

Mean-Variance Econometric Analysis of Household Portfolios

Sergio Pastorello*

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Abstract

I investigate households' portfolio choice using a microeconomic approach derived from mean-variance optimization. I assume that households can't take short positions in the risky assets. Assuming two such assets, I derive an explicit solution of the model which gives rise to structural probit and tobit specifications characterized by two observable and three latent variables. Both specifications are estimated by maximum likelihood on a cross section of Italian households. The tobit specification is simulated in order to evaluate the regressors' effects on regimes probabilities and asset demands, and its correctness is tested using the conditional moments approach.

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*Dipartimento di Scienze Economiche, Università di Bologna, Piazza Scaravilli, 2, 40126 Bologna, Italy. Email: pastorel@economia.unibo.it. I thank Giovanni D'Alessio of Bank of Italy for providing me with the data, and Pravin Trivedi and seminar participants at the Department of Economics at the University of Bologna, the ASSET 2004 Annual Meeting (Barcelona), and the 2005 ICEEE (Venezia) for helpful comments and suggestions. Any remaining errors are my own. Financial support from the MIUR prin 2005 program is gratefully acknowledged.

1 Introduction

The analysis of household financial portfolios has been a hot topic of research in recent years, also thanks to the fact that reliable datasets have become increasingly available for a number of countries in the last 10 to 20 years. As surveyed in the excellent book by Guiso, Haliassos and Jappelli (2002), the analysis of household portfolios poses to economists a number of interesting issues. Just to cite a few of them, the frequently observed extreme lack of diversification, the limited participation in holding risky assets, some important differences in portfolios held by similar households in different countries, and the inability of the standard portfolio models to account for some empirical stylized facts. Apart from these microeconomic puzzles, the study of the household portfolio choice mechanism is interesting also for other reasons, e.g. its close link with retirement saving, which makes it a key component of the analysis of the effectiveness of economic policies aiming at improving the level and the distribution of retirement income, as well as the efficiency of financial markets.

The study of the households' portfolio choice is complicated also because it is likely subject to the effect of many factors, all of whom can be loosely reunited under the collective label of "heterogeneity", be it between individuals, time periods or countries. These factors include, at the individual level, the presence of a variable fraction of the households' wealth is invested in housing, or, more generally, in real illiquid assets (Hochguertel and van Soest, 2001; Flavin and Yamashita, 2002; Pelizzon and Weber, 2003), the presence of labor income risk (Guiso, Jappelli and Terlizzese, 1996) or of entrepreneurial risk (Heaton and Lucas, 2000). Between countries and time periods, relevant factors are given, among others, by tax effects (Hochguertel, Alessie and van Soest, 1997), institutional arrangements for retirement saving and capital markets imperfections (Kapteyn and Panis, 2003).

In this paper I focus on cross-sectional phenomena, such as the apparent lack of di-

versification in household portfolios, and the low fraction of households investing in risky assets. Many authors have tried to justify this observation using some of the factors cited above, such as various structures of fixed and variable transaction, holding and monitoring costs, borrowing and liquidity constraints, or short sale constraints. Most of these studies, however, share the common feature of being based on reduced form specifications which do not completely exploit the restrictions placed by the theoretical utility maximization model on the econometric specification and on the link between the discrete and the continuous components of portfolio choice.

There are two notable exceptions to this observation. The first is Perraudin and Sørensen (2000), who base their qualitative and quantitative econometric investigation on an application of Roy's identity. The latter, which is a standard result in consumer theory, states that the demand functions for risky assets are given by ratios of partial derivatives of the indirect utility function. They consider three asset categories, given by money (M), bonds (B) and stocks (S). Each household necessarily invests in the former, but it can freely choose whether to invest in the latter two. Among the resulting four possible regimes (M, MB, MS and MBS), the one composed by money and stocks is dropped because it is very rarely observed in their sample. Each household chooses between the remaining three regimes by comparing the associated indirect utilities, which depend on a number of household's demographic and economic indicators according to an econometric specification obtained by slightly generalizing the one deriving from HARA preferences. To model zero holdings, Perraudin and Sørensen (2000) introduce monitoring costs in the form of a linear parametric function of the household demographic characteristics. They also allow for endogenous selection by generalizing the approach in Dubin and McFadden (1984). This is a crucial point in their analysis, because it allows to add an error term in the demands equations for risky assets, which are derived from the indirect utilities using Roy's identity. The result is a discrete-continuous econometric model which has a multinomial

logit qualitative component, and a nonlinear regression quantitative component. They implement their approach on a cross-section of U.S. households from the 1983 Survey of Consumer Finances. As this is a random sample of the U.S. population, they are able to simulate the model to estimate the aggregate effects on money, bonds and stocks demands of demographic and economic shocks.

The second exception is given by Fougère, Gouriéroux, Tiomo and Trognon (1997), who develop their analysis in the well known mean-variance paradigm which is the basis of Markowitz (1959) standard portfolio efficiency theory. Zero holdings can be modelled in this framework by assuming that households can not take short positions on risky assets. By explicitly solving the model in the case of three assets, and by assuming that the expected excess returns on the risky assets are positive, it is possible to show that both the regime choice and the asset demands depend on two latent variables, identified as the ratio of the Sharpe performances of the two risky assets (τ), and the correlation coefficient between their excess returns (ρ), given by the difference between their end of period payoff and the initial price capitalized at the riskless asset. Given an econometric specification for these two latent variables, they derive a completely structural discrete-continuous model for regime choice and risky asset demands. Their approach is based on the assumption that each household chooses its portfolio on the mean-variance efficient frontier defined by its preferred values of τ and ρ . The problem is to estimate how these key quantities depend on the demographic and economic characteristics of the household.

This strategy exhibits the usual trade-off of structural econometrics: it allows less flexibility w.r.t. a fully reduced form analysis, but the results it provides are much more easy to interpret in terms of household behavior. I find that this is a non negligible advantage. For this reason, this paper builds upon Fougère, Gouriéroux, Tiomo and Trognon (1997), and extends their analysis in several directions. First of all, I relax the assumption of positive expected excess returns, mainly because under this assumption it

would be impossible to explain the very high fraction of households holding neither risky asset commonly observed in most samples. However, it is also quite reasonable to imagine that some households (temporarily) expect them to provide a negative risk premium, especially if expected excess returns are evaluated net of any fixed and variable transaction, holding and monitoring costs. With this modification, I show that regime choice and risky assets demands are driven by three latent variables: the Sharpe performances of the two risky assets divided by the risk aversion coefficient (π_1 and π_2), and the correlation coefficient between their excess returns, ρ . Second, I model these three latent variables as (up to a nonlinear transformation for ρ) functions of the household demographic and economic characteristics which are linear in some unknown parameters, plus three error terms which are allowed to be correlated conditionally on the characteristics. Under this assumption, the qualitative econometric specification is a structural probit model, and the quantitative one is a structural tobit model. Although both specifications are characterized by a three dimensional vector of latent variables, it is easy to check that one of them plays the role of an heterogeneity factor which has an observable and an unobservable component; in the notation introduced in the following section, this is typically the case of ρ . Both models are estimated on a cross-section of Italian households from the 2000 Survey of Households Income and Wealth. Finally, I simulate the model to evaluate the effect of aggregate demographic and economic shocks on participation patterns and asset demands, and I perform a variety of model specification tests.

The paper is organized as follows. The next section outlines the theoretical model, and explicitly solves it in the two risky assets case. In section 3 I derive two alternative econometric specifications of the portfolio choice model: a purely discrete (structural probit) one, and a mixed discrete-continuous (structural tobit) alternative. I compute the individual likelihood contributions and briefly discuss the parametric identification in both cases. The models have been estimated by maximum likelihood using the data

described in section 4, and the results obtained are illustrated in section 5. Finally, model simulations and specification tests are reported in sections 6 and 7. Section 8 concludes.

2 The mean-variance portfolio selection model

In this section I present the problem faced by a household wishing to choose the optimal allocation of its wealth among the available assets, and assuming that risky assets can not be sold short. This section is based on section 2 of Fougère, Gouriéroux, Tiomo and Trognon (1997).

2.1 The problem

I consider a variant of the standard Markowitz (1959) mean-variance single period framework. At the initial date, the household chooses the optimal allocation of its wealth W_0 among one riskless asset and N risky assets. I assume that households are price takers, and that they are subject to two kinds of constraint: a standard budget constraint, and N nonnegativity constraints corresponding to the shares of wealth allocated to the risky assets. This means that households can not take a short position in the risky assets.

Let r_0 denote the interest rate on the riskless asset, and $\mathbf{z} = (z_1, \dots, z_N)'$ be the random (as of the initial date) vector of the excess returns on the risky assets. If $\mathbf{a} = (a_1, \dots, a_N)'$ is the vector of shares of financial wealth invested in the risky assets, the random end of period wealth is given by

$$W_1 = W_0(1 + r_0 + \mathbf{a}'\mathbf{z}). \quad (1)$$

The investors' preferences are assumed to be described by a standard mean-variance expected utility function:

$$\mathbb{E}[U(\mathbf{a})] = \mathbb{E}(W_1) - \frac{\eta}{2} \text{Var}(W_1),$$

where $\eta > 0$ is a risk aversion index, which varies across households. Inspection of (1)

shows that the moments of W_1 depend on those of excess returns vector \mathbf{z} . If we let

$$\boldsymbol{\mu} = \mathbb{E}(\mathbf{z}) \quad \text{and} \quad \boldsymbol{\Omega} = \text{Var}(\mathbf{z}),$$

then the expected utility can be rewritten as:

$$\mathbb{E}[U(\mathbf{a})] = (1 + r_0)W_0 + W_0 \left(\mathbf{a}'\boldsymbol{\mu} - \frac{\eta W_0}{2} \mathbf{a}'\boldsymbol{\Omega}\mathbf{a} \right).$$

The household portfolio selection problem can hence be formally stated as follows:

$$\max_{\mathbf{a} \in \mathbb{R}_+^N} \left(\mathbf{a}'\boldsymbol{\mu} - \frac{\eta W_0}{2} \mathbf{a}'\boldsymbol{\Omega}\mathbf{a} \right). \quad (2)$$

2.2 The solution

2.2.1 The general N assets case

To solve the constrained problem (2), it is convenient to consider the set of unconstrained problems associated to the 2^N possible subsets of the available N risky assets. Let I denote a generic subset of the N risky assets, and let \mathbf{a}^I , $\boldsymbol{\mu}^I$ and $\boldsymbol{\Omega}^I$ be the corresponding subsets of \mathbf{a} , $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$. Consider now the associated unconstrained problem:

$$\max_{\mathbf{a}^I} \left[(\mathbf{a}^I)'\boldsymbol{\mu}^I - \frac{\eta W_0}{2} (\mathbf{a}^I)'\boldsymbol{\Omega}^I\mathbf{a}^I \right].$$

The solution of this problem is given by:

$$\mathbf{a}_*^I = \frac{1}{\eta W_0} (\boldsymbol{\Omega}^I)^{-1} \boldsymbol{\mu}^I, \quad (3)$$

with the associated optimal value of the objective function:

$$\frac{1}{2\eta W_0} (\boldsymbol{\mu}^I)'(\boldsymbol{\Omega}^I)^{-1}\boldsymbol{\mu}^I. \quad (4)$$

Notice that $(\boldsymbol{\mu}^I)'(\boldsymbol{\Omega}^I)^{-1}\boldsymbol{\mu}^I$ is the squared maximum Sharpe performance which can be attained using the assets in I . It follows from (3) and (4) that the solution of the constrained problem (2) can be expressed as:

$$I_* = \arg \max_{I \in \mathcal{J}} (\boldsymbol{\mu}^I)'(\boldsymbol{\Omega}^I)^{-1}\boldsymbol{\mu}^I \quad \text{with: } \mathcal{J} = \{I : \mathbf{a}_*^I \geq \mathbf{0}\},$$

where the inequality in the definition of \mathcal{J} is intended to hold element-by-element.

In practice, finding the optimal subset of the original N assets does not require to examine all of the 2^N possible subsets, thanks to the so called *Law of conservation of the (squared) Sharpe performances*. This property, which is easily demonstrated with an application of the rule of the inverse of a partitioned matrix, states that if I and J are two subsets of the set of N assets, and if $J \subset I$, then the squared Sharpe performance of J can not be greater than the squared Sharpe performance of I :

$$\text{If } J \subset I \quad \Rightarrow \quad (\boldsymbol{\mu}^J)'(\boldsymbol{\Omega}^J)^{-1}\boldsymbol{\mu}^J \leq (\boldsymbol{\mu}^I)'(\boldsymbol{\Omega}^I)^{-1}\boldsymbol{\mu}^I.$$

This result allows to solve the constrained problem hierarchically, starting from the unconstrained problem associated to the whole set of N assets, and dropping one asset at a time. Whenever a subset I is found such that the optimal shares invested \mathbf{a}_*^I are nonnegative, it is unnecessary to examine any other subset included in I .

2.2.2 The $N = 2$ asset case

To explicitly solve the problem I consider the special case of $N = 2$ assets. Let

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Omega} = \begin{pmatrix} \omega_1^2 & \omega_{12} \\ \omega_{12} & \omega_2^2 \end{pmatrix}$$

be the expected value and the covariance matrix of the excess returns associated to asset 1 and 2. Further, let

$$\rho = \frac{\omega_{12}}{\omega_1\omega_2}$$

be the correlation coefficient between the two excess returns, and

$$\pi_1 = \frac{\mu_1}{\eta\omega_1} \quad \text{and} \quad \pi_2 = \frac{\mu_2}{\eta\omega_2}$$

be the respective Sharpe performances divided by the risk aversion coefficient η . With two assets, there are four possible regimes, that I shall denote with $(0, 0)$, $(1, 0)$, $(0, 1)$ and

(1, 1), where a 0 in the first (second) position indicates that the first (second) asset is not included in the portfolio, while 1 indicates inclusion.

Consider first the solution of the unconstrained problem for the set of two assets, i.e. regime (1, 1). The optimal shares invested are given by:

$$\mathbf{a}_*^{(1,1)} = \frac{1}{\eta W_0 (\omega_1^2 \omega_2^2 - \omega_{12}^2)} \begin{bmatrix} \omega_2^2 \mu_1 - \omega_{12} \mu_2 \\ -\omega_{12} \mu_1 + \omega_1^2 \mu_2 \end{bmatrix} = \frac{1}{W_0 (1 - \rho^2)} \begin{bmatrix} (\pi_1 - \rho \pi_2) / \omega_1 \\ (-\rho \pi_1 + \pi_2) / \omega_2 \end{bmatrix}. \quad (5)$$

The (1, 1) regime is the solution of the constrained problem (2) if the two quantities above are strictly positive. This is equivalent to:

$$\pi_1 > \rho \pi_2 \quad \text{and} \quad \pi_2 > \rho \pi_1. \quad (6)$$

Notice that these two conditions can not hold at the same time if both π_1 and π_2 are negative, irrespective of the sign of ρ .

Consider now the (1, 0) regime. In this case,

$$\mathbf{a}_*^{(1,0)} = \begin{bmatrix} \mu_1 / (\eta W_0 \omega_1^2) \\ 0 \end{bmatrix} = \begin{bmatrix} \pi_1 / (W_0 \omega_1) \\ 0 \end{bmatrix}. \quad (7)$$

This regime is the solution of the constrained problem if at least one of the conditions in (6) is violated, but at the same time $\pi_1 > 0$. A similar reasoning applies to the analysis of the (0, 1) regime, for which:

$$\mathbf{a}_*^{(0,1)} = \begin{bmatrix} 0 \\ \mu_2 / (\eta W_0 \omega_2^2) \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_2 / (W_0 \omega_2) \end{bmatrix}.$$

This regime is the solution of (2) if (6) is violated, but $\pi_2 > 0$. Finally, the (0, 0) regime holds whenever both π_1 and π_2 are negative.

The decision diagram for the $N = 2$ assets case is depicted in figure 1, where the horizontal axis measures π_1 and the vertical axis measures π_2 . In the lower left quadrant both π_1 and π_2 are negative, and hence the (0, 0) regime will prevail, irrespective of ρ . The

areas corresponding to the other three regimes depend on ρ through the position of the two straight lines starting at the origin with equation $\pi_2 = \rho\pi_1$ and $\pi_1 = \rho\pi_2$. Specifically, the area comprised between these two lines is associated to the (1, 1) regime. The width of this region is inversely related to ρ . It vanishes for $\rho = 1$, when the two lines coincide, and it progressively grows as ρ decreases, reaching its maximum width when $\rho = -1$, when the two lines jointly form the bisecting line of the upper left and lower right quadrant. Notice that when $\rho = 0$ the two lines coincide with the axis, and hence the decision diagram is identical to that of a standard bivariate discrete or censored dependent variable model. Finally, the regions lying on the right of the vertical axis and below the $\pi_2 = \rho\pi_1$ line, and above the horizontal axis and on the left of the $\pi_1 = \rho\pi_2$ line, respectively correspond to the (1, 0) and the (0, 1) regime.

3 Econometric analysis

In this section I will focus on the econometric implications of the analysis outlined above. As households differ in various respects, such as initial wealth W_0 , risk aversion η , investment horizon, perceived expected excess returns $\boldsymbol{\mu}$ and variances and covariances between excess returns in $\boldsymbol{\Omega}$, so does the solution of (2), which will in general closely depend on these characteristics. Inspection of the results above show that the choices of the regime and of the shares of wealth invested depend on π_1 , π_2 , ρ , ω_1 and ω_2 . Hence, it seems natural to introduce some heterogeneity in the model through these quantities. In this paper I assume that ω_1 and ω_2 are constant unknown parameters, while π_1 , π_2 and ρ differ across households because they depend on observable and unobservable characteristics.

More specifically, let

$$y = \tan\left(\frac{\pi}{2}\rho\right) \quad \Leftrightarrow \quad \rho = \frac{2}{\pi} \arctan y,$$

and assume that

$$\begin{pmatrix} \pi_1 \\ \pi_2 \\ y \end{pmatrix} \Bigg| \mathbf{x} \sim \mathcal{N} \left[\begin{pmatrix} \mathbf{x}'_1 \boldsymbol{\beta}_1 \\ \mathbf{x}'_2 \boldsymbol{\beta}_2 \\ \mathbf{x}'_y \boldsymbol{\beta}_y \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & r_{12} \sigma_1 \sigma_2 & r_{1y} \sigma_1 \sigma_y \\ r_{12} \sigma_1 \sigma_2 & \sigma_2^2 & r_{2y} \sigma_2 \sigma_y \\ r_{1y} \sigma_1 \sigma_y & r_{2y} \sigma_2 \sigma_y & \sigma_y^2 \end{pmatrix} \right], \quad (8)$$

where \mathbf{x} is a basis for $(\mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_y)'$. Also, let $\boldsymbol{\theta} = (\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2, \boldsymbol{\beta}'_y, \sigma_1, \sigma_2, \sigma_y, r_{12}, r_{1y}, r_{2y}, \omega_1, \omega_2)'$ be the vector collecting all the parameters in this specification of the model. Notice that, contrary to Fougère, Gouriéroux, Tiomo and Trognon (1997), I do not assume that all expected excess returns in $\boldsymbol{\mu}$, and consequently that π_1 and π_2 are strictly positive. The reason for this is that in this framework zero holdings in *all* risky assets can be justified only by allowing negative expected excess returns for *all* assets. As it will become apparent in the empirical application, the former regime is by far the most frequently observed in the sample I use.

3.1 Qualitative analysis

The sample I use in the empirical analysis (which is described in section 4) refers a cross-section of Italian households at the end of 2000. For each household, I observe the investment regime, which is a qualitative information, along with the shares of financial wealth invested in the risky assets, which is a quantitative one. Although it might seem natural to exploit both of them, there are at least two important reasons suggesting to also consider a specification using only the qualitative dimension in the data. The first one is that the comparison between the estimation results of two specifications (based on purely qualitative and on qualitative/quantitative observations, respectively) allows to derive several versions of the Hausman specification discussed in section 7. The second one is that the qualitative information may be less affected than the quantitative one by the non-reporting and under-reporting phenomena usually affecting survey data on the households wealth and investment behavior, and may hence provide more robust results.

When only the qualitative information in the observed data is used for estimation purposes, only π_1 , π_2 and ρ are relevant, and hence it is impossible to estimate ω_1 and ω_2 . Under assumption (8), the probabilities of observing each one of the four possible regimes can be easily computed, and are outlined below. To simplify the notation, let $\Phi_2(u_1, u_2; r)$ and $\phi(u_1, u_2; r)$ denote the cdf and the pdf of a bivariate Normal distribution with zero means, unit variances and correlation coefficient r .

To evaluate the probabilities of observing each one of the four possible regimes under assumption (8), it is convenient to first compute the relevant probabilities conditionally on y (or, equivalently, on ρ), and then integrate the resulting expressions w.r.t. the marginal distribution of y , which is easily derived from (8). Notice however that this is not necessary for the probability of regime $(0, 0)$. As the region over which density (8) has to be integrated ($\pi_1 < 0, \pi_2 < 0$) is independent on ρ , it is possible to directly compute the desired probability using the joint marginal cdf of (π_1, π_2) . The result is then given by:

$$p^{(0,0)}(\boldsymbol{\theta}) = \Phi_2 \left(-\frac{\mathbf{x}'_1 \boldsymbol{\beta}_1}{\sigma_1}, -\frac{\mathbf{x}'_2 \boldsymbol{\beta}_2}{\sigma_2}; r_{12} \right). \quad (9)$$

Here and in the following discussion I simplify the notation by omitting the conditioning on \mathbf{x} when this is not cause of confusion.

To evaluate the remaining three terms, I begin by deriving the distribution of the π_1 and π_2 conditional on y :

$$\begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} \Big| (\mathbf{x}, y) \sim \mathcal{N} \left[\begin{pmatrix} \mu_{1|y} \\ \mu_{2|y} \end{pmatrix}, \begin{pmatrix} \sigma_{1|y}^2 & r_{12|y} \sigma_{1|y} \sigma_{2|y} \\ r_{12|y} \sigma_{1|y} \sigma_{2|y} & \sigma_{2|y}^2 \end{pmatrix} \right], \quad (10)$$

where:

$$\begin{aligned} \mu_{i|y} &= \mathbf{x}'_i \boldsymbol{\beta}_i + \frac{r_{iy} \sigma_i}{\sigma_y} (y - \mathbf{x}'_y \boldsymbol{\beta}_y), \quad i = 1, 2, \\ \sigma_{i|y}^2 &= \sigma_i^2 (1 - r_{iy}^2), \quad i = 1, 2, \\ r_{12|y} &= \frac{r_{12} - r_{1y} r_{2y}}{\sqrt{(1 - r_{1y}^2)(1 - r_{2y}^2)}}. \end{aligned}$$

Conditionally on y , the probabilities of regimes (1, 0), (0, 1) and (1, 1) have expressions analogous to (9), albeit with the moments conditional on \mathbf{x} and y above substituting those conditional only on \mathbf{x} . To drop the conditioning on y , I must evaluate the expectation of these expressions w.r.t. the latter variable. If E^y denotes the latter operator, then:

$$p^{(1,0)}(\boldsymbol{\theta}) = E^y[\text{Prob}(\pi_1 > 0, \pi_2 < \rho\pi_1|y)] \quad (11)$$

$$= E^y \left[\Phi_2 \left(\frac{\mu_{1|y}}{\sigma_{1|y}}, -\frac{\mu_{2|y} - \rho\mu_{1|y}}{\sqrt{\sigma_{2|y}^2 + \rho^2\sigma_{1|y}^2 - 2\rho r_{12|y}\sigma_{1|y}\sigma_{2|y}}}; \frac{-r_{12|y}\sigma_{2|y} + \rho\sigma_{1|y}}{\sqrt{\sigma_{2|y}^2 + \rho^2\sigma_{1|y}^2 - 2\rho r_{12|y}\sigma_{1|y}\sigma_{2|y}}} \right) \right],$$

$$p^{(0,1)}(\boldsymbol{\theta}) = E^y[\text{Prob}(\pi_1 < \rho\pi_2, \pi_2 > 0|y)] \quad (12)$$

$$= E^y \left[\Phi_2 \left(-\frac{\mu_{1|y} - \rho\mu_{2|y}}{\sqrt{\sigma_{1|y}^2 + \rho^2\sigma_{2|y}^2 - 2\rho r_{12|y}\sigma_{1|y}\sigma_{2|y}}}, \frac{\mu_{2|y}}{\sigma_{2|y}}; \frac{-r_{12|y}\sigma_{1|y} + \rho\sigma_{2|y}}{\sqrt{\sigma_{1|y}^2 + \rho^2\sigma_{2|y}^2 - 2\rho r_{12|y}\sigma_{1|y}\sigma_{2|y}}} \right) \right],$$

$$p^{(1,1)}(\boldsymbol{\theta}) = E^y[\text{Prob}(\pi_2 > \rho\pi_1, \pi_1 > \rho\pi_2|y)] \quad (13)$$

$$= E^y \left[\Phi_2 \left(-\frac{\rho\mu_{1|y} - \mu_{2|y}}{\sqrt{\rho^2\sigma_{1|y}^2 + \sigma_{2|y}^2 - 2r_{12|y}\rho\sigma_{1|y}\sigma_{2|y}}}, -\frac{-\mu_{1|y} + \rho\mu_{2|y}}{\sqrt{\sigma_{1|y}^2 + \rho^2\sigma_{2|y}^2 - 2r_{12|y}\rho\sigma_{1|y}\sigma_{2|y}}}; \right. \right.$$

$$\left. \frac{r_{12|y}\sigma_{1|y}\sigma_{2|y}(1 + \rho^2) - \rho(\sigma_{1|y}^2 + \sigma_{2|y}^2)}{\sqrt{\rho^2\sigma_{1|y}^2 + \sigma_{2|y}^2 - 2r_{12|y}\rho\sigma_{1|y}\sigma_{2|y}}\sqrt{\sigma_{1|y}^2 + \rho^2\sigma_{2|y}^2 - 2r_{12|y}\rho\sigma_{1|y}\sigma_{2|y}}} \right).$$

Inspection of these probabilities shows that β_1 , β_2 , σ_1 and σ_2 are not separately identified. For this reason, I fix $\sigma_1 = 1$. Notice that β_y and σ_y are separately identified due to the nonlinearity in the relation between ρ and y . Given the above expressions for the probabilities of the single regimes, it is straightforward to derive the loglikelihood function. Let M denote the number of households in the sample, and let:

$$d_m^{(0,0)} = \begin{cases} 1 & \text{if household } m \text{ chooses regime } (0, 0) \\ 0 & \text{otherwise} \end{cases},$$

and similarly for the other three regimes. The loglikelihood function of the sample is then given by:

$$\ell(\boldsymbol{\theta}) = \sum_{m=1}^M \left[\sum_{h=0,1} \sum_{k=0,1} d_m^{(h,k)} \log p^{(h,k)}(\boldsymbol{\theta}) \right]. \quad (14)$$

3.2 Quantitative analysis

I now turn to the quantitative implications of the model in section 2. I will proceed as in the previous section, with the difference that I will estimate the parameters using also the observed shares invested in each asset. The basic assumption about heterogeneity between individuals is still given by (8).

Clearly, the contribution to the likelihood of every observed $(0, 0)$ regime is $p^{(0,0)}(\boldsymbol{\theta})$ defined in (9), because for these observations the observed invested shares are zero. For the remaining three regimes I proceed as in the previous subsection, i.e. I first compute the relevant probabilities/densities conditionally on y , and then drop the conditioning by integrating the resulting expressions w.r.t. the marginal distribution of y . For the $(1, 0)$ regime, and conditionally on y , the likelihood contribution is the product of the density of the optimal share invested in the first asset, denoted by $a_{*1}^{(1,0)}$, evaluated at the observed share a_1 , and the probability that $\pi_2 < \rho\pi_1$, conditional on $a_{*1}^{(1,0)} = a_1$. To evaluate these two quantities, it is convenient to derive first the joint distribution of $(a_{*1}^{(1,0)}, \pi_2)'$ using (10) and noticing that the analysis in section 2.2.2 has shown that $a_{*1}^{(1,0)} = \pi_1/(W_0\omega_1)$:

$$\left(\begin{array}{c} a_{*1}^{(1,0)} \\ \pi_2 \end{array} \right) \Big| (\mathbf{x}, y) \sim \mathcal{N} \left[\left(\begin{array}{c} \mu_{1|y}/(W_0\omega_1) \\ \mu_{2|y} \end{array} \right), \left(\begin{array}{cc} \sigma_{1|y}^2/(W_0^2\omega_1^2) & r_{12|y}\sigma_{1|y}\sigma_{2|y}/(W_0\omega_1) \\ r_{12|y}\sigma_{1|y}\sigma_{2|y}/(W_0\omega_1) & \sigma_{2|y}^2 \end{array} \right) \right].$$

This result allows to derive the two elements of the contribution of the $(1, 0)$ regime to the total loglikelihood. If $\phi(\cdot)$ and $\Phi(\cdot)$ respectively denote the pdf and the cdf of a standard Gaussian variate, then the pdf of $a_{*1}^{(1,0)}$ evaluated at a_1 can be expressed as:

$$f^{(1,0)}(a_1, y; \boldsymbol{\theta}) = \frac{1}{\sigma_{1|y}/(W_0\omega_1)} \phi \left[\frac{a_1 - \mu_{1|y}/(W_0\omega_1)}{\sigma_{1|y}/(W_0\omega_1)} \right],$$

while the probability that $\pi_2 < \rho\pi_1 = \rho W_0\omega_1 a_1$, conditional on $a_{*1}^{(1,0)} = a_1$ and y , is given by:

$$p^{(1,0)}(a_1, y; \boldsymbol{\theta}) = \Phi \left[\frac{\rho W_0\omega_1 a_1 - (\mu_{2|y} + r_{12|y}\sigma_{2|y}/(\sigma_{1|y}/(W_0\omega_1))(a_1 - \mu_{1|y}/(W_0\omega_1)))}{\sigma_{2|y}(1 - r_{12|y}^2)^{1/2}} \right].$$

To drop the conditioning on y , I must take the expectation of the product of these two quantities w.r.t. the marginal distribution of y , which leads to the following likelihood contribution:

$$l^{(1,0)}(\boldsymbol{\theta}; a_1) = \mathbb{E}^y[p^{(1,0)}(a_1, y; \boldsymbol{\theta}) f^{(1,0)}(a_1, y; \boldsymbol{\theta})]$$

In the same way, I obtain the expression of the likelihood contribution of the (0, 1) observed regimes:

$$l^{(0,1)}(\boldsymbol{\theta}; a_2) = \mathbb{E}^y[p^{(0,1)}(a_2, y; \boldsymbol{\theta}) f^{(0,1)}(a_2, y; \boldsymbol{\theta})],$$

where a_2 denotes the observed invested share in asset 2, and where

$$f^{(0,1)}(a_2, y; \boldsymbol{\theta}) = \frac{1}{\sigma_{2|y}/(W_0\omega_2)} \phi \left[\frac{a_2 - \mu_{2|y}/(W_0\omega_2)}{\sigma_{2|y}/(W_0\omega_2)} \right]$$

$$p^{(0,1)}(a_2, y; \boldsymbol{\theta}) = \Phi \left[\frac{\rho W_0\omega_2 a_2 - (\mu_{1|y} + r_{12|y}\sigma_{1|y}/(\sigma_{2|y}/(W_0\omega_2)))(a_2 - \mu_{2|y}/(W_0\omega_2))}{\sigma_{1|y}(1 - r_{12|y}^2)^{1/2}} \right]$$

respectively denote the (conditionally on y) density of $a_{*2}^{(0,1)}$ evaluated at a_2 , and the probability (conditionally on y and on $a_{*2}^{(0,1)} = a_2$) that $\pi_1 < \rho\pi_2$.

It remains to derive the likelihood contribution of the (1, 1) regime. In this case, two strictly positive invested shares are observed. Since their relation with π_1 and π_2 is given by (5), I get:

$$\mathbf{a}_*^{(1,1)} | (\mathbf{x}, y) \sim \mathcal{N} \left[\begin{pmatrix} \frac{\mu_{1|y} - \rho\mu_{2|y}}{W_0\omega_1(1 - \rho^2)} \\ \frac{-\rho\mu_{1|y} + \mu_{2|y}}{W_0\omega_2(1 - \rho^2)} \end{pmatrix}, \begin{pmatrix} \frac{\sigma_{1|y}^2 + \rho^2\sigma_{2|y}^2 - 2\rho r_{12|y}\sigma_{1|y}\sigma_{2|y}}{W_0^2\omega_1^2(1 - \rho^2)^2} & \frac{r_{12|y}\sigma_{1|y}\sigma_{2|y}(1 + \rho^2) - \rho(\sigma_{1|y}^2 + \sigma_{2|y}^2)}{W_0^2\omega_1\omega_2(1 - \rho^2)^2} \\ \frac{r_{12|y}\sigma_{1|y}\sigma_{2|y}(1 + \rho^2) - \rho(\sigma_{1|y}^2 + \sigma_{2|y}^2)}{W_0^2\omega_1\omega_2(1 - \rho^2)^2} & \frac{\rho^2\sigma_{1|y}^2 + \sigma_{2|y}^2 - 2\rho r_{12|y}\sigma_{1|y}\sigma_{2|y}}{W_0^2\omega_2^2(1 - \rho^2)^2} \end{pmatrix} \right].$$

For the (1, 1) regime, and conditionally on y , the likelihood contribution is given by this

bivariate gaussian pdf evaluated at the observed shares invested, $(a_1, a_2)'$, given by:

$$f^{(1,1)}(a_1, a_2, y; \boldsymbol{\theta}) = \phi_2 \left[\frac{W_0 \omega_1 (1 - \rho^2) a_1 - \mu_{1|y} + \rho \mu_{2|y}}{\sqrt{\sigma_{1|y}^2 + \rho^2 \sigma_{2|y}^2 - 2\rho r_{12|y} \sigma_{1|y} \sigma_{2|y}}}, \frac{W_0 \omega_2 (1 - \rho^2) a_2 + \rho \mu_{1|y} - \mu_{2|y}}{\sqrt{\rho^2 \sigma_{1|y}^2 + \sigma_{2|y}^2 - 2\rho r_{12|y} \sigma_{1|y} \sigma_{2|y}}}; \right. \\ \left. \frac{r_{12|y} \sigma_{1|y} \sigma_{2|y} (1 + \rho^2) - \rho (\sigma_{1|y}^2 + \sigma_{2|y}^2)}{\sqrt{\sigma_{1|y}^2 + \rho^2 \sigma_{2|y}^2 - 2\rho r_{12|y} \sigma_{1|y} \sigma_{2|y}} \sqrt{\rho^2 \sigma_{1|y}^2 + \sigma_{2|y}^2 - 2\rho r_{12|y} \sigma_{1|y} \sigma_{2|y}}} \right].$$

Integration of this expression leads to the likelihood contribution of the observed (1, 1) regimes:

$$l^{(1,1)}(\boldsymbol{\theta}; a_1, a_2) = \mathbb{E}^y[f^{(1,1)}(a_1, a_2, y; \boldsymbol{\theta})].$$

Inspection of these expressions shows that all these probabilities and densities remain unchanged if β_1 , β_2 , σ_1 , σ_2 , ω_1 and ω_2 are multiplied by the same constant. For this reason, I fix $\sigma_1 = 1$ as in the qualitative specification. Under this constraint, all the parameters in $\boldsymbol{\theta} = (\beta_1', \beta_2', \beta_y, \sigma_2, \sigma_y, r_{12}, r_{1y}, r_{2y}, \omega_1, \omega_2)'$ are identified. Taken all things together, the loglikelihood function is given by:

$$\ell(\boldsymbol{\theta}) = \sum_{m=1}^M \sum_{i=1}^2 \sum_{j=1}^2 d_m^{(i,j)} \log l^{(i,j)}(\boldsymbol{\theta}; \mathbf{a}_m)$$

where $l^{(0,0)}(\boldsymbol{\theta}; \mathbf{a}) = p^{(0,0)}(\boldsymbol{\theta})$.

3.3 Modelling heteroskedasticity

As a first step, I estimated the structural probit and tobit models above assuming homoskedasticity of the multivariate normal distribution (8). The associated quantitative specification was soundly rejected by the conditional moment tests outlined in section 7, in particular by the test statistics investigating the presence of heteroskedasticity in the generalized residuals (see below for details on the estimation of the generalized residuals in the structural tobit model). To overcome this issue, I allow for a simple form of heteroskedasticity correction whereby the covariance matrix in (8) is multiplied by $1 + |W|$, where W denotes the household total net wealth (defined as the sum of financial and

real wealth, minus total financial liabilities – see below for a definition of these variables) expressed in million €. This is the same form of heteroskedasticity adopted by Perraudin and Sørensen (2000), although their econometric specification is somewhat simpler than the one considered here. More complicated forms of heteroskedasticity (especially those with parameters included in the heteroskedasticity term) would likely further complicate the inference.

4 The data

The probit and tobit models in this paper were estimated using the data collected in the Survey on Household Income and Wealth (SHIW) conducted by Bank of Italy for the year 2000. Similar surveys have been conducted for the years 1987, 1989, 1991, 1993, 1995 and 1998. The sampling scheme is a rotating panel; for example, the 2000 survey includes a total of 8,001 households, of which 3,873 were also interviewed in a previous survey (the panel component), while the remaining 4,128 were interviewed for the first time.

In this paper, I will neglect the panel component of the sample, and I will instead focus on the cross-sectional dimension. The survey collects detailed information about social and demographic characteristics of the household members, as well as their incomes and consumption expenditures. Furthermore, data about real and financial assets and liabilities are also collected at the household level.

As is usually the case, answers to survey questions concerning wealth are much less reliable than those about income and consumption expenditures. Comparisons with national accounts figures shows that underestimation of the current income is roughly of the order of 30%, about 20% for real assets (albeit this figure is much higher for non-owner occupied houses, which account for a significant fraction of real assets), and 73% for aggregate financial assets. The latter underestimation rate is the aggregate result of rates

of 49% for transaction and savings accounts, 59% for government bonds, and 84% for stocks, private bonds and mutual funds. (More information on the surveys can be found in Brandolini, Cannari, D'Alessio and Faiella (2002), and in the references cited therein.) This phenomenon is due to three distinct kinds of behavior on the part of the surveyed households, non-response, non reporting and under-reporting, which are obviously not independent of wealth. Luckily, Cannari e D'Alessio (1993) have examined the problem and suggested methods to correct the resulting biases, based on a two steps procedure. In the first one, a comparison is made between the SHIW data with others, stemming from a survey carried out by the Banca Nazionale del Lavoro (BNL) Italian retail bank on a sample of its customers in 1993. It should be noted that BNL has not repeated its survey after 1993. Hence, the reliability of the adjustment procedure is likely lower in 2000 than in 1993, when it was originally proposed. In the second step, a proportional adjustment is made on the declared amount invested in each financial instrument to align the resulting aggregate estimate to the corresponding figure in the national accounts. The final outcome of the adjustment procedure are the estimates of the amounts invested in (*i*) bank and post deposits, (*ii*) government bonds, and (*iii*) stocks, private bonds, mutual funds, and other instruments.

To prepare the data for the econometric analysis, I first discarded the 663 households with zero holdings of bank and post deposits, which are seen as the riskless asset. To also discard some probable outliers, I subsequently chose to drop 281 more households because its total, net financial or real wealth is higher than 1 million €, or its income is higher than 400,000 €. This leaves me with 7,058 records. Finally, I identify asset 1 with government bonds, and asset 2 with stocks, private bonds and mutual funds. This classification can be justified by the financial characteristics of the assets (such as expected return, risk and liquidity), but it is also dictated by the structure of the results of the adjustment procedure illustrated above.

Table 1 presents the marginal fractions of households choosing each one of the four possible investment regimes according to a variety of socio-economic and demographic characteristics. Of these, age, marital status, education, occupation and sex actually refer to the head of the household. The table also reports the percentage frequency of occurrence of some specific values of these characteristics in the sample. Inspection of the table immediately suggests the existence of strong relationships between household characteristics and portfolio choice. This intuition is especially significant for characteristics such as wealth (both financial and real), income, geographical area and education. Actually, a Pearson chi squared test of independence overwhelmingly rejects the null hypothesis for all the variables in the table (this and other results cited in the following are not reported for the sake of brevity, and are available at request).

In the econometric analysis I used the same explanatory variables for the three state variables; hence, $\mathbf{x}_1 = \mathbf{x}_2 = \mathbf{x}_y = \mathbf{x}$. These are given by:

- Cost (the constant)
- Age (of the head of the household, in years divided by 100)
- Age2 (Age squared times 100)
- Ncomp (= 1 if the household has at least 4 members)
- Married (= 1 if the household head is married)
- Grad (= 1 if the household head has a university degree)
- Self (= 1 if the household head is manager or self employed)
- Ret (= 1 if the household head is retired)
- Pop (= 1 if the household resides in a town with at least 40,000 inhabitants)

- Central (= 1 if the household lives in central Italy)
- South (= 1 if the household lives in southern Italy)
- Sex (= 1 if the household head is female)
- Homeown (= 1 if the household owns the house where it lives)
- Rw (household’s real wealth, in million €)
- Fw (household’s financial wealth, in million €)
- Fl (household’s financial liabilities, in million €)
- Income (household’s income, in million €)

5 Estimation results

In this section I present the estimation results obtained on the SHIW 2000 sample for the two econometric models. All relevant calculations were made in Fortran. The log-likelihoods were maximized using NAG subroutine E04UCF, with analytical derivatives explicitly coded. Details on the their evaluation are presented in appendix A.1.

Table 2 reports the results of the qualitative and quantitative versions of the model, respectively introduced in sections 3.1 and 3.2. The table is divided in two vertical panels. The first, labelled “Probit” on the first line, contains the results for the qualitative model, while the second panel, labelled “Tobit”, those for the quantitative model. Each panel contains the estimated parameters values and (in parentheses) the associated asymptotic t ratios, computed using the standard errors resulting from the inverse of the outer product of the gradients. The last row reports the maximized loglikelihood values for each specification. Both specifications contain the parameters in the conditional expectations

$(\beta_1, \beta_2$ and $\beta_y)$, the standard errors σ_2 and σ_y and the correlation coefficients r_{12} , r_{1y} and r_{2y} . The quantitative model also allows to estimate ω_1 and ω_2 .

Before examining the results in detail, a couple of remarks are in order about the interpretation of the estimated coefficients in the π_1 and π_2 equations. First, these state variables were defined as the ratios of the perceived Sharpe performances of the two assets μ_i/ω_i divided by the individual risk aversion index η . On one side, this means that it is impossible to disentangle the effects of an independent variable on these two quantities – i.e., it is impossible to separately identify the effect on the numerator and the denominator; presumably, however, many of the factors determining the perceived Sharpe performances also affect the risk aversion index, and generally with the opposite sign, so that the overall effect on the ratio should be reinforced. On the other side, since η appears both in π_1 and π_2 , and since it is likely that the perceived Sharpe performances tend to share the same determinants, it is natural to expect the estimates of β_1 and β_2 to be close to each other, and the estimates of σ_2 and r_{12} to be close to 1.

Second, it might be tempting to explicitly establish a connection between the size and sign of the estimated coefficients and the decision about if and how to invest in asset 1 and 2, respectively. It should be noted, however, that while this would be completely correct in a standard probit or tobit framework, in the nonlinear specifications considered in this paper it would miss an important point, i.e. that, *ceteris paribus*, a shift in π_i not only affects the decision concerning asset i , but also that of the alternative asset. Consider for example the effect of a decrease in π_1 for a household characterized by positive and fixed values of π_2 and ρ . If π_1 is sufficiently high, the preferred regime is (1,0), but moving to the left leads first to switch to regime (1,1), and only subsequently to drop asset 1 and move to regime (0,1); in this case, it is also easy to check that the share invested in asset 1 (2) is a non increasing (non decreasing) piecewise linear function of π_1 . Notice however that these patterns may be different if either π_2 or ρ or both become zero or negative. This feature

obviously complicates the interpretation of the coefficient estimates. To better understand the overall effects of the independent variables, it is necessary to turn to model simulation; see section 6 for a discussion of the results.

I start by examining the results concerning the qualitative (Probit) model. Apart from those in y equation, all the parameters seem to be well identified; moreover, as expected, the explanatory variables in \mathbf{x} tend to have a similar effect on π_1 and π_2 . Notice that the estimates of the coefficients of Age and Age2 clearly suggest a hump shape effect of age on both π_1 and π_2 , which is maximum at:

$$\text{Age}_i^* = -50 \frac{\widehat{\beta}_{i,\text{Age}}}{\widehat{\beta}_{i,\text{Age}2}}, \quad i = 1, 2$$

For the probit model, this formula implies $\text{Age}_1^* = 41.5$ and $\text{Age}_2^* = 34.8$; these values suggest that π_1 and π_2 start to decrease with the household age fairly early, in particular for the more risky assets. The professional status indicators (Self and Ret) have a strongly significant effect on π_2 only, with negative coefficients; for self employed household heads, this result is in line with those reported elsewhere, e.g. Heaton and Lucas (2000). Both Central and South, the dummy indicators of the area in Italy where the household lives, have a negative and very significant estimated coefficient, which is a frequently observed result on Italian data. Higher education (Grad) raises both π s, but only the effect on π_1 is significantly different from zero. The parameters for the indicators for the civil status (Married), the gender (Sex) of the household head, the number of household components (Ncomp) and the size of the town where the household resides (Pop) are not significantly different from zero. The most interesting results come from the estimates of parameters related to households income (Income), real wealth (Rw), homeownership (Homeown), financial wealth (Fw) and liabilities (Fl). First, homeownership positively impacts on π_1 and π_2 , although the effect is somewhat stronger on the latter, but at the same time real wealth has a strong negative effect on both π_1 and π_2 . Second, the effects of financial

wealth are obviously significant and positive everywhere; financial liabilities do not seem to have an impact. Finally, a higher income tends to significantly decrease π_1 , while leaving π_2 only marginally lower. This result might seem puzzling, but it should be kept in mind that this is a marginal result holding real and financial wealth constant, and that, due to the nonlinearity of the structural specification, a decrease in π_1 might actually lead to invest in asset 2, as discussed above and in section 6.

The parameters in β_y are never significantly different from zero, according to the asymptotic t ratios; the estimate of σ_y is quite high, but it is only marginally significant. This result is hardly surprising, given that the separate identifiability for the mean and the variance parameters β_y and σ_y is crucially dependent on the nonlinearity of the transform used to map the latent variable y into a valid correlation coefficient ρ . Although such nonlinearity might guarantee identifiability in theory, one would expect that identifiability in the sample would be poor (a point also made by Fougère, Gouriéroux, Tiomo and Trognon, 1997).

As expected, the estimate of r_{12} is very close to 1. A formal test of $H_0 : r_{12} = 0$ is complicated because under the null r_{12} lies on the boundary of the parameter space, but this problem could be addressed using the results in Andrews (2001); for simplicity, I simply remark that, given that its asymptotic standard error of the estimate of r_{12} is slightly lower than 0.05, the upper bound lies inside the 95% confidence interval. The other two correlation coefficients, r_{1y} and r_{2y} , are also estimated to be positive, but slightly lower. This results, along with the previous discussion of its probable causes, highlights the importance of letting the correlation coefficients be free parameters, instead of constraining them to zero as in Fougère, Gouriéroux, Tiomo and Trognon (1997).

I now turn to the second panel in table 2, which reports the estimation results for the structural tobit model. Overall, the estimates of the parameters in the equations for π_1 and π_2 in the quantitative specification agree fairly closely both in sign and in statistical

significance with the corresponding ones in the probit model. This is for example the case for the regressors related to the economic condition of the household (Homeown, Rw, Fw, Fl and Income) and the geographical indicators (Central and South). Any difference in sign generally arises when the estimates are not significantly different from zero, such as Pop for π_2 , Sex for π_1 , or Grad for both. The effects of age on π_1 and π_2 are still hump shaped, with maxima located at 40.0 and 39.6 years, respectively.

As it should be expected, the estimates of the coefficients in the y equation tend to be much more precise when using the quantitative information. However, only two out of 16 (excluding the constant) explanatory variables have a statistically significant coefficient at the 5% level, namely South and Fl. This result is disappointing, but nevertheless it is interesting to notice that the signs of the estimates are frequently the opposite of those in the π s equations. For example, increasing the financial wealth decreases the perceived correlation coefficient, and the same is true (albeit not significantly) for the other economic characteristics, apart from Income. ρ is also monotonically increasing over positive values of Age, and tends to be higher for households with low education head and located in southern Italy. I defer to the following section a more sensible assessment of the effect of these variables on market participation patterns and asset demands.

Both variance parameters seem to be well identified; the estimate of σ_2 is slightly higher than 1, and that of σ_y is much lower than in the probit model, but much more precise. For the correlation parameters, the estimate of r_{12} is even closer to 1 than in the qualitative specification, but its asymptotic standard error is now as small as 0.0033, which makes the unit upper bound quite unlikely. The estimates of r_{1y} and r_{2y} are much lower, and neither one is significantly different from 0 at the 5% level. Finally, the estimated standard errors of the risky assets excess returns ω_1 and ω_2 are quite high and significant. Their values might seem puzzling, but it should be kept in mind that they are a consequence of the identifying constraint $\sigma_1 = 1$.

6 Model simulation and partial effects

Contrary to the standard univariate tobit setup, the structural tobit model developed in this paper is characterized by a fairly complicated relation between the latent (π_1 , π_2 and y) and the observable variables (a_1 and a_2). This makes it hard to interpret the estimated coefficients in terms of partial effects. For this reason, I turn to model simulation to get a more intuitive measure of the relevance of each regressor from the point of view of the regime probabilities and the asset demands. Moreover, since the sample I work on is drawn randomly from the population of Italian households, the same exercise would provide a way to measure the aggregate impact of changes in the socio-economic profile of the population.

To do this, I computed the percentage changes in regime probabilities and conditional (on the regime) and unconditional expected asset demands for each household in the sample following a variation in one of the explanatory variables, and keeping all the other regressors constant. The individual changes were then aggregated using the sample average and standard deviation. Both measure are weighted using the sampling weights provided in the SHIW files. The results are reported in Tables 3 and 4. Finally, the changes in the regressors used to perform these simulations are the following:

- *On Age*: 5 years added to every household head.
- *On Ncomp, Married, Grad, Self, Ret, Pop, Central, South, Sex, Homeown*: the percentage variation is measured by first setting the regressor to zero, and then setting it to 1, for all households.
- *On Rw, Fw, Fl and Income*: Each regressor is increased by 50%.

Details on the formulas used to compute the conditional and unconditional expected asset demands are reported in appendix A.2.

The results in table 3 and 4 show that in some cases the average differences and their sample standard deviations are quite high. Among the economic regressors, financial wealth has generally the strongest effect, followed by homeownership and income. Higher financial wealth and homeownership increase the probability of market participation, i.e. of the regimes including one or both risky assets, while the opposite is true for real wealth. Households with higher income tend to move away from states including asset 1. For the socio-demographic characteristics, the main effects refer to the geographical location (residing in Central or Southern Italy makes the household much less likely to invest in risky assets) and the professional status (self employed and retired tend to shy away from regime (0,1), in favor of regimes (0,0) and (1,0)). A higher age discourages investment in asset 2, as expected. Also, higher education encourages market participation, while households living in larger towns tend to choose more often the risky asset only regime.

The regressors whose changes have important effects on the expected asset demands tend to be the same as those illustrated in the case of regime probabilities; in many cases, moreover, the impact has the same direction. This is for example the case of age and the dummy regressors associated to the professional status, which lower the demand of the more risk asset, while the opposite is true for the size of the town of residence, Pop, and the number of components of the household, Ncomp. The geographical dummies effect shows that households located in central and southern Italy tend not only to abstain from market participation, but also to have lower demands for both assets when they participate. Other dummy indicators have a positive effect on the demands of both assets, such as the marital status indicator, Married, or the education level of the household head, Grad.

Overall, a 50% increase in real wealth tends to increase the investment in the less risky asset, and to decrease the investment in the more risky one. It is interesting to observe that, on the contrary, homeownership increases both asset demands, a conclusion which is obviously shared, to a much larger extent, by financial wealth. This observation, coupled

to the one highlighted above about the simulated effects on regime probabilities, suggests that real wealth impacts on household portfolio choice in a distinctly different manner than financial wealth and homeownership. Several explanations of this result are possible, such as the different degree of liquidity characterizing the two forms of wealth, or special status of real wealth in the form of homeownership due to the provision of housing services. The problem of portfolio choice in the presence of illiquid allocations has been studied, among others, by Gouriéroux and Jouneau (1999), whose results have been exploited by Pelizzon and Weber (2003) to study the efficiency of Italian households portfolios.

7 Specification tests

Specification tests for the structural qualitative and quantitative models analyzed in this paper can be derived following the conditional moments approach set up in Pagan and Vella (1989). These tests are based on orthogonality conditions for the error terms of the latent model which can be checked using the associated generalized residuals, i.e. the expected value of these error terms conditionally on the observable variables. Details on the computation of these quantities are provided in appendix A.3. For the structural tobit model I focus on, these quantities are not so easy to compute as in the textbook univariate setup, but they can nevertheless be evaluated using one dimensional quadrature methods.

A variety of specification tests can be derived using the conditional moments approach. Following Pagan and Vella (1989), I focus on RESET tests of linearity of the conditional expectations of π_1 , π_2 and y w.r.t. \mathbf{x} , and on tests of homoskedasticity and normality of the error terms of the latent variables model. As an example, let $u_1 = \pi_1 - \mathbf{x}'\boldsymbol{\beta}_1$ be the error term in the equation for the (unobservable) latent variable π_1 , and consider the tests based on the following orthogonality conditions:

- RESET tests:

$$E[(\mathbf{x}'_1 \widehat{\boldsymbol{\beta}}_1)^h E(u_1 | \mathbf{a}) | \mathbf{x}] = 0, \quad h = 2, 3$$

- Homoskedasticity tests:

$$E[x_{1k}(E(u_1^2 | \mathbf{a}) - \sigma_1^2) | \mathbf{x}] = 0, \quad k = 1, \dots, K$$

- Normality tests:

$$E[E(u_1^3 | \mathbf{a})] = 0, \quad E[E(u_1^4 | \mathbf{a}) - 3\sigma_1^4] = 0$$

These orthogonality conditions, and the similar conditions formulated on the error terms of the remaining two latent variables, π_2 and y , may be tested separately or jointly. Also, notice that other null hypotheses of interest may be tested using the conditional moments approach. A particularly interesting example would be a test of exogeneity of some of the regressors in the latent models, such as real and financial wealth, homeownership and income. Suitable test statistics may be derived by exploiting the orthogonality condition between the generalized residuals and the fitted residuals of the regression of the variable of interest on some other variables (see Pagan and Vella, 1989, p. S38, for details). This test statistic is equivalent, under the null hypothesis of weak exogeneity, to the one derived by Smith and Blundell (1986). This opportunity, however, is not pursued here.

Let us denote the general formulation of these orthogonality conditions with $E[\mathbf{m}(\mathbf{a}, \mathbf{x}; \boldsymbol{\theta}_0)] = \mathbf{0}$, where \mathbf{m} denotes a vector of q known functions, and let $\frac{1}{N} \sum_{n=1}^N \widehat{\mathbf{m}}_n = \frac{1}{N} \sum_{n=1}^N \mathbf{m}(\mathbf{a}_n, \mathbf{x}_n; \widehat{\boldsymbol{\theta}})$ be the sample counterpart of the theoretical condition. Pagan and Vella (1989) show that, under the assumption of independent observations, these hypothesis may be easily tested using the following test statistic:

$$CM = \boldsymbol{\iota}' \mathbf{M} [\mathbf{M}' \mathbf{M} - \mathbf{M}' \mathbf{D} (\mathbf{D}' \mathbf{D})^{-1} \mathbf{D}' \mathbf{M}]^{-1} \mathbf{M}' \boldsymbol{\iota} \quad (15)$$

where $\boldsymbol{\iota}$ is the $(N \times 1)$ unit vector, \mathbf{M} is the $(N \times q)$ matrix with generic n -th row given by $\widehat{\mathbf{m}}_n$, and \mathbf{D} is the $(N \times p)$ matrix whose n -th row is the gradient of the tobit loglikelihood

for the n -th observation evaluated at $\widehat{\boldsymbol{\theta}}$, and p is the total number of parameters in the model. The asymptotic distribution of ξ is χ_q^2 under the relevant null hypothesis. Under the null hypothesis, an asymptotically equivalent version of this test requires to verify the nullity of the vector of intercepts in the multivariate regression of \mathbf{M} on $(\boldsymbol{\iota}, \mathbf{D})$.

Alternatively, a global specification test of the structural tobit model may be developed following Hausman (1978), by comparing the parameter estimates in the quantitative and the qualitative models. This strategy has been suggested by Ruud (1984), and it has been implemented in Fougère, Gouriéroux, Tiomo and Trognon (1997). Their conclusion is that the structural model cannot be rejected, although the same test applied parameter by parameter rejects the null for some specific variables. The results in Ruud (1984) show that the number of degrees of freedom of the asymptotic distribution of the joint test statistic equals the number of free parameters in the qualitative model. This property holds because the likelihood of a tobit model can be factored in two likelihoods: the one of a probit model, and the one of a censored regression model. Ruud (1984) shows that this factorization, which is also possible in the structural context studied in this paper, allows to derive several interesting results about ML estimators and test statistics based on them. Notice that in the structural framework analyzed in this paper the quantitative model has two parameters more than the qualitative model. Let $\boldsymbol{\theta}_p$ and $\boldsymbol{\theta}_t$ denote the vectors of free parameters in the structural probit and tobit models, respectively, and let \mathbf{S} be the matrix which extracts from $\boldsymbol{\theta}_t$ the parameters also present in $\boldsymbol{\theta}_p$ (i.e. $\mathbf{S}\boldsymbol{\theta}_t$ drops ω_1 and ω_2). Also, let $\widehat{\mathbf{V}}_p$ and $\widehat{\mathbf{V}}_t$ be the asymptotic variance matrices of the corresponding estimates $\widehat{\boldsymbol{\theta}}_p$ and $\widehat{\boldsymbol{\theta}}_t$. The Hausman test statistic can then be written as:

$$H = N(\widehat{\boldsymbol{\theta}}_p - \mathbf{S}\widehat{\boldsymbol{\theta}}_t)'(\widehat{\mathbf{V}}_p - \mathbf{S}\widehat{\mathbf{V}}_t\mathbf{S}')^{-1}(\widehat{\boldsymbol{\theta}}_p - \mathbf{S}\widehat{\boldsymbol{\theta}}_t) \quad (16)$$

I implemented the Hausman test for the structural quantitative model, but I found it difficult to constrain the difference of variance matrices to be positive definite, as it

should be asymptotically. Luckily, there exists an asymptotically equivalent version of this test based on a conditional moment restriction in which \mathbf{m} is the gradient of the probit loglikelihood evaluated at $\mathbf{S}\hat{\boldsymbol{\theta}}_t$. Hence, the same global specification test may be conducted by suitably adapting the *CM* statistic defined in (15). See also Ruud (1984) for other equivalent versions of the Hausman test statistic. Finally, in the standard univariate tobit model Chesher, Lancaster and Irish (1985) suggested to test the correctness of the assumed specification following White (1982) information matrix test. This approach, however, would be extremely cumbersome in the structural specification investigated here, because it would involve a huge number of moment conditions ($58 \times 59/2 = 1711$), and because the number of degrees of freedom of the asymptotic distribution of the resulting test statistic would be in general unknown.

It is well known that in finite samples the conditional moment tests illustrated above tend to be characterized by an actual size much lower than the theoretical one, and are hence prone to overreject the null hypothesis, sometimes very severely. This is due to the outer product of gradients form of the estimate of the asymptotic variance matrix of the test statistic. A possible solution would be to parametrically bootstrap the conditional moment tests, but this is unrealistic for the model I consider, given the degree of difficulty characterizing its estimation. Aside from this issue, the fact that the test procedure is so easy to compute makes it nevertheless very attractive.

For the structural tobit model, the p -values of the specification tests just outlined were essentially zero in all cases, leading to a clear rejection of the model specification; the same conclusion is suggested by the global specification test based on the probit score, which is asymptotically equivalent to the Hausman test. Although this might be justified by the poor small sample performance just cited, I believe that this possibility can at the very best represent a partial explanation of the rejection of the model. Notice that these kind of specification tests are rare in empirical analysis of households market

participation and investment decisions, and, more generally, in empirical applications of discrete and limited dependent variables. An exception for the problem at hand is provided by Perraudin and Sørensen (2000), who do not consider conditional moment tests, but perform a skewness-kurtosis normality test on the residuals of the asset demand functions they estimate (which are explicitly observed in their framework), and a Chow test of parameter constancy between two subsamples formed by individuals with wealth higher or lower than the median level. These test statistics also lead to the rejection of the model specification.

To get a more intuitive understanding of the shortcomings of the model, table 5 reports some summary statistics of the individual estimated expected state variables. $\hat{\pi}_1$, $\hat{\pi}_2$ and $\hat{\rho}$ were computed as expected values conditional on the observed investment regime and asset demands, evaluated at the parameter estimates in the tobit model with heteroskedasticity correction. Ideally, it would be natural to compare these quantities to the corresponding figures computed on time series of excess returns of the two risky assets on some interval ending in December 2000. This is possible for the correlation coefficient ρ , but not for the other two state variables, π_1 and π_2 , because (i) they are defined as the ratios of the Sharpe performances on the individual risk aversion index η , and (ii) they are subject to the identifying restriction $\sigma_1 = 1$. The ratio $\tau = \pi_1/\pi_2$, however, avoids both issues. In table 5, τ is approximated by the ratio $\hat{\pi}_1/\hat{\pi}_2$.

Estimating τ and ρ on time series of excess returns is not easy, as several problems immediately arise. The first one relates to the indexes to be used to approximate the returns of the risky assets. For risky asset 1, I use the Bank of Italy index of Treasury bills and bonds. For risky asset 2, the choice is more difficult, because this asset includes domestic and foreign stocks, mutual funds, corporate bonds, and other instruments. To keep things simple, I consider a couple of Italian stocks indexes (MIBTEL and MSCI), and a general mutual funds index provided by Fideuram, an Italian private bank. These

time series were used to compute monthly log returns and excess returns, using the return on the three-month Treasury bill as the risk free rate. It should be noted that all these indexes include taxes and transaction costs. A second problem relates to the length of the interval used to compute the historical estimate of τ and ρ . I consider three intervals ending in December 2000: 18, 30 and 60 months. Unluckily, the Fideuram mutual fund index is only available for the shortest interval. Table ?? illustrates the results.

The comparison of the figures in the two tables allows to draw some interesting conclusions. For slightly more than 40% of the households in the sample, the estimated Sharpe performances divided by the risk aversion index is negative. This can be possible only if the perceived risk premium on the assets is negative. Table ?? shows that this was indeed the case over the shortest interval for the treasury index approximating risky asset 1, and for the general mutual fund index. The remaining historical estimates are always positive, reflecting the development of the Internet bubble in the case of equity indexes, and the sharp decline in interest rates following the introduction of the Euro in the case of the fixed income index. Although taking into account taxes and transaction costs would reduce the historical estimates of the Sharpe performances of both assets, it is unlikely that this would suffice to justify the high percentage of negative estimated perceived Sharpe performances reported in table 5.

The historical estimate of τ exhibits fairly wide fluctuations, depending on the time interval used for its estimation. It appears, however, that a reasonable estimate of its value, taking into accounts the biases in the data, would be positive and not too far from 1. This value is quite close to most of the individual estimates of τ provided by the tobit model. On the contrary, there is a huge difference between that historical and the estimated individual correlation coefficients: the former is generally small, whereas the latter is extremely high and close to the upper bound 1. A possible conclusion of this comparison could hence be that, according to the tobit model derived from the mean-

variance portfolio choice model, most of the households behave as if they were capable of correctly measure the relative risk-return trade-off characterizing the two risky assets, but they were also unable to accurately appreciate the diversification opportunities between them.

8 Conclusions

This paper builds on Fougère, Gouriéroux, Tiomo and Trognon (1997) to derive structural qualitative and quantitative econometric models of household portfolio choice. Under the assumption of mean-variance expected utility and no short selling constraints, and considering only two risky assets, the observed individual portfolio allocations allow to estimate the relation between the implicit state variables driving the household investment choice, namely the Sharpe performances of the assets divided by the individual risk aversion coefficient, and the correlation among their excess returns, and the household socio-economic characteristics. This makes it possible to disentangle the impact of each regressor on the asset (risk corrected) performances and on the degree of diversification available.

The econometric models are more complicated, non linear extensions of the classical multivariate probit and tobit setups. In either model the observed portfolio regime is described by a two dimensional vector, and both structural specifications exhibit a three dimensional latent vector. One of the latent variables (most naturally, the correlation coefficient between the excess returns, ρ) can be seen as a structural heterogeneity component which contains an observable and an unobservable component, and which is integrated away to compute the loglikelihood and other relevant quantities.

To get a better intuition on the effect of each regressor on portfolio choice I computed the average partial effects following a change in the independent variables. I also tested the structural tobit model using the conditional moments approach surveyed in Pagan

and Vella (1989). Unluckily, the various specification tests (including the Hausman test) strongly reject the structural specification. These results are hardly surprising, given the huge degree of heterogeneity typically observed in samples of households, and the stringent assumptions characterizing the portfolio choice model. According to the model, most households correctly value the risk-return trade-off associated to the two risky assets, but highly overestimate the degree of correlation among them. This, together with the fairly large fraction of households expecting negative risk premia on the assets, appears to be the only way to reconcile the main features of the data (namely the large sample proportion of households holding only the risk free asset and the small fraction investing in both risky assets) with the theoretical framework of mean-variance optimization. I leave to future research the development of a parsimonious, theoretically founded and yet sufficiently flexible econometric model that might overcome the limitations of the simple mean-variance framework explored in this paper.

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A Computation details

This appendix provides some details about the formulas used to evaluate several expected values of interest in the structural probit and tobit model presented in the paper. To simplify the notation, in the following discussion I drop the dependence of all the relevant expectations on \mathbf{x} and $\boldsymbol{\theta}$.

A.1 Loglikelihood and score numerical evaluation

The numerical evaluation of the different likelihood contributions derived in section 3 involve the evaluation of a bivariate Gaussian cdf, and one - dimensional quadrature techniques. (The latter is not necessary for the probability of the first regime.) For the first task, the NAG Fortran 77 Library provides the highly accurate subroutine G01HAF, which is based on a high order polynomial approximation to Mills ratios. For the second task, I used the subroutine D01BAF with 64 (the maximum allowed) Gauss - Hermite weights and abscissae, which is especially well suited to evaluate integrals with both limits infinite, and with integrands whose behavior at $\pm\infty$ closely resembles that of a Gaussian pdf.

The loglikelihoods were numerically maximized using subroutine E04UCF. Although the high number of abscissae used in the quadrature step made the objective functions fairly smooth, some numerical investigation showed that the performance of the optimizer was dramatically improved by the use of the analytical derivatives. The latter are rather tedious, but overall straightforward, to code. The expression of the score is too lengthy to be reported here, but the main steps in its derivation may be described as follows. The first one is obviously the transfer of the derivative w.r.t. θ inside the external integral w.r.t. y . If the integrand only involves univariate Gaussian pdfs or cdfs, or bivariate Gaussian pdfs, computing its derivative w.r.t. θ is straightforward. If it involves a bivariate Gaussian cdf, then the derivative may be computed by first redefining it through an application of the change of variables formula in such a way that the limits of integration do not depend on θ , and then by noticing that the derivative of a bivariate Gaussian pdf w.r.t. the parameters is given by the same pdf times a quadratic function of the two random variables. The final result is that the score is given by the integral w.r.t. y of the moments up to the second order of a truncated bivariate Gaussian r.v. The analytical expression of

the latter quantities is given by formula (7) in Weiler (1959).

A.2 Regime probabilities and expected asset demands

The expressions used to evaluate the regime probabilities obviously coincide with (9), (11), (12) and (13) provided in section 3.1. To evaluate the expected asset demands, I need to consider first the demands conditioned on the portfolio regime. The reason for this is that unconditional expected asset demands may be computed using the law of iterated expectations as the weighted average of conditional expected asset demands, with weights given by the probabilities of the different regimes. Let me start with the (1, 0) regime, and the associated demand for asset 1. Using the results in section 2.2.2, I obtain:

$$\begin{aligned} E[a_1|(1, 0)] &= \frac{1}{W_0\omega_1} E(\pi_1|\pi_1 > 0, \pi_2 < \rho\pi_1) \\ &= \frac{1}{W_0\omega_1} E^y[E(\pi_1|\pi_1 > 0, \rho\pi_1 - \pi_2 > 0, y)] \end{aligned} \quad (17)$$

Notice that the inner integral is taken w.r.t. the joint distribution of (π_1, π_2) conditional on y , provided by (10). Its analytical evaluation is possible using formula (7) in Weiler (1959). Hence, the numerical evaluation of (17) only requires one dimensional quadrature.

The same applies to the remaining conditional expected asset demands:

$$\begin{aligned} E[a_2|(0, 1)] &= \frac{1}{W_0\omega_2} E^y[E(\pi_2|-\pi_1 + \rho\pi_2 > 0, \pi_2 > 0, y)] \\ E[a_1|(1, 1)] &= \frac{1}{W_0\omega_1(1 - \rho^2)} E^y[E(\pi_1 - \rho\pi_2|\pi_1 - \rho\pi_2 > 0, -\rho\pi_1 + \pi_2 > 0, y)] \\ E[a_2|(1, 1)] &= \frac{1}{W_0\omega_2(1 - \rho^2)} E^y[E(-\rho\pi_1 + \pi_2|\pi_1 - \rho\pi_2 > 0, -\rho\pi_1 + \pi_2 > 0, y)] \end{aligned}$$

Notice that these expected asset demands are expressed in terms of shares of the financial wealth W_0 .

A.3 Generalized residuals

In this section, I provide the formulas used to compute the generalized residuals for the structural tobit model. These quantities form the basis on the conditional moments specification tests discussed in section 7. Let the error terms of the latent variables model be defined by

$$u_1 = \pi_1 - \mathbf{x}'\boldsymbol{\beta}_1, \quad u_2 = \pi_2 - \mathbf{x}'\boldsymbol{\beta}_2, \quad u_y = y - \mathbf{x}'\boldsymbol{\beta}_y.$$

To perform the desired specification tests, I need to evaluate $E(u_i^h|\mathbf{a})$, for $i = 1, 2, y$ and $h = 1, 2, 3, 4$, where \mathbf{a} denotes the vector of observed asset holdings. From (8), it is easy to derive the joint distribution of these error terms, as well as every marginal or conditional distribution of either the original state variables or the error terms (all the distributions being conditional on \mathbf{x}). To simplify the notation, I will generically denote with $f(\cdot)$ these Gaussian marginal or conditional distributions derived from (8).

In the (0, 0) regime:

$$\begin{aligned} E[u_1^h|(0, 0)] &= \frac{1}{p^{(0,0)}} \int_{-\infty}^0 f(\pi_2) \left(\int_{-\infty}^{\mathbf{x}'\boldsymbol{\beta}_1} u_1^h f(u_1|\pi_2) du_1 \right) d\pi_2 \\ E[u_2^h|(0, 0)] &= \frac{1}{p^{(0,0)}} \int_{-\infty}^0 f(\pi_1) \left(\int_{-\infty}^{\mathbf{x}'\boldsymbol{\beta}_2} u_2^h f(u_2|\pi_1) du_2 \right) d\pi_1 \\ E[u_y^h|(0, 0)] &= \frac{1}{p^{(0,0)}} E^y \left[u_y^h p^{(0,0)}(y) \right] \end{aligned}$$

where $p^{(0,0)}(y) = \text{Prob}\{\pi_1 \leq 0, \pi_2 \leq 0|y\}$. These expressions only require one dimensional quadrature techniques to be evaluated, since the inner integral in the first two formulas can be evaluated recursively using the following generalization of the Lee and Maddala (1985) formula (which is valid for the classical univariate tobit setup): if $z \sim \mathcal{N}(m, v^2)$, and if $g(z) = (1/v)\phi[(z - m)/v]$ is the corresponding density, then

$$\int_{-\infty}^b z^h g(z) dz = m \int_{-\infty}^b z^{h-1} g(z) dz + v^2(h-1) \int_{-\infty}^b z^{h-2} g(z) dz - vb^{h-1} \phi\left(\frac{b-m}{v}\right)$$

which can be initialized by setting:

$$\int_{-\infty}^b z^0 g(z) dz = \Phi\left(\frac{b-m}{v}\right) \quad \text{and} \quad \int_{-\infty}^b z^1 g(z) dz = m\Phi\left(\frac{b-m}{v}\right) - v\phi\left(\frac{b-m}{v}\right).$$

This recursion is also necessary to simplify the evaluation of some of the following expressions.

In the (1, 0) regime:

$$\begin{aligned} \mathbb{E}[u_1^h | (a_1, 0)] &= (\omega_1 a_1 - \mathbf{x}'_1 \boldsymbol{\beta}_1)^h \\ \mathbb{E}[u_2^h | (a_1, 0)] &= \int_{-\infty}^{+\infty} f(y) \left(\int_{-\infty}^{\rho\omega_1 a_1 - \mathbf{x}'_2 \boldsymbol{\beta}_2} u_2^h f(u_2 | \pi_1 = \omega_1 a_1, y) du_2 \right) dy \\ \mathbb{E}[u_y^h | (a_1, 0)] &= \frac{1}{l^{(1,0)}(a_1)} \mathbb{E}^y \left[u_y^h p^{(1,0)}(a_1, y) f^{(1,0)}(a_1, y) \right] \end{aligned}$$

In the (0, 1) regime:

$$\begin{aligned} \mathbb{E}[u_1^h | (0, a_2)] &= \int_{-\infty}^{+\infty} f(y) \left(\int_{-\infty}^{\rho\omega_2 a_2 - \mathbf{x}'_1 \boldsymbol{\beta}_1} u_1^h f(u_1 | \pi_2 = \omega_2 a_2, y) du_1 \right) dy \\ \mathbb{E}[u_2^h | (0, a_2)] &= (\omega_2 a_2 - \mathbf{x}'_2 \boldsymbol{\beta}_2)^h \\ \mathbb{E}[u_y^h | (0, a_2)] &= \frac{1}{l^{(0,1)}(a_2)} \mathbb{E}^y \left[u_y^h p^{(0,1)}(a_2, y) f^{(0,1)}(a_2, y) \right] \end{aligned}$$

Finally, in the (1, 1) regime:

$$\begin{aligned} \mathbb{E}[u_1^h | (a_1, a_2)] &= \mathbb{E}^y \left[(\omega_1 a_1 - \mathbf{x}'_1 \boldsymbol{\beta}_1 + \rho\omega_2 a_2)^h \right] \\ \mathbb{E}[u_2^h | (a_1, a_2)] &= \mathbb{E}^y \left[(\rho\omega_1 a_1 + \omega_2 a_2 - \mathbf{x}'_2 \boldsymbol{\beta}_2)^h \right] \\ \mathbb{E}[u_y^h | (a_1, a_2)] &= \frac{1}{l^{(1,1)}(a_1, a_2)} \mathbb{E}^y \left[u_y^h f^{(1,1)}(a_1, a_2, y) \right] \end{aligned}$$

Figure 1: Decision diagram of risky asset holdings.

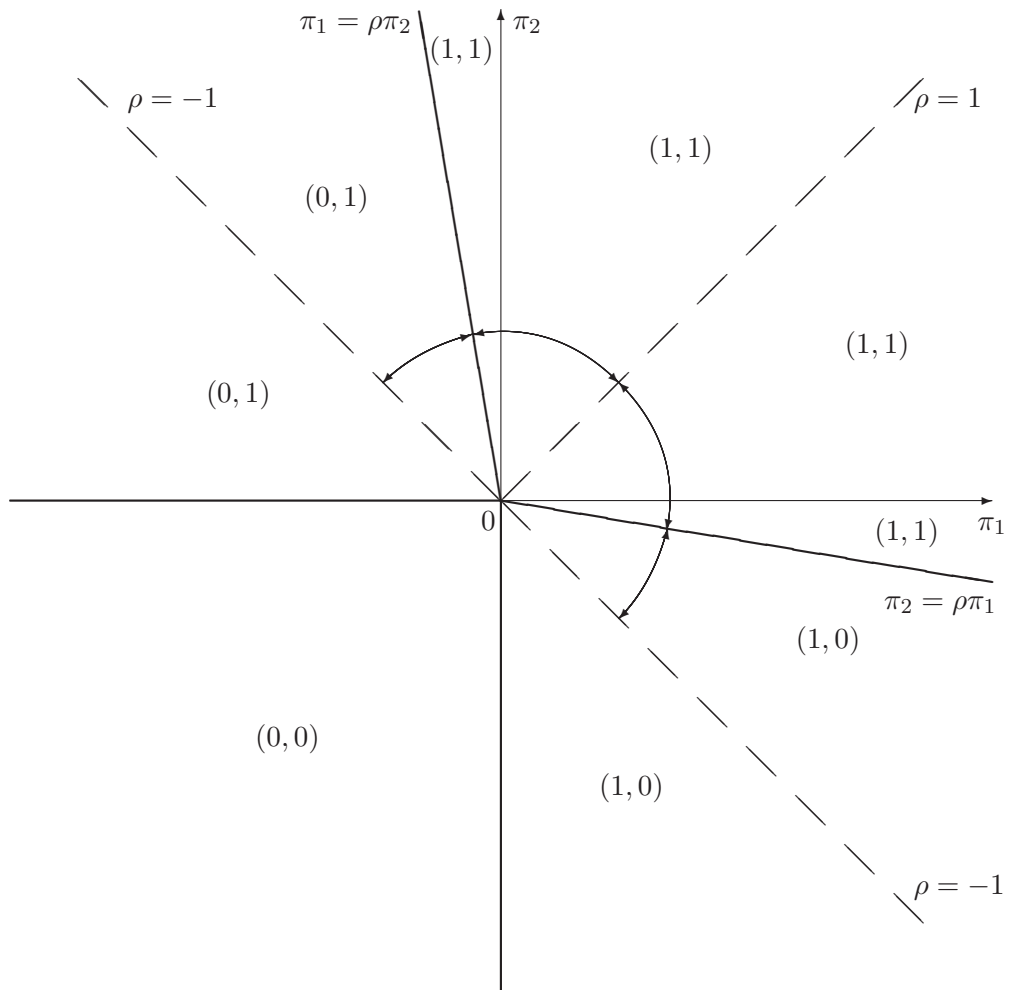


Table 1: Marginal holding rates for households with different characteristics.

Variable	Value	% in sample	Asset holding regime			
			(0,0)	(1,0)	(0,1)	(1,1)
Age quartile	≤ 42	25.6	60.8	10.9	20.9	7.4
	43 – 54	26.2	55.3	14.5	22.4	8.5
	55 – 67	23.3	54.5	18.3	16.8	1.04
	≥ 68	24.9	59.5	21.9	10.8	7.7
Number of components	≤ 3	68.1	56.0	18.1	16.6	9.3
	≥ 4	31.9	60.6	12.1	20.5	6.8
Marital status	Not married	30.0	60.2	19.2	12.9	7.7
	Married	70.0	56.2	14.9	20.0	8.8
Education	\leq Primary	35.0	61.9	22.2	8.8	7.1
	Junior high school	27.0	62.2	13.7	17.6	6.6
	High school	29.2	52.0	12.0	25.6	10.3
	\geq Graduate	8.8	43.8	13.8	29.00	13.4
Occupation	Blu collar	15.9	66.2	13.6	15.7	4.5
	White collar	15.8	51.7	9.8	28.2	10.2
	Manager or self-empl.	16.6	51.4	13.8	25.1	9.7
	Retired	40.7	56.8	20.4	13.4	9.3
	Other	11.0	65.0	17.0	11.2	6.8
City population	$\leq 40,000$	91.7	56.9	16.5	18.0	8.6
	$> 40,000$	8.3	64.2	12.5	16.1	7.2
Geographical area	North	45.6	42.7	19.9	24.4	13.1
	Center	21.3	62.2	11.2	19.5	7.1
	South	33.1	74.8	14.3	7.8	3.0
Sex of head	Male	67.2	56.0	15.6	19.6	8.8
	Female	32.8	60.6	17.3	14.3	7.8
Homeowner	No	30.6	64.3	15.4	14.4	5.8
	Yes	70.2	54.5	16.5	19.3	9.6
Real wealth	≤ 10.3	25.0	66.5	16.6	12.1	4.8
	10.3 – 94.5	25.1	65.9	16.4	11.4	6.3
	94.5 – 170.4	24.9	52.0	16.9	21.5	9.7
	> 170.4	25.0	45.5	14.9	26.4	13.2
Financial wealth ^a	≤ 7.1	25.0	98.0	1.1	0.9	0.0
	7.1 – 20.5	25.0	84.2	10.0	5.0	0.8
	20.5 – 61.8	25.0	40.0	38.7	16.7	4.9
	> 61.8	25.0	8.0	14.9	48.8	28.3
Financial liabilities ^a	= 0	74.8	57.7	17.6	16.1	8.6
	> 0	25.2	57.0	12.0	22.9	8.1
Income ^a	≤ 15.0	25.0	73.1	19.0	5.0	2.8
	15.0 – 22.8	25.0	61.4	18.7	12.8	7.1
	22.8 – 33.4	25.0	54.7	15.4	21.5	8.4
	> 33.4	25.0	40.8	11.7	32.1	15.5
Total		100.0	57.5	16.2	17.8	8.5

^aThousands of Euros.

Table 2: Parameter estimates of the trivariate qualitative and quantitative model with heteroskedasticity correction. Asymptotic t -ratios in parenthesis.

Model	Probit			Tobit		
Parameter	β_1	β_2	β_y	β_1	β_2	β_y
Cost	-1.480 (-5.856)	-1.601 (-5.845)	15.942 (1.288)	-1.669 (-7.750)	-1.795 (-7.786)	16.647 (2.880)
Age	3.356 (3.663)	3.819 (3.871)	9.868 (0.222)	4.151 (5.370)	4.507 (5.482)	7.827 (0.403)
Age2	-4.039 (-5.043)	-5.479 (-5.920)	5.597 (0.140)	-4.937 (-7.555)	-5.692 (-8.051)	8.282 (0.467)
Ncomp	-0.041 (-0.730)	0.014 (0.245)	2.841 (1.123)	-0.005 (-0.102)	0.014 (0.300)	0.236 (0.240)
Married	0.037 (0.632)	0.073 (1.191)	1.643 (0.637)	0.047 (1.021)	0.058 (1.201)	-0.431 (-0.414)
Grad	0.165 (2.175)	0.124 (1.649)	-1.562 (-0.539)	0.062 (1.030)	0.066 (1.056)	-0.410 (-0.373)
Self	-0.140 (-1.825)	-0.288 (-3.713)	-0.263 (-0.079)	-0.168 (-2.650)	-0.234 (-3.533)	1.282 (0.975)
Ret	-0.026 (-0.391)	-0.163 (-2.276)	-4.199 (-1.299)	-0.076 (-1.387)	-0.134 (-2.293)	0.811 (0.708)
Pop	-0.141 (-1.879)	-0.093 (-1.240)	-2.103 (-0.591)	0.019 (0.262)	0.049 (0.666)	-0.361 (-0.257)
Central	-0.489 (-8.675)	-0.377 (-6.274)	-0.095 (-0.038)	-0.357 (-8.133)	-0.343 (-7.527)	0.995 (0.930)
South	-0.562 (-10.559)	-0.686 (-11.486)	2.552 (0.859)	-0.509 (-11.775)	-0.589 (-12.751)	4.274 (2.781)
Sex	-0.018 (-0.332)	-0.026 (-0.470)	-1.420 (-0.617)	-0.011 (-0.265)	-0.014 (-0.302)	-0.544 (-0.587)
Homeown	0.139 (2.590)	0.172 (3.038)	0.190 (0.075)	0.129 (3.013)	0.149 (3.297)	-1.025 (-0.905)
Rw	-0.921 (-4.465)	-1.177 (-5.507)	-1.379 (-0.143)	-0.932 (-6.143)	-1.049 (-6.653)	-1.218 (-0.325)
Fw	28.748 (36.737)	31.866 (39.652)	0.659 (0.053)	36.239 (115.370)	38.365 (79.866)	-17.050 (-3.890)
Fl	-0.749 (-0.632)	0.425 (0.376)	7.433 (0.169)	0.199 (0.223)	0.712 (0.850)	-0.632 (-0.040)
Income	-5.367 (-3.303)	-1.884 (-1.142)	-7.300 (-0.101)	-6.623 (-5.368)	-5.780 (-4.499)	8.577 (0.281)
Other parameters						
σ_2	0.967 (36.427)			1.043 (93.081)		
σ_y	27.150 (2.610)			5.364 (4.522)		
r_{12}	0.920 (19.102)			0.989 (299.572)		
r_{1y}	0.437 (3.778)			0.130 (1.932)		
r_{2y}	0.668 (10.081)			0.066 (0.969)		
ξ_1	-			38.653 (43.690)		
ξ_2	-			37.639 (70.119)		
ℓ	-5309.67			-5517.11		

Table 3: Simulated partial effects of the regressors on the different regimes probabilities for the structural tobit model with heteroskedasticity correction. The partial effects are measured as changes in regime probabilities multiplied by 100. For each explanatory variable and regime probability, the first row reports the sample weighted average, and the second (in parentheses) the weighted sample standard deviation, of the individual partial effects. See the text for a description of the changes in the regressors chosen to compute the partial effects.

	$p^{(0,0)}$	$p^{(1,0)}$	$p^{(0,1)}$	$p^{(1,1)}$
Age	1.367 (1.914)	0.341 (2.341)	-1.398 (1.623)	-0.310 (1.107)
Ncomp	-0.017 (0.082)	-1.160 (0.976)	1.801 (1.640)	-0.625 (1.405)
Married	-1.042 (0.603)	-0.048 (0.808)	0.839 (0.743)	0.251 (0.375)
Grad	-1.337 (0.759)	0.604 (0.658)	0.240 (0.716)	0.493 (0.530)
Self	3.835 (2.235)	1.752 (4.002)	-5.498 (4.127)	-0.088 (2.449)
Ret	1.944 (1.141)	2.553 (3.283)	-4.955 (3.793)	0.458 (2.531)
Pop	-0.586 (0.372)	-1.575 (1.645)	2.565 (2.049)	-0.403 (1.425)
Central	7.826 (4.156)	-5.468 (3.181)	0.358 (3.889)	-2.717 (2.833)
South	11.320 (6.127)	-2.156 (7.393)	-6.004 (5.838)	-3.158 (3.412)
Sex	0.251 (0.145)	-0.075 (0.107)	-0.547 (0.742)	0.370 (0.783)
Homeown	-2.785 (1.614)	0.445 (1.782)	1.553 (1.628)	0.787 (0.886)
Rw	0.824 (1.286)	-0.075 (0.904)	-0.802 (1.311)	0.054 (0.894)
Fw	-7.927 (8.520)	1.207 (8.236)	3.940 (5.994)	2.780 (5.660)
Fl	-0.013 (0.103)	-0.077 (0.316)	0.138 (0.651)	-0.049 (0.412)
Income	1.549 (1.354)	-1.528 (1.174)	0.835 (1.716)	-0.856 (1.415)

Table 4: Simulated partial effects of the regressors on the conditional and unconditional expected asset demands for the structural tobit model with heteroskedasticity correction. The partial effects are measured as changes in asset demands expressed in units of 1,000 Euro. For each explanatory variable and asset demand, the first row reports the sample weighted average, and the second (in parentheses) the weighted sample standard deviation, of the individual partial effects. See the text for a description of the changes in the regressors chosen to compute the partial effects.

	$E[a_1 (1, 0)]$	$E[a_1 (1, 1)]$	$E[a_1]$	$E[a_2 (0, 1)]$	$E[a_2 (1, 1)]$	$E[a_2]$
Age	0.087 (1.893)	-0.028 (3.688)	0.131 (3.068)	-0.576 (1.628)	-1.180 (3.294)	-1.319 (3.005)
Ncomp	-0.328 (0.789)	-0.699 (1.333)	-0.829 (1.588)	0.011 (0.700)	0.115 (0.833)	0.756 (1.419)
Married	0.022 (1.693)	0.144 (2.039)	0.061 (1.602)	0.410 (0.889)	0.664 (1.708)	0.693 (1.290)
Grad	0.368 (1.524)	0.465 (2.132)	0.431 (1.737)	0.600 (0.737)	0.682 (1.714)	0.492 (1.221)
Self	0.225 (1.842)	0.364 (6.116)	0.856 (5.101)	-1.543 (2.005)	-2.599 (5.846)	-3.542 (5.262)
Ret	0.717 (1.960)	0.864 (4.135)	1.294 (3.801)	-0.626 (1.466)	-1.691 (4.090)	-2.779 (4.035)
Pop	-0.520 (1.611)	-0.603 (1.892)	-0.809 (1.880)	0.101 (0.927)	0.707 (1.874)	1.319 (1.984)
Central	-3.034 (2.485)	-2.871 (6.481)	-2.876 (6.106)	-3.873 (2.770)	-2.949 (5.577)	-1.757 (4.750)
South	-1.745 (3.374)	-2.008 (18.480)	-1.515 (15.200)	-4.790 (5.645)	-5.973 (15.650)	-5.818 (12.330)
Sex	-0.255 (1.850)	0.279 (2.786)	0.301 (2.298)	-0.308 (0.913)	0.134 (1.944)	-0.183 (1.430)
Homeown	0.385 (1.904)	0.592 (5.063)	0.436 (4.053)	1.188 (1.623)	1.608 (4.216)	1.517 (3.115)
Rw	-0.192 (1.377)	0.616 (2.521)	0.328 (1.730)	-0.413 (1.598)	-0.128 (1.715)	-0.751 (1.971)
Fw	14.470 (27.962)	12.903 (124.820)	9.702 (108.622)	21.381 (40.424)	28.745 (135.812)	28.157 (118.721)
Fl	-0.045 (0.392)	-0.061 (0.467)	-0.052 (0.377)	-0.002 (0.181)	0.008 (0.205)	0.056 (0.348)
Income	-0.956 (1.725)	-0.874 (1.431)	-0.932 (1.562)	-1.006 (1.219)	-0.606 (1.105)	-0.177 (1.032)

Table 5: Summary statistics for the individual estimated expected state variables. $\hat{\pi}_1$, $\hat{\pi}_2$ and $\hat{\rho}$ are computed as expected values conditional on the observed investment regime and asset demands, evaluated at the parameter estimates in the tobit model with heteroskedasticity correction. $\hat{\tau} = \hat{\pi}_1/\hat{\pi}_2$ approximately measures the ratio of the perceived Sharpe performances of the two risky assets. “Q $p\%$ ” is the p -th sample percentile of the variables.

	$\hat{\pi}_1$	$\hat{\pi}_2$	$\hat{\tau}$	$\hat{\rho}$
Average	0.62	0.56	1.06	0.97
Std.Dev.	3.40	3.69	10.20	0.01
Q 2.5%	-1.85	-2.19	0.66	0.95
Q 25%	-1.42	-1.60	0.86	0.97
Median	-1.05	-1.18	0.92	0.97
Q 75%	1.30	1.23	0.97	0.97
Q 97.5%	10.62	11.24	1.71	0.98
% positive	42.2	41.5	98.7	100

Table 6: Historical Sharpe performances and correlation coefficients for several indexes approximating the excess returns of the two risky assets over three different time intervals. See the text for a description of the data used to compute these estimates.

Risky asset 1	Risky asset 2			Correlation with risky asset 1		
Bank of Italy	MIBTEL	MSCI	Fideuram	MIBTEL	MSCI	Fideuram
July 1999 - December 2000 (18 months)						
-0.24	0.21	0.01	-0.04	0.00	-0.10	-0.09
July 1998 - December 2000 (30 months)						
0.14	0.22	0.12	n.a.	0.06	0.11	n.a.
January 1996 - December 2000 (60 months)						
0.33	0.24	0.14	n.a.	0.30	0.37	n.a.