ON PRICE-INCREASING (MONOPOLISTIC) COMPETITION

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ABSTRACT

We introduce a new class of “increasing elasticity of substitution” (IES) preferences to model product differentiation. In a monopolistic competition setting à la Dixit – Stiglitz (1977) we find that, even under constant returns to scale and complete information, a rise in the number of firms can be price-increasing. This result extends to the case of Cournotian competition. Despite the price increase, consumers benefit from a rise in the number of monopolistic competitors because of higher product diversity. Higher prices are therefore associated with higher consumer welfare. Our results suggest a possible explanation to the empirical puzzle posed by the countercyclical movements of price-cost margins behaviour following globalisation and market reforms. In addition, they should be of interest for the real business cycle literature which investigates the impact of an endogenous market structure.

Keywords: monopolistic competition, endogenous mark up, elasticity of substitution

JEL Classification: D43, D11, L11, L16.

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1 Introduction

A maintained hypothesis of competition policy is that, absent increasing returns to scale, an “increase in competition” (properly measured\(^1\)) should deliver an equilibrium with lower prices, larger quantities and (possibly) higher product variety. Formally inspired by the standard results of Cournotian models with homogenous products, the economic intuition behind this commonplace is that an increase in the number of firms should lessen their market power and reduce the distortions associated with imperfect competition (see e.g. Tirole, 1988: chapter 5). This wisdom is deeply rooted and it has not been challenged neither by some contrary casual empirical evidence from assorted industries, nor by a few theoretical counterexamples (see e.g. the references given in Chen & Riordan, 2006: Introduction).

The aim of this paper is to question the robustness of the previous commonplace with respect to the introduction of product differentiation. Namely, in a model of Chamberlinian “monopolistic competition”, we introduce a new class of preferences compatible with price-increasing competition. However, we show that the rise of product variety produced by an increase in the number of monopolistic competitors more than compensate consumers for the related price increase: higher prices are therefore associated with higher consumer welfare. As proved in Appendix 2, our results do not occur solely in a monopolistic competition setting, but they can also arise in an oligopoly model à la Cournot. In particular, we study the way preferences affect the elasticity of residual demand in the setting introduced by Dixit & Stiglitz (1977) and Krugman (1979). The intuition for our results is the following. A rise in the number of firms shifts downwards the residual demand curve and, given the representative consumer’s disposable income, reduces the equilibrium consumption of each variety. The overall effect on prices is contingent on how demand elasticity changes according to the scale of consumption. It turns out that demand elasticity only depends on the functional form chosen to model the consumers’ “love for variety”. In particular, in a symmetric equilibrium, the elasticity of demand (in absolute value) coincides with the elasticity of substitution between any two varieties. The main issue is then how the scale of consumption affects commodity substitutability.

Notice that in monopolistic competition models an increase in the number of firms is also an increase in the number of varieties. Therefore, the possibility that consumers have to pay more for a richer set of products should not come as a complete surprise. However, the widely adopted hypothesis of a representative consumer with CES preferences (see e.g. Dixit & Stiglitz, 1977: section 1, and Krugman, 1980) gives rise to an isoelastic residual demand. Under these preferences

\(^{1}\) On the difficulty of measuring competition see e.g. Boone (2000).
competition does not affect demand elasticity, the equilibrium price does not depend on the number of firms and consumers benefit from an increase in the number of firms through higher product diversity. In the somewhat less known case of varieties with a decreasing (with respect to the scale of consumption) elasticity of substitution (see e.g. Krugman, 1979, and Bertolletti, 2006), more competition also lowers the equilibrium price.

The class of (symmetric) preferences that we introduce in this paper and call “Increasing Elasticity of Substitution” (IES) is characterized by an increasing elasticity of substitution with respect to the level of (symmetric) consumption. Therefore, the lower the scale of consumption of each variety, the lower (in absolute value) the residual demand elasticity that each firm faces. Even if IES preferences are deeply related to the widely used CES preferences\(^2\), one could wonder how realistic they are as a description of consumer attitudes towards product variety. Here we offer the following example. Consider a child who owns many pencils of different colours/varieties. With IES preferences she would regard any two pencils of different colours to be closer substitutes when she has many of them (the same number for each colour) than when she has a few. On the contrary, with CES preferences the substitutability would not change with the number of pencils of each colour.

We regard the contribution of this paper as being twofold: on the one hand, it adds to the recent literature on price-increasing competition under product differentiation. The possibility that competition could raise prices has in fact been considered in a couple of papers by Chen & Riordan (2006) and Cowan & Yin (2006). These papers show that, in discrete choice models of product differentiation, the symmetric duopoly price can be higher than the monopoly price.\(^3\) The latter paper also shows that, in a Hotelling model with a monopolistic firm, consumers can actually be made worse off by the entrance of a new competitor if the monopolist does not fully cover the market and two-part tariffs can be adopted.

On the other hand, providing an endogenous (preference-based) countercyclical mark up, our work also contributes to the literature that looks for explanations of the price-cost margin behaviour. The monopolistic competition setting we consider has indeed been widely adopted both in macroeconomic and international trade models. In particular, our paper directly relates to the many contributions that followed Krugman (1979) and (1980) in discussing the possibly pro-competitive effects of international trade: see e.g. Boulhol (2006) for a recent example. Moreover, our results might be of some interest for the vast literature on the cyclical behaviour of prices (see e.g. the

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\(^2\) CES utility function arises under a special parametric choice of the IES utility function: see next section.

\(^3\) This result appears to have been somehow anticipated in a passage by Wicksell (1901: pp. 87-8).
review in Rotemberg & Woodford, 1999), and in particular for the recent stream concerned with the so-called “endogenous market structure”: see e.g. Colciago & Etro (2007).

The paper is organized as follows: section 2 introduces the model, and section 3 studies its welfare implications. Section 4 summarizes the results. Appendix 1 illustrates the cases of monopolistic competition with the alternative CES and CARA preferences. Appendix 2 deals with the case of Cournotian competition under IES preferences.

2 The model

Following Dixit & Stiglitz (1977) and Krugman (1979), we consider a market with \( n \) identical firms, each producing a different variety of a commodity. Let \( x_i \) be the quantity of variety \( i \), produced by firm \( i \) with a (positive) marginal cost \( c \). If variety \( i \) is actually sold, its market price is \( p_i \). Assume that the representative consumer has the utility function \( U(x) = \sum u(x_i) \) (that is, her preferences are symmetric and additive\(^4\)), defined over a large number \( N > n \) of potential varieties \( (i = 1, \ldots, N) \). It is also assumed that \( u(\cdot) \) is a well-behaved (sub-utility) function with \( u(0) = 0 \) and \( u''(\cdot) < 0 < u'(\cdot) \).\(^5\) Let \( Y \) denote the disposable income of the representative agent. Then her budget constraint in any symmetric equilibrium (i.e., in any equilibrium in which the price of varieties, \( p_i \), is the same and hence also the quantity, \( x_i \), is the same) can be written as:

\[
p = \frac{Y}{nx}.
\]

The other equilibrium condition is given by firms’ profit maximization. In order to compute that, we first have to consider the FOCs for the maximization of the representative consumer’s utility:

\[
p_i = \frac{u'(x_i)}{\lambda},
\]

for \( i = 1, \ldots, n \), where \( \lambda > 0 \) is her marginal utility of income.

One can prove that, if prices are not disproportionate, the elasticity of \( \lambda \) with respect to each price is of the same order of magnitude as \( 1/n \) (see e.g. Deaton & Muellbauer, 1980: section 5.3). Thus, under the assumption of many varieties (i.e., if \( n \) is large enough), one can assume that each firm

\(^4\) See e.g. Deaton & Muellbauer, 1980: section 5.3.

\(^5\) Note that, being strictly concave with respect to \( x \) and increasing with respect to \( n \), \( U(\cdot) \) embodies a Chamberlinian “taste for variety”. Moreover, it is well defined over the positive orthant of the relevant Euclidean space: according to standard results, this implies regular and well-behaved demand functions for (strictly) positive prices and income.
ignores the price interaction with the others, that is, each firm considers \( \lambda \) as a constant (this is the monopolistic competition hypothesis popularised by Dixit & Stiglitz, 1977). Accordingly, the inverse demand function for variety \( i \), is given by \( p_i(x_i) = u'(x_i)/\lambda \). Therefore, demand elasticity can be written as:

\[
\varepsilon_i(x_i) = \frac{p_i(x_i)}{p_i'(x_i)x_i} = \frac{u'(x_i)}{u''(x_i)x_i}.
\] (3)

Notice that \( \varepsilon(\cdot) \) does not depend on \( \lambda \).\(^6\) It can be showed that, in any symmetric equilibrium (thanks to the properties of symmetric additive preferences), \( \varepsilon_{ij} = -u'(x)/(nxu''(x)) = - \varepsilon_{ij}/(n-1) \), where \( \varepsilon_{ii} \) and \( \varepsilon_{ji} \) are respectively the direct and cross elasticities of the “compensated” (Hicksian) demand for variety \( i \). It follows that, for a symmetric consumption, demand elasticity (in what follows we use the shorthand to omit the suffix \( i \)), \( \varepsilon(\cdot) \), equals in absolute terms the (partial) elasticity of substitution between any two varieties demand elasticity, i.e., \( \varepsilon(x) = - \sigma(x) \).\(^7\)

The profit-maximizing first and second order conditions for each firm under monopolistic competition can be written as follows:

\[
p = -\frac{\varepsilon(x)}{1+\varepsilon(x)} c = m(x)c,
\] (4)

\[
u''(x)x + 2u''(x) < 0.
\] (5)

The symmetric equilibrium is then given by a couple \((p, x)\), such that equations (1), (4) and (5) are simultaneously met. Equation (1) – the budget constraint – is an equilateral hyperbola in the space \((p, x)\), whose distance from the origin depends on the disposable income per variety \( Y/n \). Equation (4) – the profit maximising FOC – depends only on \( \varepsilon(\cdot) \) (it requires \( |\varepsilon(x)| > 1 \), i.e. \( u''(x)x + u'(x) > 0 \)), that is on the elasticity of the marginal utility \( u'(\cdot) \). Equation (5) – the profit maximising SOC – is just a decreasing marginal revenue condition which must be satisfied in our setting.

In order to study the effect of an increase in competition, we consider an exogenous decrease in the disposable income per variety, \( Y/n \). This affects only equation (1): from a graphical point of view, the equilateral hyperbola simply shifts towards the origin. In economic terms, this implies that the

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\(^6\) \( \varepsilon(\cdot) \) equals the reciprocal of the elasticity of the marginal utility \( u'(\cdot) \), i.e. minus the so-called “coefficient of relative risk aversion” of \( u(\cdot) \).

\(^7\) More precisely, it can be showed that the so-called “Morishima” elasticity of substitution between varieties \( i \) and \( j \), \( \sigma_{ij} = \varepsilon_{ij} - \varepsilon_{ii} \), is equal to - \( u'(x_i)/(u''(x_i)x_i) \) whenever \( p_i = p_j \) and then \( x_i = x_j \) (on the different partial elasticity of substitution measures see Blackorby and Russell, 1989).
revenues of each active firm, $px$, must decrease. Since (5) implies that $m(x)x = [u'(x)x]/[u''(x)x + u'(x)]$ is an increasing function, in this setting also $x$ must get smaller. However, $p$ might increase or decrease depending on the properties of $\varepsilon(\cdot)$. Figure 1 provides a graphical representation of how the effects of a rise in competition might differ according to the slope of $m(x)$, which in turn depends on the characteristics of $\varepsilon(\cdot)$.

![Figure 1: A non-monotonic elasticity case](image)

In Appendix 1 we illustrate the alternative cases of CARA (Constant Absolute Risk Aversion: see Bertoletti, 2006 and Behrens & Murata, 2007) and CES preferences (see e.g. Dixit & Stiglitz, 1977: section 1), leading respectively to a result of price decrease and of no change in price after an increase in competition.

Notice that the condition for demand elasticity to grow (locally) is easily derived to be:

$$\frac{1}{u'(x)} - \frac{1}{u''(x)x} < \frac{u'''(x)}{u''(x)x},$$

and that this inequality requires $u''' > 0$ in the relevant interval; in other terms, a convex individual demand curve for the single firm under monopolistic competition. To satisfy (5) and (6), we assume that preferences can be described by a functional form for $u(\cdot)$ in the class:

$$u(x) = \frac{x^\rho}{\rho} + \frac{x^\gamma}{\gamma},$$

where $0 < \rho < \gamma \leq 1$, or

$$u(x) = \ln x + \frac{x^\gamma}{\gamma}.$$
where $0 < \gamma < 1$, for $i = 1, ..., N$. We call the preferences represented by $U(\cdot)$ under (7) or (8) “Increasing Elasticity of Substitution” (IES) preferences. Notice that if $\rho = \gamma$ in (7) preferences would be CES: indeed (7) and (8) are combinations of two “CES expressions”, respectively with elasticity of substitution $1/(1 - \rho)$ and $1/(1 - \gamma)$ (lnx in (8) corresponds to the “Cobb-Douglas case” with $\rho = 0$ and unit elasticity of substitution). In what follows, without loss of generality, to illustrate the case of IES preferences we use (7) and assume $\gamma = 1$: i.e.:

$$u(x_i) = \frac{x_i^\rho}{\rho} + x_i, \quad (9)$$

for $i = 1, ..., N$.

Under (9), we obtain:

$$p_i(x_i) = \frac{1 + x_i^{\rho-1}}{\lambda}, \quad \varepsilon(x) = \frac{1 + x^{1-\rho}}{\rho - 1}, \quad m(x) = \frac{1 + x^{1-\rho}}{\rho + x^{1-\rho}}. \quad (10)$$

Notice that the elasticity of substitution $\sigma(x) = -\varepsilon(x)$ is increasing with respect to the scale of consumption, with:

$$\sigma(0) = \frac{1}{1-\rho}, \quad \lim_{x \to \infty} \sigma(x) = \infty. \quad (11)$$

Whenever the representative consumer has IES preferences, an increase in the number of firms/varieties, by shifting down the residual demand of each variety makes it less elastic for any given price. This, in turn, implies an increase in the equilibrium price $p(n)$. Conversely, when the number of firms (varieties) decreases, the scale of consumption increases and the market price tends to marginal cost. i.e., $p'(n) > 0$ and:

$$m(0) = \frac{1}{\rho}, \quad \lim_{x \to \infty} m(x) = 1. \quad (12)$$

Are the equilibrium values unique and can they be given an explicit expression? The equilibrium value $x(n)$ is implicitly given by the condition:

$$\frac{Y}{nc} = m(x)x = \frac{x + x^{2-\rho}}{\rho + x^{1-\rho}}, \quad (13)$$

More generally: under preferences given by (7), one gets that as $x$ goes to infinite $\sigma$ goes to $1/(1 - \rho)$ and $m$ goes to $1/\gamma$; while under preferences (8) $\sigma(0) = 1$ and $m$ goes to infinite as $x$ goes to zero.
and it is easily proved to be unique since the function in (1) is steeper than the one given by (4) at any point such that (13) is satisfied (i.e., the equilibrium loci (1) and (4) only cross once under IES preferences). The situation is described in Figure 2.

Accordingly, an increase in the number of competitors does increase the equilibrium price, while it decreases each firm’s revenue and profit, and the consumption of each variety. The latter fact implies a decrease in the equilibrium elasticity of substitution between any two varieties, which provides a rationale for the result. What happens to consumers’ welfare is not obvious, since consumers pay higher prices but also enjoy a higher product variety, and it is investigated in next section. Interestingly, the case of price-increasing competition under IES preferences for the representative consumer can also be extended to an oligopoly (à la Cournot) version of the previous setting, in which the number of competitors directly affects the mark up. The proof of this result is sketched in Appendix 2.

3 Welfare implications

Following Dixit & Stiglitz (1977: section 2), we can compare the (long-run) equilibrium that would arise in our setting under market free entry if the production of each potential variety also involves some fixed set-up cost $F > 0$, with a constrained (no lump-sum transfers) social optimum. The market equilibrium must satisfy the zero-profit condition:

$$p = c + \frac{F}{x}. \quad (14)$$
Together with (1) this gives the condition:

$$m(x^e) = \frac{cx^e + F}{cx^e}, \tag{15}$$

which characterizes the free entry market equilibrium. The latter has to be compared with the social optimum which maximizes $U = nu(x)$, under the constraint that $Y = ncx + nF$. The FOCs for the stated problem imply:

$$\frac{1}{\phi(x^e)} = \frac{cx^e + F}{cx^e}, \tag{16}$$

where $\phi(x) = u'(x)x/u(x)$ is the elasticity of utility $u(\cdot)$. Since $m(x) < \phi(x)^1$ if the representative consumer has IES preferences, it can be easily proved that (15) and (16) uniquely define $x^e$ and $x^c$, with $x^e > x^e$. Accordingly, by (14), under the free entry hypothesis the number of varieties ($n^e$) and their price ($p^e$) are lower than in the social optimum ($n^c$ and $p^c$). Therefore, a social planner would introduce more varieties, expand less their production and price them higher with respect to the free entry market equilibrium.

An intuition for these results can be grasped by looking at the sign of $\phi'(\cdot)$, as suggested by Dixit & Stiglitz (1977: pp. 303-4). As defined above, $\phi(x)$ is the ratio between $u'(x)x$ and $u(x)$. The numerator is proportional to each firm’s revenue, while the denominator measures the contribution of each variety to consumer welfare. Therefore, if $\phi'(\cdot) > 0$, at the margin each firm finds it profitable to produce more than the social optimum. This is indeed the case under IES preferences, since

$$\frac{\phi'(x)x}{\phi(x)} = \frac{1}{m(x)} - \phi(x). \tag{17}$$

This result relates to the debate between Pettengill (1979) and Dixit & Stiglitz (1979) on having too few firms in the market equilibrium with respect to the social optimum. Dixit & Stiglitz (1977: p. 304) based their presumption on the expectation of a positive correlation between $\phi(\cdot)$ and $|\varepsilon(\cdot)^1|$. We show that under IES preferences the free entry market equilibrium has too little product diversity even if $\phi(\cdot)$ and $|\varepsilon(\cdot)^1|$ are not positively related.

The previous welfare result also suggests that the entry of a new competitor (out of the long-run equilibrium) can raise consumer welfare, even if associated with a price increase. This is what actually happens under IES preferences. Indeed, by using (1) and (4) it is easily computed that:
\[
x'(n)n = - \frac{m(x(n))}{m'(x(n))x(n) + m(x(n))}.
\]

It follows that an increase in the number of competitors increases consumer welfare if and only if:

\[
\frac{1}{\phi(x(n))} > \frac{m(x(n))}{m'(x(n))x(n) + m(x(n))}.
\]

Since profit maximisation, even under price-increasing competition, implies that an increase in the number of competitors reduces firm’s profits, the following condition must hold:

\[
m'(x(n))x(n) + m(x(n)) > 1.
\]

Thus \(\phi' > 0\) is a sufficient condition for more monopolistic competition to deliver an improvement of consumers’ welfare.

### 4 Conclusions

In this paper we introduce IES preferences and show that in such a case: (a) more monopolistic competition results in an increase of prices; (b) because of higher product diversity, competition more than compensate consumers of the associated price increase; (c) the constrained Pareto optimum would have more varieties, higher prices and smaller quantities than the market long-run equilibrium of free entry. It remains an open question whether an increase in the number of firm could actually make consumers worse-off in a monopolistic competition setting. We also show that a case for price-increasing competition can be made also in an oligopoly setting à la Cournot. Finally, we mention that if firms have different (constant) marginal cost (i.e., with \(c_j \neq c_i\)), IES preferences naturally generate monopolistic competition Lerner indexes \(L_j = (p_j - c_j)p_j = -1/\xi_j(x_j)\) which are lower for more efficient firms and associated with larger production levels, very much as in the (asymmetric) Cournot equilibrium.

Our results add to the small set of recent papers (Chen & Riordan, 2006 and Cowan & Yin, 2006) which have considered the case for price-increasing competition in models with differentiated products. However, our setting differs from their on several grounds. First, we use symmetric “non-address” product differentiation; second, by using the Chamberlinian model of monopolistic competition, we have firms that compete non-strategically; third, we consider a market with a large number of competitors and measure the degree of competition by the ratio between consumer expenditure and the number of firms.
By proposing an explicit (and simple) micro-foundation for a countercyclical mark up based on consumer preferences, our model provides an explanation for the empirical puzzle posed by non-decreasing price-cost margins following globalisation and market reforms which is alternative to those based on cost factors: see e.g. Griffith, Harrison & Simpson (2006), Boulhol (2006) and (2008). The endogenous mark up generated by IES preferences should also be of interest to the macroeconomic literature concerned with the impact of an endogenous market structure on the standard real business cycle framework: see e.g. Colciago & Etro (2007).

One could wonder how peculiar IES preferences are. Dixit & Stiglitz (1977: p. 304) write: “... we would normally expect that as the number of commodities produced increases, the elasticity of substitution between any pair of them should increase. In the symmetric equilibrium, this is just the inverse of the elasticity of marginal utility. Then a higher x would correspond to a lower n, and therefore a lower elasticity of substitution ...”. This suggestion is taken up by Krugman (1979), who assumes a decreasing elasticity of substitution and comments this way: “... [this assumption] seems to be necessary if this model is to yield reasonable results, and I make the assumption without apology.” (Krugman, 1979: p. 476). However, the previous intuition seems misplaced. As a matter of fact, due to additive symmetric preferences, in a symmetric equilibrium the elasticity of substitution between any two varieties does not depend directly on the number of varieties actually consumed. The relevant question is rather how the elasticity of substitution might change according to the scale of consumption. We argue that IES preferences are not less likely than the popular CES ones, or than the CARA preferences, and deserve to be investigated. Indeed, this paper shows that the assumption of a non increasing elasticity of substitution is not necessary for monopolistic competition to yield “reasonable results”.

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9 This was first noticed by Pettengill (1979: p. 960), and acknowledged by Dixit & Stiglitz (1979: pp. 962-3) while pointing out that, in their setting, an increase in the number of commodities/firms does not increase the degree of crowding in the commodity space.
Appendix 1: Examples of different monopolistic competition effects on prices

In this Appendix we show two examples of utility functions leading respectively to no change and a decrease in the equilibrium price after an increase in the number of monopolistic competitors.


Suppose that:

\[ u(x_i) = \frac{x_i^\rho}{\rho} \]  

for \( i = 1, \ldots, N \), with \( 0 < \rho < 1 \): i.e., \( u(\cdot) \) is a “constant relative risk aversion” (sub-)utility function. Preferences are CES, and demand elasticity is constant and given by \( \varepsilon = 1/(\rho - 1) \), with \( m = 1/\rho \).

Figure 1 shows the effect of an increase in the number of varieties:

Notice that in Figure A.1 an increase in the number of firms/varieties for a given level of nominal income decreases the amount of each of them but leaves unchanged the equilibrium price. The obvious reason is that the optimal mark up does not depend in such a case on the amount to be produced. Since the equilibrium values can be easily computed, the CES case has been tremendously popular in applications (especially in international trade and macroeconomic models).
2. The CARA case (Bertoletti, 1998 and 2006 and Behrens & Murata, 2007)

Assume that

\[ u(x_i) = -\frac{e^{-\alpha x_i}}{\alpha}, \quad (A1.2) \]

for \( i = 1, \ldots, N \), with \( \alpha > 0 \). Therefore, \( u(\cdot) \) is a “constant absolute risk aversion” (sub-)utility function, preferences are quasi-homothetic and \( \varepsilon(x) = -1/(\alpha x) \), with \( m(x) = 1/(1 - \alpha x) \) (this requires that \( \alpha \) is small enough with respect to \( x \)). Then demand elasticity is increasing along a given individual demand curve (the elasticity of marginal utility \( u'(\cdot) \) is decreasing) and a smaller consumption is associated to lower prices in a symmetric equilibrium. The situation can be graphically represented as follows:

Notice that an increase in the number of varieties reduces both the consumption level of the single variety and its equilibrium price. The mark up varies “procyclically”, so that an increase in competition also benefits consumers through lower prices.
Appendix 2: The case of price-increasing Cournot competition

In this Appendix we show that, under IES preferences, price-increasing competition can also arise in an oligopoly (à la Cournot) version of our setting. By using (1) and (2) one can easily derive the complete inverse demand function of the representative consumer:

\[ p_i(x) = \frac{u'(x_i)Y}{\sum_j u'(x_j)x_j}, \]  

(A2.1)

which is decreasing with respect to \( x_i \). Given (A2.1), it is straightforward to compute that the FOC for Cournot profit maximization is given by (it requires \( u''(x)x + u'(x) > 0 \)):

\[ \frac{\partial R_i(x)}{\partial x_i} = Y \left[ u''(x_i)x_i + u'(x_i)\left(\sum_{j\neq i} u'(x_j)x_j\right)\right] \left(\sum_j u'(x_j)x_j\right)^2 = c, \]  

(A2.2)

and that the (global) satisfaction of (5) ensures that the “marginal revenue” \( \partial R_i/\partial x_i \) is decreasing with respect to \( x_i \).

It follows that in any symmetric equilibrium à la Cournot for which (A2.2) holds it must be that:

\[ x = \frac{(n-1)[u''(x)x + u'(x)]}{n^2u'(x)c} Y = \frac{(n-1)}{n^2m(x)c} Y. \]  

(A2.3)

with

\[ p = \frac{n}{(n-1)} m(x)c. \]  

(A2.4)

Thus, by using (1), the symmetric oligopoly equilibrium quantity \( x_o(n) \) is defined by:

\[ \frac{(n-1)Y}{n^2c} = m(x) x. \]  

(A2.5)

Since the left-hand-side of (A2.5) does not increase with respect to \( n \) if \( n \geq 2 \), and the right-hand-side is increasing with respect to \( x \), it follows that an increase in the number of competitors/varieties decreases \( x_o(n) \). Such an increase in competition also raises the equilibrium price \( p_o \) if and only if it decreases \( nx \), i.e., if and only if the elasticity of \( x_o(n) \) is less than – 1. Since:

\[ \frac{x_o'(n)n}{x_o(n)} = \frac{2-n}{n-1} \frac{m(x_o(n))}{m'(x_o(n))x_o(n) + m(x_o(n))}, \]  

(A2.6)
the condition for price-increasing oligopoly competition is equivalent to:

\[- \frac{m'(x_o(n))x_o(n)}{m(x_o(n))} > \frac{1}{n-1}. \tag{A2.7}\]

Condition (A2.7) is not obviously satisfied in any case of IES preferences (for any value of \(Y/n\)); however, consider the (limiting) case of (8). Computation shows that, under (8):

\[m(x) = \frac{1 + x^{\gamma}}{x^{\gamma}}; \tag{A2.8}\]

and

\[m'(x)x = \gamma \frac{x}{1 + x^{\gamma}}. \tag{A2.9}\]

Accordingly, under (8), (A2.7) is equivalent to:

\[(n-1)\gamma - 1 > x_o(n)^{\gamma}, \tag{A2.10}\]

which must hold when \(n\) is large enough.

References


