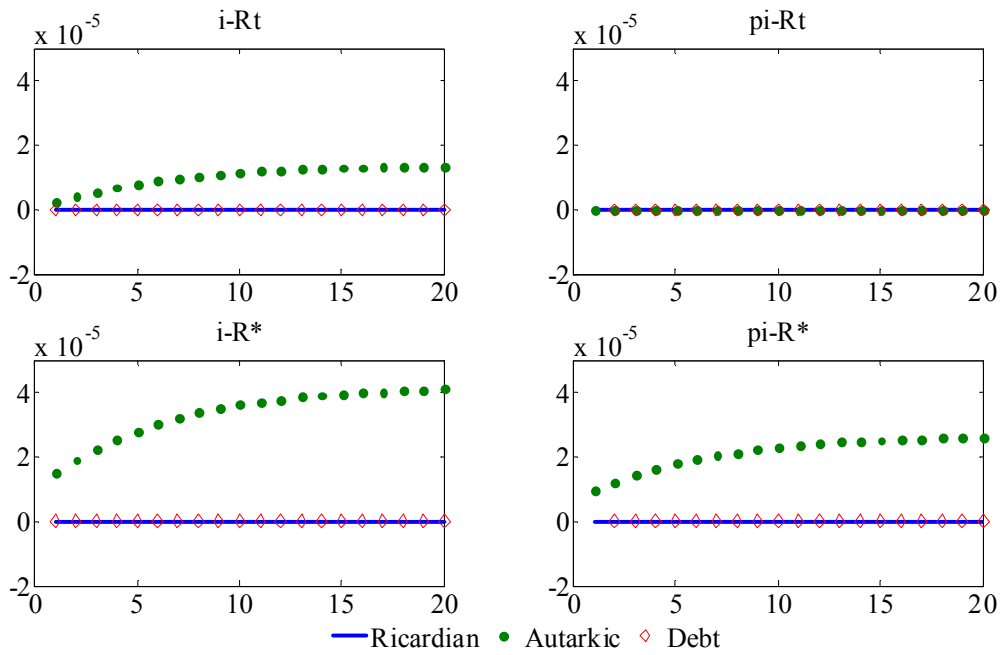
²⁰Fiscal Theory of the Price Level determination.

level in the first period. All these informations are summarized in Figure 5:

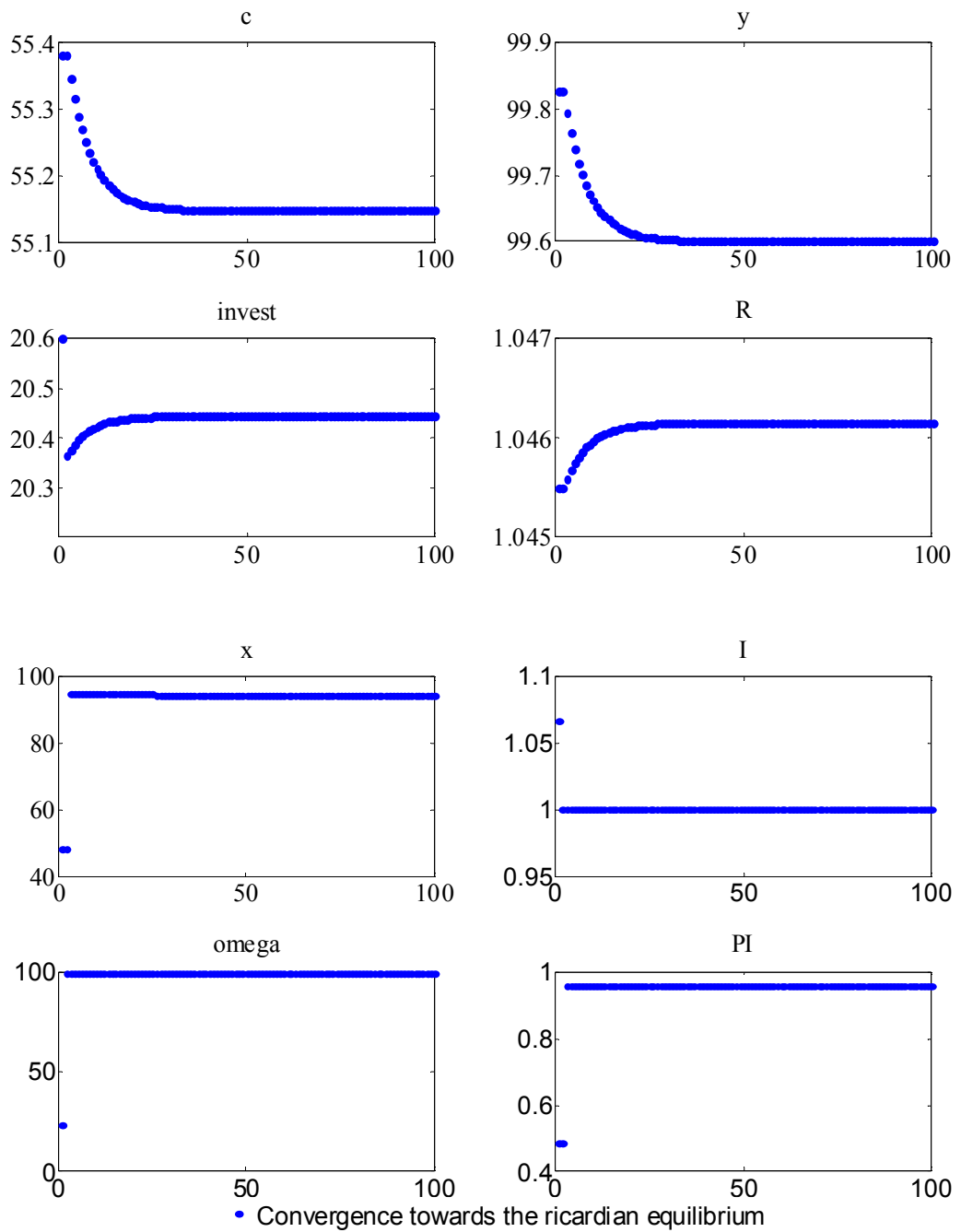


The use of a more traditional (but non linear) Taylor rule, $\Phi(R^*, \Pi_t; \bar{\Pi})$, does not significantly modify the evolution of real variables. However, we observe one difference, in the Autarkic Equilibrium, concerning the evolution of inflation and the nominal interest rate. The use of a constant target, $\bar{R}_t = R^*$, appears to be less efficient for the stabilization of inflation. By

targeting a real interest rate lower than the natural one ($R^* < R_t$), the rule creates an unnecessary inflationary bias that has lasted as long as the interest rate has not regained its stationary level. This bias clearly appears on the graphs of figure 6 where the first period jump of the inflation rate in the Debt Equilibrium has not been represented so as not to "crush" the representation of the dynamics of the Autarkic Equilibrium.



5.0.1 Convergence towards the Debt equilibrium



Appendix 1

Proposition 1 : *The real value of the per capita public debt is positive in the "debt" equilibrium if and only if the associated real interest rate is greater than the "autarkic" real interest rate, i.e.:*

$$R^{**} \geq R^* \iff \omega^{**} \geq 0.$$

Proof : By reminding the concavity of the production function $f(k)$, and (consequently) the decrease of the function $\tilde{R}(k)$, we verify that :

$$R^{**} \geq R^* \implies k^{**}/c^{**} \leq k^*/c^*$$

By using (44) that we remind:

$$R^* = \tilde{R}(k^*) = \frac{\beta^{-1}}{1 - \Psi(k^*/c^*)}$$

we easily find :

$$R^{**} \geq R^* \iff \frac{\beta^{-1}}{1 - \Psi(k^{**}/c^{**})} \leq \frac{\beta^{-1}}{1 - \Psi(k^*/c^*)} \quad (\text{A1.1})$$

Now, by using the equation (41) :

$$\tilde{R}(k) = \frac{\beta^{-1} + \Psi(\omega/c)}{1 - \Psi(k/c)}$$

evaluated in the autarkic equilibrium and in the debt equilibrium, one observes that :

$$R^{**} \geq R^* \iff \frac{\beta^{-1} + \Psi(\omega^{**}/c^{**})}{1 - \Psi(k^{**}/c^{**})} \geq \frac{\beta^{-1}}{1 - \Psi(k^*/c^*)} \quad (\text{A1.2})$$

By collecting the inequalities (A1.1) and (A1.2), we obtain finally :

$$R^{**} \geq R^* \iff \frac{\beta^{-1}}{1 - \Psi(k^{**}/c^{**})} \leq \frac{\beta^{-1}}{1 - \Psi(k^*/c^*)} \leq \frac{\beta^{-1} + \Psi(\omega^{**}/c^{**})}{1 - \Psi(k^{**}/c^{**})}$$

Or, more simply:

$$R^{**} \geq R^* \iff \omega^{**} \geq 0$$

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Appendix 2

Proposition 2 : Under the assumptions (H2) and (H3), the autarkic equilibrium associated to the inflation rate target, $\bar{\Pi}$, is locally determined when $\phi_\pi > 1$. The trap equilibrium is locally indeterminate.

Proof : *i)* We show, at first, that the condition (H3) is sufficient so that the matrix $J_2(k^*, c^*)$ admits one and a single eigenvalue lower than the unity in absolute value. Let us remind, by convenience, $J_2(k^*, c^*)$:

$$J_2(k^*, c^*) = \begin{pmatrix} \frac{R^* - gf_k^*}{1+n} & -\frac{1}{1+n} \\ \frac{R^* - gf_k^*}{1+n} \left(c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^* \right) & \beta R^* - \frac{c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^*}{1+n} \end{pmatrix}$$

Its characteristic polynomial is given by :

$$P^*(\lambda) = \left(\frac{R^* - gf_k^*}{1+n} - \lambda \right) \left(\beta R^* - \frac{c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^*}{1+n} - \lambda \right) + \frac{R^* - gf_k^*}{1+n} \left(\frac{c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^*}{1+n} \right)$$

Let us calculate the critical values of $P^*(\lambda)$. We find:

$$\begin{aligned} P^*(-1) &= (1 + \beta R^*) \left(1 + \frac{R^* - gf_k^*}{1+n} \right) - \frac{\left(c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^* \right)}{(1+n)} > 0 \\ P^*(0) &= \frac{R^* - gf_k^*}{1+n} \beta R^* > 0 \\ P^*(1) &= \left(1 - \frac{R^* - gf_k^*}{1+n} \right) (1 - \beta R^*) + \frac{\left(c^* \frac{f_{kk}^*}{R^*} - \Psi \beta R^* \right)}{(1+n)} \end{aligned}$$

The signs of $P^*(-1)$ and of $P^*(0)$ are evident. Notice that $R^* > \beta^{-1}$; a sufficient condition to guaranty that $P^*(1)$ is negative is given by : $R^* - gf_k^* < 1 + n$. Using the fact that : $R^* = 1 - \delta + f_k^*$, the previous inequality can also be written :

$$R^* < \frac{(1+n) - (1-\delta)g}{1-g} \quad (\text{H3})$$

The polynomial $P^*(\lambda)$ is of degree 2, the condition $P^*(1) < 0$ implied by (H3), jointly with $P^*(-1) > 0$ and $P^*(0) > 0$ is sufficient to guaranty the uniqueness of the eigenvalue inside the unit circle.

ii) Notice that the eigenvalue of J_1' (ω^*) verify : $R^*/R^{**} < 1$, one conclude that the initial matrix $J_4^* = J_4(k^*, \omega^*, c^*, \pi^*)$ has at least two eigenvalues less than the unit and one eigenvalue greater than the unit (in absolute value). According to the sign of $\phi_\pi - 1$, the equilibrium is either locally determinate ($\phi_\pi > 1$), or locally indeterminate ($\phi_\pi < 1$).

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Appendix 3

Proposition 3 : Under the assumption (H2) and (H3), the "debt" equilibrium associated to the higher inflation rate, $\bar{\Pi}$ or Π^{**} , is locally overdetermined when $\phi_\pi > 1$. The trap equilibrium is locally determinate.

Proof : We demonstrate that the matrix $J_3^{**} = J_3(k^{**}, \omega^{**}, c^{**})$ admits one eigenvalue in the absolute value less than the unit and two eigenvalues greater than the unit. We show that its characteristic polynomial $\mathcal{P}^{**}(\lambda)$ has one root in the interval $[-1, 1]$. Let us remind, by convenience, J_3^{**} :

$$J_3^{**} = \begin{pmatrix} \frac{R^{**} - gf_k^{**}}{1+n} & 0 & -\frac{1}{1+n} \\ \frac{R^{**} - gf_k^{**}}{1+n} f_{kk}^{**} \frac{\omega^{**}}{R^{**}} & 1 & -\frac{1}{1+n} f_{kk}^{**} \frac{\omega^{**}}{R^{**}} \\ \frac{R^{**} - gf_k^{**}}{1+n} \left(c^{**} \frac{f_{kk}^{**}}{R^{**}} - \Psi \beta R^{**} \right) & -\Psi \beta & \beta R^{**} - \frac{c^{**} \frac{f_{kk}^{**}}{R^{**}} - \Psi \beta R^{**}}{1+n} \end{pmatrix}$$

Its characteristic polynomial is given by :

$$\mathcal{P}^{**}(\lambda) = -\lambda^3 + T^{**}\lambda^2 - S^{**}\lambda + D^{**}$$

where T^{**} , S^{**} and D^{**} represents the trace, the diagonal minor sum and the determinant of the matrix J^{**} , respectively, which are given by:

$$\begin{aligned} T^{**} &= \frac{R^{**} - gf_k^{**}}{1+n} + (1 + \beta R^{**}) - \frac{c^{**} \frac{f_{kk}^{**}}{R^{**}} - \Psi \beta R^{**}}{1+n} > 0 \\ S^{**} &= \beta R^{**} - \frac{c^{**} \frac{f_{kk}^{**}}{R^{**}} - \Psi \beta R^{**}}{1+n} + \frac{R^{**} - gf_k^{**}}{1+n} (1 + \beta R^{**}) - \Psi \beta \frac{f_{kk}^{**}}{(1+n)} \frac{\omega^{**}}{R^{**}} > 0 \\ D^{**} &= \frac{R^{**} - gf_k^{**}}{1+n} \beta R^{**} > 0 \end{aligned}$$

Let us calculate the critical values and the derivative of $\mathcal{P}^{**}(\lambda)$. We find:

$$\begin{aligned} \mathcal{P}^{**}(-1) &= 1 + T^{**} + S^{**} + D^{**} > 0 \\ \mathcal{P}^{**}(0) &= D^{**} > 0 \\ \mathcal{P}^{**}(1) &= \Psi \beta \frac{f_{kk}^{**}}{(1+n)} \frac{\omega^{**}}{R^{**}} < 0 \end{aligned}$$

and :

$$\mathcal{P}_\lambda^{**}(\lambda) = -3\lambda^2 + 2T^{**}\lambda - S^{**}$$

We show, at first, that $\mathcal{P}^{**}(\lambda)$ does not admit a root in $[-1, 0]$; then we prove that it admits only one root in $[0, 1]$.

i) $\mathcal{P}_{3,\lambda}^{**}(\lambda)$ is strictly negative in $[-1, 0]$, the polynomial $\mathcal{P}^{**}(\lambda)$ is strictly decreasing in $[-1, 0]$. Given the sign of $\mathcal{P}^{**}(-1)$ and of $\mathcal{P}^{**}(0)$, we deduce that $\mathcal{P}^{**}(\lambda) \neq 0$ in $[-1, 0]$. The polynomial $\mathcal{P}^{**}(\lambda)$ does not admit a root in $[-1, 0]$.

ii) Observing that the polynomial $\mathcal{P}^{**}(\lambda)$ changes of sign between 0 and 1, it can have, either one, or three roots in $[0, 1]$. In the last case, its derivative should cancel twice in $[0, 1]$. Now, we have :

$$\mathcal{P}_\lambda^{**}(0) = -S^{**} < 0$$

and

$$\mathcal{P}_\lambda^{**}(1) = \left(1 - \frac{R^{**} - gf_k^{**}}{1+n}\right)(\beta R^{**} - 1) - \frac{c^{**} \frac{f_{kk}^{**}}{R^{**}} - \Psi \beta R^{**}}{1+n}$$

of which sign is positive when the hypothesis H3 is verified. $\mathcal{P}_\lambda^{**}(\lambda)$ cancels only once in $[0, 1]$ and the polynomial $\mathcal{P}^{**}(\lambda)$ admits only one root in $[0, 1]$.

We deduce that the matrix J_4^{**} has one eigenvalue in the absolute value less than the unit and two, greater than the unit. According to the sign of $\phi_\pi - 1$, the equilibrium is either locally determinate ($\phi_\pi < 1$), or locally overdeterminate ($\phi_\pi > 1$).

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Appendix 4

In this appendix, we derive the state-space form of the model composed of the variables \hat{c}_t , $\hat{\pi}_t$, \hat{k}_t and \hat{x}_t (rather than $\hat{\omega}_t$). We remind, by convenience, the equations (30) to (35):

$$c_t = \beta^{-1} \frac{c_{t+1}}{R_{t,t+1}} + \Psi \left[\frac{\omega_{t+1}}{R_{t,t+1}} + \frac{\tilde{R}(k_{t+1})}{R_{t,t+1}} k_{t+1} \right] \quad (\text{A4.1})$$

$$k_{t+1} = \frac{1}{1+n} [(1-\delta)k_t + (1-g_t) \cdot f(k_t) - c_t] \quad (\text{A4.2})$$

$$E_t \left(\frac{\omega_{t+1}}{R_{t,t+1}} \right) = \frac{1}{1+n} [(1-\theta)\omega_t + (g_t - z_t) f(k_t)] \quad (\text{A4.3})$$

$$E_t \left(\frac{1}{R_{t,t+1}} \right) = \frac{1}{\tilde{R}(k_{t+1})} \quad (\text{A4.4})$$

$$E_t \left(\frac{1}{R_{t,t+1}\Pi_{t+1}} \right) = \frac{1}{1+i_t} \quad (\text{A4.5})$$

$$1+i_t = \Phi(\bar{R}_t, \Pi_t) \quad (\text{A4.6})$$

From (A4.1), we express the value of $R_{t,t+1}$:

$$R_{t,t+1} = \beta^{-1} \frac{c_{t+1}}{c_t} + \Psi \left[\frac{\omega_{t+1}}{c_t} + \frac{\tilde{R}(k_{t+1})}{c_t} k_{t+1} \right]$$

that we inject in (A4.3), (A4.4) and (A4.5). By using the value of $1+i_t$ given by the equation, defining the predetermined variable $x_t = \Pi_t \omega_t = \frac{M_{t-1} + B_{t-1}}{N_t P_{t-1}}$, and rearranging the equations, we get:

$$\begin{aligned} c_t &= \left[E_t \left[\beta^{-1} \frac{c_{t+1}}{\tilde{R}(k_{t+1})} + \Psi \left(\frac{x_{t+1}}{\tilde{R}(k_{t+1})\Pi_{t+1}} + k_{t+1} \right) \right]^{-1} \right]^{-1} \\ k_{t+1} &= \frac{1}{1+n} [(1-\delta)k_t + (1-g_t) \cdot f(k_t) - c_t] \\ x_{t+1} &= \frac{\Phi(\bar{R}_t, \Pi_t)}{(1+n)} \left[(1-\theta) \frac{x_t}{\Pi_t} + (g_t - z_t) f(k_t) \right] \\ \Phi(\bar{R}_t, \Pi_t) &= \frac{\tilde{R}(k_{t+1}) E_t \left[\beta^{-1} c_{t+1} + \Psi \left(\frac{x_{t+1}}{\Pi_{t+1}} + \tilde{R}(k_{t+1}) k_{t+1} \right) \right]^{-1}}{E_t [\Pi_{t+1}]^{-1} \left[\beta^{-1} c_{t+1} + \Psi \left(\frac{x_{t+1}}{\Pi_{t+1}} + \tilde{R}(k_{t+1}) k_{t+1} \right) \right]^{-1}} \end{aligned}$$

which constitute a system of four dynamics, stochastic and non linear equations, with 2 predetermined variables, k_t and x_t , and non predetermined

variables, c_t and Π_t . It is necessary to clarify the processes followed by g_t and z_t - 2 additional predetermined variables - as well as the form of the function $\Phi(\cdot)$ and the value held for the real interest target \bar{R}_t to obtain a completely specified system.

By linearizing the previous system around a some steady state, one obtains :

$$\begin{aligned}\hat{c}_t &= \beta^{-1} \frac{E_t \hat{c}_{t+1}}{R} + \Psi \frac{E_t \hat{x}_{t+1}}{\Pi R} - \Psi \omega \frac{E_t \hat{\pi}_{t+1}}{\Pi R} + \left(\Psi R - \frac{\beta^{-1} c + \Psi \omega}{R} f_{kk} \right) \frac{E_t \hat{k}_{t+1}}{R} \\ \hat{k}_{t+1} &= \frac{1}{1+n} \left([R - g f_k] \hat{k}_t - \hat{c}_t - f \cdot \hat{g}_t \right) \\ E_t \hat{x}_{t+1} &= \omega E_t \hat{\pi}_{t+1} + \frac{\Pi \omega f_{kk}}{R^{**}} \hat{k}_{t+1} + \frac{R}{R^{**}} \hat{x}_t - \frac{R}{R^{**}} \omega \hat{\pi}_t + \frac{\Pi R}{1+n} f \cdot (\hat{g}_t - \hat{z}_t) \\ E_t \hat{\pi}_{t+1} &= \phi_\pi \hat{\pi}_t + (\phi_R - 1) \frac{\Pi}{R} f_{kk} \hat{k}_{t+1}\end{aligned}$$

where $\phi_\pi = \Pi \Phi_\Pi / \Phi$ and $\phi_R = R \Phi_R / \Phi$ are the elasticity of the function $\Phi(\cdot)$ and where we used $R^{**} = \frac{1+n}{1-\theta}$.

By denoting $\hat{Y}_{x,t} = [\hat{k}_t \ \hat{x}_t \ \hat{c}_t \ \hat{\pi}_t]'$, the vector of the endogenous variables and $\epsilon_t = [\hat{g}_t \ \hat{z}_t]'$, the vector of shocks, the previous equations can be combined to obtain the state-space form as follows :

$$E_t \hat{Y}_{x,t+1} = J_x \cdot \hat{Y}_{x,t} + J_\epsilon \cdot \epsilon_t$$

where the Jacobian matrix J_x is given by:

$$J_x = \begin{pmatrix} \frac{R-gf_k}{1+n} & 0 & -\frac{1}{1+n} & 0 \\ \frac{\omega \Pi}{R^{**}} f_{kk} \frac{R-gf_k}{1+n} \phi_R & \frac{R}{R^{**}} & -\phi_R \frac{\Pi}{R} \omega \frac{f_{kk}}{1+n} & \left(\phi_\Pi - \frac{R}{R^{**}} \right) \omega \\ \frac{R-gf_k}{(1+n)} \left(c \frac{f_{kk}}{R} - \Psi \beta R \right) & -\Psi \beta \frac{R}{R^{**}} & \beta R - \frac{c \frac{f_{kk}}{R} - \Psi \beta R}{1+n} & \beta \Psi \frac{R}{R^{**}} \frac{\omega}{\Pi} \\ \frac{R-gf_k}{(1+n)} (\phi_R - 1) \frac{\Pi}{R} f_{kk} & 0 & -(\phi_R - 1) \frac{\Pi}{R} \frac{f_{kk}}{1+n} & \phi_\Pi \end{pmatrix}$$

and J_ϵ by :

$$J_\epsilon = \begin{pmatrix} -\frac{f(k)}{1+n} & 0 \\ \Pi \left(R - \omega \frac{\phi_R}{R} f_{kk} \right) \frac{f(k)}{(1+n)} & -\Pi \frac{R}{1+n} f(k) \\ -\frac{f_{kk} f(k)}{R(1+n)} c & R \beta \frac{\Psi}{1+n} f(k) \\ -(\phi_R - 1) \frac{\Pi}{R} \frac{f_{kk}}{(1+n)} f(k) & 0 \end{pmatrix}$$

The evolution of the variable $\hat{\omega}_t$ is obtained in a residual way:

$$\hat{\omega}_t = \frac{1}{\Pi} \hat{x}_t - \frac{\omega}{\Pi} \hat{\pi}_t$$

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