

Optimal saving in the presence of two risks

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Abstract. We examine optimal saving in the presence of two small risks: income risk and a background risk. First, we compute the necessary and sufficient condition for a positive precautionary saving, showing that it depends on two terms capturing respectively the direct effect of income risk and the interaction between the two risks. Secondly, we examine the necessary and sufficient condition for a positive extra-saving due to the contemporaneous presence of the two risks. We show that this condition also depends on a term capturing the direct effect of background risk and that it can hold independently of the previous one.

Key words: Precautionary saving, Background risk, Uncertainty

JEL Classification: D11, D81, E21.

1. Introduction

The theory of saving under uncertainty was firstly studied by Leland (1968), Sadmo (1970) and Drèze and Modigliani (1972). These well-known works showed that a positive third derivative of the utility function implies that uncertainty about future income causes an increase in saving, generating an extra-saving called “precautionary saving”. Kimball (1990) shows the parallelism of this condition with the condition ensuring risk aversion and introduces the concept of prudence. Finally, Menegatti (2001) shows that the condition on the third derivative of the utility function can be weakened under other plausible assumptions on first and second derivatives.

Many recent papers re-examine different issues in the theory of decision under uncertainty in a two-risk framework where income risk is flanked by a second uninsurable background risk. This framework takes into account that consumers make their decisions while facing different risks at the same time, and that some of the risks are uninsurable. Many of the works cited suggest that neglecting backward risk can lead to misleading conclusions on agents’ optimal behaviour.

The introduction of an uninsurable background risk into the theory of optimal choice under uncertainty can be implemented in different forms. Some works (e.g. Gollier and Pratt, 1996, Eeckhoudt et al., 1996) introduce an uninsurable background risk in a one-argument utility function. Others (e.g. Pratt, 1988; and Finkelshtain et al., 1999) introduce a non-financial background risk using a two-argument utility. This

second approach can distinguish between those risks which affect income and those, such as health or environmental risk, which directly determine a reduction in utility due to non-financial reasons.¹

In a recent paper, Courbage and Rey (2007) analyse the conditions for the existence of a positive precautionary saving in the presence of a non-financial background risk in some specific cases. Courbage and Rey show that the usual condition on the third derivative of the utility function is still necessary and sufficient for a positive precautionary saving when the two risks are independent. In two other cases, when either the two risks are Bernoulli distributed variables or they exhibit a first degree stochastic dominance, they compute a more complex set of sufficient conditions. Menegatti (2008) provides the correct interpretation of some of these conditions whose implications were partially misunderstood in Courbage and Rey's work.

The effect of background risk on the precautionary saving motive is an important issue to be studied. Examining saving determination in a two-risk framework is, however, potentially related to a more complex analysis. In fact the presence of two risks implies not only that we have different contemporaneous effects on saving, but also that we have different sources of saving related to different problems. In order to illustrate this issue, we briefly describe the usual structure of a two-risk model.

We consider a two-period general framework. The consumer has a von Neumann-Morgenstern utility function $u(y, x)$ in period 0 and $v(y, x)$ in period 1, where y is income and x is a non-financial variable (as, for instance, health status or environmental quality). We have a sure income y_0 in period 0 and an uncertain income in period 1 $\tilde{y}_1 = \bar{y}_1 + \tilde{u}$ where \tilde{u} is a random variable and \bar{y}_1 is not random. In addition the individual faces a non-financial background risk $\tilde{\varepsilon}$ in the second period. This implies that x has a sure value x_0 in period 0 while it has the random value $\tilde{x}_1 = \bar{x}_1 + \tilde{\varepsilon}$ in period 1, where the term \bar{x}_1 is not random. We finally assume $E[\tilde{u}] = 0$ and $E[\tilde{\varepsilon}] = 0$ implying $E[\tilde{y}_1] = \bar{y}_1$ and $E[\tilde{x}_1] = \bar{x}_1$.

The above general framework can be used to study different problems derived from it. In particular, we can potentially examine four cases introducing different assumptions on y_1 and x_1 :

- a) income risk and background risk (both y_1 and x_1 random);
- b) income risk and no background risk (y_1 random and $\tilde{x}_1 = \bar{x}_1$);
- c) background risk and no income risk ($\tilde{y}_1 = \bar{y}_1$ and x_1 random);
- d) no uncertainty ($\tilde{y}_1 = \bar{y}_1$ and $\tilde{x}_1 = \bar{x}_1$).

Comparing these four cases in pairs we can examine different problems of saving determination and we can thus get information about different "sources" of saving under uncertainty.

Comparing cases b) and d) we examine the effect of the income risk on saving neglecting the presence of the background risk. The question we ask in this case is: what is the effect of income risk on saving when this is the unique source of uncertainty? Or, more precisely, what is the necessary and sufficient condition ensuring that income risk increases saving when the agent does not bear other risks? This is the analysis of precautionary saving in the one-risk framework and it is the issue studied in

¹ Obviously risks such as health risk and environmental risk also have a financial dimension which is included in income risk. On this point see the discussion in section 4.

the traditional literature on precautionary saving starting from Leland (1968), Sadmo (1970) and Drèze and Modigliani (1972).

Comparing cases a) and c) we study the effect of income risk on saving taking into account the presence of a non-financial background risk. The question we ask in this case is: what is the effect of income risk on saving when there is also a non-financial background risk? Or, more precisely, what is the necessary and sufficient condition ensuring that income risk increases saving when the agent also bears another non-financial uninsurable risk? This is the analysis of precautionary saving in the two-risk framework, studied, until now, by Courbage and Rey (2007) in some specific and restrictive cases.

Finally, comparing cases a) and d) we examine the effect on saving of income risk and a non-financial background risk together. The question we ask in this case is: what is the joint effect on saving of income risk and a non-financial background risk? Or, more precisely, what is the necessary and sufficient condition ensuring that, under both income risk and background risk, the level of saving is larger than when there is no uncertainty?²

Examining this last issue, it is easy to see that it is different from the previous one. The comparison of a) and c) studies the eventual extra-saving (called precautionary saving) *due to income risk given the presence of a non-financial background risk*. The comparison of a) and d) studies the extra-saving *due to the contemporaneous presence of both income risk and a non-financial background risk*. So the first problem considers background risk only as regards its interaction with income risk while the second one considers its autonomous direct effect on the saving decision too. Finally this means, in turn, that the first problem focuses on a *partial effect* of uncertainty on saving while the second one examines its *total influence* determining the total extra-saving due to uncertainty. In order to distinguish between these two kinds of extra-saving due to uncertainty, we call them respectively: “one-source precautionary saving” (or simply “*precautionary saving*” following the previous literature) and “*two-source precautionary saving*”.

In line with this reasoning it is important to distinguish between precautionary saving and two-source precautionary saving for at least two reasons. First, the difference is relevant to the distinction between what precautionary saving says and what it does not. In particular, since the precautionary saving motive does not consider some effects of uncertainty on saving, the information on agents’ reaction to risk can be misleading if we interpret the strength of that motive as a measure of the total influence of risk. Secondly, precautionary saving and two-source precautionary saving feature different aspects of consumers’ behaviour. This implies that they should be used to examine different economic issues and for inquiry into different elements of saving determinants. Some examples in this field are given in Section Four.

On the basis of the previous reasoning, this work examine optimal saving under uncertainty in a two-risk framework, analysing two main issues.

The first aim of this work is to reconsider the issue of precautionary saving in a two risk-framework in the general case. In this, our goal is to find the general necessary and

² For completeness, two other comparisons are possible between cases c) and d) and cases a) and b). These two comparisons examine the effect of the background risk, respectively neglecting and taking into account income risk (i.e. either in a one-risk or in a two-risk framework). The potential relevance of this issue, not studied in this work, seems however limited to the analysis of certain specific problems. Finally a comparison between b) and c) is economically meaningless.

sufficient condition for positive precautionary saving in the presence of a non-financial background risk. We thus aim to determine the condition ensuring a positive precautionary saving in the presence of a background risk without introducing assumptions on the form of the two risks, and we try to provide an economic interpretation for it.

The second aim of this paper is to study two-source precautionary saving. Our goals in this field are to find the necessary and sufficient condition for the presence of a positive extra-saving due to uncertainty, to compare this with the condition ensuring a positive precautionary saving, and to examine sources and implications of the differences between them.

Finally, the last part of the paper illustrates some possible applications of the results.

Analysing the above issues, we focus on the case of small risks. As it is well known, this choice is not unusual since, starting from the seminal paper by Pratt (1964), the analysis of small risks, which studies the optimal choice when the variability of random variables generating uncertainty is not great, has been an important field in the literature on decision making under uncertainty.

The paper proceeds as follows. Section Two presents the model and examines precautionary saving in the two-risk framework. Section Three analyses the conjunct effects of income risk and background risk studying two-source precautionary saving. Section Four presents some simple applications of results. Section Five concludes.

2. Precautionary saving in the two-risk framework

We consider the two-risk framework previously described and we assume that consumer preferences are represented by functions $u(y, x)$ and $v(y, x)$ introduced in Section One. Functions u and v are also assumed to be strictly increasing with respect to each argument, strictly concave and three times continuously differentiable. Functions u_i, u_{ij}, u_{ijt} (respectively v_i, v_{ij}, v_{ijt}) denote the first second and third partial derivatives of u (respectively v). For simplicity, we assume that the intertemporal discount rate and the interest rate are null.

In this context, when both y_1 and x_1 are random, the consumer maximisation problem is given by

$$\text{Max}_s u(y_0 - s, x_0) + E[v(\tilde{y}_1 + s, \tilde{x}_1)] \quad (1)$$

Optimal saving under uncertainty s^* in the presence of the two introduced risks is determined by the condition

$$u_1(y_0 - s, H_0) = E[v_1(\tilde{y}_1 + s, \tilde{x}_1)] \quad (2)$$

For our purposes it is useful to consider a second-order two-variable Taylor approximation of $v_1(\tilde{y}_1 + s, \tilde{x}_1)$ around the point (\bar{y}_1, \bar{x}_1) . This approximation is given by

$$\begin{aligned}
v_1(\tilde{y}_1 + s, \tilde{x}_1) &\approx v_1(\bar{y}_1 + s, \bar{x}_1) + v_{11}(\bar{y}_1 + s, \bar{x}_1)(\tilde{y}_1 - \bar{y}_1) + \\
&+ v_{12}(\bar{y}_1 + s, \bar{x}_1)(\tilde{x}_1 - \bar{x}_1) + \frac{1}{2} \left[v_{111}(\bar{y}_1 + s, \bar{x}_1)(\tilde{y}_1 - \bar{y}_1)^2 + \right. \\
&\left. + v_{122}(\bar{y}_1 + s, \bar{x}_1)(\tilde{x}_1 - \bar{x}_1)^2 + 2 v_{112}(\bar{y}_1 + s, \bar{x}_1)(\tilde{y}_1 - \bar{y}_1)(\tilde{x}_1 - \bar{x}_1) \right]
\end{aligned} \tag{3}$$

Given this premise we can now examine the conditions for positive precautionary saving and for positive extra-saving due to uncertainty in the presence of two small risks.

We start studying the condition ensuring that the uncertainty on income generates a positive precautionary saving given the contemporaneous presence of a background risk. This can be done by comparing the level of saving s^* determined in (2) when both y_1 and x_1 are uncertain, with the level of saving (s°) that occurs when there is no uncertainty on future income (i.e. when $\tilde{y}_1 = \bar{y}_1$ and x_1 is random).

When there is no income risk the optimal level of saving is determined by a condition similar to (2) where \tilde{y}_1 is replaced by \bar{y}_1 . Comparing this optimality condition with (2) we have that $s^* \geq s^\circ$ if and only if

$$E[v_1(\tilde{y}_1 + s, \tilde{x}_1)] - E[v_1(\bar{y}_1 + s, \tilde{x}_1)] \geq 0 \tag{4}$$

In the case of small risks, we can substitute equation (3) into equation (4). We make this substitution twice: both in term $E[v_1(\tilde{y}_1 + s, \tilde{x}_1)]$ and in term $E[v_1(\bar{y}_1 + s, \tilde{x}_1)]$, in this second case with $\tilde{y}_1 = \bar{y}_1$. Computations yield

$$v_{111}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{y}_1] + 2 v_{112}(\bar{y}_1 + s, \bar{x}_1) \text{cov}[\tilde{y}_1, \tilde{x}_1] \geq 0 \tag{5}$$

where $\text{var}[\tilde{y}_1]$ is the variance of \tilde{y}_1 and $\text{cov}[\tilde{y}_1, \tilde{x}_1]$ is the covariance between \tilde{y}_1 and \tilde{x}_1 . We have thus that

Proposition 1.

Condition (5) is necessary and sufficient to have a positive precautionary saving ($s^ \geq s^\circ$).*

Furthermore, since $(s^* - s^\circ)$ is an increasing function of the left-hand side of (4) we also have that

Corollary 1

The precautionary saving ($s^ - s^\circ$) is an increasing function of the value of $v_{111}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{y}_1] + 2 v_{112}(\bar{y}_1 + s, \bar{x}_1) \text{cov}[\tilde{y}_1, \tilde{x}_1]$.*

These results show the general condition for a positive precautionary saving in the presence of a background risk. The condition involves two terms. The first term depends on the sign of v_{111} and favours precautionary saving if v_{111} is positive. It captures the “direct” effect of income risk which characterises precautionary saving in the usual

one-risk framework. A new economic interpretation of this condition in the one-risk framework is provided by Menegatti (2007), who shows that $v_{111} > 0$ is a necessary and sufficient condition to get that a larger wealth implies a reduction in the disutility due to income risk, generating an incentive to increase saving in order to raise consumption in the future, when income is uncertain.

The second term involves the covariance between the two risks and the cross derivative of the marginal utility function, and captures the interaction between the two risks. This interaction depends on two elements. The sign of the covariance indicates if the shocks on \tilde{x}_1 tend to reinforce the shocks on \tilde{y}_1 (positive covariance) or to counteract them (negative covariance). The sign of v_{112} indicates how this first element acts on marginal utility v_1 .

An interpretation of the sign of v_{112} is provided by Eeckhoudt *et al.* (2007), who show that $v_{112} > 0$ is necessary and sufficient for “cross-prudence in the background variable”, meaning that a higher level of the background variable mitigates the detrimental effect of the income risk.

In order to clarify condition (5), consider the following case. Assume v_{111} and v_{112} positive and $\text{cov}[\tilde{y}_1, \tilde{x}_1]$ negative. In this case it is possible that, although we have a positive direct effect of income risk on saving, there is no positive precautionary saving. This can occur because the interaction term can generate a reduction in the variability of marginal utility v_1 since positive (negative) shocks on \tilde{y}_1 are counteracted by negative (positive) shocks on \tilde{x}_1 .

The opposite occurs if we consider the same set of assumptions but $\text{cov}[\tilde{y}_1, \tilde{x}_1]$ positive. In this case, the interaction term strengthens the direct effect. Indeed the interaction between the two risks causes an increase in the variability of marginal utility v_1 since positive (negative) shocks on \tilde{y}_1 are reinforced by positive (negative) shocks on \tilde{x}_1 .

This example also suggests a further result related to condition (5):

Corollary 2

The presence of positive precautionary saving in the one risk framework does not imply the presence of positive precautionary saving in the two-risk framework and vice versa.

Corollary 2 indicates that the conditions for a positive precautionary saving in the one-risk framework ($v_{111} \geq 0$) and the condition in the two-risk framework (expressed by (5)) can hold independently. This implies that neglecting background risk can produce either an underestimation or an overestimation of the precautionary saving motive. This result confirms the conclusions of many other works showing the potential misleading consequences of neglecting the presence of background risks in the analysis of decision making under uncertainty in different frameworks (e.g. Gollier and Prat, 1996; Eeckhoudt et al., 1996 and Dana and Scarsini, 2006).

Finally, as mentioned in the introduction, Courbage and Rey (2007) examine the conditions for a positive precautionary saving in the presence of two risks in three specific cases: first when income risk and background risk are independent, second when income risk and background risk are Bernoulli distribute variables, and third when income risk and background risk exhibit a first degree stochastic correlation.

Courbage and Rey show in particular that, in the case of independent risks, $v_{111} \geq 0$ is a necessary and sufficient condition for a positive precautionary saving. In the other two cases they show that the following sets of sufficient conditions for a positive precautionary saving hold.³

a) Bernoulli distributed variables:

- $v_{111} \geq 0$;
- either $v_{112} \geq 0$ and positive correlation between the two risks or $v_{112} \leq 0$ and negative correlation between the two risks.

b) First degree stochastic dominance

- $v_{111} \geq 0$;
- either $v_{112} \geq 0$ and positive first-degree stochastic correlation between the two risks or $v_{112} \leq 0$ and negative first-degree stochastic correlation between the two risks.

It is easy to see that the result in Proposition 1 generalises Courbage and Rey's conclusions. Examining condition (5) we firstly see that, since, for independent risks, we have $\text{cov}[\tilde{y}_1, \tilde{x}_1] = 0$, the condition indicates the same result as Courbage and Rey. Furthermore, again from condition (5), we have that

Corollary 3

The two conditions $v_{111}(\bar{y}_1 + s, \bar{x}_1) \geq 0$ and $v_{112}(\bar{y}_1 + s, \bar{x}_1)\text{cov}[\tilde{y}_1, \tilde{x}_1] \geq 0$ are sufficient to have positive precautionary saving ($s^ \geq s^\circ$).*

The formulation of Corollary 3 is equivalent to that used by Courbage and Rey in the two other specific cases they study. In fact, the additional sufficient conditions shown by Courbage and Rey are simply the specification of the general condition $v_{112}(\bar{y}_1 + s, \bar{x}_1)\text{cov}[\tilde{y}_1, \tilde{x}_1] \geq 0$ in Corollary 3 referred to each specific case they examine.

3. Two-source precautionary saving

Section 2 looks at precautionary saving due to a small income risk in the presence of a small non-financial background risk. As stated in the introduction, however, the presence of a background risk not only alters the effect of income risk on saving but also has an autonomous influence on it. Taking this effect into account means studying the joint influence of income risk and background risk and determining the total variation in saving due to uncertainty (i.e. two-source precautionary saving). Here too our analysis focuses on small risks, as our analysis of precautionary saving.

In order to examine this issue we consider a formulation which differs from the previous one and compares the optimal saving s^* (determined when there is uncertainty on both y and x) and the optimal saving (s°) when we have no uncertainty at all (i.e. when both $\tilde{y}_1 = \bar{y}_1$ and $\tilde{x}_1 = \bar{x}_1$).

³ These conclusions take into account the corrections in Menegatti (2008).

When there is no uncertainty the optimal level of saving is determined by a condition similar to (2) where \tilde{y}_1 is replaced by \bar{y}_1 and \tilde{x}_1 is replaced by \bar{x}_1 . Comparing this optimality condition with (2) we have that $s^* \geq s^{\circ\circ}$ if and only if

$$E[v_1(\tilde{y}_1 + s, \tilde{x}_1)] - v_1(\bar{y}_1 + s, \bar{x}_1) \geq 0 \quad (6)$$

In the case of small risks, we can substitute equation (3) into equation (6). Again we make this substitution twice: both in term $E[v_1(\tilde{y}_1 + s, \tilde{x}_1)]$ and in term $v_1(\bar{y}_1 + s, \bar{x}_1)$, in this second case with $\tilde{y}_1 = \bar{y}_1$ and $\tilde{x}_1 = \bar{x}_1$. Computations yield

$$\begin{aligned} & v_{111}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{y}_1] + 2 v_{112}(\bar{y}_1 + s, \bar{x}_1) \text{cov}[\tilde{y}_1, \tilde{x}_1] + \\ & + v_{122}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{x}_1] \geq 0 \end{aligned} \quad (7)$$

which implies that:

Proposition 2

Condition (7) is necessary and sufficient to have a positive two-source precautionary saving, i.e. , i.e. an extra-saving due to both income risk and background risk ($s^ \geq s^{\circ\circ}$).*

Furthermore, for a reason similar to that in Section Two we also have that

Corollary 4

The two-source precautionary saving ($s^ - s^{\circ\circ}$) is an increasing function of the value of $v_{111}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{y}_1] + 2 v_{112}(\bar{y}_1 + s, \bar{x}_1) \text{cov}[\tilde{y}_1, \tilde{x}_1] + v_{122}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{x}_1]$.*

Condition (7) includes three terms. The first two are the same we have in condition (5) while the third one, absent in condition (5), involves the sign of v_{122} and the variance of \tilde{x}_1 . This term measures the direct effect of background risk on saving and exhibits a clear isomorphism with the first term in (5) and in (7), which measures the direct effect of income risk. A positive v_{122} implies that the direct effect of background risk is positive (exactly as a positive v_{111} implies a positive direct effect of income risk).

This interpretation is reinforced by Eeckhoudt *et al.* (2007), who show that $v_{122} > 0$ is a necessary and sufficient condition for “cross-prudence in wealth”, meaning that a higher wealth reduces that detrimental effect of the background risk. It is easy to see that this generates a positive incentive to increase saving in order to raise consumption in the future, when the level of the background variable is uncertain.

A comparison between (5) and (7) also suggests a further result partially related to the previous reasoning:

Corollary 5

A positive two-source precautionary saving ($s^ \geq s^{\circ\circ}$) does not imply a positive precautionary saving ($s^* \geq s^{\circ}$).*

In the presence of a positive precautionary saving ($s^* \geq s^\circ$), the condition $v_{122} \geq 0$ is sufficient to ensure a positive two-source precautionary saving ($s^* \geq s^{\circ\circ}$).

Corollary 5 suggests that, in general, conditions (5) and (7) are independent since each can hold if the other one does not.

Given this general conclusion, we can examine an interesting specific example. Assume that v_{111} , v_{122} and v_{112} are positive and $\text{cov}[\tilde{y}_1, \tilde{x}_1]$ is negative and assume that

$$\begin{aligned} v_{111}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{y}_1] &\leq 2v_{112}(\bar{y}_1 + s, \bar{x}_1) |\text{cov}[\tilde{y}_1, \tilde{x}_1]| \leq \\ &\leq v_{111}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{y}_1] + v_{122}(\bar{y}_1 + s, \bar{x}_1) \text{var}[\tilde{x}_1] \end{aligned} \quad (8)$$

In this case we can obtain different results by considering different problems. If we examine the effect of income risk taking into account the presence of a background risk (i.e. if we examine precautionary saving in a two-risks framework), we get a negative effect on saving. On the contrary, if we analyse the total influence of uncertainty on saving, by considering the effect of both income risk and background risk, we get a positive effect. We have thus a negative precautionary saving and a positive two-source precautionary saving.

It is important to emphasise that these different conclusions do not conflict since they refer to different problems and thus to different variables.⁴ However, they indicate that, when considering a two-risk framework, it is relevant to clearly distinguish between precautionary saving and two-source precautionary saving and remember these two variables provide information on different aspects of the choice of saving level.

4. Some simple applications

The effect of a mean-preserving spread

A very simple application of the previous results can easily be obtained by studying the effects on saving of a mean-preserving spread, which increases the variability of future income.⁵ Does the spread increase precautionary saving? The intuitive answer is affirmative. Since the spread raises the variability of future income it should also increase the extra-saving due to income risk.

The traditional one-risk framework confirms this conclusion. Indeed a mean-preserving spread implies that the distribution of future income is described by a

⁴ See Section 1.

⁵ The concept of mean-preserving spread was firstly introduced by Rothschild and Stiglitz (1970). From this work we can derive the following informal description. Given a random variable with a density function f , a mean-preserving spread is obtained “by taking some of the probability weight from the centre of f and adding to each tail in such a way as to leave the mean unchanged” [Rothschild and Stiglitz, 1970, p. 226]. For a formal definition see Rothschild and Stiglitz.

random variable \hat{y}_1 with $E[\tilde{y}_1] = E[\hat{y}_1]$ and $\text{var}[\hat{y}_1] > \text{var}[\tilde{y}_1]$.⁶ Under the assumption that $v_{111} > 0$, this implies, in turn, an increase in precautionary saving.

A two-risk framework gives a different conclusion. As shown by Corollary 1, the effect of shock on precautionary saving depends not only on the comparison between $\text{var}[\tilde{y}_1]$ and $\text{var}[\hat{y}_1]$ but also on the comparison between $\text{cov}[\tilde{y}_1, \tilde{x}_1]$ and $\text{cov}[\hat{y}_1, \tilde{x}_1]$. In particular, if either $v_{112} > 0$ and $\text{cov}[\hat{y}_1, \tilde{x}_1] < \text{cov}[\tilde{y}_1, \tilde{x}_1]$ or $v_{112} < 0$ and $\text{cov}[\hat{y}_1, \tilde{x}_1] > \text{cov}[\tilde{y}_1, \tilde{x}_1]$, the level of precautionary saving after the shock can be lower.

This implies the counterintuitive result that in a two-risk framework a mean-preserving spread increasing income variability can reduce precautionary saving. This can occur since the spread raises the direct effect of income risk but it can contemporaneously reduce the effect of uncertainty due to income variability on marginal utility v_1 because of the interaction between income risk and background risk.

A second relevant case can be obtained by introducing a mean-preserving spread increasing the variability of x_1 . What are the effects of this second shock?

A first result concerns two-source precautionary saving. The mean-preserving spread implies that the distribution of x_1 is described by the new random variable \hat{x}_1 with $E[\tilde{x}_1] = E[\hat{x}_1]$ and $\text{var}[\hat{x}_1] > \text{var}[\tilde{x}_1]$. By Corollary 4, under the assumption that $v_{122} > 0$, the increase in the variability of x_1 raises two-source precautionary saving. However, the total variation in extra-saving also depends on the comparison between $\text{cov}[\tilde{y}_1, \tilde{x}_1]$ and $\text{cov}[\tilde{y}_1, \hat{x}_1]$. If either $v_{112} > 0$ and $\text{cov}[\hat{y}_1, \tilde{x}_1] < \text{cov}[\tilde{y}_1, \tilde{x}_1]$ or $v_{112} < 0$ and $\text{cov}[\hat{y}_1, \tilde{x}_1] > \text{cov}[\tilde{y}_1, \tilde{x}_1]$, it is possible that the mean-preserving spread on x_1 reduces two-source precautionary saving.

A second result concerns precautionary saving. In fact, since the mean-preserving spread affects the covariance between income risk and background risk, we see, by Corollary 1, that the level of precautionary saving will also change. This result implies that in a two-risk framework the level of precautionary saving can change even if the distribution of future income does not.

Saving and Insurance

Sections Two and Three show that the presence of a small non-financial uninsurable background risk affects the level of saving. This conclusion provides new insight into the effects on saving of risks such as health risk and environmental risk. In fact, these kinds of risk clearly exhibit both a financial and non-financial dimension. For instance the risk of catching a disease implies the risk of paying for medicines. This financial risk is included in the income risk (as a possible negative realisation of $\tilde{\epsilon}$) and its effect on saving is described by traditional precautionary saving theory. The health risk, however, also has a non-financial dimension due to the risk of physical suffering and other disadvantages of illness. This second dimension is measured by the background risk. The results in Sections Two and Three show that this dimension too has an effect on saving. This means that the non-financial dimension of the health risk and the

⁶ Obviously these conditions alone are not sufficient for a mean-preserving spread, while a mean-preserving spread is sufficient to ensure an increase in variance.

environmental risk generate a change in optimal intertemporal allocation of monetary resources.

This reasoning also has implications for the effect of insurance on saving. Assume for instance that an agent can buy a full insurance against the variability of future income by paying in period 0 the appropriate insurance premium. What is the effect of this insurance on saving? It is easy to see that the insurance will completely nullify precautionary saving since it will remove any variability of y_1 . However it would be mistaken to conclude from this that the level of saving is now the same as in the absence of uncertainty. Indeed, as shown by (7), the variability of x_1 , which is not nullified, is sufficient to ensure a positive extra-saving. The presence of a non-financial background risk, such as a non-financial health or environmental risk, implies that the extra-saving due to uncertainty is positive even when future income is fully insured.

The same conclusion can be applied to specific insurance against health or environmental costs. This kind of insurances removes the financial dimension of health or environmental risk but not the non-financial one. This implies that, even in the presence of specific insurances, risks such as health or environmental risks generate variations in the saving level.

5. Conclusions

This work examines optimal saving under uncertainty in the presence of two small risks: income risk and non-financial background risk. Two main issues are studied.

First we determine the necessary and sufficient condition for a positive precautionary saving in the presence of a non-financial background risk in the general case. We show that this condition depends on two terms. The first term captures the direct effect of income risk (similar to that present in the one-risk framework) and the second term captures the interaction between the two risks. We also show that this result generalises, for small risks, the analysis by Courbage and Rey (2007) who study the same problem under some specific and restrictive assumptions on the form of the two risks. Finally we suggest that neglecting background risk can cause either an underestimation or an overestimation of the precautionary saving motive.

Second we study the total effect of uncertainty on saving by analysing the joint influence of income risk and a non-financial background risk. The analysis shows that, in this problem, the two previously described effects are flanked by a third one capturing the direct effect of background risk. We also show that the condition for a positive precautionary saving and the condition for a positive two-source precautionary saving can hold independently. This means that we can have a negative precautionary saving together with a positive increase in saving due to the contemporaneous effects of income risk and background risk and vice versa.

This work thus extends to the theory of saving under uncertainty the conclusion that neglecting background risk can generate misleading conjectures about agent's optimal behaviour. In the case of saving this conclusion has a double dimension. On one hand, taking into account the background risk changes the rule determining precautionary saving, so that the results obtained on precautionary saving in a one-risk framework can be wrong. On the other hand, the presence of a background risk implies that precautionary saving does not coincide with two-source precautionary saving. For this

reason, the results on precautionary saving could be misleading if they were interpreted as conclusions on the entire effect of uncertainty on saving.

Finally, the results have implications for many economic problems. Some of these are discussed in the paper: the consequences of a mean-preserving spread on either income risk or background risk and the effect on saving of insurance against future income variability.

6. References

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