Cost Channel and the Price Puzzle: The Role of Interest Rate Smoothing*

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Abstract

A new-Keynesian DSGE model embedding the cost-channel is able to generate an on impact *positive* reaction of the inflation rate to a monetary policy shock. Such prediction appears to offer a rationale for the "price puzzle" typically found when estimating VAR impulse responses. We show that this prediction is overturned when the model features interest rate smoothing and past-dependent policies in general. In particular, we study the effect of monetary policy gradualism on the inflation impulse response function, and we relate such effect to the behavior of inflation expectations. Bayesian estimations with U.S. data support the relative strength of the demand channel, i.e. the estimated model produces a negative inflation reaction to an unexpected interest rate hike even in presence of an active cost-channel.

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1 Introduction

What is the short-run reaction of inflation to an unexpected and temporary monetary policy tightening? The conventional view suggests that inflation should react negatively to such a monetary policy move (Woodford (2003a)). However, empirical investigations based on the VAR-methodology cast doubts on this prediction. Figure 1 makes this point.[1] In a trivariate VAR for the U.S. that considers real GDP, inflation, and the federal funds rate, an unexpected one-shot increase in the policy rate leads to a *significantly positive* inflation reaction. As said, this result hardly squares with the predictions offered by standard monetary models of the business cycle. This is the reason why Eichenbaum (1992) labeled as "price puzzle" this empirical conditional correlation.

[Figure 1 about here]

A growing body of the empirical literature has supported the relevance of the "cost-channel" in determining inflation.[2] The idea is simple. Cash-constrained firms must borrow money from financial intermediaries to pay the wage-bills to the workers employed in the production process. Consequently, the interest rate paid on borrowings is one of the elements of firms' marginal costs. This creates a direct and positive link between monetary policy moves, oscillations in marginal costs and movements in prices charged by monopolistically competitive firms. In aggregate terms, this translates into movements in the inflation rate. Therefore, monetary policy affects inflation throughout two channels. One is the standard "demand channel", which hinges upon monetary policy-makers' ability to influence agents' intertemporal allocation decisions. The second one is the "supply channel" (or "cost-channel"), based on the direct impact that the policy rate exerts on inflation via firms' marginal costs.[3] If the latter is stronger than the former, the model is able to produce a conditional correlation between inflation and the policy rate in line with the one in Figure 1.

[1]Impulse responses related to a trivariate VAR with log real GDP, GDP deflator inflation, and federal funds rate, sample: 1966Q1-2005Q4. The monetary policy shock was identified via a Cholesky decomposition (ordering of the variables: log real GDP, inflation, federal funds rate). Similar evidence is reported by Stock and Watson (2001). Hanson (2004) and Castelnuovo and Surico (2006) show that this result is robust to the introduction of commodity prices as well as a variety of other inflation predictors in the VAR.

[2]Barth and Ramey (2001) analyze different U.S. sectors and find that sectorial differences in the working capital may rationalize the heterogeneous impact across sectors of a monetary policy shock. Similar results are obtained by Gaiotti and Secchi (2005) for Italy, and Dedola and Lippi (2005) for France, Germany, Italy, and the United Kingdom. Christiano et al (2005) find empirical support in favor of the working capital hypothesis for the U.S. in a DSGE framework. In a single-equation framework, Ravenna and Walsh (2006) support the presence of the cost-channel for the U.S. economy, Tillmann (2007) for the U.S., U.K., and Euro Area, and Chowdhury et al (2006) for Canada, France, Italy, the U.K. and the United States.

[3]For a comprehensive introdution to the monetary transmission mechanism, see Ireland (2008).

Clearly, central banks and academic practitioners are very interested in understanding the sign (as well as the magnitude) of the inflation reaction to a monetary policy shock. This information requires the employment of a fully specified model. Along this line, Chowdhury et al (2006) add to an estimated Phillips curve embedding the cost-channel a calibrated IS curve along with a Taylor rule without interest rate smoothing motive. Their findings point towards a positive reaction of the inflation rate to a monetary policy shock for Italy, the U.K., and the United States. Christiano et al (2005) estimate a model featuring several nominal and real rigidities by impulse-response functions matching, and find evidence in favor of a positive reaction of inflation after an unexpected policy rate hike. With the same econometric strategy, Henzel et al (2007) obtain similar results for the Euro Area.

This paper concentrates on the standard three-equation framework a la Woodford (2003a) modified to embed the cost-channel. We begin our analysis with a fully forward looking version of the model in order to derive a necessary and sufficient condition for the "model consistent price puzzle" to realize.[4] Then, we allow for several sources of persistence (price indexation, habit formation, interest rate smoothing) and resort to simulations to evaluate the inflation impulse response function to a monetary policy shock. Finally, we assess the sign and the magnitude of the inflation impulse response to a monetary policy shock in model estimated with U.S. data.

Our findings read as follows. First, we show that a calibrated model that does *not* feature a *past-dependent monetary policy* may actually predict a positive reaction of the inflation rate to a monetary policy shock. Interestingly, when investigating the role of interest rate smoothing in affecting inflation dynamics, we find a monotonically negative relationship between the degree of interest rate smoothing and the on-impact inflation reaction to a monetary policy tightening. This relationship builds upon the influence exerted by interest rate smoothing on inflation expectations (Woodford (1999,2003b)). Several different estimated versions of the model offer strong support to the conventional view in scenarios in which the systematic monetary policy is allowed to be past-dependent.

The paper closest to ours is the one by Rabanal (2007), who investigates the importance of the cost-channel in a medium-scale model a la Christiano et al (2005). Rabanal (2007) also finds evidence supporting the relative strength of the standard demand channel. In his paper, the key-drivers for this result are i) a less than full wage indexation, ii) a moderate wage stickiness, and iii) a high price stickiness. Our paper differs with respect to Rabanal (2007) along several dimension. First, we derive the necessary and sufficient condition for the existence of the price puzzle in a simple, purely forward looking model. Second, we highlight the role played by past-dependence

[4] We label as "model consistent price puzzle" a positive inflation reaction to a monetary policy shock predicted by the new-Keynesian model.

in fostering the strength of the demand channel, i.e. a different driver with respect to those identified by Rabanal (2007). Third, given our different interpretation of the cost-channel parameter, i.e. "risk-free to lending rate pass-through" in our paper, as opposed to "share of financially constrained firms" in Rabanal (2007)'s paper, such parameter may assume values larger than one, so putting the relative strength of the demand channel to a very hard test. Overall, we believe that our contribution complements and corroborates Rabanal (2007)'s.

This paper develops as follows. Section 2 presents the model, *de facto* borrowed from Chowdhury et al (2006) and Dennis (2005). Section 3 investigates the role played by interest rate smoothing in determining the on impact inflation reaction to a policy rate hike. In Section 4 we present and comment our estimation results. Finally, a discussion on the misspecification of the policy shock likely to affect VAR exercises is entertained. Section 5 concludes.

2 A microfounded model with the cost channel

As mentioned above, we concentrate on a simple DSGE model allowing for (but not imposing) the cost-channel.

Supply side

We model the supply side of the economy as in Chowdhury et al (2006) and Tillmann (2006). There exists a continuum of monopolistically competitive firms employing labor for producing goods with a constant returns to scale technology. Firms must pay workers before the goods market opens, i.e. they are financially constrained. Each firm i must borrow funds $Z_{it} \geqslant P_t w_t l_{it}$ from financial intermediaries, where P_t is the aggregate price level, w_t stands for the economy-wide real wage, and l_{it} is the firm-specific amount of labor employed.

Firms' real total costs read $TC_t = (1 + R_t)^{\psi} w_t l_t$, where ψ is "pass-through" parameter relating the risk-free net interest rate R_t to the lending rate R_t^l, i.e. $(1 + R_t^l) = (1 + R_t)^{\psi}$. The parameter ψ can be larger than one depending on financial market frictions or costly financial intermediation (Chowdhury et al (2006)).[5] It is then possible to derive the following expression for firms' real marginal costs (in log-deviation with respect to their steady-state values): $mc_t = w_t + \psi R_t$. Interestingly, Ravenna and Walsh (2006) and Rabanal (2003,2007) show that one may obtain they same expression by interpreting the parameter ψ as the share of financially constrained firms operating in the monopolistic market.[6]

[5] The average ratio between the U.S. bank prime loan rate and the federal funds rate amounts to 1.50 in the sample 1966-2005.

[6] Alternatively, Ravenna and Walsh (2006) interpret it as the fraction of the wage bill received by workers at the beginning of each period.

The price-setting of monopolistic firms is regulated by a Calvo-type mechanism, i.e. each period firms face a constant, exogenous probability θ of not being able to re-optimize their prices. When able to re-optimize, a fraction ω of firms follow a rule of thumb, i.e. it re-optimizes on the basis of the past inflation rate π_{t-1}; firms not able to re-optimize adjust their prices with the average (steady-state) inflation rate $\overline{\pi}$. Log-linearization around steady-state inflation leads to the following Phillips curve:[7]

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \chi mc_t \tag{1}$$

where mc_t stands for real marginal costs, $E_t\pi_{t+1}$ stands for today's rational forecast on tomorrow's inflation, $\gamma_f \equiv \frac{\beta\theta}{\theta+\omega[1-\theta(1-\beta)]}, \gamma_b \equiv \frac{\omega}{\theta+\omega[1-\theta(1-\beta)]}, \chi \equiv \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta+\omega[1-\theta(1-\beta)]}$, and β represents firms' discount factor.

As explained by Chowdhury et al (2006) and Dennis (2005), firms' real marginal costs may be expressed as a function of the output gap as follows:

$$mc_t = [\eta + \sigma(1-h)^{-1}]x_t - \sigma h(1-h)^{-1}x_{t-1} + \psi R_t \tag{2}$$

where x_t is a measure of the output gap (i.e. the log-difference between the actual output and the level of output under flexible prices), σ and η represent respectively the degree of relative risk aversion and the (inverse of the) labor elasticity, and h identifies the degree of external habits of the representative consumer.[8] As stressed by Dennis (2005), habits influence the marginal rate of substitution between consumption and leisure. In fact, yesterday's consumption augments the marginal utility of today's consumption and induces the representative consumer to increase labor supply, so exerting a downward pressure on firms' marginal costs.[9] As suggested by expression (2), as long as $\psi > 0$ the risk-free rate R_t is part of firms' marginal costs, and monetary policy exerts a *direct* impact on inflation in addition to the indirect effect working via the standard demand channel.

By merging eqs. (1) and (2), we obtain the output-gap based Phillips curve

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \chi \left[(\eta + \sigma)x_t - \sigma h(1-h)^{-1}x_{t-1} + \psi R_t\right] + \varepsilon_t^{\pi} \tag{3}$$

where the "cost-push" shock ε_t^{π} could be rationalized by assuming a shock to the time-varying mark-up as in Smets and Wouters (2003). We assume this shock to be serially correlated, i.e. $\varepsilon_t^{\pi} = \rho_\pi \varepsilon_{t-1}^{\pi} + \eta_t^{\pi}$.

Demand side

[7] The variables in the curves presented in this paper are expressed in percentage deviation with respect to their steady state values.

[8] The contemporaneous utility function of the representative household is given by $U(c_t, l_t) = (1-\sigma)^{-1}(c_t - h_t)^{(1-\sigma)} - (1+\eta)^{-1} l_t^{(1+\eta)}$, where $h_t = hc_{t-1}$ stands for habit consumption.

[9] For further discussions, see Dennis (2005).

The demand side of the economy is standard. In particular, the law of motion of the output gap is the following:

$$x_t = (1+h)^{-1}E_t x_{t+1} + h(1+h)^{-1}x_{t-1} - (1-h)[\sigma(1+h)]^{-1}(R_t - E_t\pi_{t+1}) + \varepsilon_t^x \quad (4)$$

Eq. (4) is obtained by log-linearizing the consumption Euler equation stemming from the household's intertemporal problem (in which consumption and bond holdings are the control variables) after imposing the equilibrium condition that consumption equals output. Output fluctuations are driven both by expectations on future realizations of the business cycle and by the ex-ante real interest rate. We model the demand shock as an AR(1) shock, i.e. $\varepsilon_t^x = \rho_x \varepsilon_{t-1}^x + \eta_t^x$.

Monetary policy

The model is closed by the following Taylor rule:

$$R_t = \phi_R R_{t-1} + (1-\phi_R)(\phi_\pi \pi_t + \phi_x x_t) + \eta_t^R \quad (5)$$

Eq. (5) postulates a systematic reaction of the policy rate to movements in inflation (identified by the parameter ϕ_π) and the output gap (via ϕ_x). Past policy decisions matter, and their impact is captured by the interest rate smoothing parameter ϕ_R, as in Clarida et al (2000). The zero-mean *i.i.d.* random shock η_t^R stands for the monetary policy innovation.[10]

3 Inflation reaction: Understanding the drivers

NSC in a purely forward-looking model

Given the model (3)-(5), what drives the *on impact* inflation reaction to a monetary policy shock? Some intuition can be gained by studying a version of the model featuring neither rule-of-thumb behavior in price setting ($\omega = 0$) nor habit formation in consumption ($h = 0$) nor monetary policy interest rate smoothing ($\phi_R = 0$), and in which the only shock hitting the economy is the monetary policy shock η_t^R. Under these constraints, the model displays no endogenous state variable, and the generic form of the solution for inflation reads $\pi_t = a\eta_t^R$, with a being a convolution of structural parameters. It is then possible to derive the following closed form solutions for the equilibrium inflation rate π_t:[11]

[10] It is worth stressing that the only shock of interest for our impulse response function exercise is the monetary policy shock. We append supply and demand shocks to the model to avoid the otherwise arising stochastic singularity problem when estimating it (see Section 4). An alternative - not entertained in this paper - would be that of assuming measurement errors for inflation and output.

[11] The derivation of this expression is provided in the Appendix.

$$\pi_t = \frac{\chi[\psi\sigma - (\eta+\sigma)]}{\sigma + \phi_x - \chi(\eta+\sigma)\phi_\pi + \chi\psi\sigma\phi_\pi}\eta_t^R \quad (6)$$

The numerator puts in evidence the tension existing between the cost-channel effect (driven by ψ) and the standard demand channel.

Given that χ is positive in the parameter space of interest, eq. (6) teaches us that, as long as $\psi \in [0, \overline{\psi} \equiv (\sigma+\phi_x+\chi(\eta+\sigma)\phi_\pi)(\chi\sigma\phi_\pi)^{-1}]$,[12] the on impact inflation reaction is $\frac{\partial\pi_t}{\partial\eta_t^R} > 0$ iff

$$\psi > \widetilde{\psi} \equiv 1 + \eta/\sigma \quad (7)$$

Condition (7) underlines the role played by the (inverse of the) labor supply elasticity η and the relative risk-aversion σ.[13] In particular, the higher (lower) η (σ), the lower the likelihood of observing a model-consistent positive inflation reaction to a monetary policy shock. After a monetary policy shock, consumers optimally postpone consumption. As a consequence, aggregate demand decreases, and so does production. Then, in response to a lower labor demand by firms, there is a drop in real wages and marginal costs. The amount of this drop is related to η: the higher η, the lower the labor supply elasticity to real wage fluctuations (i.e. the higher the slope of labor supply), the higher the real wage and marginal cost drop, the more intense the negative push on inflation exerted by the demand channel.[14] As regards the relative risk aversion σ, the higher σ, the lower the intertemporal elasticity of substitution, the weaker the demand channel.[15]

Inflation reactions in a model with inflation and output endogenous persistence

From a positive perspective, it is more interesting to study a model featuring some amount of endogenous persistence. This renders to model analytically intractable, we resort to simulations to evaluate inflation dynamics.[16] We calibrated the structure of

[12] This condition holds true under a very large variety of plausible calibrations.

[13] Notice that our assumption of constant returns to scale technology implies a more "favorable" condition for the price puzzle to arise. In fact, a decreasing returns to scale production function $y_{it} = Al_{it}^{(1-\alpha)}$ would imply $\psi > \widetilde{\psi} \equiv 1 + (\eta+\alpha)/[\sigma(1-\alpha)]$, which would call for a higher value for the pass-through parameter to produce a "model consistent price puzzle".

[14] We thank Pau Rabanal for a useful exchange on this point.

[15] Condition (7) implies that in this model the "price puzzle" cannot be induced by a share ψ of financially constrained firms (borrowing at a rate R_t). In fact, such interpretation would imply $\psi \in [0,1] \leq \widetilde{\psi} \equiv 1 + \eta/\sigma$ for each plausible parameterization η and σ. This observation was already put forward by Ravenna and Walsh (2006) as regards a model similar to the one presented here. However, this condition is model-specific, e.g. in the Christiano et al (2005) framework a value of ψ lower than one may still induce a positive inflation reaction (Rabanal (2007)).

[16] We solve the model numerically by searching for the unique equilibrium under rational expectations with the algorithm elaborated by Sims (2001). For a discussion of the uniqueness conditions in the model (1)-(3) with no endogenous persistence of any sort and with a central bank just reacting to inflation fluctuations, see Brueckner and Schabert (2003). For a study on the Taylor principle in a scenario featured by a Taylor rule also embedding the output gap, see Surico (2005). Llosa and

the model with fairly standard values: $\omega = 0.5, h = 0.7, \theta = 0.5, \sigma = 2, \eta = 0.5$. As regards systematic monetary policy, we impose $\phi_\pi = 1.5$ and $\phi_x = 0.125$ as in Christiano et al (2005). For the moment, we do not incorporate interest rate smoothing into the picture, i.e. $\phi_R = 0$.

We compute impulse response functions to a monetary policy shock.[17] To highlight the importance of the parameter ψ - and to understand how robust are our results to the variation of such a key parameter - we run alternative simulations with the values $\psi \in \{0, 1, 1.5, 2.7\}$. The first value identifies the "no cost channel scenario". The second value is the one considered by Christiano et al (2005). The third value is the average value of the ratio of the bank prime loan rate over the federal funds rate over the last fourthy years, and it is statistically in line with the estimates provided by Chowdhury et al (2006) and Ravenna and Walsh (2006) for the United States. The fourth value is equal to the 95th percentile of the bank prime loan rate/federal funds rate ratio, and it is intended to put the relative strength of the demand channel to a hard test.[18] Figure 2 depicts the impulse responses of the so-parameterized model to a standard deviation policy innovation. Under the "no cost channel scenario" the inflation reaction to a policy tightening is largely negative (due to the usual demand channel effect). When $\psi = 1$, the inflation reaction is still negative, even if much closer to zero. Once more, this is in line with our expectations, given that the adopted calibration implies $\widetilde{\psi} = 1.25$. Then, it does not come as a surprise that, when employing values of ψ higher than $\widetilde{\psi}$, the inflation reaction turns positive. Indeed, the last two simulated responses produce an evidence in line with estimated VARs for several industrialized countries (Sims (1992)).

[Figure 2 about here]

The role of interest rate smoothing

At this stage, we aim at understanding the role that interest rate smoothing may play in this context. As pointed out by Woodford (1999, 2003b), interest rate smoothing implies a past-dependent monetary policy. In other words, if a central bank follows a Taylor rule with interest rate smoothing and the private sector knows it, then agents will expect a monetary policy move to be followed by several other moves in the same

Tuesta (2007) investigate the effects of the cost-channel on determinacy and learnability of the rational expectations equilibrium. For an analysis on uniqueness in a set-up in which real balances enter the production function, see Benhabib et al (2001).

[17] The amount of the shock is equal to the standard deviation of the residual of the interest rate equation in the estimated VAR, i.e. 0.237.

[18] When considering the Commercial and Industrial Loan Rate (all loans, spread), the 95th percentile of the ratio of interest turns out to be equal to 2.50 (sample: 1986Q3-2007Q3). For a VAR study on the effects of monetary/non-monetary policy shocks on bank loans with C&I bank loans and loan rates, see den Haan et al (2007).

direction, and will adjust inflation expectations accordingly. This mechanism strengthens the demand channel, and may consequently exert a strong impact on the inflation reaction to a monetary policy shock.

To shed some light on the marginal impact of interest rate smoothing, we progressively augment ϕ_R and store per each different parameterization the value of the *on impact inflation reaction* to a policy shock. As previously done, we repeat this exercise for four different calibration of the cost-channel parameter. Figure 3 displays the *on impact* inflation reactions plotted against different degrees of interest rate smoothing. Evidently, interest rate smoothing reinforces the relative importance of the demand channel. In fact, the larger the former, the lower the on-impact inflation rate. Moreover, for values of the interest rate smoothing typically found in the literature (Clarida et al (2000), English et al (2003), Castelnuovo (2003)), inflation assumes negative values. Figure 4 plots the on impact effect of a monetary policy shock to inflation *expectations*. It is immediate to notice that the larger the interest rate smoothing degree, the lower the level of expected inflation.[19] These simulations support Woodford (1999, 2003b)'s analysis on the impact of past-dependent policies on inflation expectations.[20]

[Figures 3 and 4 about here]

4 Bayesian estimation

The results obtained so far are conditional to the particular parameterizations we employed. To assess the relevance of interest rate smoothing for the U.S. economy, we take the model to the data. We estimate the model (3)-(5) with Bayesian methods (for an extensive survey, see An and Schorfheide (2007)). We consider U.S. quarterly data on the output gap, inflation, and a nominal short-term interest rate for the sample 1966Q1-2005Q4. The beginning of the sample is justified by the fact that the federal funds rate

[19] Habit formation also exerts an impact on the inflation impulse response function: the higher the degree of habits, the lower the intertemporal elasticity of substitution, the weaker the demand channel. However, some simulations showed us that the impact of habit formation on the inflation reaction is actually negligible under plausible calibrations. Another set of some simulations confirmed us that the impact of interest rate smoothing on inflation expectations induces a more sustained path for the ex-ante real interest rate and a more severe recession. We thank Eric Mayer for raising this point.

[20] An extreme case might be of interest. Suppose $\omega = h = \phi_x = \rho_\pi = \rho_x = 0$, i.e. the only source of persistence in the model is the interest rate smoothing implemented by the monetary policy authorities. Then, the equilibrium value of inflation assumes the generic form $\pi_t = aR_{t-1} + b\eta_t^R$, while, the equilibrium policy rate value is given by $R_t = cR_{t-1} + d\eta_t^R$, where a, b, c, d are convolutions of the parameters of the model. Therefore, $E_t\pi_{t+1} = aR_t = a(cR_{t-1} + d\eta_t^R)$, which highlights the dependence of inflation expectations on past monetary policy. By contrast, in absence of interest rate smoothing (i.e. $a = 0$), $E_t\pi_{t+1} = 0$. We refer to Woodford (1999, 2003b) for a more detailed explanation on the effect of the lagged interest rate on expectations and current realizations in a framework similar to the one employed here.

has been consistently traded above the discount rate since 1966, so suggesting the use of the former as the primary instrument of monetary policy (Fuhrer (2000)). The output gap is computed as percentualized log-deviation of the real GDP with respect to potential output. As regards the latter, we rely on the Congressional Budget Office's estimates of the potential output level.[21] The inflation rate is the quarterly growth rate of the GDP deflator P_t, i.e. $\pi_t = 100(P_t - P_{t-1})/P_{t-1}$. Finally, for the short-term nominal interest rate we consider the quarterly federal funds rate (averages of monthly values).[22] All the transformed data are demeaned before estimation.

4.1 Priors

We impose some dogmatic priors before estimation. In particular we fix the discount factor $\beta = 0.99$ (corresponding to an annual discount rate of approximately 4%). Preliminary attempts to estimate the Calvo-parameters θ and the price-indexation parameter ω showed us that the posterior distribution does not display an appreciable curvature around the posterior mode, a signal suggesting lack of identification for those parameters. We then fixed $\theta = 0.65$ as in Smets and Wouters (2007), and $\omega = 0.80$. This last value is higher than the one obtained by Galí et al (2003), but appears to be favored by the data.[23]

As regards our priors, we assume $\eta \sim Gamma(1, 0.3^2)$ and $\sigma \sim Gamma(3, 0.9^2)$. This implies that, on average, $\widetilde{\psi}$ (from condition (7)) will take a value equal to $4/3 \approx 1.33$. With the aim of not taking any strong a-priori position in favor or against the "model consistent price puzzle", we assume $\psi \sim Beta(1.34, 0.55^2)$.[24]

Notice that this support covers all the point estimates for the U.S. case offered by the literature so far, e.g. the values obtained by Rabanal (2007) (0.15 in a full sample analysis, 0.56 for the '80s and '90s), the benchmark point estimate 1.276 provided by Ravenna and Walsh (2006), and the 1.3 reported by Chowdhury et al (2006). In order to reach a robust conclusion regarding the sign of the inflation reaction to a monetary

[21] Ravenna and Walsh (2006) point out that under an active cost-channel, the welfare relevant output gap is the one computed by considering flexible output conditional on a constant and positive nominal interest rate. Admittedly, there might be a mismatch between our empirical measure of the output gap and the one present in the theoretical framework. As a robustness check, we also allowed the Fed not to respond to any measure of output (gap). The results presented in this paper are robust to this perturbation of the model.

[22] The data on real GDP, potential output, GDP deflator, and federal funds rate were downloaded from the Federal Reserve Bank of St. Louis' website.

[23] We comparing the marginal likelihoods (computed via Laplace approximation) of three different models estimated alternatively under $\omega = 0.3, \omega = 0.5$, and $\omega = 0.8$, we verified that the last one is favored by the data.

[24] Notice that we imposed bounded domains for η, σ and ψ, i.e. $\eta \in [0.5, 1.5], \sigma \in [1, 5], \psi \in [0, 2.68]$. Preliminary attempts of estimating ψ with a uniform prior led to convergence problems when searching for the posterior mode.

policy shock (i.e. a conclusion not driven by identification issues concerning the cost-channel parameter ψ), we estimate other four different versions of the model featured by the following values of the cost-channel parameter: $\psi \in \{0, 1, 1.5, 2.7\}$.

The remaining priors are fairly standard. The habit formation parameter $h \sim Beta(0.7, 0.1^2)$. The parameters of the Taylor rule ϕ_π and ϕ_x follow a Gamma distribution having respectively mean 1.75 and 0.125 and standard deviations 0.125 and 0.05, while the interest rate smoothing degree ϕ_R is Beta distributed with mean equal to 0.75 and standard deviation equal to 0.1. As regards the roots of the AR(1) structural shocks, $\rho_\pi \sim Beta(0.3, 0.1^2)$, and $\rho_x \sim Beta(0.7, 0.1^2)$. Finally, the standard errors of the three structural innovations are assumed to follow an Inverse-Gamma distribution with a mean equal to 0.1 and a standard deviation equal to 0.25.

4.2 Posteriors

Given the vector $\xi = (\beta, \theta, \omega, \eta, \sigma, \psi, h, \phi_\pi, \phi_x, \phi_R, \rho_\pi, \rho_x, \sigma_{\eta\pi}, \sigma_{\eta x}, \sigma_{\eta r})'$ of structural parameters, the vector of endogenous variables $z_t = [x_t, \pi_t, R_t]'$, the vector of exogenous shocks $\varepsilon_t = [\varepsilon_t^x, \varepsilon_t^\pi]'$, the vector of innovations $\eta_t = [\eta_t^x, \eta_t^\pi, \eta_t^R]'$, and the vector of observable variables we aim at tracking $Y_t = [x_t, \pi_t, R_t]'$, we write the model in state space form, we relate the latent processes to the observable variables via the measurement equation (without assuming any measurement errors), and we employ the Kalman filter to evaluate the likelihood $L(\{Y_t\}_{t=1}^T \mid \xi)$. The posterior distribution $p(\xi \mid \{Y_t\}_{t=1}^T)$ is then proportional to the product of the likelihood function $L(\{Y_t\}_{t=1}^T \mid \xi)$ and the priors $\Pi(\xi)$.[25]

Figure 5 displays our prior and posterior distributions. It is immediate to see that all the posteriors are unimodal and smooth. Moreover, priors and posteriors do not overlap, a signal supporting the informativeness of the data. Focusing on the parameter ψ, we see that the support of the posterior distribution is relatively wide, and comprises

[25] To perform our Bayesian estimation we employed Dynare 4.0, available at http://www.cepremap.cnrs.fr/dynare/. The mode of each parameter's posterior distribution was computed by using the 'csminwel' algorithm elaborated by Chris Sims. A check of the posterior mode, performed by plotting the posterior density for values around the computed mode for each estimated parameter in turn, confirmed the goodness of our optimizations. We then exploited such modes for initializing the random walk Metropolis-Hastings algorithm to simulate the posterior distributions. In particular, the inverse of the Hessian of the posterior distribution evaluated at the posterior mode was used to define the variance-covariance matrix of the chain. The initial VCV matrix of the forecast errors in the Kalman filter is set to be equal to the unconditional variance of the state variables. We initialized the state vector in the Kalman filter with steady-state values. We simulated two chains of 250.000 draws each, and discarded the first 50% as burn-in. To scale the variance-covariance matrix of the random walk chain we used a factor equal to 0.6, which implied an acceptance rate of about 0.30. To assess the stationarity of the chain, we considered the graphical outcome of the Brooks and Gelman (1998)'s convergence checks. We concentrated on the draws that imply a unique solution of the system at hand. The 90% support of the priors imposed in the benchmark estimation delivers uniqueness in the 98.8% of the draws.

most of the point-estimates in the literature. Table 1 reports some statistics regarding the computed posterior distributions. By concentrating on the posterior means, we verify that $\widehat{\psi} = 1.04 < \widehat{\widehat{\psi}} \approx 1.47$. According to condition (7), this suggests that the model is likely to return a negative inflation reaction to an unexpected monetary policy tightening. Figure 6 plots the Bayesian impulse responses of output, inflation, and the federal funds rate to a MP shock.[26] Indeed, when considering the 5th and 95th percentile, one can state that the estimated model offers no support to the "price puzzle" VAR evidence even if featuring the cost-channel.

The other structural parameter of interest display posteriors in line with those presented by several other authors in the literature. Finally, the shocks in the economy display a fair amount of persistence (larger for the demand shock with respect to the cost-push shock), but the estimated autoregressive parameters values are far from unity. This suggests that in the model there is a propagation mechanism of the shocks capable to replicate the unit-root like dynamics of output and inflation without the need of imposing unit-root (or almost unit-root) shocks.

[Figures 5 and 6, Table 1 about here]

Table 2 collects the values of the log-marginal likelihoods of eight different versions of our model.[27] Along with the "Benchmark" model, we estimated the log-marginal likelihoods of four models featured by different constraints over ψ. For the sake of comparison, we also estimated the log-marginal likelihoods of three models displaying some lack of endogenous persistence.[28]

Interestingly, it turns out that the likelihood of the model a la Woodford (2003a) with $\psi = 0$ is not penalized with respect to the one with the freely estimated cost-channel parameter. In fact, the marginal likelihood favors the standard model, even if the difference between the two marginal likelihoods is very mild.[29] When assuming

[26] We produce impulse response functions by drawing 500 different vectors of parameters from the posterior distributions and plotting the mean response along with the 5th and 95th percentile.

[27] We computed the log-marginal likelihoods both by means of the Laplace approximation around the posterior mode (based on a normal distribution) and via the modified harmonic mean estimator (Geweke (1998)), which exploits the draws from the posterior distribution. The two methods deliver virtually identical results. This is due to the close-to-normal distribution of all the estimated posteriors. Given the large computational gains implied by the Laplace approximation, we employ this approximation for our model comparison.

[28] In performing our model comparisons we keep fixed across models the priors on unobservables such as the autoregressive parameters or the variances of the structural shocks. In other words, we treat these priors as subjective priors. For an alternative approach based on "calibrating" such priors on the basis of external information with respect to the sample employed for estimating the model, see Del Negro and Schorfheide (2006).

[29] Notice that an advantage of the marginal likelihood criterion is that it penalizes overparameterization, i.e. models with a more sophisticated economic structure are not necessarily better ranked if extra-parameters (with respect to those already present in a given benchmark model) are not helpful in

uniform priors on the two models "$\psi = 0$" and "$\psi = 2.7$", the Bayes factor amounts to $exp(-300.86 - (-304.44)) \approx 35.87$, a difference that might be judged as "strong" but not "very strong".[30] In other words, while implying a deterioration of the marginal likelihoods, high values of the cost-channel parameter are not necessarily "rejected" by the model posteriors. Notice that this is not an intrinsic feature of this model. In fact, when dropping ingredients such as price indexation, habit formation, or interest rate smoothing, the deterioration of the likelihood is remarkably much higher.

[**Table 2 about here**]

As said, one may still believe that the model with $\psi = 2.7$ is statistically supported by the data, even if economically it appears to be a very high value. Then, to put the demand channel at a very hard test, we re-estimate the model by fixing $\psi = 2.7$. Moreover, to assess the importance of interest rate smoothing for the sign of the inflation dynamic response, we estimate a model under $\psi = 2.7$ and $\phi_R = 0$.

Figure 7 depicts the dynamics responses produced by these two models. It is immediate to notice that inflation reacts positively to the shock under the "without interest rate smoothing" scenario. The impact exerted by the interest rate smoothing motive is clear when comparing the top-middle panel to the bottom-middle one: the sign of the inflation reaction significantly changes. In fact, impulse responses under the "with interest rate smoothing" scenario look more plausible: the reaction of output and inflation is hump-shaped, while that of the policy rate displays a persistence similar to the one coming from the VAR-evidence. Moreover, the comparison of the estimated marginal likelihoods (-477.83 in the "without interest rate smoothing scenario", -304.44 in the alternative one) clearly favors the framework modeling policy gradualism. This leads us to conclude that the data do not support the cost-channel interpretation of the VAR price puzzle evidence.

[**Figure 7 here**]

4.3 Robustness checks

We check the robustness of our results by varying our benchmark set-up along several dimensions.

explaining the data. As explained by e.g. Rabanal and Rubio-Ramírez (2005, 2007), this happens because the marginal likelihood considers all the regions of the parameter space, and it takes the average of both relatively large and small values of the likelihood function.

[30] According to Kass and Raftery (1995), a Bayes factor between 1 and 3 is "not worth more than a bare mention", between 3 and 20 suggests a "positive" evidence in favor of one of the two models, between 20 and 150 suggests a "strong" evidence against it, and larger than 150 "very strong" evidence.

Subsample analysis. The analysis developed so far has relied on the assumption of stability of the estimated parameters in the sample at hand, as in Smets and Wouters (2007). However, some authors have testified a break in the U.S. economic dynamics at the end of the '70s, i.e. the "Great Moderation" (McConnell and Perez-Quiros (2000)). Moreover, the appointment of Paul Volcker as Chairman of the Fed has also been associated to a break in the U.S. monetary policy conduct (Clarida et al (2000), Lubik and Schorfheide (2004), Cogley and Sargent (2005), Boivin and Giannoni (2006), Belaygorod and Dueker (2007)). To control for these breaks, we focus on the sample 1985Q1-2005Q4, and re-estimate the benchmark model under the constraint $\psi = 2.7$.

Monetary policy reaction to first-differences of inflation and output. Smets and Wouters (2003) model the systematic monetary policy by allowing a reaction both to the levels of inflation and the output gap and to their *first differences*. This modified Taylor rule reads as follows:

$$R_t = \phi_R R_{t-1} + (1 - \phi_R)(\phi_\pi \pi_t + \phi_x x_t + \phi_\pi^\Delta \Delta\pi_t + \phi_x^\Delta \Delta x_t) + \eta_t^R$$

Notice that this rule postulates a past-dependent behavior by the monetary authorities (due to lagged inflation and output) even under $\phi_R = 0$. As regards prior densities, we assume that $\phi_\pi^\Delta \sim Gamma(1.75, 0.125^2)$ and $\phi_x^\Delta \sim Gamma(0.125, 0.05^2)$.

Autocorrelated monetary policy shocks. Rudebusch (2002) states that the smooth behavior of the policy rate observed in the U.S. (and a variety of other countries) is not intentionally implemented by the Fed, but it is instead caused by serially correlated monetary policy shocks. The Taylor rule then reads as follows:

$$R_t = \phi_\pi \pi_t + \phi_x x_t + \varepsilon_t^R, \varepsilon_t^R = \rho_\varepsilon \varepsilon_{t-1}^R + \eta_t^R$$

Also according to this policy rule monetary authorities behave in a past-dependent fashion under $\phi_R = 0$. This is so because the shock ε_t^R is a state variable of the system. We assume the prior density $\rho_\varepsilon \sim Beta(0.75, 0.1^2)$.

Lower share of rule-of-thumb firms. We re-estimate the model by considering a degree of price indexation $\omega = 0.3$, as in Galí et al (2001, 2003).

The results of our robustness checks, portrayed by Figure 8, confirm that past-dependent monetary policies play a key role in turning the sign of the inflation impulse response negative. There are two cases in which such response is negative even under $\phi_R = 0$. One is the case in which the Fed reacts also to first differences in inflation and output. The other case is the one in which $\phi_R = 0$ but the monetary policy shock is autocorrelated. In these scenarios, past realizations of the policy rate "anchor" current monetary policy, therefore influencing inflation expectations and driving current inflation to assume negative values after a monetary policy shock (see panels (2,1) and (3,2) in Figure 8).

[Figure 8 here]

5 Why do we observe the "price puzzle" in VARs?

Our exercise sheds light on the reasons why an estimated small-scale structural model produces a negative reaction of inflation to a monetary policy shock. Rabanal (2007) reaches the same conclusion by focusing on the importance of different sources of persistence in the Christiano et al (2005) model. Then, if estimated models do not offer a rationale for the price puzzle, why do we observe it in VARs?

Sims (1992) was the first to point out that the price puzzle is likely to be due to a misspecification of the monetary policy shock. In fact, if the central bank reacts to expected inflation, then a predicted upcoming surge in inflation will be followed by an increase in the policy rate, a decrease in the output gap, and - as long as the monetary policy tightening is not such to fully offset the inflationary shock - a rise in current inflation. If the VAR omits expected inflation, and if expected inflation and current inflation are not strictly linked (i.e. current inflation is not a "sufficient statistic" for expected inflation), then the supposed-to-be monetary policy shock in a trivariate VAR in inflation, output gap, and policy rate will somewhat naturally capture the positive correlation between inflation and the policy rate, i.e. it will produce a price puzzle. Sims (1992) proposed to add an indicator of nascent inflation (commodity prices) to the vector of variables of interest. While not solving the price puzzle problem, this trick ameliorates the picture.

Interestingly, some of the best predictors of future inflation turn out to be useless for correcting the bias in the dynamics of inflation (Hanson (2004)). However, theoretical models may provide important insights in this direction. Castelnuovo and Surico (2006) show that the price puzzle evidence is actually limited to the pre-Volcker subsample. By contrast, when running a VAR with U.S. quarterly data for the '80s and '90s, one does not find any trace of it. Similar evidence is provided by Barth and Ramey (2001), Hanson (2004) and Boivin and Giannoni (2006). What can explain such a change in the inflation impulse response? Castelnuovo and Surico (2006) show that by simulating a standard DSGE model under passive monetary policy and estimating a structural VAR on simulated data, one finds a price puzzle even if the model does not feature any supply-side channel. Interestingly, they show that an augmented VAR including model-consistent inflation expectations produces the correct inflation impulse response featuring a negative inflation reaction to a monetary policy shock. Some exercises conducted with actual data with Greenbook inflation expectations corroborate Castelnuovo and Surico (2006)'s intuition on the importance of introducing inflation expectations in VARs estimated over subsamples featured by a passive monetary policy conduct.[31]

Other recent contributions have pointed towards other types of VAR misspecifications. Leeper and Roush (2003) show that money is important for well specifying the monetary policy shock when studying economies in which a double-causal link between money and interest rate might have occurred. In particular, if the central bank reacts contemporaneously to monetary aggregates, and if money demand is contemporaneously driven by the nominal interest rate, then the omission of money would lead to a misspecification of the monetary policy shock. A different issue is raised by Giordani (2004) who shows that the omission of potential output in standard trivariate VARs may severely bias impulse responses and be the responsible of the price puzzle. In fact, potential output appears in all the equations of a standard new-Keynesian AD/AS model. Hence, its omission will lead supposed-to-be shocks to be residuals correlated across the VAR equations, and consequently to produce biased impulse response functions. The omitted variable issue is also tackled by Bernanke et al (2005) and Boivin et al (2007), who show that by allowing some factors extracted by a large panel of variables to enter the vector autoregression (as "endogenous variables") the price puzzle tends to disappear. Romer and Romer (2004) stick on a standard trivariate VAR but produce a careful measure of the monetary policy shocks based on changes in the intended federal funds rate and the Fed's expectations on future inflation and output. Such new measure of monetary policy shock does not imply any price puzzle in their estimated VARs.

It seems to us that the research conducted so far has pointed towards interesting avenues for the solution of what appears to be a statistical artifact more than a truly structural response of inflation to a monetary policy shock.

6 Conclusions

This paper showed that new-Keynesian model embedding the cost-channel may hardly offer a rationale for the price puzzle typically observed in VAR analysis. Under some particular parameterizations of the model, a positive inflation reaction to an unexpected, restrictive monetary policy may actually arise. However, when allowing even for a moderate amount of interest rate smoothing by the central bank, the sign of the inflation reaction to a monetary policy tightening turns out to be invariably negative. The impact exerted by systematic monetary policy on inflation expectations when allowing for monetary policy gradualism explains this result. When taken to the U.S. data, the model predicts an inflation reaction in line with the conventional view even under a very large cost-channel pass-through. This result is robust under different subsamples

[31] A somewhat complementary paper is the one by Belaygorod and Dueker (2007), who show that a positive inflation reaction to a monetary policy shock may be model-consistent under indeterminacy when selecting particular equilibrium paths of the endogenous variables.

and several alternative parameterizations of the non-estimated part of the model. We think of this result as being important for understanding the sign (and the magnitude) that monetary policy actions should take in response to shocks leading the inflation rate off target.

We stress that this paper does *not* offer evidence against the existence of the cost-channel. In fact, our empirical results may also be interpreted as supporting the existence of a supply channel, i.e. the nominal interest rate may actually be an important component of firms' marginal costs. In this sense, the structural role of the interest rate in the Phillips curve calls for a serious re-thinking of optimal monetary policy in presence of supply effects, as recently shown by Ravenna and Walsh (2006). Moreover, given the uncertainty surrounding the magnitude of the cost-channel parameter, more research is needed both for the quantification of the importance of the cost-channel and for the design of an optimal monetary policy in presence of cost channel uncertainty. Along this line, a recent investigation has been put forward by Tillmann (2007).

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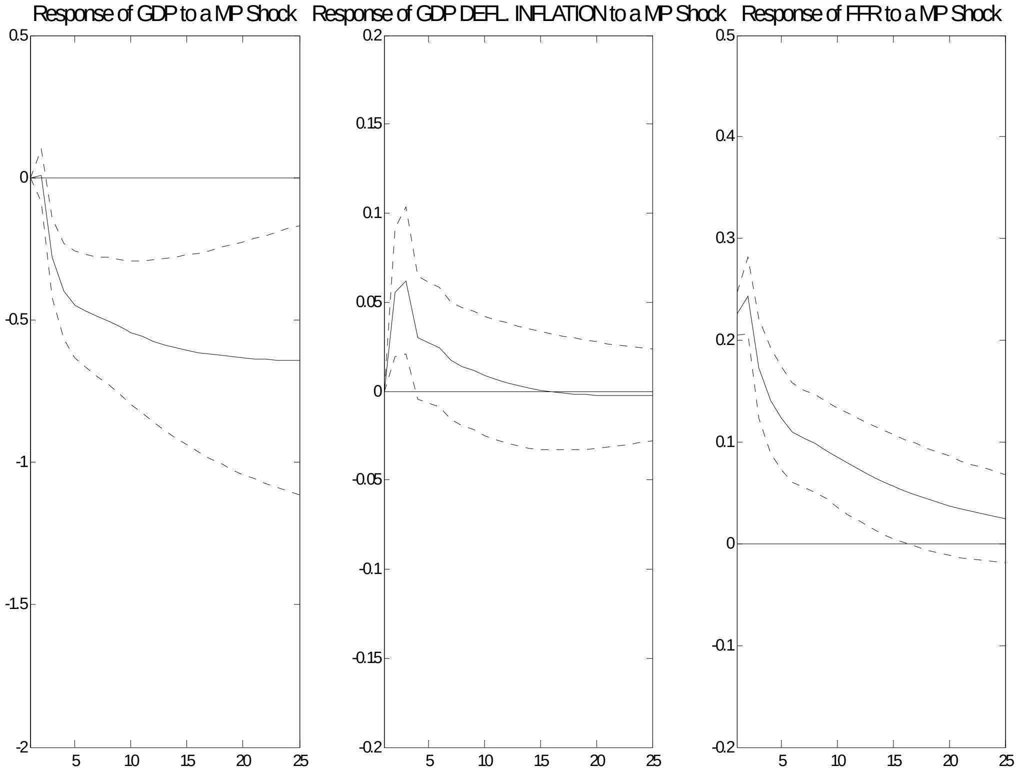

Figure 1: IMPULSE RESPONSE FUNCTIONS TO A MONETARY POLICY SHOCK. Sample: 1966Q1-2005Q4. Solid line: Median response; dotted lines: 90% confidence bands (analytically computed). Shock size: One standard deviation. Number of lags: 3. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: real GDP, inflation, federal funds rate). Data: 100log(real GDP), quarterly GDP deflator inflation, quarterly federal funds rate (more information in the text).

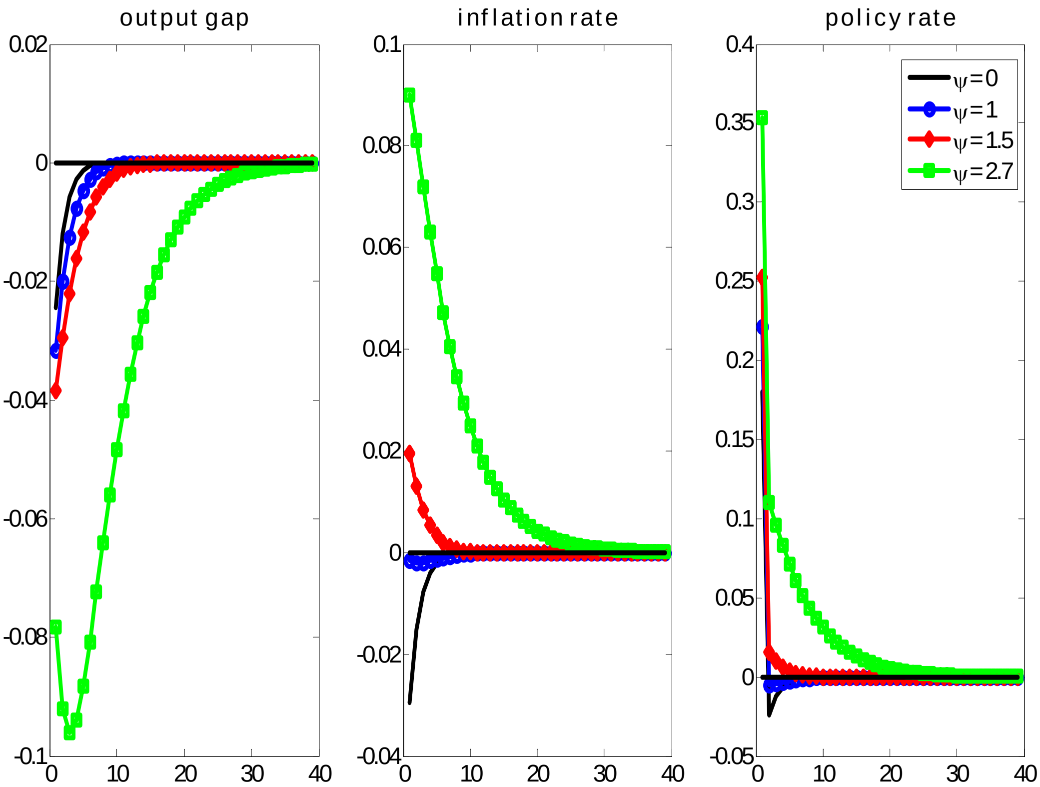

Figure 2: MODEL-CONSISTENT IMPULSE RESPONSES TO A MONETARY POLICY SHOCK. Calibration: $\theta = 0.5, \omega = 0.5, \sigma = 2, \eta = 0.5, h = 0.7, \phi_\pi = 1.5, \phi_x = 0.125, \phi_R = \rho_x = \rho_\pi = 0$.

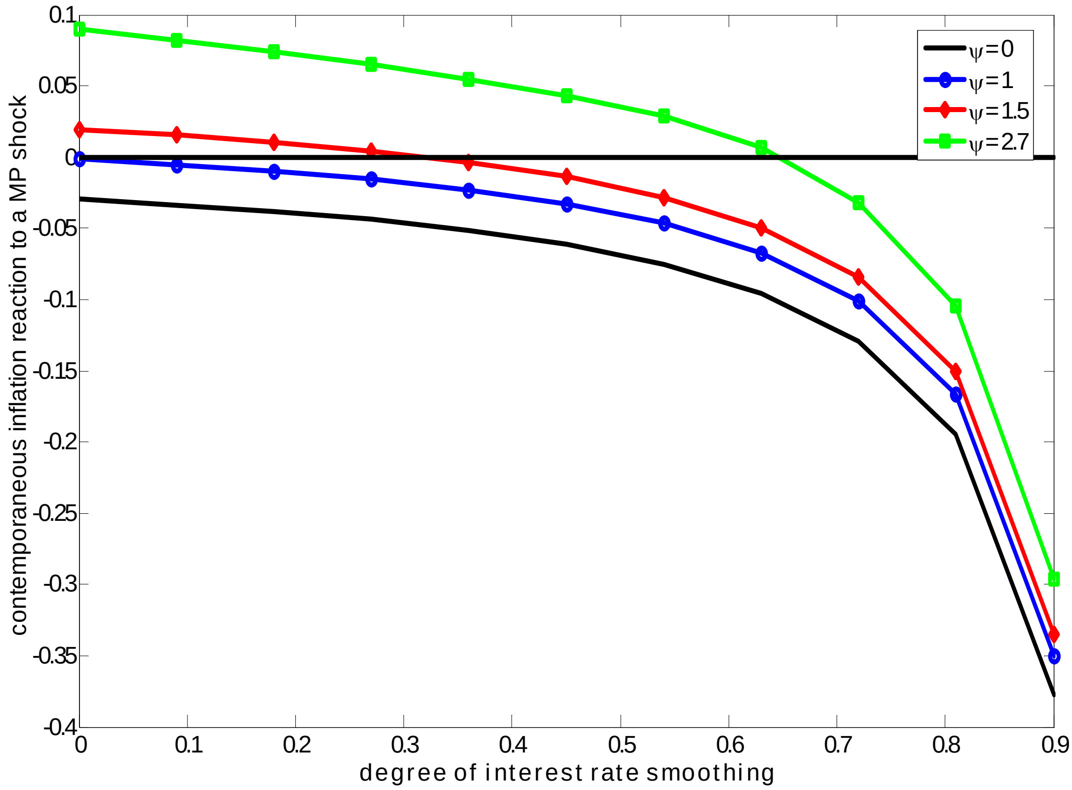

Figure 3: ON IMPACT INFLATION REACTION TO A MONETARY POLICY SHOCK: ROLE OF INTEREST RATE SMOOTHING. Calibration: $\theta = 0.5, \omega = 0.5, \sigma = 2, \eta = 0.5, \phi_\pi = 1.5, \phi_x = 0.125, \phi_R = \rho_x = \rho_\pi = 0$.

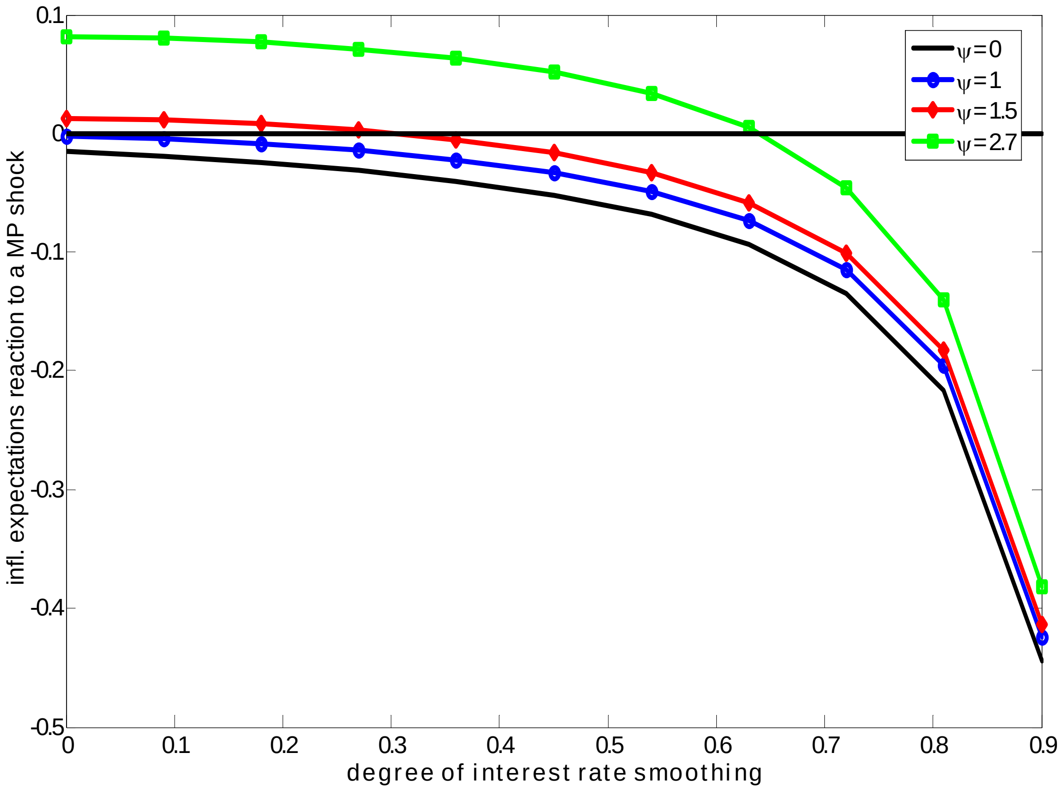

Figure 4: ON IMPACT INFLATION EXPECTATIONS REACTION TO A MONETARY POLICY SHOCK: ROLE OF HABIT FORMATION AND INTEREST RATE SMOOTHING. Calibration: $\theta = 0.5, \omega = 0.5, \sigma = 2, \eta = 0.5, \phi_\pi = 1.5, \phi_x = 0.125, \phi_R = \rho_x = \rho_\pi = 0$.

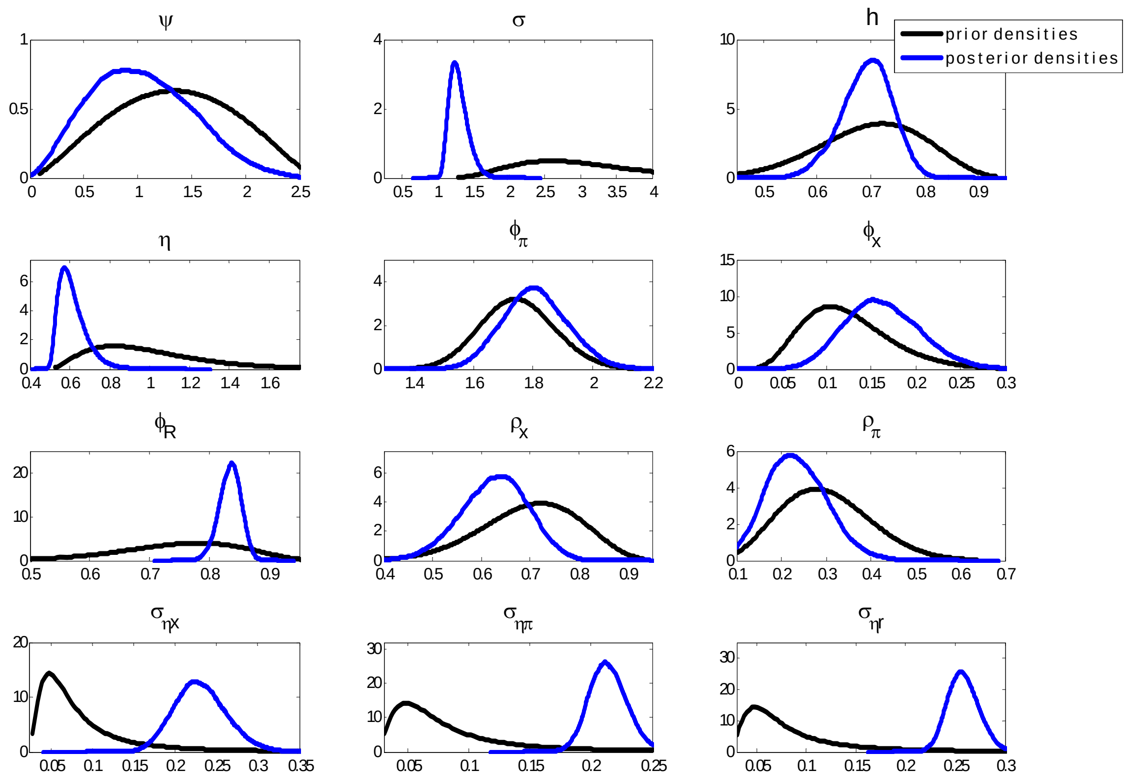

Figure 5: PRIORS AND POSTERIOR DENSITIES: BENCHMARK MODEL. Sample: 1966Q1-2005Q4. Prior densities discussed in the text.

Param.	*PriorDistrib.*	*Prior Mean* (*St.dev.*)	*Posterior Mean* (*St.dev.*)	*Posterior* [5%, 95%]
ψ	Beta*	1.34 (0.55)	1.04 (0.56)	[0.28, 1.78]
h	Beta	0.70 (0.10)	0.69 (0.05)	[0.62, 0.77]
σ	Gamma**	3.00 (0.90)	1.30 (0.11)	[1.09, 0.49]
η	Gamma***	1.00 (0.30)	0.61 (0.05)	[0.51, 0.71]
ϕ_π	Gamma	1.75 (0.125)	1.81 (0.11)	[1.63, 1.98]
ϕ_x	Gamma	0.125 (0.05)	0.16 (0.04)	[0.09, 0.23]
ϕ_R	Beta	0.75 (0.10)	0.83 (0.02)	[0.80, 0.86]
ρ_x	Beta	0.70 (0.10)	0.62 (0.07)	[0.51, 0.74]
ρ_π	Beta	0.30 (0.10)	0.23 (0.07)	[0.12, 0.34]
$\sigma_{\eta x}$	Inv_Gamma	0.10 (0.25)	0.23 (0.03)	[0.18, 0.38]
$\sigma_{\eta \pi}$	Inv_Gamma	0.10 (0.25)	0.21 (0.01)	[0.19, 0.24]
$\sigma_{\eta r}$	Inv_Gamma	0.10 (0.25)	0.26 (0.01)	[0.23, 0.28]

Table 1: PRIOR AND POSTERIOR DISTRIBUTIONS, BENCHMARK MODEL. Sample: 1966Q1-2005Q4. Dogmatic priors: See description in the text. * Beta with [0, 2.68] support; ** Gamma with [1,5] support; *** Gamma with [0.5,1.5] support.

Model	$Log(\widehat{L})$
Benchmark	-301.47
$\psi = 0$	-300.86
$\psi = 1$	-301.26
$\psi = 1.5$	-301.82
$\psi = 2.7$	-304.44
$\omega = 0$	-466.56
$h = 0$	-337.49
$\phi_R = 0$	-462.24

Table 2: MODEL COMPARISON. Sample: 1966Q1-2005Q4. Log-marginal likelihoods computed via Laplace approximation. The Table indicates departures from the Benchmark model.

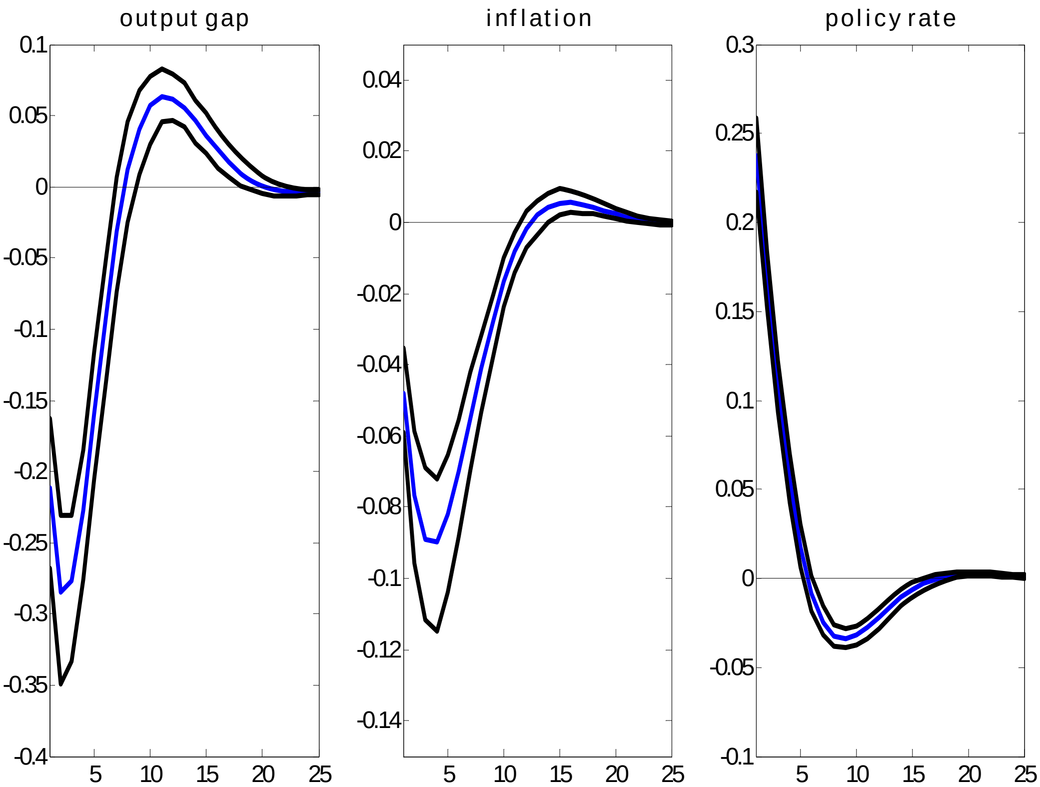

Figure 6: BAYESIAN IMPULSE RESPONSES: BENCHMARK MODEL. Blue lines: mean impulse response. Black lines: 5th and 95th percentiles.

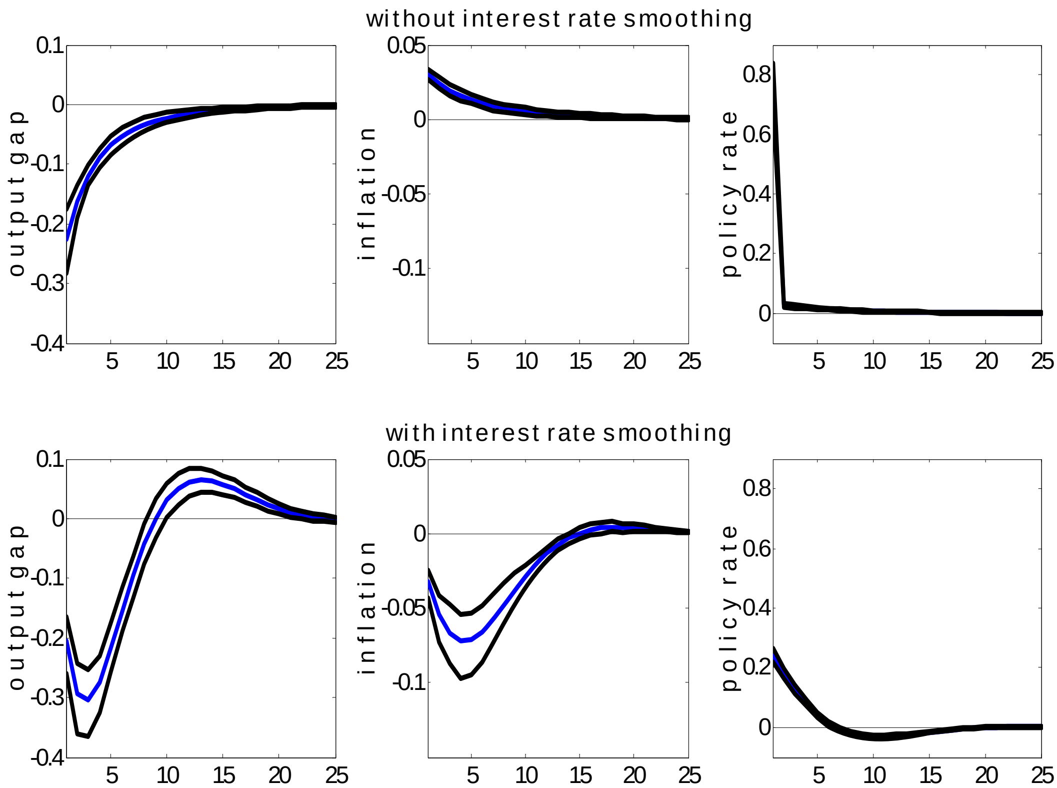

Figure 7: IMPULSE RESPONSE FUNCTIONS: THE INTEREST RATE SMOOTHING EFFECT. All models conditional to $\psi = 2.7$. Top panels: impulse responses from a model estimated under $\phi_R = 0$. Explanations of the models in the text. Blue lines: mean impulse response. Black lines: Black lines: 5th and 95th percentiles.

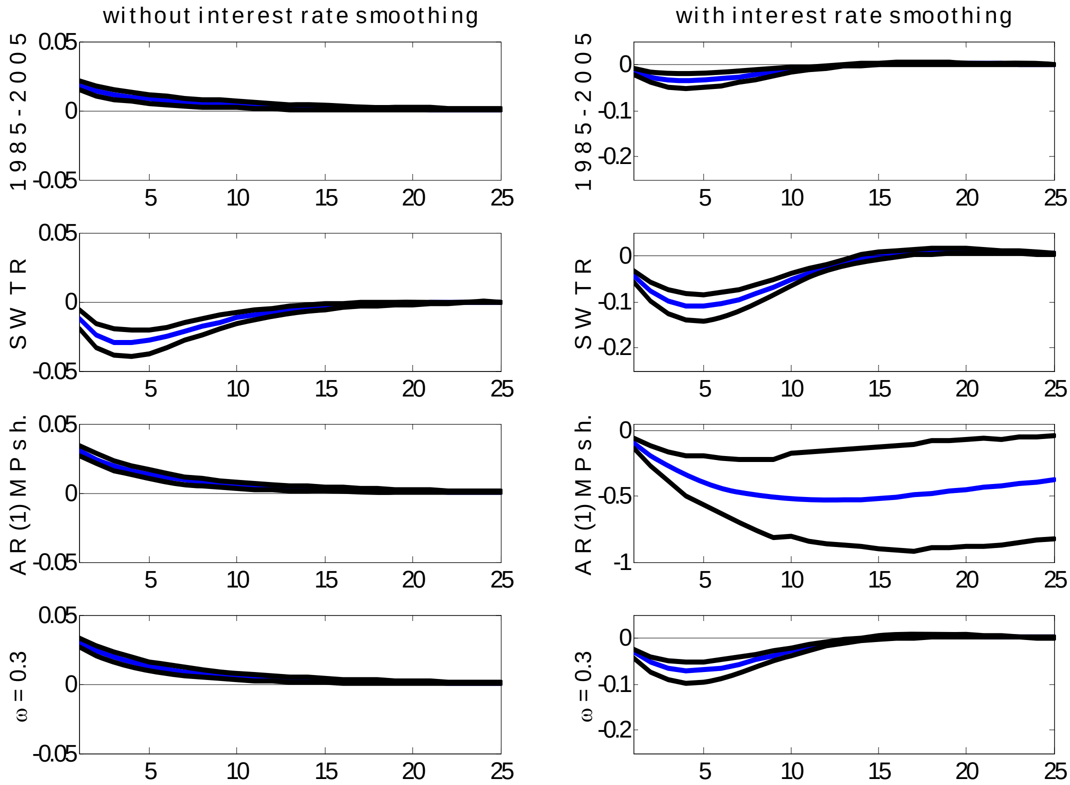

Figure 8: INFLATION IRFs: ROBUSTNESS CHECKS. All models conditional to $\psi = 2.7$. "1985-2005" refers to the models estimated over the sample 1985Q1-2005Q4; "SW TR" refers to the models estimated with the Taylor rule a la Smets and Wouters (2003, 2007); "AR(1) MP sh." refers to the models allowing for an autoregressive monetary policy shock (in this case, "without interest rate smoothing" should be understood as "with white noise monetary policy shock", and "with interest rate smoothing" as "with autoregressive monetary policy shock"); "$\omega = 0.3$" indicates the results related to the models estimated conditional on a calibration of the price indexation parameter in line with Galí et al (2001, 2003). Blue lines: mean impulse response. Black lines: 5th and 95th percentiles.

7 Appendix

7.1 Computation of the equilibrium inflation rate

Proposition. *Given the model (3)-(5) under $\omega = h = \phi_R$, the on-impact inflation reaction to a monetary policy shock reads as follows:* $\pi_t = \frac{\chi[\psi\sigma-(\eta+\sigma)]}{\sigma+\phi_x+\chi(\eta+\sigma)\phi_\pi+\chi\psi\sigma\phi_\pi}\eta_t^R$.

Proof. Given that we are studying a monetary policy shock, all other shocks in the model are at their unconditional mean, which is zero. Then, the only shock driving the economy is the monetary policy shock η_t^R, which is a white noise stochastic element. Then, we can guess the solutions $\pi_t = a\eta_t^R$ and $x_t = b\eta_t^R$. Given the nature of the policy shock, these solutions imply $E_t\pi_{t+1} = E_t x_{t+1} = 0$. Consequently, the model simplifies as follows:

$$\pi_t = \chi\left[(\eta+\sigma)x_t + \psi R_t\right]$$

$$x_t = -\frac{1}{\sigma}R_t$$

$$R_t = \phi_\pi\pi_t + \phi_x x_t + \eta_t^R$$

Working with the IS equation and the Taylor rule, it is easy to show that

$$x_t = -(\sigma+\phi_x)^{-1}(\phi_\pi\pi_t + \eta_t^R)$$

and

$$R_t = (\sigma+\phi_x)^{-1}\sigma(\phi_\pi\pi_t + \eta_t^R)$$

By plugging these last two expressions into the Phillips curve and performing some manipulations, one may derive the expression

$$\pi_t = \frac{\chi[\psi\sigma-(\eta+\sigma)]}{\sigma+\phi_x-\chi(\eta+\sigma)\phi_\pi+\chi\psi\sigma\phi_\pi}\eta_t^R$$

which verifies the guess $\pi_t = a\eta_t^R$, enables us to verify the guess $x_t = b\eta_t^R$, and proves the Proposition.

■

7.2 Bayesian estimation

Bayesian estimation has been widely adopted for taking DSGE models to the data in the last few years (for an extensive survey, see An and Schorfheide (2007)). This type of estimation allows a researcher to exploit a-priori information regarding the parameters' values (coming e.g. from previous micro or macroeconometric studies, or being the researcher's subjective beliefs). Moreover, it is system-based and fits the solved DSGE model to the data, so allowing to exploit all the cross-equation restrictions coming from the theoretical model under consideration. The possibility of computing the marginal likelihood for each estimated model offers a way to perform meaningful comparisons across competing set ups even if they are non-nested. Furthermore, Férnandez-Villaverde and Rubio-Ramírez (2004) demonstrate that Bayesian estimations and model comparisons are consistent even in the case of misspecified models.For estimating our baseline model, we proceed as follows. Given the vector of structural parameters $\xi = (\beta, , \theta, \omega, \eta, \sigma, \psi, h, \phi_\pi, \phi_x, \phi_R, \rho_\pi, \rho_x, \sigma_{\eta\pi}, \sigma_{\eta x}, \sigma_{\eta r})'$, the vector of endogenous variables $z_t = [x_t, \pi_t, R_t]'$, the vector of exogenous shocks $\varepsilon_t = [\varepsilon_t^x, \varepsilon_t^\pi]'$, the vector of innovations $\eta_t = [\eta_t^x, \eta_t^\pi, \eta_t^R]'$, and the vector of observable variables we aim at tracking $Y_t = [x_t, \pi_t, R_t]'$, the solution of the system (3)-(5) can be written in terms of a transition equation

$$\begin{bmatrix} z_t \\ \varepsilon_t \end{bmatrix} = A(\xi) \begin{bmatrix} z_{t-1} \\ \varepsilon_{t-1} \end{bmatrix} + B(\xi)\eta_t$$

and a measurement equation

$$Y_t = C(\xi) \begin{bmatrix} z_t \\ \varepsilon_t \end{bmatrix}$$

where $E(\eta_t \eta_t') = \Sigma(\xi)$, and A, B, and C are conformable matrices.[32] Once the system is solved, we can employ the Kalman filter to evaluate the likelihood $L(\{Y_t\}_{t=1}^T \mid \xi)$. The posterior distribution is then proportional to the product of the likelihood function and the priors.

To compare models estimated with Bayesian techniques, we employ the Bayes factor, which is computed on the basis of the models' marginal likelihoods. The marginal likelihood of a given model averages all possible likelihoods across the parameter space by employing the parameters' priors as weights. Notice that an advantage of the marginal likelihood criterion is that it penalizes overparametrization, i.e. models with a more sophisticated economic structure are not necessarily better ranked if the extra-parameters (with respect to those already present in a given benchmark model) are not helpful in explaining the data. As explained by e.g. Rabanal and Rubio-Ramírez

[32] The measurement equation may allow for a vector of measurement errors v_t, i.e. $Y_t = C(\xi)[zt, \varepsilon_t]' + Dv_t$, often added to overcome stochastic singularity. As regards the model at stake, $D = 0$.

(2005, 2007), this happens because the marginal likelihood considers all the regions of the parameter space, and takes the average of both relatively large and small values of the likelihood function.

The marginal likelihood of a model m reads

$$L(\{Y_t\}_{t=1}^T \mid m) = \int_{\xi \in \Xi} L(\{Y_t\}_{t=1}^T \mid \xi, m)\Pi(\xi, m)d\xi$$

Then, for two different models $m1$ and $m2$, the posterior odds ratio is defined as

$$\frac{P(m1 \mid \{Y_t\}_{t=1}^T)}{P(m2 \mid \{Y_t\}_{t=1}^T)} = \frac{\Pi(m1) \int_{\xi \in \Xi} L(\{Y_t\}_{t=1}^T \mid \xi, m1)\Pi(\xi, m1)d\xi}{\Pi(m2) \int_{\xi \in \Xi} L(\{Y_t\}_{t=1}^T \mid \xi, m2)\Pi(\xi, m1)d\xi}$$

where $\Pi(m1)$ and $\Pi(m2)$ represent the researcher's priors on - respectively - $m1$ and $m2$. If one sets $\Pi(m1) = \Pi(m2) = 1/2$, the posterior odds ratio equals the Bayes factor, i.e. the ratio of the marginal likelihoods, which is the metric we employ for comparing the models we estimate. Since a closed-form solution is not available, we estimate its value by employing the modified-harmonic mean estimator proposed by Geweke (1998).

We assess the "significance" of the difference signalled by the Bayes factor on the basis of the indications reported in Kass and Raftery (1995). According to Kass and Raftery (1995), a Bayes factor between 1 and 3 is "not worth more than a bare mention", between 3 and 20 suggests a "positive" evidence in favor of one of the two models, between 20 and 150 suggests a "strong" evidence against it, and larger than 150 "very strong" evidence.