# Price Indexation and Optimal Simple Rules in a Medium Scale New Keynesian Model

**Guido Ascari*** — *University of Pavia*

**Nicola Branzoli** — *University of Pavia*

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**Abstract**

Indexation was the subject of a substantial literature in the era of high inflation, but it was then neglected. Recently, it has becoming standard to assume backward looking indexation in prices in the New Keynesian DSGE models. Given the backward looking indexation scheme, we study how the *degree* of indexation affects the optimal simple policy rule in a medium-scale model. We show that both the shape and the parameters of the simple policy rule are affected by the degree of indexation. In particular, indexation helps in stabilizing price dispersion, thus optimal monetary policy does not need to focus on stabilizing inflation. We also show that full indexation is a very peculiar assumption, eliminating the price dispersion mechanism in the model. The central message of this paper is that one should be very careful in introducing *ad hoc* features, and hence use these models for normative analysis.

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* *Corresponding author*: Guido Ascari, Department of Economics and Quantitative Methods, University of Pavia, Via San Felice 5, 27100 Pavia, Italy. *Tel: +39 0382 986211 ; Fax: +39 0382 304226; e-mail:*guido.ascari@unipv.it.

# 1 Introduction

Indexation of wages and prices was the subject of substantial literature in macro in the era of high inflation (see the seminal paper by Gray, 1976). However, after the disinflation in the '80s, indexation vanished from the public policy and scientific debates. Quite recently many authors reintroduced indexation in the New Keynesian models, within the Calvo price staggering framework (e.g., Yun, 1996, Christiano et al., 2005, CEE henceforth). These models are then used for both positive and normative policy analysis, that is, to describe the functioning of the monetary transmission mechanism and to investigate the optimal monetary policy, performing welfare analysis. However, it should be clear that the results of both the analysis do depend on the assumed indexation scheme.

In this branch of literature indexation is introduced in a completely *ad hoc* manner. When indexation is introduced into a model, two issues arise: the *type* and the *degree* of indexation. With respect to the first issue, the nowadays most popular form of indexation embedded in these models is the so-called backward-looking indexation, firstly suggested by CEE. The main reason is empirical in order to have a lagged term in the New Keynesian Phillips Curve to match the inflation persistence in the data. With respect to the second issue, indexation is most of the times assumed to be full, even if there are neither positive nor normative reasons for such an assumption. Indeed, in the empirical estimates of these New Keynesian models, indexation is usually found to be only partial.[1] Moreover, full indexation is not optimal in many settings (e.g., Ascari and Branzoli, 2007). The assumption of full indexation is, however, convenient because it avoids the problems arising with positive inflation in steady state (see Ascari, 2004).

This paper looks at the second issue. So we do not address the normative question about which is the optimal indexation scheme.[2] We rather take the by now standard backward-looking indexation scheme proposed by CEE and ask the following question: given the backward looking indexation scheme, how the *degree* of indexation affects the optimal policy rule?

[1] See, e.g., Sbordone (2005) and Levin. et al. (2006) for the U.S., and Smets and Wouters (2003) for the Euro area.

[2] This is an obviously important question that we leave to further research. A first interesting attempt in this direction is Mash (2007).

We think this is an interesting question to answer, because backward looking indexation has many, and potentially conflicting, effects both on the dynamics and the long-run properties of the model (see Section 3). Even if this kind of indexation scheme is very commonly used, to the best of our knowledge no paper so far assesses the implications of different degrees of indexation for the shape of the optimal monetary policy rule.

In answering this question, we need to make several modelling choices. First, we decide to use an "operational" medium-scale model, more precisely the model in Schmitt-Grohé and Uribe (2004a) (SGU henceforth). Once backward-looking indexation is introduced in the model, then the order of the dynamic system increases such that no analytical results are possible even in the simplest New Keynesian model. As a consequence, given that in any case we need to resort to simulation, we decide to use a model with a rich set of both nominal and real frictions. This model is indeed becoming a sort of benchmark model for the literature and CEE show it to be able to replicate sufficiently well the behavior of the US business cycle.

Second, we confine our analysis to optimal, "simple and implementable" monetary policy rules, following closely SGU. This insightful paper characterizes the optimal coefficients in simple "operational" Taylor rules. For a rule to be "operational", it has to satisfy the following three properties: (i) it must be a function of a few readily observable macroeconomic variables; (ii) it must deliver an unique rational expectation equilibrium; (iii) it must induce an equilibrium that satisfy a constraint on the lower bound on the nominal interest rates. As in SGU, we consider nine different cases, combining on the one hand backward-looking, current-looking and forward-looking Taylor rules and, on the other hand, no inertia, inertial and superinertial Taylor rules. This allows us to focus on the implications of the *degree* of backward-looking indexation for both the shape and the coefficients of the optimal simple rule from the point of view of the stochastic steady state. The bad side of the coin is that this way of proceeding is computationally very costly.

Third, we just focus on the *degree of indexation of prices*, and not of wages. The first reason is that, given our research strategy, the curse of dimensionality is very high, so we could not perform both the analysis[3]. The second reason is that assuming full

[3] As explained in Section 4, we performed 677448 simulations, each of one take 90 seconds (on a standard Pentium IV (R) 3GHz), so it is about 16936.2 computer hours (almost 2 years).

wage indexation seems more compelling from both an empirical and theoretical point of view[4], and also from anecdotical evidence.

Our results should be first compared with the analysis in SGU. Their main findings are that: (i) the optimal simple monetary policy rule is a forward-looking one with no inertia; (ii) the optimal and simple rule takes the form of a real interest rate targeting rule, i.e., it should not respond to output and approximately reacts one-to-one to inflation; (iii) there is actually not much difference in terms of welfare evaluation across optimal rules for the different shapes; (iv) the optimal rule calls for significant inflation variability. We show that many of these results are not robust to changing the degree of indexation.

In general, the degree of price indexation changes the trade-offs monetary policy is facing. Indeed, we show that the variance of price dispersion decreases, while the one of inflation increases with indexation. Moreover, a general prescription for monetary policy is that the lower is the degree of indexation, the larger should be the response of the monetary policy to inflation gap.

Our results show that the difference in conditional welfare across the various cases is mainly driven by the first order steady state effects, suggesting that the literature often ends focusing on second-order analysis (as the shape of optimal policy in an approximated model), overlooking important first-order aspects (as the calibration of the degree of indexation) that strongly influence the shape of the optimal policy and have important welfare effects.

Finally, one of the main message of this paper is that full indexation is a very special assumption, since it nullifies the effects of price dispersion, which is one of the key variables in the Calvo type price staggering models. When prices that are not repotimized are automatically anchored to inflation, price dispersion become irrelevant not only from a long-run perspective (as it is obvious), but also in the dynamics of the model. Full indexation is a very special case, almost like a discontinuity point, in this sense, because it cancels one of the main mechanism of the model.

The central message of this paper, thus, is that one should be very careful in in-

[4] For example Levin. et al. (2006) estimates the average price indexation between 0.11 and 0.19 while the average wage indexation between 0.77 and 0.86. Furthermore Ascari and Branzoli (2007) shows that full wage indexation maximizes the steady state welfare for every level of price indexation.

troducing *ad hoc* features as backward-looking indexation, in a microfounded model to match some empirical evidence and hence use these models for normative analysis. The results of such an analysis are going to depend on the *degree* of indexation, and thus one needs to think deeply about the calibration of such a parameter, before drawing policy conclusions.[5]

## 2 The Model

The basic setup is a medium-scale macroeconomic model, obtained by augmenting the standard New Keynesian model with nominal and real frictions that are proved to be crucial in replicating the dynamics of US business cycle. Since the model is exactly the one described in many papers such as SGU and CEE[6], we will briefly introduce here the key elements, leaving to Appendix all the details about the model and calibration.

The model displays both real and nominal frictions. The real features of the model are monopolistic competition, habit persistence in consumption, fixed cost in an otherwise standard Cobb-Douglas production function that generates increasing return to scale and guarantees zero profit in equilibrium, variable capacity utilization and adjustment costs in investment. Money is introduced into the model via real balances in the utility function and cash-in-advance constraint on wage payments of firms. Wages and prices are sticky à la Calvo-Yun. Furthermore, as in CEE, prices and wages, that are not reoptimized each period, are indexed to past inflation. CEE argues that all these frictions appear to be crucial in replicating the dynamics of macroeconomic variables along the business cycle.

As in SGU, the long-run level of inflation is set equal to the average inflation of the US in the post World-War II period, i.e. 4.2%. While this complicates considerably the analytics of the model, it is an important feature of the data and will be also crucial for our results, since our analysis needs a non zero steady state inflation.

Finally, monetary policy follows a Taylor-type rule, expressed in log deviation from

[5] Leaving aside the important question of the optimal type of indexation scheme. Ths is an important question, that is beyond the scope of this paper. See footnote 2.

[6] Other empirical papers use analogous set-up such as Smets and Wouters (2003) and Schmitt-Grohé and Uribe (2005).

steady state's values (denoted with a hat):

$$\hat{R}_t = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R E_t \hat{R}_{t-i}, \tag{1}$$

where $i \in \{-1, 0, 1\}$ indicates respectively forward, current and backward looking policies. We assume the existence of a full commitment technology. As discussed in SGU this specification considers only policies that are simple in the sense that they depend only on a few simple and readable macroeconomics variables.

# 3 The Macroeconomic Effects of Price Indexation

Before looking in depth at the various results of the simulations, it is useful to think about the effects of indexation on the equilibrium of the medium-scale macroeconomic model. Price indexation reduces the degree of price dispersion in the economy. Indeed, in a standard Calvo pricing scheme without indexation, a positive trend inflation implies that there will be some firms with very low relative prices (i.e., the ones that set the price long ago in the past) and firms with high relative prices (i.e., the ones that instead just reset their prices). The degree of price dispersion is thus increasing in the level of trend inflation. It is well known that price dispersion is inefficient because it leads to a misallocation of resources across different firms and it acts as a negative shift in productivity (see e.g., Schmitt-Grohé and Uribe, 2004, 2005 and Ascari, 2004). Indexation counteracts this effect, because it allows also the non price-resetting firms to keep up with the pace of inflation. Partial indexation, thus, decreases price dispersion for any given trend inflation level. Both CEE and SGU assume that both wages and prices are fully indexed, that is indexation is 100%, so that there is no price dispersion in steady state.

Ascari and Branzoli (2007) shows that full indexation does not maximize the steady state welfare of the representative household in the SGU medium-scale model. In particular, they find the maximum steady state welfare level to be attained at $\tilde{\chi} = 1$ and $\chi = 0.8788$, that is, full wage indexation, but only partial price indexation (where $\tilde{\chi}$ and $\chi$ measure the degree of wage and price indexation respectively, see Appendix). The reason why full price indexation is not optimal lies in the effect of price indexation on the average mark-up in the economy. On the one hand, the lower $\chi$, the lower the ratio of the general price level to the price charged by resetting firms, and the lower the average mark-up in the economy. On the other hand, the lower $\chi$, the higher the mark-up of the resetting firms, and the higher the average mark-up in the economy. These two counteracting effects compensate exactly at $\chi = 0.8788$, while for lower levels of $\chi$, the second effect dominates such that the average mark-up decreases with $\chi$, and vice versa for higher levels of $\chi$. Moreover, Ascari and Branzoli (2007) show that full wage indexation is always optimal in steady state, whatever the value of price indexation. No full wage indexation in this medium scale model results in very high costs in terms of welfare both in steady state and in the dynamics of the model. It follows that there are quite compelling normative reasons for assuming full wage indexation. There are instead neither normative, nor positive reasons (see footnote 1) to assume full indexation in prices. As a result, we decide to focus on the effects on price indexation in this paper.

It is important to stress that partial price indexation has first order effects on the model. In other words, assuming full rather than partial indexation substantially affects welfare, and generally will do it more than other features of the model that have second order effects, like demand and supply shocks under optimized policy rules. It follows that the degree of indexation is a key and important variable in determining the welfare ranking of different policies. This is what this paper is about.

The degree of indexation strongly affects also the dynamics of NK models. The lower the degree of indexation, the more trend inflation would impact on the adjustment of aggregate variables and on the forward-looking decision of agents.[7]

First, as shown by SGU, the dynamics of price dispersion would not affect the log-linearized version of the model if one assumes full indexation, while it matters at first-order with partial indexation. Partial indexation introduces thus a further backward-looking variable in the model that would tend to increase the inertia in the dynamic adjustment. Loglinearizing (27) and (34) in Appendix, and then substituting the term referring to the newly reset price, it yields the following expression for the log-linearized

[7] See Ascari and Ropele (2007) for a thourough discussions of the effects of trend inflation in New Keynesian models.

dynamics of price dispersion, i.e., $s$ :[8]

$$\hat{s}_t = \left[ \frac{\eta\alpha\bar{\pi}^{(\eta-1)(1-\chi)}\left(\bar{\pi}^{1-\chi}-1\right)}{1-\alpha\bar{\pi}^{(\eta-1)(1-\chi)}} \right] (\hat{\pi}_t - \chi\hat{\pi}_{t-1}) + \alpha\bar{\pi}^{\eta(1-\chi)}\hat{s}_{t-1}. \quad (2)$$

Note that the lower the level of $\chi$, the higher is the inertia in the price dispersion term, and thus the more, *ceteris paribus,* deviations of price dispersion from its steady state value are going to be persistent.

To visualize the intuition behind the effect of indexation on price dispersion and inflation, Figure 1 and 2 plot the impulse responses of price dispersion after a 1% increase in the TFP under a standard Taylor Rule with $\alpha_\pi = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ for three different levels of indexation. These figures show that indexation influences the dynamics of price dispersion both qualitatively and quantitatively. Indeed comparing Figure 1 and 2 shows that partial indexation induces a negative rather than a positive response of price dispersion to such a shock. Moreover, also quantitively the response is of an other order of magnitude with partial indexation (such that it seems that there is no response with full indexation in Figure 1). Finally, also the degree of indexation matters: moving from $\chi = 0.8788$ to $\chi = 0$ results in a much larger response of price dispersion and also a greater inertia.[9]It is clear from eq.(2) that lower levels of indexation increase the effect of inflation on price dispersion changing both qualitatively and quantitatively the dynamics of the model.

Second, the degree of indexation affects the first-order condition of the price resetting firms. Similarly to the analysis of the effects of trend inflation in Ascari and Ropele (2007), a lower degree of indexation makes the optimal decision of price setters more forward-looking, because they will give higher weights to future economic conditions, trying to keep up with the trend, rather than responding to current transitory level of demand and costs.

Third, also the dynamics of the aggregate price level depends upon the degree of indexation. In particular, past inflation indexation, by construction, induce inertial adjustment in the price level, by making the inflation rate in the log-linearized version of (34) a function of its own lagged value. The higher the level of indexation, the more

[8] In (2), $\hat{\pi}_t$ is the log-deviation of inflation, $\chi$ is the degree of price indexation, $\bar{\pi}$ is trend inflation, and the rest are parameters described in the Appendix.

[9] Note that price dispersion decreases after such a shock. Clearly this is true after shocks that reduce the inflation rate, as shown in Figure 3.

backward-looking is the inflation rate, through (34).

The second and third effect point in the same direction regarding the effect of the level of indexation on the inertia of the inflation rate. Indeed it is easy to show (see Sahuc, 2006) that the higher the degree of past inflation indexation, the higher, quite intuitively, is the inertia in the New Keynesian Phillips Curve (NKPC). The first effect through price dispersion, instead, works in the opposite direction: the lower the level of indexation, the more price dispersion is biting in the model and the more inertial is its dynamics. Figure 3 clarifies how the net result of these opposite effects shapes the response of inflation: positive indexation induce the hump-shaped response of inflation that we observe in the data and delivers a more inertial adjustment.

In sum, it is clear that the dynamics of the model, and hence the trade-offs monetary policy is facing, are quite different depending on the level of indexation. The two main effects of indexation on the dynamics of the model goes through price dispersion and inflation. Partial indexation affects both qualitatively and quantitatively the dynamics of price dispersion, which is a quite important variable because it induces very high output costs in these kind of models. Partial indexation also affects the degree of persistence of inflation. The dynamics of price dispersion and inflation are very much related to each other, as from (2), and because price dispersion affects the marginal costs. It is not clear what are the implications for monetary policy design. The higher the degree of indexation, the lower and the less responsive and inertial is price dispersion, and the more persistent is inflation dynamics. In what follows we assess the importance of these arguments in shaping optimal operational monetary policies.

## 4 Optimal Operational Monetary Rules

As stated in the introduction, the aim is to analyze simple and implementable Taylor rules, such that these policies can be actually "operational" for policy makers. Simple rules, as the ones considered here, are very easy to communicate and to be understood by the public, helping the transparency of central bank behavior, and easy to implement since they are functions of few readable economic variables. An operational rule should also be implementable in the sense that should both deliver a unique rational expectation equilibrium and satisfy the lower bound on the nominal interest rate.[10] As in SGU, we looked for the optimal monetary policy numerically discretizing the support $[-3, 3]$ in intervals of length 0.0625 for $\alpha_\pi$ and $\alpha_y$ in the particular classes of rule of the form (1). Moreover, as in SGU, in (1): (i) $i$ can take three different values, i.e., $i \in \{-1, 0, 1\}$ corresponding to forward- looking, current-looking and backward-looking policies respectively; (ii) $\alpha_R \in \{0, 1, 2\}$, corresponding to no inertial, inertial and super inertial rules respectively. On top of that, we also allow the price indexation parameter to vary across the following levels of indexation: $\chi \in \{0, .75, .80, .85, .8788, .90, .95, 1\}$. Given the curse of dimensionality of our grid-search method, we investigate only 8 levels of indexation. We hence performed 677448 simulations, each of one take 90 seconds (on a standard Pentium IV (R) 3GHz), so it is about 16936.2 computer hours (almost 2 years). This was made possible by optimizing the functioning of MATLAB symbolic toolbox, and clustering 30 computers. Given the great number of cases to look at, we organize the presentation of results as follows. First, we illustrate how indexation changes the optimal rule and the dynamic response of the economy *across all the different type of considered rules.* Second, we analyze in detail how indexation changes the optimal rule and the dynamic response of the economy in the case of *one particular rule*, i.e., forward-looking no inertia. Third, we will illustrate how the optimal rule changes with indexation *within a particular class of policy rule*, for all the different cases considered in this paper.

# 5 Price Indexation and the Optimal Simple Rule

In this section we investigate the effect of indexation on the overall optimal simple monetary policy rules. Table 1 shows the type of policy rule, the optimal values of the coefficients and the corresponding welfare levels, for different values of the degree of indexation. Table 2 displays the corresponding unconditional moments for some variables of interest: consumption, output, price dispersion and inflation.

**Welfare**

[10] Following SGU, formally, we require the logarithm of the equilibrium nominal interest rate not to be lower than two times the variance of the nominal interest rate, i.e., $ln(R^*) \geq 2\sigma_{\hat{R}_t}$ . If the equilibrium nominal interest rate was normally distributed around its target value, then this constraint would ensure a positive nominal interest rate 98 percent of the time.

Table 1 shows two different welfare measures: steady state welfare and conditional welfare. Our operational rules are expressed in deviations from the steady state, thus the welfare level of the deterministic steady state does not depend on the policy rule. The steady state welfares instead do depend on the degree of indexation, since the value of $\chi$ determines the price dispersion and thus affects the variables of the model. The different levels of steady state welfare, therefore, can be thought as a measure of the magnitude of the first-order effects of changing the degree of indexation.

The conditional welfare instead take into account the stochastic steady state of the economy, and therefore the second-order effects deriving from the volatility of the shocks. It follows that the stochastic steady state depends on the particular monetary policy rule considered[11].

Looking at each conditional welfare and at the correspondent steady state welfare, Table 1 shows that the conditional welfare is always lower, since both the transitional dynamics (from the deterministic to the stochastic steady state) and the second order effects due to the volatility of the variables are taken into account. The difference between the two is sizeable and basically invariant across indexation levels. Indeed, the two different welfare measures imply the same ranking across indexation levels.

Table 1 shows that the loss across optimal policies due to different degrees of indexation is basically determined by the steady state one. In other words, looking at the difference of welfare levels across the degrees of indexation, the difference in conditional welfare is basically equal to the steady state one: the optimal simple rule can modify it only by a tiny fraction. For example, given our calibration, the best indexation degree is 0.8788. If instead, the economy features full indexation the steady state welfare loss amounts to 0.0023%, while the loss in terms of conditional welfare amounts to 0.0020%. If instead, the economy features no indexation the steady state welfare loss amounts to 0.14%, and the loss measured in terms of conditional welfare is basically the same. Hence, the first order effects of indexation are much more important than the second-order ones. This suggests that an optimal simple monetary policy does a good job in stabilizing the cycle around the deterministic steady state, regardless the degree

[11] The conditional measure of welfare assumes a initial state of the economy and takes into consideration the transitional dynamics from that initial condition to the stochastic steady state implied by the policy rule. We will assume that the initial condition is always the deterministic steady state (recall that the deterministic steady state varies with the degree of indexation).

of indexation, but cannot do much in compensating the first order effects deriving from different degree of indexation.

The same point is visualized in Figure 4, that displays the percentage welfare gain of the different indexation levels with respect to zero indexation. Each bar displays the steady state welfare gains and the overall conditional welfare gain net of the steady state one.[12] The graph shows that an increase in the level of indexation reduces both the steady state losses and the losses associated with the stochastic steady state under the optimal rule, since indexation acts as a partial correcting mechanism for those firms that can not optimize their price. However, the effects of indexation on losses due to the stochastic volatility of the model are very small, compared with the losses induced in the deterministic steady state. In other words, indexation has a first order effect on steady state because it affects price dispersion. Moreover, it has also effect on the ability of the economy to respond to shocks. The optimal rule can partly offset the latter, but it can not improve on the former, which, for realistic calibration, is much larger in magnitude.

Furthermore this key result is robust across types of policies. SGU shows that in the full indexation case, the difference between the conditional welfare levels associated to the optimal rules in the different cases is very small. That is, conditional on choosing the optimal rule for each one of the different class of policies, the differences in conditional welfare levels are extremely tiny. This result holds also for any given level of $\chi$ considered.[13] Therefore, *conditional on choosing the optimal policy, indexation matters much more for welfare than the actual form (current, backward, inertial etc..) of the monetary policy rule.* The literature, however, focuses more on the second aspect (i.e., the optimal policy type), while it is rather careless when it comes to define the calibrated value for the degree indexation or to develop normative analysis on the indexation level. However, that is proven to be much more important in terms of welfare effects.

**Policy Rule**

Table 1 shows that the type of optimal operational policy changes with indexation.

[12] That is, define $ssw_\chi$ and $cw_\chi$ the steady state welfare and the conditional welfare, respectively, associated with a given value of $\chi$. Then, for all levels of $\chi$ analyzed, the percentage conditional welfare gain is defined as: $\frac{cw_\chi - cw_0}{cw_0}$, and the percentage steady state gain (normalized over the conditional one) as: $\frac{ssw_\chi - ssw_0}{cw_0}$. Then the black area in the graph is $\frac{cw_\chi - cw_0}{cw_0} - \frac{ssw_\chi - ssw_0}{cw_0}$.

[13] That is, for a given level of $\chi$, conditional on choosing the optimal rule for each one of the different class of policies, the differences in conditional welfare levels are low. Results are available upon requests.

While the forward looking rule with no inertia is found to be optimal for the highest level of indexation, it turns out that lowering the degree of indexation leads the forward-looking rule to be substituted by the current-looking one. The backward-looking policy is never optimal. When there is no indexation then the forward looking inertial policy is optimal.

**The Importance of Price Dispersion**

Table 2 has one clear message: the lower is indexation, the higher is the variance of price dispersion and the lower the one of inflation. First, the column $E(s)$ reports the average of the deviation of $s$ from steady state. This is very low, meaning that the mean value of price dispersion is its steady state value, that we know decreases with indexation. Second, the unconditional variance, $\sigma_s$, do not change very much across different degrees of indexation (except for full indexation) and it is very small. *This means that the main task of the optimal operational rule is to stabilize the degree of price dispersion* around the steady state value. As shown by SGU, price dispersion is the main inefficiency associated with inflation in New Keynesian models, because it acts like a negative productivity shift in this economy, and thus the optimal policy response calls for its stabilization.

A temporary surge in inflation generates an increase in price dispersion, that needs to be stabilized by monetary policy. As we argued above, moving away from full indexation increases significantly the inertia in price dispersion. Furthermore the lower the degree of indexation, the more current inflation is going to affect current price dispersion. It follows that the lower the degree of indexation, the more important is to stabilize inflation. As a matter of fact under optimal rules the variance of inflation reduces as the degree of indexation decreases.[14]

Table 2 also shows that full indexation is a very special case. Not only price dispersion is the lower, the degree of indexation is the higher, but when there is full indexation this cost is of second order magnitude.[15] Indeed, despite the rather high volatility in inflation, the volatility of price dispersion is infinitesimal, that is price dispersion is almost always zero in this rather peculiar economy. It is interesting to note that, even moving away

[14] See section 6.3 for a further discussion of this point.

[15] If $\chi = 1$, there is no effect of current inflation on price dispersion at first order, see (2). In this case, at first order the dynamics of price dispersion is autonomous from the model and does not influence it.

only slightly from full indexation, i.e. $\chi = 0.95$, considerably worsens the trade-offs monetary policy is facing. Indeed, the volatility of inflation drops by roughly a half, while the one of output increases by one third. Despite the lower volatility of inflation induced by a higher $\alpha_\pi$, the volatility of price dispersion is higher by a factor $10^{10}$. For the other values of indexation we analyze, instead, the volatility of price dispersion are of similar order of magnitude.[16] This indeed signals that full indexation is a quite special case. Unfortunately, this is the case the literature often focus on, just to avoid any long-run effect of inflation. Assuming full indexation, however, undoes the role of price dispersion, which is the main mechanism in New Keynesian models. The full indexation assumption, hence, strongly affects the functioning of the economy, making the task of monetary policy easier.

As SGU shows, full backward-looking indexation, in fact, does not imply price stability, since the variance of inflation is about 2 per cent per annum, that is, half of its steady state value. However, as said above, the degree of indexation turns out to be very important in affecting the optimal operational policy, because it affects the long-run and short-run properties of the model. In particular, partial indexation calls for a tighter control of inflation, as a way to stabilize price dispersion.

How does the monetary policy stabilize inflation, and thus, price dispersion? By increasing $\alpha_\pi$ for lower indexation levels, that is, getting tougher on inflation deviations from target, as showed by the first 4 forward-looking policies in Table 1. The increase in $\alpha_\pi$is only modest, possibly due to the fact that a lower indexation makes inflation more forward looking (Sahuc, 2005 and Ascari and Ropele, 2007), and, thus, easier to control by a credible forward-looking rule. Moreover, despite the increase in $\alpha_\pi$, the ability of the optimal policy to stabilize price dispersion worsen with indexation. For values of $\chi$ lower than 0.8788, the optimal policy becomes first current-looking and then forward-looking inertial, so we can not just compare the coefficients. It may surprise that this last inertial policy features a very low $\alpha_\pi$, but an inertial policy means a permanent change in the nominal interest rate in response to inflation. Indeed, it is interesting to

[16] Note that the volatility of price dispersion in the no indexation case is roughly one hundred times bigger than when $\chi = 0.95$. In this sense also $\chi = 0$ is an extreme case. But while the no indexation case is changing the dynamics of the model quantitatively (i.e., strengthening the effects of price dispersion), the full indexation case is changing the dynamics of the model also qualitatively (i.e., cancelling the price dispersion mechanism).

note that, for the inertial forward looking optimal policy when $\chi = 0$, the sum of $\alpha_\pi$ and $\alpha_R$ is the same as the value of $\alpha_\pi$ for the forward looking no inertial optimal policy for high values of $\chi$. Moreover, in the no indexation case, inflation is basically kept fix at the steady state level, since the variance is extremely low. Note that this would be the case also if the optimal policy when $\chi = 0$ (i.e., forward looking, $\alpha_\pi = 0.1875, \alpha_y = 0$ and $\alpha_R = 1$) is implemented in the full indexation case. $\sigma_\pi$ would then be very small and equal to 0.2416[17]. This is exactly the task accomplished by the inertial policy: stabilize inflation. Hovewer, such a policy is not chosen in the full indexation case, because there is no need to stabilize price dispersion: full indexation keeps it basically at zero. In other words, indexation plays the role of an extra instrument that can take care of the problem of stabilizing price dispersion. Then stabilizing inflation is no more a fundamental issue for monetary policy.

One the one hand, inertial policy stabilizes inflation because the unit root in the policy affects expectations of future inflation and the long-run interest rate, helping monetary policy to stabilize inflation more efficiently. On the other hand, an inertial policy lacks flexibility and therefore would also entail some welfare costs. For high degree of indexation, these costs are higher than the gains in controlling the variance of price dispersion through inflation, while when indexation is null (or very low) then the gains in stabilizing inflation overcome the costs of an inertial policy rule. This may explain why in this case inertial policy is the best choice. Table 3 provides some further evidence in this direction, showing how the optimal value of $\alpha_\pi$ in a forward-looking rule changes with the value of $\alpha_R$ and $\chi$ : $\alpha_\pi$ decreases with indexation, unless $\alpha_R$ assumes values close to 1.

A very robust feature of the optimal policies is that $\alpha_y$ is zero. Therefore, the result that the optimal policy should not respond to output (as in SGU) is independent from the degree of indexation.

## 6 Price indexation and the forward-looking rule

In this section we concentrate on a particular rule: the forward-looking rule with no inertia (FLNI), i.e., $i = -1$ and $\alpha_R = 0$. We look at this particular rule because it turns

[17] In this case the conditional welfare is equal to -156.7254 and $\sigma_s$ = 4.8039e-016.

out to be the globally optimal one both in SGU and in our simulations. In doing so, we can focus on the effects of indexation on the parameters of the policy rule and on the dynamics of our economy within a single policy rule, leaving the comparison across rules to the next sections.

### 6.1 Implementability

Figure 5 shows how changes in the level of indexation can affect the determinacy and implementability regions for the FLNI. The graphs visibly display an increase in both the determinacy and implementability areas with indexation. Low levels of indexation tend thus to reduce the parameter space available for policy options. In particular, if indexation is not full then the Taylor principle ($\alpha_\pi > 1$) does not define a condition for determinacy, and also the value of $\alpha_y$ is very much restricted to be around zero. The effect of varying the degree of indexation on the implementability region are qualitatively similar to the effect of changing the trend inflation level, as in Ascari and Ropele (2007). The main effect of a lower indexation, thus, is to increase price dispersion in the economy, which in turn increases the likely of sunspots fluctuations. *Ceteris paribus*, in fact, a sunspot increase in inflation leads to an increase in price dispersion, which in turn rises the marginal costs, and hence inflation. This mechanism gets stronger the lower is the indexation, and therefore the policy response needs to be tougher to induce determinacy of the rational expectation equilibrium.

### 6.2 Welfare Surfaces

As in SGU, we can plot the level of conditional welfare as a function of $\alpha_y$ and $\alpha_\pi$ for those policies that implies a welfare loss of 1% from the one attained by the best operational policy within this FLNI class of policies. We can do that for different values of the indexation parameter. Figure 6, however, does not look very revealing. The welfare surfaces look quite smooth, but they are not, and this is the reason to cut them focusing only to the policies with small deviation from the best. Otherwise, the surfaces would display multiple local maxima and minima, and the graph would be just very confusing. This is important in motivating the particular painstaking grid search method that SGU and we followed. A clever alternative would have been to develop an optimization algorithm that maximizes the objective function over the parameter space.

The choice of a grid-search approach is instead justified by the shape of the welfare surfaces, that would stuck the optimization algorithm.

### 6.3 Indexation, Optimal Policy and Unconditional Moments

Table 4 and 6 are equivalent to Table 1 and 3 for the FLNI policy rule. They show the optimal values of the coefficients of the FLNI policy, the corresponding welfare levels and unconditional moments for some variables of interest. The results are qualitatively the same but much more clear-cut. A lower degree of indexation calls for a policy that further reduce the variance of inflation. The optimal policy does it in a straightforward way: by increasing the response to inflation, i.e., $\alpha_\pi$, from 1.125 to 2.6875. If the policy $\alpha_\pi = 2.6875$ and $\alpha_y = 0.1875$ is implemented in the full indexation case, then $\sigma_\pi = 1.03$. Again in the full indexation case monetary policy could stabilizes inflation through a higher $\alpha_\pi$, but it chooses not to do so, because price dispersion is zero thanks to full indexation. Note, however, that, when $\chi = 0$, despite a value of $\alpha_\pi$ that is more than double than the one in, say, $\chi = 0.85$, the variance of inflation is a bit higher. This signals that price dispersion inertia induced by lower indexation makes inflation more difficult to control. As said above, this may explain why for sufficiently low levels of indexation, the inertial policy rule may be preferred. Furthermore, optimal policies are not responding to the output gap. Under full indexation, the optimal policy rule resembles an almost real interest rate targeting rule, while, as indexation decreases, the optimal policy rule shift to a pure inflation targeting rule with a stronger reaction to inflation deviation from target. Moreover, both the steady state and the conditional welfare levels are maximized when $\chi = 0.8788$, and again the difference across welfare levels is mainly driven by the the steady state effect.

Finally, as Table 2, Table 5 again shows that full indexation is a rather special case. While the partial indexation cases are all similar in terms of order of magnitude of the second moments of the variables, the full indexation tends to cancel the effects of price dispersion, as evident from the variances of $s, \pi$ and $y$.

## 7 Price Indexation and the simple optimal rule within different class of policies

In this section we present how indexation affects the optimal operational rule also for each of the other policy class: current looking, backward looking and inertial policies.

Tables 6 to 8 display the results for the optimal operational simple policy rules within each different class of policies. Given the large number of policies we analyzed, Tables 6 to 8 shows the optimal policy rules for just 3 levels of indexation: full indexation (i.e., $\chi = 1$), no indexation (i.e., $\chi = 0$) and the optimal steady state indexation level (i.e., $\chi = 0.8788$)[18]. Moreover, Table 6 presents the no inertial policies, Table 7 the inertial ones (i.e., $\alpha_R = 1$) and Table 8 the super-inertial ones (i.e., $\alpha_R = 2$).

The no inertial policy rules exhibit the same features explained above. In particular: (i) the variance of price dispersion decreases, while the one of inflation increases, with indexation; (ii) the difference in conditional welfare across the various cases is mainly driven by the first order steady state effects; (iii) the case $\chi = 1$ eradicate the effects of price dispersion from the model (iv) the optimal rule is not responding to output; (v) the lower the degree of indexation, the larger $\alpha_\pi$. The points (i)-(iv) basically hold true also for the inertial and super inertial policies[19]. The inertial and super inertial policies, instead, exhibit quite a different pattern regarding the parameter $\alpha_\pi$. In particular, $\alpha_\pi$ is surprisingly decreasing with the degree of indexation. In the case of super inertial policy rules and no indexation it even becomes substantially negative. Since the value of $\alpha_R$ is different for inertial and super inertial policy rules, it may not surprise to find different values for $\alpha_\pi$ and $\alpha_y$, but actually we do not have an intuition of the effects of indexation on $\alpha_\pi$ in these cases.[20]

Regarding conditional welfare, the no inertial policy rules always perform the best for high degree of indexation, while the inertial ones generally perform the best when indexation is zero. This suggests that the results in SGU are not robust to a change in

[18] For the other level of indexation analyzed results are available upon requests.

[19] There are two exceptions among the superinertial policy rules with full indexation: the current looking and forward looking policy rules, where $\alpha_y$ is respectively equals to 0.3125 and 0.625, respectively.

[20] Moreover, while the policy rules in the Tables satisfy the requirements for an operational policy, they are quite close to the boundaries of the determinacy frontiers. In particular, it is clear that a combinations of value for $\{\alpha_\pi, \alpha_y, \alpha_R\}$ as {0,0,1} or {0,0,2} would immediately lead to an explosive path for the nominal interest rate.

the indexation parameter.

## 8 Unconditional Welfare

Table 6 to 8 also display the unconditional welfare levels implied by the policies. The unconditional welfare measure is the most commonly employed in the literature. Opposite to the conditional welfare measures, that is conditional on the initial point, the unconditional one by design 'integrates away' the role of the initial state. The unconditional welfare measure is just the weighted average of the conditional welfare levels associated with all possible values of the initial state vector with weights given by their unconditional probabilities. It is evident that unconditional welfare may imply a different ranking with respect to conditional welfare. As stressed in SGU, the different ranking implied by the two measures demonstrates the importance of considering the transitional dynamics and the initial condition. A ranking based on a conditional welfare measure is a "conditional ranking", that is, it depends upon the particular initial condition used in calculating the transitional dynamics and thus the conditional welfare. It follows that different initial condition can indeed reverse a ranking of policies. As stressed by SGU, this result is indicative of the fact that the optimal operational rule lacks time consistency.

For the sake of completeness, Table 9 presents the optimal policy for each level of indexation, according to unconditional welfare. Table 10 displays some unconditional moments. A first result is that unconditional welfare is substantially higher than steady state welfare. Moreover, unconditional welfare is always increasing with indexation. Figure 7 replicates Figure 1 for the unconditional welfare ranking. It shows that the unconditional welfare gains, net of the steady state ones, are still lower than the steady state one, but quite sizeable. Furthermore, the optimal policies are different from the ones implied in Table 1, not only quantitatively, but also qualitatively. The current looking no inertial policy is optimal for high level of indexation, while the backward looking no inertia is optimal for low ones. Recall that the backward looking policy was never optimal according to conditional welfare. Besides, all the optimal policies are very close to the upper bound for $\alpha_\pi$ in our set of values for the grid search (i.e., $\alpha_\pi \in [-3, 3]$). It is very likely that the optimal policy would have implied an higher level of $\alpha_\pi$ for $\chi < 0.8$, so the results should be interpreted with caution. Indeed, $\alpha_\pi$ is then on the boundary, and the control on the volatility of inflation has to be made by lowering the value of $\alpha_y$.[21] Furthermore the order of magnitude of price dispersion volatility is the same as under optimal policies ranked according to conditional welfare, suggesting that full indexation is a very special case whatever welfare measure assumed. Finally, the volatilities implied by these optimal policies according to unconditional welfare are higher with respect the ones implied by the optimal policies under conditional welfares.

## 9 Conclusions

In this paper we analyze how the optimal, simple and implementable monetary policy rule changes with the degree of price indexation in a standard medium-scale New Keynesian model. The effects of the assumed degree of price indexation is a quite neglected issue in the literature, since the vast majority of paper focus on full indexation. However, full indexation is a very special case, because it eradicates the effects of price dispersion from the model. Monetary policy hence faces a very different economic structure and trade-offs with respect to a world with partial indexation. Moreover, the degree of indexation has also important first order welfare effects.

We show that indexation strongly affects monetary policy: both the kind of policy rule and its parameters do depend on the degree of indexation. In particular, we show that indexation acts as an instrument to stabilize price dispersion. As a consequence, lower degree of indexation increases the size and the variance of price dispersion, calling for monetary policy to control it by stabilizing inflation, which indeed is not a task of monetary policy when indexation is full.

The central message is that when *ad hoc* features are introduced in microfounded models, one should be careful in drawing normative implications, without assessing the effects of such a feature on the mechanics of the model, on welfare and hence on the shaping of the optimal policy.

[21] The volatility of inflation is quite sensitive to the value of $\alpha_y$, in the sense that small increases in $\alpha_y$ rises inflation volatility substantially. This is the main reason why $\alpha_y$ is always close to zero in the optimal policies, except in Table 10.

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# 10 Appendix: The Christiano-Eichenbaum-Evans Model for US business Cycle

## 10.A Households

Agents live forever and discount future at a rate $\beta$. There is a continuum of households that seek to maximize their expected utility function, given by:

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) \right\}. \tag{3}$$

$E_0$ defines the mathematical expectation operator conditional on the information set available at time 0. The function $u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right)$ is well-behaved and increasing in consumption $c_t$ and money holdings $m_t^h$, decreasing in hours supplied $h_t^s$. Preferences display habit in consumption levels, measured by the parameter $b$.

There is a continuum of final goods, indexed by $i \in [0, 1]$, that enter in the consumption bundle $c_t$ through the usual Dixit-Stiglitz aggregator:

$$c_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}, \tag{4}$$

where the parameter $\eta$ indicates the elasticity of substitution between different varieties of goods. The standard household problem defines the optimal demand of good $i$, given by $c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t$, where $P_t$ is the general price index given by $P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$.

We assume a continuum of labour services $h_{jt}$, $j \in [0, 1]$, that are combined according to the following technology

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj\right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}},$$

where $\tilde{\eta}$ is the elasticity of substitutions of labour types. Given production plans, a firm that minimizes costs has a labour-specific demand function given by $h_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\tilde{\eta}} h_t^d$, where $W_{jt}$ is the wage prevailing in labour market $j$ and $W_t$ is a wage index defined as $W_t = \left[\int_0^1 W_{it}^{1-\tilde{\eta}} di\right]^{\frac{1}{1-\tilde{\eta}}}$. Integrating labour-specific demand functions one obtains $h_t$ defined as

$$h_t \equiv \int_0^1 h_{jt} dj = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj. \tag{5}$$

Agents owns physical capital $k_t$ that depreciates at rate $\delta$. The capital accumulation equation is:

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right], \tag{6}$$

where the function $S$ introduce a cost of varying the level of investment and satisfies the properties that $S(1) = S'(1) = 0$, $S''(1) > 0$.

Variable capacity utilization of physical capital is denoted by $u_t$, with an associated cost implicitly defined by $a(u_t)$. Agents owns firms and rent capital at a real interest rate $r_t^k$, earn profits and decide also over the utilization rate. Money is injected via lump-sum transfer $\tau_t$. Finally the existence of complete markets on state contingent assets $x_t^h$ assure that all agents choose the same level of consumption independently of the hours supplied. The budget constraint is then:

$$E_t r_{t,t+1} x_{t+1}^h + c_t + i_t + m_t^h + a(u_t) k_t = \frac{x_t^h}{\pi_t} + h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj + r_t^k u_t k_t + \phi_t - \tau_t \frac{m_{t-1}^h}{\pi_t} \quad (7)$$

Given wage stickiness à la Calvo wage dispersion $\int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$ can be written as:

$$w_t = \tilde{\alpha} w_{t-1} \frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t} + (1 - \tilde{\alpha}) \tilde{w}_t \quad (8)$$

where $\tilde{w}_t$ is the optimal wage set at time $t$.

The problem is to maximize (3) under eqs.(5)-(8). Household's first order conditions are hence given by

$$u_{c_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + u_{c_t}\left(c_{t+1} - bc_t; h_{t+1}^s; m_{t+1}^h\right) = \lambda_t \quad (9)$$

$$u_{h_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) = -\lambda_t \frac{w_t}{\tilde{\mu}_t} \quad (10)$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t}\left[q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - a(u_{t+1})\right] \quad (11)$$

$$q_t \lambda_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left[S_i\left(\frac{i_t}{i_{t-1}}\right)\right] i_t\right] - \beta q_{t+1} \lambda_{t+1} S_i \left(\frac{i_{t+1}}{i_t}\right) i_{t+1} = \lambda_t \quad (12)$$

$$a_{u_t}(u_t) = r_t^k \quad (13)$$

$$u_{m_t^h}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + \beta \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t. \quad (14)$$

Wages are sticky à la Calvo, and $1 - \tilde{\alpha}$ is the probability of being able to reset wages next period. With probability $\tilde{\alpha}$ wages can not be re-optimized, and thus they are updated with past inflation, more precisely they vary according to $w_{j,t+1} = w_{j,t} \pi_t^{\tilde{\chi}}$ where $\tilde{\chi}$ is the degree of indexation to past inflation. Define $\tilde{w}_t$ as the optimal real wage set every period $t$. A union chooses the optimal wage maximizing its the utility function given by equation (3), subject to demand of labour in the specific market $h_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} h_t^d$ and the probability of not being able to re-optimize in future periods. The resulting first order condition is:

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right] = 0. \tag{15}$$

Equation (15) states that optimal real wage must equate the future stream of marginal revenues from working to the expected sum of marginal cost of suppling labour. Given the structure of the model, all the reset optimal wages are identical in all labour markets in which the household can re-optimize. SGU shows how to write condition (15) in recursive form.

## 10.B Firms

Each good is produced by a firm which monopolistically supplies its own variety using a production technology of the form:

$$z_t F(k_{it}, h_{it}) - \psi,$$

where $z_t$ is an aggregate exogenous technology factor that follow an AR(1) process.$\psi$ represent a fixed cost of production that generates increasing return to scale and guarantees zero profits in equilibrium. The production function $F(k_{it}, h_{it})$ is well-behaved and the same for all firms. Final goods can be used for consumption, investment, public expenditure and to pay cost of capital utilization. Each firm faces the following demand function:

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t, \tag{16}$$

where:

$$y_t = c_t + i_t + g_t + a(u_t) k_t. \tag{17}$$

We assume that firms can access a centralized market for capital, and must pay a fraction $\nu$ of wages at the beginning of the period by cash. Therefore their money demand function is:

$$m_{it}^f = \nu w_t h_{it} \tag{18}$$

Firms maximizes the expected value of future profits, under their demand function (16) and the cash-in-advance constraint (18). Firms' first order condition with respect to capital and labour services are:

$$mc_{it} z_t F_{k_{it}}(k_{it}, h_{it}) = r_t^k \tag{19}$$

$$mc_{it} z_t F_{h_{it}}(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. \tag{20}$$

If we assume that all firms have access to the same factor markets and $F$ is homogeneous of degree one, equation (19) and equation (20) imply that all firms have the same marginal costs and aggregation across firms is straightforward.

Prices are sticky a la Calvo. Every period each firm can choose a new price of its own good with a probability $1 - \alpha$. Those firms who can not reset their price update their price according to past inflation. Specifically their new price is $P_{it} = P_{it-1}\pi_{t-1}^{\chi}$ where $\chi$ is the degree of price indexation. The optimal price solve the first order condition:

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} y_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}}\right)^{\eta} \left[\frac{\eta - 1}{\eta} \frac{\tilde{P}_t}{P_t} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{i,t+s}\right] = 0 \tag{21}$$

These expression states that optimizing firms choose a price $\tilde{P}_t$ that equates the expected sum of future marginal costs with the expected sum of marginal revenues, conditional on not being able to re-optimize in the future. Given the structure of the model, all the reset optimal prices are identical in all good markets in which firms can re-optimize. SGU shows how to write condition (21) in recursive form.

## 10.C The Government

The government has two policy instruments: public expenditure and the nominal interest rate. Government expenditure is financed through lump-sum taxes and seigniorage:

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t}. \tag{22}$$

We assume an optimizing government that minimizes costs of acquiring the composite good, hence given public expenditure, government's absorption of a single variety of goods is $g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} g_t$.

## 10.D Equilibrium

Equilibrium on money market is simply: $m_t = m_t^h + m_t^f$. Equilibria in labour and capital markets imply that:

$$\begin{aligned} \int_0^1 h_{it}^d di &= h_t^d && (23) \\ \int_0^1 k_{it} di &= u_t k_t. && (24) \end{aligned}$$

Consider equilibrium in the final goods' markets. The assumption that both government and agents minimize their expenditure choosing the optimal quantity of each variety of good implies the following condition:

$$z_t F(k_{it}, h_{it}) = (c_t + g_t + i + a(u_t) k_t) \left( \frac{P_{it}}{P_t} \right)^{-\eta}. \tag{25}$$

Integrating the right side of the equation considering that the capital-labour ratio is the same among firms and imposing equations (23) and (24), yields:

$$z_t h_t^d F\left( \frac{u_t k_t}{h_t^d}, 1 \right) = (c_t + g_t + i + a(u_t) k_t) \int_0^1 \left( \frac{P_{it}}{P_t} \right)^{-\eta} di, \tag{26}$$

where $s_t \equiv \int_0^1 \left( \frac{P_{it}}{P_t} \right)^{-\eta}$ constitutes a wedge between aggregate supply and aggregate absorption and represents the price dispersion generated by price staggering. Then:

$$s_t = (1 - \alpha) \tilde{p}_t^{-\eta} + \alpha \left( \frac{\pi_{t-1}^{\chi}}{\pi_t} \right)^{-\eta} s_{t-1} \tag{27}$$

and the equilibrium on final goods' markets is given by:

$$z_t F\left( u_t k_t, h_t^d \right) = (c_t + g_t + i + a(u_t) k_t) s_t. \tag{28}$$

Using the same properties, we aggregate equations (19) and (20) obtaining:

$$\begin{aligned} mc_t z_t F_{k_t}\left( u_t k_t, h_t^d \right) &= r_t^k && (29) \\ mc_t z_t F_{h_t^d}\left( u_t k_t, h_t^d \right) &= w_t \left[ 1 + \nu \frac{R_t - 1}{R_t} \right]. && (30) \end{aligned}$$

Finally, the expression for wage dispersion closely follows its price counterpart. Using labour-specific demand function we can write:

$$h_{jt}^d = \left( \frac{w_{jt}}{w_t} \right)^{-\tilde{\eta}} h_t^d \tag{31}$$

and integrating both sides and using equation (23) yields:

$$h_t = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj \tag{32}$$

Then defining $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$, it is easy to show that:

$$\tilde{s}_t = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\eta}} \tilde{s}_{t-1} \tag{33}$$

Finally the definition of the price and the wage index generates a low of motion for the aggregate price and wage levels:

$$P_t^{1-\eta} = \alpha\left(P_{t-1}\pi_{t-1}^{\chi}\right)^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta} \tag{34}$$

$$w_t^{1-\tilde{\eta}} = \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}}\left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\eta}} + (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\eta}} \tag{35}$$

## 10.E Functional forms

As in SGU we assume the following functional forms:

$$\begin{aligned}
u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) &= \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2}h_t^2 + \phi_1 \frac{\left(m_t^h\right)^{1-\sigma_m}}{1-\sigma_m} \\
F\left(u_t k_t, h_t^d\right) &= (u_t k_t)^{\theta}\left(h_t^d\right)^{1-\theta} \\
S\left(\frac{i_t}{i_{t-1}}\right) &= \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 \\
a(u_t) &= \gamma_1 (u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2 .
\end{aligned}$$

The calibration is as in SGU and follows CEE's estimation results. One period in the model is interpreted as a quarter. The steady state displays full capital utilization. Furthermore the quantities of money held by households and firms are set to match the empirical distribution[22], likewise for long run inflation, 4.2% annual, and public expenditure, 52.44%. The rest of the calibration is listed in Table 11: the reader is referred to SGU and CEE for a discussion of exogenous processes and values use therein.

We solve the model with the perturbation method developed in Schmitt-Grohé and Uribe (2004b) and we rank policies using a measure of welfare based on second order approximation of the model around the non-stochastic steady state. We used both an unconditional and a conditional welfare measure, the latter to take into account of transitional dynamics.

[22] That is households hold 44% of money in steady state while fims hold the remaining 56%.

## 11 Tables

Table 1. Optimal Operational Monetary Policies Rules

| $\chi$ | Policy Class | $\alpha_R$ | $\alpha_\pi$ | $\alpha_y$ | SS Welf. | Conditional Welf. |
|---|---|---|---|---|---|---|
| 1 | Forward looking | 0 | 1.1250 | -0.0625 | -156.7143 | -156.7220 |
| .95 | Forward looking | 0 | 1.1875 | 0 | -156.7119 | -156.7199 |
| .90 | Forward looking | 0 | 1.1875 | 0 | -156.7107 | -156.7189 |
| .8788 | Forward looking | 0 | 1.1875 | 0 | -156.7106 | -156.7188 |
| .85 | Current looking | 0 | 1.0625 | 0 | -156.7108 | -156.7191 |
| .80 | Current looking | 0 | 1.0625 | 0 | -156.7122 | -156.7205 |
| .75 | Current looking | 0 | 1.0625 | 0 | -156.7149 | -156.7232 |
| 0 | Forward looking | 1 | 0.1875 | 0 | -156.9351 | -156.9428 |

Table 2. Unconditional Moments under Optimal Operational Rules (x$10^{-2}$)

| $\chi$ | $E(c)$ | $E(y)$ | $E(s)$ | $E(\pi)$ | $\sigma_c$ | $\sigma_y$ | $\sigma_s$ | $\sigma_\pi$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.13 | -0.19 | 3.9926(x$10^{-6}$) | 4.06 | 1.6722 | 3.7602 | 1.22(x$10^{-15}$) | 2.0483 |
| .95 | 0.17 | -0.38 | 5.9022(x$10^{-6}$) | 4.20 | 1.9147 | 5.1102 | 6.9(x$10^{-4}$) | 1.2012 |
| .90 | 0.17 | -0.38 | 1.2596(x$10^{-5}$) | 4.19 | 1.9127 | 5.0978 | 1.7(x$10^{-3}$) | 1.2808 |
| .8788 | 0.17 | -0.38 | 7.9551(x$10^{-6}$) | 4.20 | 1.9080 | 5.0661 | 2.0(x$10^{-3}$) | 1.2635 |
| .85 | 0.16 | -0.36 | 3.5347(x$10^{-6}$) | 4.19 | 1.9016 | 4.9235 | 1.6(x$10^{-3}$) | 0.7360 |
| .80 | 0.16 | -0.36 | 4.1604(x$10^{-6}$) | 4.19 | 1.8989 | 4.9050 | 2.4(x$10^{-3}$) | 0.7371 |
| .75 | 0.16 | -0.37 | 4.9830(x$10^{-6}$) | 4.19 | 1.8964 | 4.8863 | 3.3(x$10^{-3}$) | 0.7371 |
| 0 | 0.15 | -0.34 | 2.9616(x$10^{-6}$) | 4.20 | 1.8065 | 4.7709 | 9.6(x$10^{-3}$) | 0.1716 |

Note: variables are expressed in deviation from steady state, except $\pi$ that is in levels and annualized.

| Table 3. Optimal Monetary Policies for different values of $\alpha_R$ Forward Looking No Inertia | | | | | | |
|---|---|---|---|---|---|---|
| | $\alpha_R$= 0.1 | | $\alpha_R$= 0.5 | | $\alpha_R$= 0.9 | |
| $\chi$ | $\alpha_\pi$ | $\alpha_y$ | $\alpha_\pi$ | $\alpha_y$ | $\alpha_\pi$ | $\alpha_y$ |
| 1 | 1 | 0.0625 | 0.5 | 0.0625 | 0.6875 | 0.1875 |
| .8788 | 1.0625 | 0.0625 | 0.5625 | 0.0625 | 0.5625 | 0.125 |
| 0 | 2.1875 | 0.1875 | 1 | 0.0625 | 0.3125 | 0.0625 |

| Table 4. Optimal Monetary Policies - Forward Looking No Inertia | | | | |
|---|---|---|---|---|
| $\chi$ | $\alpha_\pi$ | $\alpha_y$ | SS Welf. | Conditional Welf. |
| 1 | 1.1250 | -0.0625 | -156.7143 | -156.7220 |
| .95 | 1.1875 | 0 | -156.7119 | -156.7199 |
| .90 | 1.1875 | 0 | -156.7107 | -156.7189 |
| .8788 | 1.1875 | 0 | -156.7106 | -156.7188 |
| .85 | 1.2500 | 0 | -156.7108 | -156.7193 |
| .80 | 1.4375 | 0 | -156.7122 | -156.7216 |
| .75 | 1.6250 | 0 | -156.7149 | -156.7267 |
| 0 | 2.6875 | 0.1875 | -156.9351 | -156.9767 |

Note: variables are expressed in deviation from steady state, except $\pi$ that is in levels and annualized.

| Table 5. Unconditional Means under Optimal Forward Looking No Inertia Rule (x$10^{-2}$) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\chi$ | $E(c)$ | $E(y)$ | $E(s)$ | $E(\pi)$ | $\sigma_c$ | $\sigma_y$ | $\sigma_s$ | $\sigma_\pi$ |
| 1 | 0.13 | -0.19 | 3.99(x$10^{-6}$) | 4.06 | 1.67 | 3.76 | 1.2195(x$10^{-15}$) | 2.05 |
| .95 | 0.17 | -0.38 | 5.90(x$10^{-6}$) | 4.20 | 1.91 | 5.11 | 6.8987(x$10^{-4}$) | 1.20 |
| .90 | 0.17 | -0.38 | 1.26(x$10^{-6}$) | 4.19 | 1.91 | 5.10 | 1.7(x$10^{-3}$) | 1.28 |
| .8788 | 0.17 | -0.38 | 7.96(x$10^{-6}$) | 4.20 | 1.91 | 5.07 | 2.0(x$10^{-3}$) | 1.26 |
| .85 | 0.16 | -0.37 | 6.40(x$10^{-6}$) | 4.20 | 1.89 | 5.01 | 2.2(x$10^{-3}$) | 1.04 |
| .80 | 0.16 | -0.37 | 4.29(x$10^{-6}$) | 4.20 | 1.87 | 4.96 | 2.4(x$10^{-3}$) | 0.75 |
| .75 | 0.17 | -0.45 | 1.18(x$10^{-5}$) | 4.26 | 1.93 | 5.4 | 4.8(x$10^{-3}$) | 1.16 |
| 0 | 0.13 | -0.56 | 2.05(x$10^{-4}$) | 4.28 | 1.95 | 5.73 | 7.78(x$10^{-2}$) | 1.16 |

Note: variables are expressed in deviation from steady state, except $\pi$ that is in levels and annualized.

| Table 6. Optimal Operational Monetary Policies - No Inertia | | | | | | |
|---|---|---|---|---|---|---|
| $\chi$ | $\alpha_\pi$ | $\alpha_y$ | $\sigma_s$ | $\sigma_\pi$ | Conditional Welf. | Unconditional Welf. |
| Forward Looking | | | | | | |
| 1 | 1.1250 | -0.0625 | 1.22(x$10^{-15}$) | 2.0483 | -156.7220 | -156.5252 |
| 0.8788 | 1.1875 | 0 | 0.0020 | 1.2635 | -156.7188 | -156.4169 |
| 0 | 2.6875 | 0.1875 | 0.0778 | 1.1574 | -156.9767 | -156.6381 |
| Current Looking | | | | | | |
| 1 | 1.0625 | 0 | 2.77(x$10^{-15}$) | 0.7278 | -156.7227 | -156.4289 |
| 0.8788 | 1.0625 | 0 | 0.0014 | 0.7645 | -156.7189 | -156.4299 |
| 0 | 1.625 | 0 | 0.0212 | 0.3353 | -156.9456 | -156.6679 |
| Backward Looking | | | | | | |
| 1 | 1.3125 | 0.0625 | 1.83(x$10^{-15}$) | 1.5648 | -156.7233 | -156.3729 |
| 0.8788 | 1.375 | 0.0625 | 0.0015 | 1.3164 | -156.7203 | -156.3810 |
| 0 | 1.3125 | 0 | 0.0180 | 0.2919 | -156.9451 | -156.4442 |

| Table 7. Optimal Operational Monetary Policies - Inertia, $\alpha_R = 1$ | | | | | | |
|---|---|---|---|---|---|---|
| $\chi$ | $\alpha_\pi$ | $\alpha_y$ | $\sigma_s$ | $\sigma_\pi$ | Conditional Welf | Unconditional Welf. |
| Forward Looking | | | | | | |
| 1 | 0.8125 | 0.1875 | 3.11(x$10^{-15}$) | 1.9911 | -156.7232 | -156.3456 |
| 0.8788 | 0.5 | 0.0625 | 0.0012 | 1.0058 | -156.7199 | -156.3975 |
| 0 | 0.1875 | 0 | 0.0096 | 0.1716 | -156.9428 | -156.6751 |
| Current Looking | | | | | | |
| 1 | 0.4375 | 0.0625 | 1.16(x$10^{-15}$) | 1.6442 | -156.7237 | -156.3639 |
| 0.8788 | 0.4375 | 0.0625 | 0.0013 | 1.1334 | -156.7203 | -156.3911 |
| 0 | 0.0625 | 0 | 0.0076 | 0.1390 | -156.9431 | -156.6733 |
| Backward Looking | | | | | | |
| 1 | 0.75 | 0.1250 | 1.54(x$10^{-15}$) | 1.3195 | -156.7243 | -156.3790 |
| 0.8788 | 0.5 | 0.0625 | 0.0011 | 0.9584 | -156.7209 | -156.3975 |
| 0 | 0.0625 | 0 | 0.0074 | 0.1355 | -156.9432 | -156.6730 |

Note: variables are expressed in deviation from steady state, except $\pi$ that is in levels and annualized.

| Table 8. Optimal Operational Monetary Policies - Super Inertia, $\alpha_R = 2$ | | | | | | |
|---|---|---|---|---|---|---|
| $\chi$ | $\alpha_\pi$ | $\alpha_y$ | $\sigma_s$ | $\sigma_\pi$ | Conditional Welf | Unconditional Welf. |
| Forward Looking | | | | | | |
| 1 | 1.6875 | 0.625 | 1.74(x$10^{-15}$) | 1.9939 | -156.7237 | -156.3435 |
| 0.8788 | 0.5625 | 0.125 | 8.40(x$10^{-4}$) | 0.6401 | -156.7203 | -156.4155 |
| 0 | -0.9375 | 0 | 0.0207 | 0.3095 | -156.9426 | -156.6931 |
| Current Looking | | | | | | |
| 1 | 0.8125 | 0.3125 | 1.03(x$10^{-15}$) | 1.3833 | -156.7248 | -156.3753 |
| 0.8788 | 0.25 | 0.0625 | 6.48(x$10^{-4}$) | 0.4206 | -156.7210 | -156.4254 |
| 0 | -0.8750 | 0 | 0.0214 | 0.3206 | -156.9450 | -156.6871 |
| Backward Looking | | | | | | |
| 1 | -0.75 | -0.0625 | 8.1(x$10^{-16}$) | 1.5262 | -156.7250 | -156.5191 |
| 0.8788 | -0.75 | -0.0625 | 0.0018 | 1.5077 | -156.7203 | -156.5230 |
| 0 | -0.6875 | 0 | 0.0182 | 0.2738 | -156.9453 | -156.6809 |

Note: variables are expressed in deviation from steady state, except $\pi$ that is in levels and annualized.

| Table 9. Optimal Monetary Policies ranked by unconditional welfare | | | | | | |
|---|---|---|---|---|---|---|
| $\chi$ | Policy Class | $\alpha_R$ | $\alpha_\pi$ | $\alpha_y$ | Unconditional Welf. | Conditional Welf. |
| 1 | Current Looking | 0 | 2.8750 | 0.6250 | -156.2653 | -156.7267 |
| .95 | Current Looking | 0 | 2.8750 | 0.6250 | -156.2654 | -156.7256 |
| .90 | Current Looking | 0 | 2.8750 | 0.6250 | -156.2669 | -156.7276 |
| .8788 | Current Looking | 0 | 2.8750 | 0.6250 | -156.2681 | -156.7277 |
| .85 | Current Looking | 0 | 2.8750 | 0.6250 | -156.2703 | -156.7298 |
| .80 | Backward Looking | 0 | 3 | 0.6875 | -156.2778 | -156.7319 |
| .75 | Backward Looking | 0 | 3 | 0.6875 | -156.2857 | -156.7398 |
| 0 | Backward Looking | 0 | 3 | 0.2500 | -156.6265 | -156.9858 |

| Table 10. Unconditional Moments under Optimal Rules (x$10^{-2}$) ranked by unconditional welfare | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\chi$ | $E(c)$ | $E(y)$ | $E(s)$ | $E(\pi)$ | $\sigma_c$ | $\sigma_y$ | $\sigma_s$ | $\sigma_\pi$ |
| 1 | 0.20 | -0.72 | 5.9189(x$10^{-6}$) | 4.54 | 2.0943 | 6.9594 | 1.97(x$10^{-15}$) | 3.2617 |
| .95 | 0.20 | -0.72 | 1.3191(x$10^{-5}$) | 4.54 | 2.0945 | 6.9605 | 9.11(x$10^{-4}$) | 3.2707 |
| .90 | 0.20 | -0.72 | 2.1045(x$10^{-5}$) | 4.53 | 2.0923 | 6.9533 | 2.5(x$10^{-3}$) | 3.2731 |
| .8788 | 0.20 | -0.72 | 2.5982(x$10^{-5}$) | 4.54 | 2.0916 | 6.9529 | 3.4(x$10^{-3}$) | 3.2755 |
| .85 | 0.20 | -0.73 | 3.4316(x$10^{-5}$) | 4.54 | 2.0908 | 6.9550 | 4.9(x$10^{-3}$) | 3.2802 |
| .80 | 0.19 | -0.74 | 5.2290(x$10^{-5}$) | 4.56 | 2.0747 | 6.9938 | 8.5(x$10^{-3}$) | 3.3466 |
| .75 | 0.18 | -0.76 | 7.8459(x$10^{-5}$) | 4.57 | 2.0745 | 7.0176 | 1.31(x$10^{-2}$) | 3.3608 |
| 0 | 0.13 | -0.61 | 2.1637(x$10^{-5}$) | 4.31 | 1.9782 | 5.9687 | 7.87(x$10^{-2}$) | 1.1346 |

Note: variables are expressed in deviation from steady state, except $\pi$ that is in levels and annualized.

| **Table 11. Calibration** | | |
|---|---|---|
| $\beta$ | $1.03^{-0.25}$ | Time discount rate |
| $\theta$ | 0.36 | Share of capital |
| $\psi$ | 0.5827 | Fixed cost (guarantee zero profits in steady state) |
| $\delta$ | 0.025 | Depreciation of capital |
| $\eta$ | 1 | Elasticity of substitution of different varieties of goods |
| $\tilde{\eta}$ | 6 | Elasticity of substitution of labour services |
| $\alpha$ | 21 | Probability of not setting a new price each period |
| $\tilde{\alpha}$ | 0.6 | Probability of not setting a new wage each period |
| $b$ | 0.64 | Degree of habit persistence |
| $\phi_0$ | 1.1196 | Preference parameter |
| $\phi_1$ | 0.5393 | Preference parameter |
| $\sigma_m$ | 10.62 | Intertemporal elasticity of money |
| $\kappa$ | 2.48 | Investment adjustment cost parameter |
| $\tilde{\chi}$ | 1 | Wage indexation |
| $\gamma_1$ | 0.0324 | Capital utilization cost function parameter |
| $\gamma_2$ | 0.000324 | Capital utilization cost function parameter |
| $z$ | 1 | Steady state value of technology shock |
| $\lambda_z$ | 0.979 | Serial correlation of technology shock (in log-levels) |
| $\eta_z$ | 0.0072 | Standard deviation of technology shock |
| $\lambda_g$ | 0.96 | Serial correlation of demand shock (in log-levels) |
| $\eta_g$ | 0.02 | Standard deviation of demand shock |
| $\sigma$ | 0.18 | Parameter scaling all exogenous shocks |

## 12 Figures

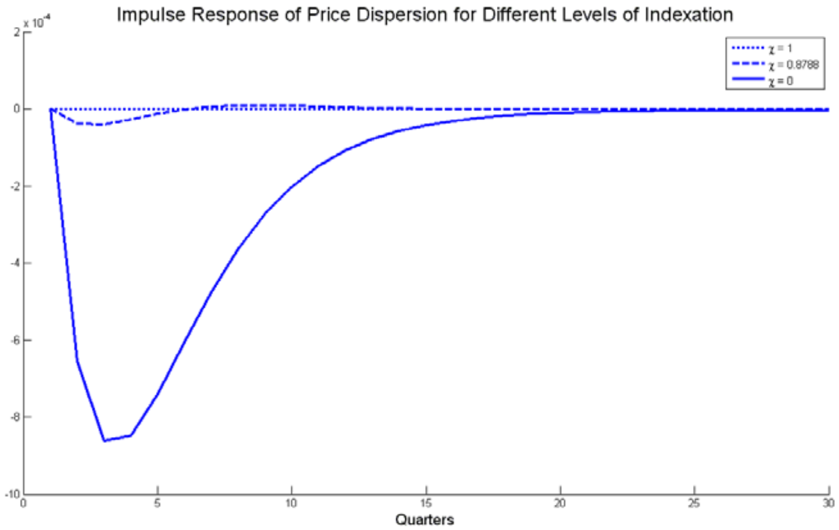


Figure 1. Impulse Response Functions of Price Dispersion after a 1% increase in the TFP for different levels of Indexation; $\alpha_\pi = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ in the Taylor Rule (1)

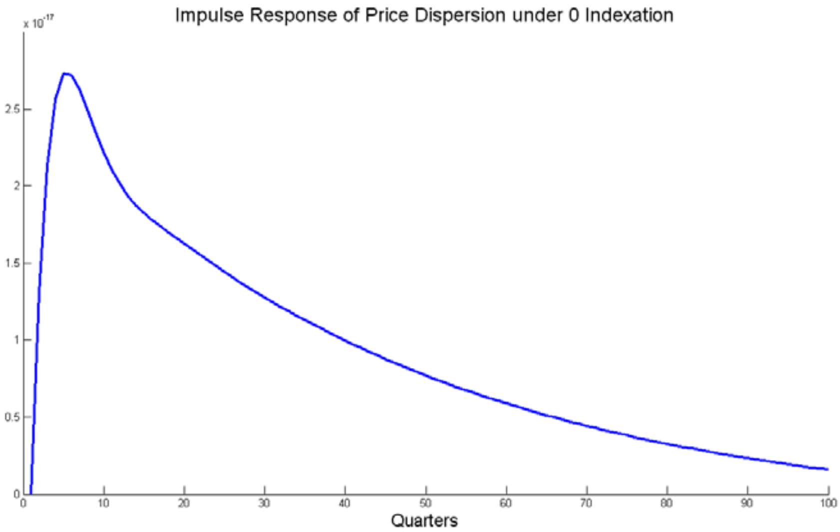


Figure 2. Impulse Response Functions of Price Dispersion after a 1% increase in the TFP in the case of full Indexation; $\alpha_\pi = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ in the Taylor Rule (1)

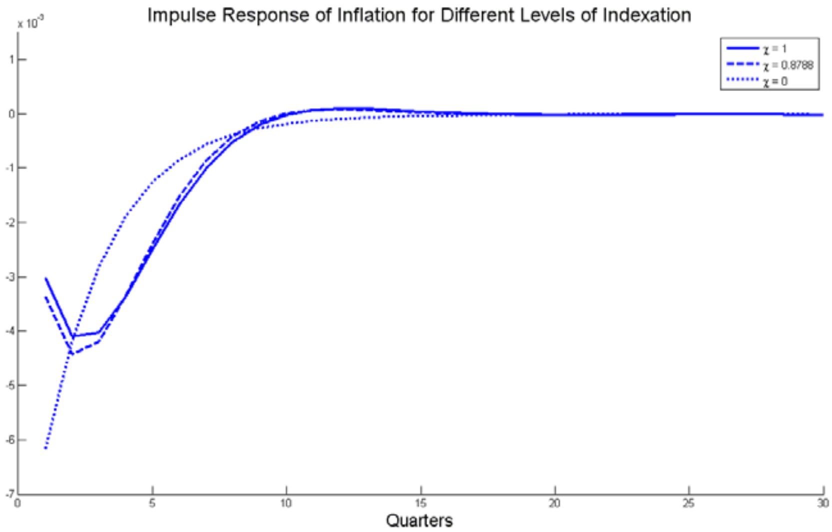


Figure 3. Impulse Response Functions of Inflation after a 1% increase in the TFP for different level of Indexation; $\alpha_{\pi} = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ in the Taylor Rule (1)

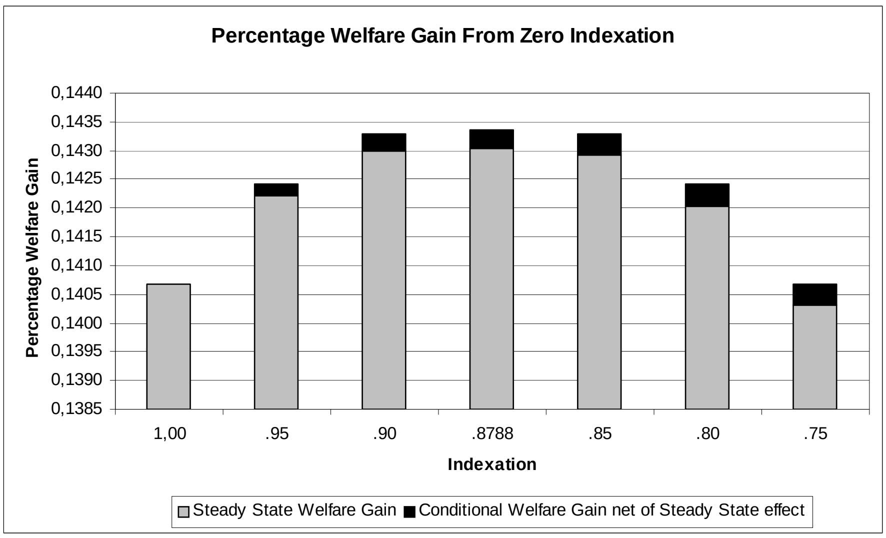


Figure 4. steady state and conditional percentage gain with respect to the 0 indexation case for the best policies ranked according to conditional welfare.

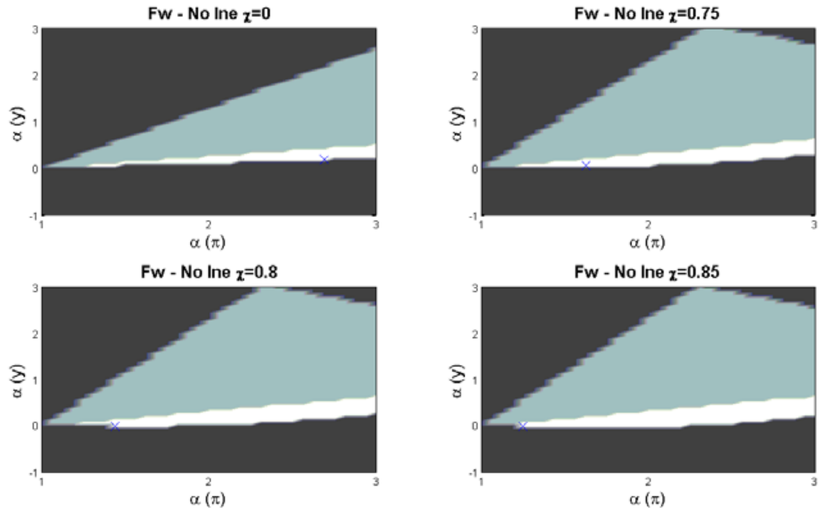


Figure 5. Indeterminacy regions

Note: Each panel shows three regions: the white one displays the values of $\alpha_y$ and $\alpha_\pi$ that deliver determinate rational expectation equilibria, the grey one signals that the equilibrium is not implementable in the sense described in footnote 10, and the black region represents indeterminate rational expectation equilibria. All the values of both $\alpha_y < -1$ and $\alpha_\pi < 1$ yield indeterminacy and are not shown in the figure.

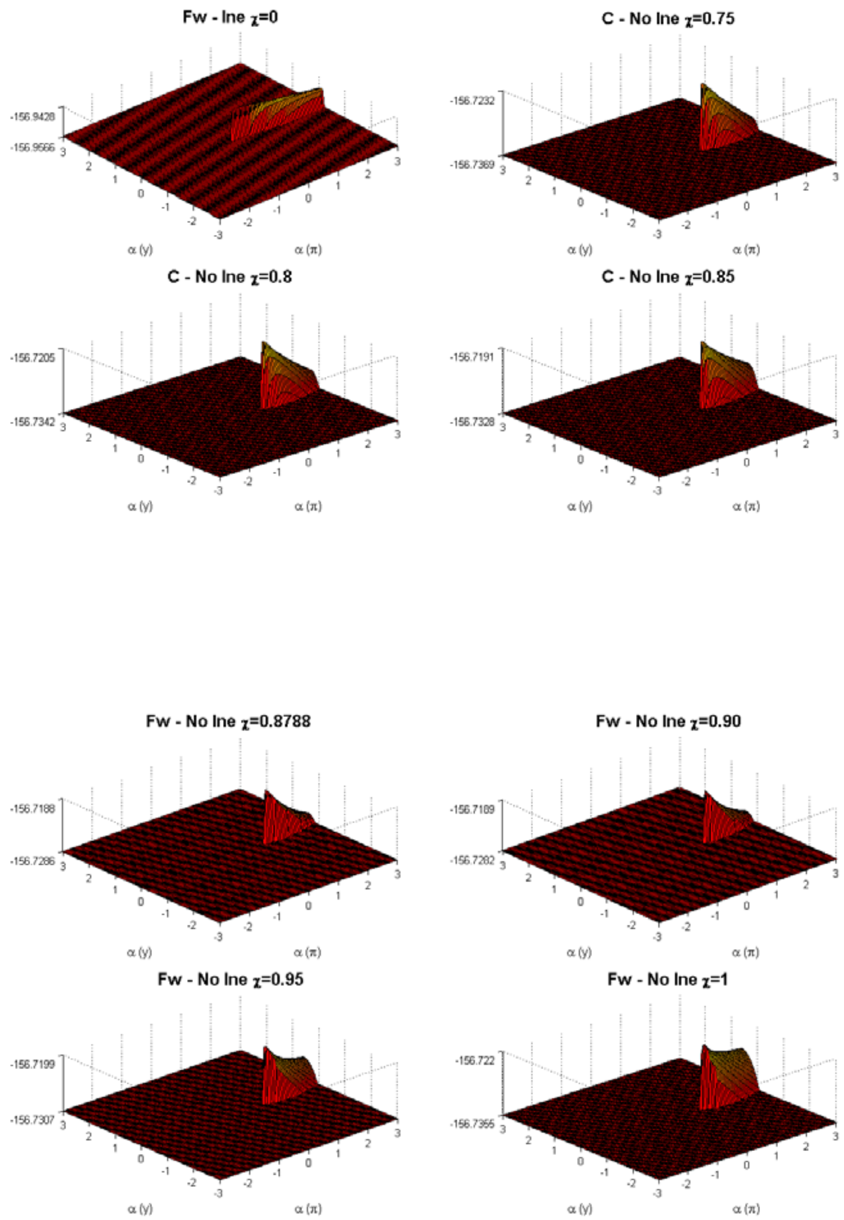


Figure 6. conditional welfare for those policies that imply less than 1% loss with respect to the best policy within the class of FLNI.

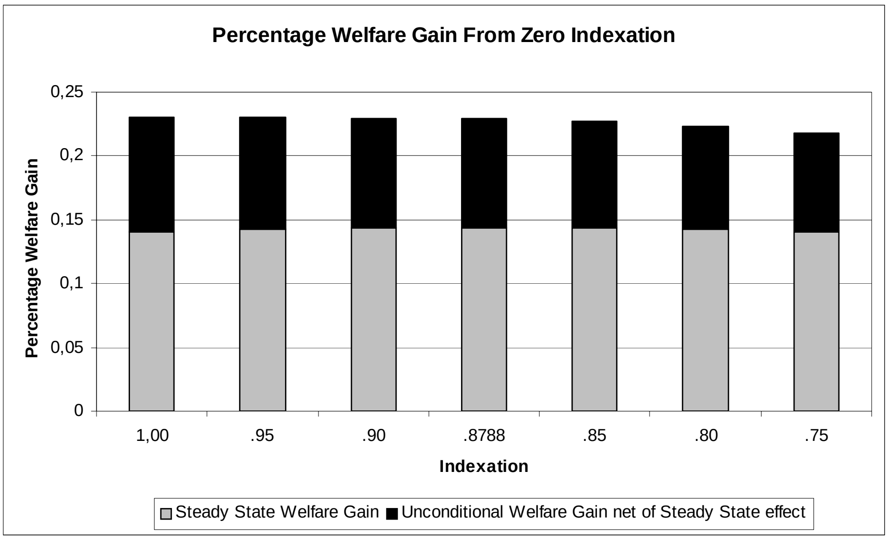


Figure 7. steady state and unconditional percentage gain with respect to the 0 indexation case for the best policies ranked according to unconditional welfare.