Rules versus Discretion in Fiscal Policy

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# 199 (07-07)
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1. Introduction

In 2008-2009 world economies faced the most serious financial and economic crisis since the Great Depression of the 1930s. As in any severe recession where aggregate demand is lower than output capacity and there is the need to restore consumer and business confidence, monetary policy tends to be ineffective, so that recovery depends on the implementation of active and extensive fiscal policy measures. But the use of the Government budget as a stabilization tool, if inherited deficits are large and the dynamics of public debts are not under full control, can be hampered by the structural fragility of public finances. The situation is particularly hard to manage in the Euro Area, where the Stability and Growth Pact imposes binding limits on Government deficits and on public debts dynamics.¹ Such constraints however tend to be violated today by the mere operation of automatic stabilizers. Moreover, when public finances deteriorate, risk aversion imposes a premium on holding Government bonds, so that long-term interest rates rise, and aggravate the burden of servicing public debt. Risk premia tend to be particularly high and rising for highly indebted countries, which are the very countries expected to experience adverse dynamics in public debt.² Hence policy-makers are facing a trade-off between output growth and fiscal stability: on one hand, they would like to implement measures to increase output and employment, but on the other hand, the deterioration of public finances forces them to stick to fixed rules.

The reasons for adopting binding rules in the conduct of fiscal policy (and thus putting debt dynamics under control) have been widely analyzed in the literature. Several strands of

¹ The Pact requires public deficits to be null on a structural basis in order to allow countercyclical fiscal policies in downturns and promote a progressive reduction of public debts towards the reference value of 60% of GDP. Specific departures from the general rules are permitted only in exceptional circumstances, such as a deep and prolonged recession.

² The recent experiences of Italy and Greece are particularly significant. In the first case the spread between Italian and German interest rates on 10-year Government bonds showed a remarkable fourfold increase in just one year, passing from 30 basis points in December 2007 to 60 points in July 2008 and finally to 140 points in December 2008. In the case of Greece, the spread showed a similar pattern between December 2007 and December 2008, but then made a spectacular jump of 360 basis points in just six months at the beginning of 2010 when the true state of Greek public finances was revealed and the resulting projections about debt growth were published.
thought may be identified. First of all, the theory of fiscal constitutionalism, attributable to the Public Choice School (Buchanan and Wagner, 1977), suggests that the Government budget should be balanced because of the distortions and costs implicit in deficit financing. The crowding out argument (Carlson and Spencer, 1975) suggests that the creation of a budget deficit ultimately leads to an increase in the interest rate that displaces productive private investment, while leaving output unchanged. The principle of Ricardian equivalence (Barro, 1974) arrives at the same final suggestion about the desirability of a balanced budget on the theoretical ground that Government deficits are wholly ineffective. The golden rule of public finance, on the other hand, admits the possibility of a public deficit, of an amount equal to the value of net public investment.\(^3\) The political economy approach outlines several reasons behind the observed emergence, in modern economies, of a “deficit bias”, the elimination of which requires the imposition of appropriate fiscal rules.\(^4\) Finally, and most relevantly for the goals of this paper, the vast literature on public debt sustainability, which can even be dated back to an old work by Domar (1944), and has returned to public attention with the seminal contribution by Sargent and Wallace (1976), suggests that Government budgets should be targeted to the goal of stabilizing the debt-GDP ratio, in order to avoid the harmful long run and short run consequences of the potential crises connected to an ever-increasing debt-GDP ratio.\(^5\)

The goal of debt stabilization, furthermore, is particularly important in the case of monetary unions, in order to prevent unfavourable debt dynamics in a single country having spillover effects on the whole union and threatening the price stability objective of monetary

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\(^3\) It has been suggested that the golden rule of public finance might be one of the rational arguments behind the Maastricht Treaty prescription of a 3% upper limit to the public deficit-GDP ratio, where this threshold figure corresponds to the average incidence of public investment on output at the time when the Treaty was signed (Butter, Corsetti and Roubini, 1993). For a recent discussion about the possibility and consequences of the application of the golden rule in EMU see Balassone and Franco (2000).


\(^5\) The original argument by Sargent and Wallace warned against the possibility that under fiscal dominance a given sequence of Government deficits would ultimately lead to debt monetization, which would inevitably generate inflation (that under rational expectations would occur immediately). The subsequent debate also emphasized the possibility of Government insolvency and thus the possibility that public debt might be ultimately consolidated or repudiated. Again, in a world of forward-looking agents, this possible long-run outcome would be anticipated and might give rise to short-run financial crises (for a recent reassessment of the debt sustainability issue, cf. Blanchard et al., 1990).
authorities. This is the fundamental rationale behind the Maastricht Treaty requirement on public
debt: in order to avoid financial crises and bailing-out, and safeguard the ECB mandate, every
member country of EMU should at least stabilize the debt-GDP ratio.\textsuperscript{6}

The previous discussion outlines the existence of several different arguments behind the
suggestion to impose rules on fiscal policy in order to control public deficits and debt growth.\textsuperscript{7}
To our knowledge, however, no attempt has been made until now to derive the prescription for
imposing constraints to fiscal policy from the “rules versus discretion” framework, so
successfully applied to the case of monetary policy (Kydland and Prescott, 1977; Barro and
Gordon, 1983).

As well known, according to this approach monetary policy should follow fixed rules,
rather than discretion, in order to avoid the inflationary bias generated by the monetary
authorities incentive to behave opportunistically by trying to stabilize output below the natural
level (first best) rather than commit themselves to maintaining a low and stable rate of inflation
(second best). In a world of rational expectations, this incentive is understood by optimizing
agents who fix their labour contracts anticipating the behaviour of monetary authorities and thus
the correct rate of future inflation. The result of this game between rational agents and monetary
authorities is the achievement of a suboptimal (third best) equilibrium where output is at its
natural level and the rate of inflation is higher than under commitment. Thus monetary policy is
dynamically inconsistent, and this provides a strong argument in favour of a rule constraining the
Central Bank to pursue a low and stable inflation rate. This conclusion is only partially mitigated
by the consideration of uncertainty: in the presence of supply shocks there is indeed a trade-off
between rigour and flexibility, i.e. between the benefits of a fixed rule in reducing the “inflation

\textsuperscript{6} Given the existing relationship between debt dynamics and deficit, this would justify the twin prescriptions in
terms of public debts and deficits (respectively 60% and 3% of GDP), under the hypothesis that inflation is around
2% and output growth 3% (cf. Buitert, Corsetti and Roubini, 1993). Finally it should be emphasized that limits to
fiscal policy have been recently put in place not only in EMU, but also in countries such as the US, the UK, New
Zealand, Chile and Brazil.

\textsuperscript{7} Also, from a practical point of view, specific criteria have been defined in order to identify the desirable features of
“good rules” (Kopits and Symansky, 1998).
bias” and the income losses due to the inability of monetary authorities to stabilize output under a more discretionary regime (Rogoff, 1985).

The purpose of this paper is to apply the Kydland-Prescott framework of dynamic inconsistency to the case of fiscal policy, by considering the trade-off between output and debt stabilization under the constraint given by the equation describing debt dynamics, and under the hypothesis of the existence of a risk premium on the yield of public bonds tied to the possibility of a default.

Within this framework our analysis will assess the effects on the economy of different cases. First we will start by comparing the loss outcomes of a discretionary equilibrium with those deriving from a commitment to debt stabilization and a commitment to an optimal fiscal rule as derived from the model. This analysis will be conducted in a static framework with full certainty. The basic model will then be extended to consider the consequences of random shocks and the implications of a dynamic framework.

It is important to note that our decision to compare discretion with different kinds of binding rules is related to the issue of the “implementability” of a commitment, by which we mean the actual ability to establish and maintain a specific rule within a given institutional framework. In this perspective, three elements are crucial. First, the rule must be sufficiently simple to be clearly stated in a legislative, institutional or statutory framework. Second, the rule must provide indications for clear responsibility and punishment in cases of violation. Finally, the rule must allow for compliance to be checked easily. These three arguments together suggest the existence of a trade-off between simple and complex rules, where complex rules are more efficient and simple rules easier to implement. As emphasised above, this conclusion is relevant to this work with regard to the choice of analyzing different rules in the basic model. It is also important for the discussion about state-contingent rules conducted at the end of the model including random shocks.

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8 We may note that, in the recent experience of EMU, the non-application of the established statutory punishment is at the root of frequent violations of the Stability Pact.
Finally, we wish to remark that, to our knowledge, this work is the first attempt to justify the introduction of a fiscal rule using a time inconsistency approach. Actually a different problem of fiscal time inconsistency has been studied in a recent work by Dixit and Lambertini (2003), where a monetary and a fiscal authority have different target levels of the same two objectives (output and inflation). The authors show that commitment by both authorities is optimal, so that discretion in fiscal policy always generates inferior outcomes. Our work differs from the Dixit-Lambertini contribution in many aspects: the problem studied (we compare the relative desirability of discretion versus commitment to stabilizing or controlling debt growth), the framework (the fiscal authority objectives and the structural equations in the two papers are different) and the conclusions (in our model discretion is preferable to commitment in some cases). We also think that focusing on the fiscal time inconsistency problem alone is more appropriate to analyze the dilemma facing policy-makers in the current situation, especially in Europe.

The rest of the paper proceeds as follows. Section Two outlines the main assumptions of the model and examines the elements characterizing Government behaviour. Section Three compares the results obtained in the case of a discretionary policy against a commitment to adopt either a debt-stabilizing rule or an optimal fiscal rule. Section Four studies the implications of random shocks. Section Five presents a preliminary analysis of the implications of a dynamic framework for Government choices. Section Six concludes.

2. The model

In this section we examine the basic features of the economy and the elements describing Government behaviour. We start by considering the equation defining public debt dynamics,

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9 The authors also show that the same result can be achieved through a combination of fiscal policy commitment and conservative central banking.
given by the usual Government budget constraint, under the hypothesis that no monetization is possible. Thus:

$$B_t - B_{t-1} = D_t + r_t B_{t-1}$$

(1)

where $B_t$ is the stock of outstanding public debt at time $t$, $D_t$ is the primary deficit (i.e. the deficit net of interest payments) and $r_t$ is real interest rate. For simplicity we assume public bonds to have a one-period maturity, so that in each period the debt stock is completely refinanced. Relating both sides of eq. (1) to the level of output $Y_t$, we get the well-known equation describing the dynamics of the debt-output ratio, given by:

$$\Delta b_t = b_t - b_{t-1} = d_t + (r_t - g_t)b_{t-1}$$

(2)

where lower case letters indicate the derived ratios, so that $b_t = B_t / Y_t$ and $d_t = D_t / Y_t$, while $g_t$ is output growth.

We assume, as usual in the literature, that there is an upper limit to the stock of public debt that private agents wish to hold, so that, if this limit is reached, the Government may default. Hence, under this hypothesis, the real interest rate paid on public bonds will incorporate a risk premium and the bonds yield will be given by:

$$r_t = \begin{cases} 
\bar{r}_t & \text{when } \Delta b_t^c \leq 0 \\
\bar{r}_t + \gamma(\Delta b_t^c) & \text{with } \gamma(\Delta b_t^c) > 0 \text{ when } \Delta b_t^c > 0 
\end{cases}$$

(3)

where $\bar{r}_t$ is the rate of interest that would prevail in the absence of a default risk and the function $\gamma(\Delta b_t^c)$ indicates the risk premium attributable to expected debt growth $\Delta b_t^c$. If agents expect public debt to be constant or decreasing over time (i.e. if $\Delta b_t^c \leq 0$) then they will exclude the

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10 It is worth noticing that all variables are expressed in real terms; in our framework we do not deal with the determination of the absolute price level which is not relevant for our purposes.

11 Eq. (2) is obtained under the usual approximation that $r_t - g_t = (r_t - g_t) / (1 + g_t)$. 

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chance of a default and $\gamma(\Delta b_t^e) = 0$. If agents instead expect public debt to increase (i.e. if $\Delta b_t^e > 0$) then there will be a default risk and $\gamma(\Delta b_t^e) > 0$. It might be plausible to assume that $\gamma$ is also an increasing function of $\Delta b_t^e$,\footnote{Indeed the recent experience of Greece, cited in Footnote 2, seems to confirm this conjecture.} this further assumption, however, is not relevant for our results, so that, for simplicity, one could assume that the function $\gamma(\Delta b_t^e)$ is linear: in this case we would simply have $\gamma(\Delta b_t^e) = \gamma \Delta b_t^e$ with $\gamma > 0$ for $\Delta b_t^e > 0$ and $\gamma = 0$ for $\Delta b_t^e \leq 0$.

Finally, agents are assumed to pursue an optimal allocation of wealth among different existing financial assets, which implies, ceteris paribus, a lower demand for public bonds when they turn out to be a risky asset.\footnote{It might be argued that the risk premium on public bonds depends not only upon the expected debt growth but also upon the debt size. This latter effect would be justified by the fact that, given debt dynamics, the probability of default is higher when the public debt is larger. If this additional influence were considered, equation (3) would become $t_t = \bar{T} + \gamma(\Delta b_t^e, b_{t-1})$; the new formulation, however, would not change the basic results of the paper, as it can be easily checked from the analysis presented in the next section.}

We assume that the primary deficit $D_t$ is given by

$$D_t = \bar{G} - \bar{T} - t_t Y_t$$

where $\bar{G}$ and $\bar{T}$ are constants and $t_t$ is Government tool used to influence the deficit level.\footnote{Since in our model the instrument used by the fiscal authority is the tax rate, we may assume that $\bar{G}$ is the uncompressible level of public expenditure connected with the institutional functions of the Government; $\bar{T}$, instead, indicates the level of lump-sum taxes.} The existence of a proportional income tax implies fiscal distortions, with a larger tax rate $t_t$ triggering a lower output level. Hence $Y_t$ is decreasing in the tax rate $t_t$, so that:

$$Y_t = f(t_t) \quad \text{with} \quad f' < 0$$

According to eqs. (4) and (5), the deficit-output ratio may be rewritten as

$$d_t = (\bar{G} - \bar{T} - t_t f(t_t))/f(t_t)$$

and the output growth rate as $g_t = \left[ f(t_t) - Y_{t-1} \right]/Y_{t-1}$. By substituting these results, and eq. (3), in eq. (2) we then get:
\[ \Delta b_t = h(t_t) + \gamma (\Delta b_t^C) b_{t-1} \]  

where \( h(t_t) = \left[ G - \bar{T} - t_t f(t_t) \right]/f(t_t) + b_{t-1} \{ \bar{r} - [f(t_t) - Y_{t-1}]/Y_{t-1} \}. \)

Let us assume now that the Government pursues two different goals: output and debt stabilization\(^{15}\). The introduction of the latter objective in the Government loss function may appear unusual; however the assumption can be easily justified if we suppose, as in the debt sustainability literature, that an economy with a growing public debt exhibits a positive risk of default. Since the chance of a default generates disutility to the society as a whole, then debt stabilization may be considered, as it actually is, a fundamental goal for the Government.

Let us define, as usual, \( Y^* \) and \( \Delta b^* \) as the desired levels of output and debt dynamics. Because of distortionary taxation, \( Y^* \) is higher than \( Y_t \),\(^{16}\) and \( Y_t = Y^* \) only if \( t_t = 0 \).\(^{17}\) In line with previous arguments, we finally assume that \( \Delta b^* = 0 \), so that the Government wants to stabilize the debt-output ratio as a sufficient condition for debt sustainability. Under these assumptions the Government loss function can be written as:

\[ L = a [\Delta b_t]^2 + [Y_t - Y^*]^2 \]  

Substituting eqs. (5) and (6) in (7), we can then express \( L \) as a function of \( t_t \) in the following way:

\[ L(t_t) = a \left[h(t_t) + \gamma (\Delta b_t^C) b_{t-1} \right]^2 + \left[f(t_t) - Y^* \right]^2 \]  

\(^{15}\) Actually one might also assume that, especially in the case of countries with a high debt-output ratio, the second goal might be represented by debt reduction; this hypothesis, however, would not alter the conclusions reached under the assumption that the Government pursues debt stabilization.

\(^{16}\) As well known, a non lump-sum taxation can distort agents choices (for instance, the choices on labour supply), pushing output below the socially desirable level (for a textbook analysis of the distortionary effects of taxation, cf. Atkinson and Stiglitz, 1980). The idea that distortionary taxation generates an undesirably low level of output is standard in time inconsistency literature (cf., for instance, Blanchard and Fischer, 1989, Ch. 11).

\(^{17}\) It might be noticed that the level of \( t_t \) associated with \( Y^* \) could even be negative if one assumed monopolistic competition among firms, implying that production should be subsidized in order to reach the desired output level (cf. Dixit and Lambertini, 2003). In the case of subsidies, in principle, it might be possible to push the level of \( Y_t \) beyond \( Y^* \). There is however no reason for doing this, since a level of \( Y_t \) higher than \( Y^* \) would imply an undesired excess of output associated with a larger debt growth. This would determine a loss greater than when \( Y_t = Y^* \), so that that choice would be suboptimal.
The properties of eq. (8) can be derived from those of function \( h(t) \), whose form depends upon that of function \( f(t) \) in eq. (5). In general, one cannot say whether \( h(t) \) is an increasing, decreasing or even non-monotone function of \( t \) since this will depend upon the effects of a reduction in the tax rate \( t \) on \( b \) and \( Y \) respectively.

From now on, however, we shall make an explicit hypothesis about the monotonicity of \( h(t) \) by assuming that \( h'<0 \) (thus implying that a reduction in the tax rate will determine an increase in the debt-output ratio). This assumption may be justified on many different grounds. First of all, if it were \( h'>0 \), then a decrease in the tax rate would cause a reduction in the debt-output ratio; this case does not seem interesting or significant, since it would imply that the Government, by reducing taxation, could at the same time raise growth and reduce the debt-output ratio. Secondly, the assumption that \( h'<0 \) seems to be confirmed both by the actual experience of a few industrial countries and by the results of some recent empirical studies concerning the effects of Government taxation on output.\(^{18}\)

If \( h'<0 \) is assumed, then minimization of the loss function \( L \) with respect to \( t \) implies:\(^{19}\)

\[
ah'(t)\left[h(t)+\gamma(\Delta b^*)b_{t-1}\right] = -f'(t)\left[f(t)-Y^*\right]
\]

Let us label \( t^{opt} \) as the optimal level of \( t \) satisfying eq. (9).\(^{20}\)

Let us define now \( t^{st} \) as the level of \( t \) that ensures public debt stabilization, i.e. \( \Delta b=0 \).

This level is given by the condition:

\(^{18}\) The function \( h(t) \) would be increasing only if a reduction in \( t \) generated an increase in output growth \( g \), so large as to counteract the increase in the primary deficit. From an empirical standpoint, however, a series of recent papers purporting to estimate the effects on output of tax reductions have come to the conclusion that these effects are small and that they also tend to reduce in size if the most recent years are considered (such as the period after 1980) and if the dynamics of the debt-output ratio is explicitly introduced into the analysis (cf. Blanchard and Perotti, 2002, Perotti, 2004 and Favero and Giavazzi, 2007).

\(^{19}\) We assume that functions \( h(.) \) and \( f(.) \) are such that the necessary second-order conditions for minimization are satisfied.

\(^{20}\) It might be noticed that, since we assume that taxation has a distortionary effect, it is possible, mainly in the case of a very high value of \( \Gamma \), that the optimal level of taxation is very high, thus determining a large distortion on output. The effect of this distortion might be particularly relevant in our model if it generated some kind of “Laffer effect” (i.e. a reduction in fiscal revenues following an increase in the tax rate). Hence we assume that, in general, the distortion is increasing in \( t \) but is not too high, implying that the Laffer effect does not occur. From an empirical standpoint, the possible occurrence of Laffer effects connected to the adoption of tax cuts is clearly disproved by the actual experience of the United States, under the Reagan and Bush administrations, and of Japan, both in the 1990s and in the most recent years of the new decade. In these cases tax reductions led to a substantial worsening of the Government deficit and a strong upsurge in the debt-output ratio.
\[ h(t^*) = -\gamma \left( \Delta b_t^c \right) b_{t-1} \]  \hspace{1cm} (10)

It is easy to see that if \( t_i = t^* \) then the left-hand side of eq. (9) is zero; hence condition (9) will be satisfied only if its right-hand side is also zero, i.e. if \( Y^* = f \left( t^* \right) \). In this case, then, the optimal level of the tax rate is also the one that ensures debt stabilization (\( t^{opt} = t^* = 0 \)).

Let us assume now that \( Y^* > f \left( t^* \right) \), i.e. that \( t^* > 0 \). This assumption introduces a trade-off between the goal of reaching the desired output level and that of stabilizing public debt. Indeed, when \( Y^* > f \left( t^* \right) \) the right-hand side of eq. (9) is negative. This implies that \( h(t^{opt}) > -\gamma \left( \Delta b_t^c \right) b_{t-1} \), implying, in turn, by eq. (10), that \( h(t^{opt}) > h(t^*) \). Since \( h(t_i) \) is a decreasing function, this entails that \( t^* > t^{opt} \), meaning that the Government does not choose the tax rate stabilizing debt growth because of its desire to push output closer to its target level. This behaviour, which stems from the assumed trade-off between the two conflicting objectives of output and debt stabilization, will however generate a case of dynamic inconsistency, as described in the next section.

It should be added that not only \( t^{opt} < t^* \), as previously shown, but also \( t^{opt} > 0 \). Indeed for \( t=0 \) the right-hand side of eq. (9) is null while the left-hand side is positive, because \( t^* > t^{opt} \geq 0 \). Since \( h(t) \) and \( f(t) \) are decreasing functions, it follows that \( t^{opt} > 0 \).\(^{21}\)

3. Rules versus discretion

Within the framework illustrated in the previous section, we now compare the outcomes of different cases. As outlined in the introduction, public debt stabilization is claimed, both theoretically and empirically, to be a good rule for ensuring fiscal sustainability; indeed it is a

\(^{21}\) Therefore the previous conclusions that \( t^* > t^{opt} \geq 0 \) and \( t^{opt} > 0 \), put together, imply that the optimal tax rate is at an intermediate level between the one stabilizing debt growth and the one enabling the economy to reach its target output.
very simple binding rule, which can be easily introduced into a legislative framework (for instance into a constitutional law) and where compliance can be easily verified ex-post. For these reasons we start with an analysis of the desirability of rules versus discretion by comparing the outcomes of a fully discretional fiscal choice with the outcomes of a rule compelling the Government to stabilize public debt. These outcomes will then be compared with those of implementing an optimal fiscal policy rule.

In order to assess the relative desirability of discretion versus a commitment to debt stabilization, we assume the following sequence of events in the one-shot game between the Government and rational agents. First the Government announces its planned fiscal policy; then public bonds are issued and the interest rate is determined on the basis of agents expectations; finally the Government chooses its actual fiscal policy.

In the case of discretion we assume that the Government declares that it will pursue public debt stabilization and hence choose the subsequent level of the tax rate given its preferences described by the loss function $L(t)$. As in the usual time inconsistency literature, we shall consider both the case in which the Government announcement is trusted (corresponding to the so-called case of “fooling”) and the case where it is not. We shall thus compare three situations:

1) Discretion and fooling (i.e. $t_t = t^{opt} = t^f$ and $\Delta b^e_t = 0$)

2) Discretion and rational expectations on public debt dynamics (i.e. $t_t = t^{opt} = t^d$ and $\Delta b^e_t = \Delta b_t = \Delta b^d_t$)

3) Commitment to stabilization (i.e. $t_t = t^{st} = t^{cs}$ and $\Delta b^e_t = \Delta b_t = 0$).

In the first case the Government announcement of debt stabilization is assumed to be believed. The Government then will choose the optimal level of $t_t$ under the condition $\Delta b^e_t = 0$. From eq. (9) it is easy to see that the optimal choice of $t_t$ (labelled in this case $t^f$) is given by
\[ ah'(t^f)h(t^f) = -f'(t^f)[f(t^f) - Y^*] \]  

(11)

As previously explained, since \( Y^* > f(t^d) \), we have that \( t^d > t^f \). This means that agents' expectations about debt stabilization (\( \Delta b^e_t = 0 \)) are wrong, since the Government choice implies that \( \Delta b_t > 0 \). If agents are supposed to have rational expectations, then, \( t^f \) cannot be an equilibrium level of \( t_t \).

If agents have rational expectations, however, they will not trust the Government announcement and will instead anticipate its actual choice and behaviour. In this case we shall have \( \Delta b^e_t = \Delta b_t \),\(^{22}\) implying that the optimal level of \( t_t \) (labelled in this case \( t^d \)) is given by the condition:

\[ ah'(t^d)[h(t^d) + \gamma(\Delta b_t)b_{t-1}] = -f'(t^d)[f(t^d) - Y^*] \]  

(12)

Comparing the loss suffered by the Government when \( t_t \) is equal to \( t^f \) or \( t^d \), it is easy to see that \( L(t^d) > L(t^f) \). Indeed, since \( \gamma(\Delta b_t)b_{t-1} > 0 \) and since \( t^f \) is the optimal choice in the case where \( \Delta b^e_t = 0 \), we have that:

\[
L(t^d) = a\left[ h(t^d) + \gamma(\Delta b_t)b_{t-1}\right]^2 + \left[ f(t^d) - Y^*\right]^2 > \\
a\left[ h(t^d)\right]^2 + \left[ f(t^d) - Y^*\right]^2 > \\
a\left[ h(t^f)\right]^2 + \left[ f(t^f) - Y^*\right]^2 = L(t^f)
\]  

(13)

The meaning of this result is straightforward. In equilibrium, agents will anticipate the growth of public debt and will ask for a risk premium that will increase the interest rate paid on

\(^{22}\) In the Kydland-Prescott framework, private agents optimal strategy is to do their best to forecast inflation in order to avoid the income losses due to an underestimation of the future change in prices that will influence their real wage in the process of wage bargaining. In our model private agents optimal strategy is to do their best to forecast the future dynamics of public debt in order to avoid the capital loss due to an underestimation of the risk premium embodied in the interest rate that will influence their real return on bonds in the process of deciding the optimal composition of their portfolios.
public bonds. This additional interest payment will raise the Government loss with respect to the case of fooling.

We can now study the case in which the Government has a binding commitment to stabilize the debt-output ratio. In this case the Government will follow the rule \( t_i = t_i^d \) (labelled in this case \( t_{cs} \)), implying \( \Delta b_t = 0 \) and, under rational expectations, \( \Delta b_t^e = 0 \). It is easy to see that the value of the loss function in this case will be:

\[
L(t_{cs}) = \left[ f(t_{cs}) - Y^* \right]^2
\]  

(14)

We can then compare the value of loss function in the case of discretion \( L(t^d) \), corresponding to the first row of inequality (13), with that in eq. (14) in order to see whether commitment to stabilization may reduce the Government loss. The difference between \( L(t^d) \) and \( L(t_{cs}) \) is given by:

\[
L(t^d) - L(t_{cs}) = a \left[ h(t^d) + \gamma(\Delta b_t) b_{t-1} \right]^2 + \left[ f(t^d) - Y^* \right]^2 - \left[ f(t_{cs}) - Y^* \right]^2
\]  

(15)

It is clear that, since \( \Delta b_t > 0 \) when \( t_i = t^d \), then the first term in the right-hand side of eq. (15) is positive while, since \( Y^* > f(t^d) > f(t_{cs}) \), the second term is negative. This result implies that, in general, we cannot say whether commitment to stabilize public debt is better or worse than discretion. This occurs because the two choices involve differentiated advantages: discretion entails larger interest payments due to the risk premium but a level of output closer to the desired value, while commitment implies lower interest payments, due to the absence of the risk premium, but also a lower level of output.

Although in general commitment to stabilize public debt can either reduce or increase the loss function, a clear conclusion as to its superiority can be drawn on the basis of the value of the inherited debt-output ratio \( b_{t-1} \). In fact, eq. (15) implies the following:
Proposition 1. \[ \exists \hat{b} : \text{if } b_{t-1} \leq \hat{b} \text{ then } L(t^d) \geq L(t^{cs}), \text{ and if } b_{t-1} \geq \hat{b} \text{ then } L(t^d) \leq L(t^{cs}) \] (see Appendix A for the proof).

This result indicates that there is a threshold level of the debt-output ratio $\hat{b}$ which, when exceeded, makes commitment to stabilization more desirable. We can therefore conclude that if the debt-output ratio in the economy is small, then discretion is preferable, while commitment to stabilization is a better alternative for highly indebted economies.²³

The interpretation of this result is straightforward. The size of the inherited debt-output ratio ($b_{t-1}$) influences the value of the additional interest payments incurred in the case of discretion. If $b_{t-1}$ is small, then the increase in interests payments will also be small and the advantage related to a larger level of output under discretion will prevail. On the contrary, if $b_{t-1}$ is high enough, then the increase in interest payments will also be large and the disadvantage connected to the risk premium paid under discretion will overcome the advantage deriving from a higher level of output.

An inspection of eq. (15) shows that the threshold value of $\hat{b}$ depends upon many features of the economy and upon Government preferences. Two elements seem however to be more important. This leads us to the following result:

Proposition 2. Ceteris paribus the value of $\hat{b}$ is decreasing in $\gamma(\Delta b_1)$ and in $a$.

A larger risk premium connected to a non-stabilized debt entails a larger interest cost for each unit of outstanding debt; this implies that a lower threshold debt is sufficient to make discretion disadvantageous. On the other hand, a Government more interested in debt stabilization (i.e. having a higher value for the constant $a$) makes the relative cost of commitment

²³ So, according to our model, the 2005 reform of the Stability and Growth Pact in Europe goes in the right direction in that it establishes more stringent rules for highly indebted countries. The architecture of the Pact, however, is still somewhat lacking in that it puts a greater emphasis on the size of deficits than on the dynamics of public debts. For a description and a critical assessment of the recent reform of the Stability and Growth Pact, the reader might usefully consult ECB (2005).
lower; this implies that discretion is disadvantageous only if the total cost due to the risk premium is also lower, i.e. if, ceteris paribus, \( \hat{b} \) is reduced.

The conclusions just reached about the relative desirability of discretion and commitment can be illustrated with the help of a graphical analysis (Figures 1 and 2). The BY schedules represent the trade-off between public debt stabilization and output implied by eqs. (5) and (6).\(^{24}\) The two schedules drawn refer respectively to the case where the expected debt growth is null (\( \Delta b_t^e = 0 : BY_0 \)) and to the case where the expected debt growth corresponds to the discretionary outcome (\( \Delta b_t^e = \Delta b_t^d : BY_1 \)); thus the distance between any two curves is given by the additional interest payments paid on Government bonds, due to a varying risk premium.\(^{25}\) Along each BY schedule agents expectations about debt dynamics are given and may be wrong; the EL schedule, instead, represents the locus of all possible equilibria under rational expectations, i.e. the locus of all combinations between \( \Delta b \) and \( Y \) for which \( \Delta b_t^e = \Delta b_t^e \).\(^{26}\) As a consequence of the assumptions made, the EL schedule will, ceteris paribus, be increasing in the size of the risk premium and the value of Government debt; its slope will be steeper than that of the BY schedules, since, under the rational expectations hypothesis, it will depend on \( \gamma(\Delta b_t) \) and the inherited debt-output ratio.\(^{27}\) In the two figures the L curves represent the Government

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\(^{24}\) Thus the BY schedules are defined by the condition \( \Delta b_t = h\left[ f^{-1}(Y_t) \right] + \gamma(\Delta b_t^e) b_{t-1} \). In the graphs, for the sake of simplicity, the schedules are assumed to be linear, as if they were of the form \( \Delta b_t = \alpha + \beta Y_t + \beta_1 \Delta b_{t-1} \).

\(^{25}\) Actually there are infinite BY schedules, one for every value of agents expectations about public debt dynamics, and the distance between any two BY schedules is given by \( \gamma(\Delta b_t^e) b_{t-1} \). In our framework, therefore, the BY schedules are the equivalent of the expectations-augmented Phillips curves used in the traditional time-inconsistency approach to monetary policy.

\(^{26}\) Thus the EL schedule is given by the condition \( \Delta b_t = h\left[ f^{-1}(Y_t) \right] + \gamma(\Delta b_t) b_{t-1} \), which, in the linear case, would take the form \( \Delta b_t = \frac{\alpha}{1-\gamma b_{t-1}} + \frac{\beta}{1-\gamma b_{t-1}} Y_t \). In our framework, therefore, the EL schedule is the equivalent of the long-run vertical Phillips curve.

\(^{27}\) In the general case the slope of the EL schedule would be given by \( \frac{h f^{-1} + \gamma b_{t-1}}{1-\gamma b_{t-1}} \), which, in the linear case, would simplify to the expression \( \frac{\beta}{1-\gamma b_{t-1}} \). Assuming \( \gamma b_{t-1} \) (or \( \gamma b_{t-1} \) in the linear case) to be less than one, the slope of the
indifference (or iso-loss) curves: along each curve the Government loss is unchanged, while a curve closer to the axes origin involves a lower loss. Finally the GRF schedule represents the Government reaction function, being the locus of all tangency points between the iso-loss curves and the BY schedules, so that all socially desirable equilibria must lie on it. The GRF schedule also stems from the origin, since that particular combination of goals may be considered the Government bliss point, according to its loss function (7).

In the case of discretion the Government declares that it will stabilize public debt and choose the level of \( t_t \) (and thus the levels of \( \Delta b_t \) and \( Y_t \)) so as to minimize its loss. If the Government is trusted, and agents may therefore be fooled, the equilibrium that will be reached is represented by point F in Figure 1 and Figure 2. In point F, however, the deficit is too large for debt stabilization and public debt grows; this implies that agents expectations \( (\Delta b_t^c = 0) \) are wrong. Since, however, rational agents will anticipate that the Government will renege on its announcement, the statement will not be trusted and agents will expect public debt to grow \( (\Delta b_t^c = \Delta b_t = \Delta b_t^d) \). This implies that equilibrium under discretion will be in point D instead of point F. The loss suffered in point D is always larger than that in point F, as shown in both Figures 1 and 2, since point F represents the Government first-best outcome.

In the case of a commitment to stabilization the Government is compelled to stabilize public debt by choosing \( t_t = t^{cs} \). Because of the commitment, the Government is trusted and the equilibrium will be in point CS along the BY_0 line. The comparison between points CS and D confirms that in general we cannot say whether commitment to stabilization is better or worse than discretion. However, as illustrated in the previous discussion and as stated in Proposition 1, the comparison between the loss suffered in points CS and D will, ceteris paribus, depend upon the size of \( b_{t-1} \). If \( b_{t-1} \) is large enough, then the slope of the EL schedule will be steeper and the distance between the two BY schedules will be wider, so that the loss in point D will be higher.

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EL schedule is necessarily steeper than that of the BY schedules, and will be the steeper the higher the value of \( \gamma' \) (or \( \gamma \) in the linear case) and \( b_{t-1} \).
than in point CS, making commitment to stabilization preferable (Figure 1). The opposite situation will occur, ceteris paribus, when $b_{t-1}$ is small, so that the slope of the EL schedule will be flatter and the distance between the two BY schedules narrower: in that case the loss in point D will be smaller than in point CS, making discretion preferable (Figure 2).  

**FIGURE 1 ABOUT HERE**

**FIGURE 2 ABOUT HERE**

The analysis developed so far compared the Government loss associated with discretion with that sustained in case of a commitment to stabilize public debt. Obviously both cases are rational expectations equilibria since they are obtained under the condition that $\Delta b_t^e = \Delta b_t$ (i.e., they are both on the EL locus). These two cases, however, are not the only possible rational expectations equilibria; all couples of values $(Y_t, \Delta b_t)$ for which $\Delta b_t^e = \Delta b_t$ (i.e. all points on the EL locus) are rational expectations equilibria too.

This fact has significant implications for Government choices since any of these equilibria can be reached by an appropriate choice of $t_t$. Furthermore, since each rational expectations equilibrium will be associated with a different level of the Government loss, it follows that the Government can choose the best fiscal strategy simply by minimizing its loss function $L$ under the constraint that agents expectations are fulfilled.

This rational expectations constraint can be derived by substituting the condition $\Delta b_t^e = \Delta b_t$ in eq. (6), thus obtaining:

$$\Delta b_t - \gamma(\Delta b_t)b_{t-1} = h(t_t)$$

which in turn implies that:

\[ \Delta b_t^e = \Delta b_t \]

\[ (i.e., they are both on the EL locus). \]

\[ These two cases, however, are not the only possible rational expectations equilibria; all couples of values $(Y_t, \Delta b_t)$ for which $\Delta b_t^e = \Delta b_t$ (i.e. all points on the EL locus) are rational expectations equilibria too. \]

28 It is worth noticing that, as in the traditional time inconsistency framework applied to monetary policy, the discretionary outcome may be derived, in a game-theory approach, as the intersection between the private agents reaction function, given by the EL schedule, and the Government reaction function, given by the GRF schedule.
\[ \Delta b_t = \varphi(t_t, b_{t-1}) \]  

(17)

with \( \frac{\partial \varphi}{\partial t_t} < 0 \) and \( \frac{\partial \varphi}{\partial b_{t-1}} > 0 \). Substituting eq. (17) in eq. (7) we get then:

\[
L = \left[ a\varphi(t_t, b_{t-1}) \right]^2 + \left[ f(t_t) - Y^* \right]^2
\]

(18)

In order to minimize this loss function we differentiate eq. (18) with respect to \( t_t \), getting the following first-order condition:

\[
a \frac{\partial \varphi}{\partial t_t} \varphi(t_t, b_{t-1}) = -f'(t_t) \left[ f(t_t) - Y^* \right]
\]

(19)

The solution of this equation is the level of \( t_t \) associated with the rational expectations equilibrium which minimizes the Government loss.\(^ {29} \)

It is clear that the Government can reach this equilibrium only by means of a commitment. Indeed, under discretion, if the Government thinks that it can announce a tax rate satisfying eq. (19) and that it can be trusted, then it will have an opportunistic incentive to deviate from its announcement reducing taxation in order to increase \( Y \). This incentive will again generate, as in the case of a debt stabilization announcement, a time-inconsistency problem. The Government behaviour would imply a fooling equilibrium that cannot be sustained under rational expectations.

Furthermore, as shown above, if the Government commits to follow the rule described by eq. (19), it will choose, by construction, the optimal level of taxation under rational expectations. This implies that \( t_t \) represents the optimal fiscal rule for commitment; we can then label this optimal level of taxation \( t_{oc} \).

\(^{29} \) It should be noticed that the optimal level of taxation obtained in eq. (19) is different from the corresponding level obtained in eq. (11). Indeed the latter is the result of the Government optimal choice for a given exogenous value of \( \Delta b^r \) and it is thus found by first minimizing the Government loss function under the constraint given by eq. (5) and then substituting in the resulting first-order condition the rational-expectations constraint (i.e. \( \Delta b^c_t = \Delta b_t \)). The optimal fiscal rule is instead the Government optimal choice under the ex-ante condition that expectations are rational; it is thus obtained by minimizing the Government loss function taking into account together both the constraint given by eq. (5) and the rational-expectations condition \( \Delta b^c_t = \Delta b_t \).
The features of $t^o$ have some interesting implications if we compare $L(t^o)$ with $L(t^d)$ and $L(t^c)$. First, since $t^o$ is, by construction, the tax rate minimizing the Government loss under rational expectations, we have $L(t^o) \leq L(t^c)$ and $L(t^o) \leq L(t^d)$. These two properties together imply the following:

Proposition 3. A Government commitment to the optimal fiscal rule $t^o$ is preferable both to a commitment to debt stabilization and to a discretionary equilibrium.

Furthermore, since $t^o$ is, in general, different from $t^c$, we also have the following:

Proposition 4. The Government commitment to the optimal fiscal rule $t^o$ implies, in general, a positive growth for public debt.

The conclusion in Proposition 3 is similar to that obtained for monetary policy in the Barro-Gordon time-inconsistency framework: there is a kind of commitment that is superior to discretion, and this occurs because it constrains Government choices, thus removing the dynamic inconsistency problem.

The conclusion in Proposition 4, however, states that the optimal Government commitment is not to debt stabilization, contrary to the common prescription that this is the best fiscal strategy to pursue. In our model, in fact, the optimal Government commitment implies a specific and predetermined positive debt growth.

The interpretation of this conclusion is straightforward. In our framework, there is a trade-off between anticipated fiscal restraint (i.e. lower expansion in debt) and output, due to the fact that a lower level of taxation increases debt growth but at the same time increases output. A commitment to debt stabilization does not exploit this trade-off at all, and thus is not an optimal choice. The optimal fiscal rule emerging in our model, instead, exploits in some way the existent trade-off, and will thus involve some (anticipated) expansion in debt and an output level lower than under discretion, though higher than with a commitment to complete debt stabilization.

30 In the Kydland-Prescott framework on the other hand there is no long-run trade-off between output and inflation, since the long-run Phillips curve is vertical. This explains why the optimal commitment of monetary authorities is always that associated with a zero rate of inflation.
The optimal fiscal rule associated with \( t^{oc} \) has an obvious graphical representation in Figures 1 and 2. Since the optimal tax rate is obtained by choosing the rational expectations equilibrium which minimizes the Government loss function, this equilibrium is represented by the point on the EL curve which is tangent to the iso-loss curve closer to the axes origin (i.e. point OC in both diagrams). It is easy to see that, by construction, the optimal equilibrium point OC is always preferable to points D and CS, whatever the level of initial debt \( b_{t-1} \). \(^{31}\)

It may be finally noted that, as anticipated in Section One, even though the rule in eq. (19) is preferable to the simpler rule determining \( t^{cs} \), it can be very difficult to implement. The rule, in fact, needs to take into account the actual features of the economic system (expressed by the functions \( f(t_i) \) and \( \varphi(t_i, b_{i-1}) \) in eq. (19)) and social preferences between debt stabilization and output growth (as specified by parameter \( a \) in the same equation). Since the form of eq. (19) is complex and its elements are difficult to estimate and can change over time, it can be very awkward to specify such a rule in an actual legislative or institutional framework.

4. A possible extension of the basic model: the effects of exogenous shocks

The model studied in the previous section assumes that there are no random shocks in the economy. The time inconsistency literature applied to monetary policy, however, shows that the presence of uncertainty is important in influencing the conclusions about the relative desirability of rules versus discretion (cf. Rogoff, 1985). In that framework, in particular, the existence of shocks provides an argument in favour of discretion since a countercyclical monetary policy can reduce the output loss triggered by an adverse supply shock. In this section we show that similar conclusions are true also with reference to fiscal policy.

\(^{31}\) The time-inconsistency problem associated with the optimal fiscal rule could also be easily illustrated graphically using Figures 1 and 2. All that is needed is to draw the BY schedule passing through point OC, where the expected debt growth is equal to \( \Delta b^{OC} \). The new fooling equilibrium would be given by the intersection between the new BY schedule and the GRF locus, so that, by construction, it would lie on an iso-loss curve closer to the origin than the curve associated with point OC (thus indicating an incentive for the Government to behave opportunistically). The Government incentive to renege on its commitment to an optimal fiscal rule, however, would be understood and anticipated by rational agents, so that the final equilibrium would settle in point D in both diagrams. This confirms that the optimal fiscal rule too is subject to time inconsistency and thus that it can be implemented only by means of a commitment.
In order to re-examine the results previously obtained under uncertainty we reconsider the basic model and introduce the possibility of exogenous random shocks. Therefore we assume that eq. (5) is substituted by:

$$Y_t = f(t_t) + \tilde{\varepsilon}_t$$

(20)

where $\tilde{\varepsilon}$ is a random variable with a null expected value and a known finite variance $\sigma^2$, which can represent either a demand or a supply shock.

The Government expected loss function in this case becomes:

$$E[L(t_t)] = E\left\{ a\left[h(t_t) + \gamma(\Delta b_t^\varepsilon)b_{t-1}\right]^2 + \left[f(t_t) - Y_* + \tilde{\varepsilon}_t\right]^2 \right\}$$

(21)

Let us suppose now, as is normally done in the time inconsistency literature, that agents form their expectations before the realization of $\tilde{\varepsilon}_t$, while the Government chooses $t_t$ after this realization. Let us label $t_t^u$ the value of $t_t$ minimizing the loss function. Its value is determined by the condition:

$$ah'(t_t^u)\left[h(t_t^u) + \gamma(\Delta b_t^\varepsilon)b_{t-1}\right] = -f'(t_t^u)\left[f(t_t^u) - Y_* + \varepsilon_t\right]$$

(22)

where $\varepsilon_t$ is the realization of the random variable $\tilde{\varepsilon}_t$.

It is worth noticing that the term $\gamma(\Delta b_t^\varepsilon)b_{t-1}$ in eq. (22) is the same as in the case of discretion without uncertainty (cf. eq. (12)), since agents do not know the realization of $\varepsilon_t$ and form their expectations by setting $E(\tilde{\varepsilon}_t) = 0$.

A comparison between eqs. (12) and (22) shows that:

$$t_t^u = t_t^d + \phi(\varepsilon_t)$$

(23)

where $\phi(0) = 0$, $\phi(\varepsilon_t)>0$ when $\varepsilon_t>0$, $\phi(\varepsilon_t)<0$ when $\varepsilon_t<0$, and $\phi'(\varepsilon_t)<0$. Obviously, when there are no shocks, the optimal Government choice is the same as in the absence of uncertainty. When there is a positive shock, instead, the output level will be larger (and closer to the desired level),
so that the Government will increase taxation in order to decrease public debt growth. When there is a negative shock, the opposite conclusion holds.

In order to assess the relative desirability of a fiscal rule versus complete discretion under uncertainty, we start, as in the previous section, by looking at a commitment to stabilize public debt. In this case we need to compare the difference \( L_{d} - L_{cs} \) with \( E[L(t^d)] - E[L(t^cs)] \), where \( E[L(t^u)] \) is the expected loss when \( t = t^u \), and \( E[L(t^cs)] \) is the expected loss under commitment to stabilization. This comparison leads to the following result:

**Proposition 5.** Ceteris paribus, the threshold level \( \hat{b} \), defined in Proposition 1, is higher under uncertainty (see Appendix B for the proof).

The explanation of Proposition 5 is quite simple. The presence of (an adverse) shock affects the level of output by generating a cost that is considered in the loss function. Under discretion the Government can reduce this cost by choosing an optimal policy that takes into account the effect of the shock; under commitment to stabilize public debt, the whole cost of uncertainty is instead borne by the economy. For this reason, random shocks introduce a new element in favour of discretion, additional to the advantage related to the output level described in Section Three. Hence the overall advantage associated with discretion is larger under uncertainty, implying that commitment to stabilization is preferable only if the loss associated with discretion is larger too, which happens if, ceteris paribus, the risk premium embodied in the interest rate is higher; this situation occurs only if the initial debt level is sufficiently high, and anyway higher than without uncertainty.\(^{32}\)

We now proceed to assess whether the additional advantage associated with discretion in the presence of uncertainty has the same implications with regards to the comparison between the Government losses under discretion or under a commitment to adopt the optimal fiscal rule.

\(^{32}\) Once again, according to our model, the 2005 reform of the Stability and Growth Pact in Europe is appropriate in that it allows wider discretion in fiscal policies during economic recession, especially if the countries involved are not too highly indebted (see also Footnote 23).
Indeed, since the commitment to adopt the optimal fiscal rule is declared before $\tilde{e}_t$ is realised and therefore before $E(\tilde{e}_t) = 0$, the optimal fiscal rule under uncertainty is still described by eq. (19). Hence the taxation level $t^o_c$ is the level that minimizes the expected Government loss under the constraint given by eq. (17) also in the presence of random shocks.

After $\tilde{e}_t$ is realized, however, the value of $L$ is not minimized. This implies that it is possible that $L(t^{oc}) > L(t^d)$. Furthermore, also the ex-ante choice between discretion and commitment to the optimal fiscal rule is different from the certainty case. Indeed, by comparing the ex-ante expected loss under discretion and under commitment to the optimal fiscal rule, we get the following:

**Proposition 6.** The sign of $E[L(t^{oc})] - E[L(t^d)]$ is ambiguous (see Appendix C for the proof).

The explanation of this result is clearly related to that of Proposition 5. The optimal discretionary choice takes into account the realization of the shock $\tilde{e}_t$. A commitment, even to an optimal fiscal rule, cannot do this; so there is a clear advantage for discretion which counteracts the benefits of a commitment to the optimal fiscal rule described in Section Three. As a conclusion, in the presence of uncertainty an optimal commitment is no longer always preferable to discretion.

This result represents the main difference between the conclusions obtained under full certainty versus the case of random shocks. It is also interesting to note that the result is similar to that obtained in the Barro-Gordon framework, where the superiority of a commitment to zero inflation for monetary policy disappears in the presence of uncertainty.

Our previous analysis examined discretion compared to the optimal deterministic rule for fiscal policy. A possible different solution could be the adoption of a state-contingent rule, incorporating the occurrence of random shocks. However, although a state-contingent rule can be envisaged in theory, it seems impossible to put into practice for two main reasons. First, a

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33 See, for instance, Walsh (2010) for a simple example in the case of monetary policy.
rule of this kind magnifies the problem, described at the end of Section Three, of inserting the rule in a legislative or institutional framework. In fact, as emphasised by Lohmann (1992, p. 273), “in practice, it is infeasible for a policy-maker to commit to a state-contingent policy rule: it is impossible or prohibitively costly to specify all possible contingencies in advance. The policymaker is constrained to make simple commitments, which are invariant to the state of the world”. Second, because of the (even ex-post) difficulties in measuring the size of random shocks, the ex-post control of compliance to a state-contingent rule is almost impossible. For both these reasons we choose to exclude the examination of a state-contingent commitment from our work.

5. Some considerations about dynamics and reputation

The analysis of the effects of different Government choices was conducted in the previous sections in a one-period static framework. A natural extension of the analysis would be to a multi-period dynamic framework. This kind of inquiry, however, is much more complex in our context than in the Barro-Gordon model. In the Barro-Gordon model, in fact, the choice made by the Central Bank about inflation in any period exhausts its effect within the same time period and does not affect the future setting of the economy. So, in that context, a multi-period analysis is reduced to the examination of the effects of a repeated game between the Central Bank and rational agents, where the game is the same in every period.

In our framework, instead, eq. (2), describing how public debt evolves over time, shows that debt growth in any one period depends upon the level of the debt in the previous period. This intertemporal linkage has two main implications. First, it implies that the choice made by the Government in one period has permanent effects on all future periods. This implication should be taken into consideration when examining the optimality of Government choices. Second, since the level of \( b_t \) changes over time, the game between the Government and rational agents is different in each time period.
The previous considerations clearly suggest that a dynamic analysis of rules versus discretion in fiscal policy needs a specific inquiry. A complete investigation of this kind is, however, beyond the scope of this paper and may be the subject of future research. In what follows, though, we wish to highlight some general implications of the introduction of dynamics, and put forward some worthwhile preliminary conclusions.

The first element to be considered is the effect of a dynamic framework on Government optimal commitment $t^{OE}$. In a multi-period context, the Government loss function (7) obviously changes, becoming:

$$L = \sum_{i=t}^{\infty} (1+\theta)^{i-1} \left\{ a \left[ \Delta b_i \right]^2 + [Y_i - Y^*_i]^2 \right\}$$  \hspace{1cm} (24)

where $\theta$ is the intertemporal discount rate. In order to get a dynamic optimal commitment, eq. (24) must be minimized under a set of period constraints similar to eq. (17). Substituting eq. (17) in eq. (24) we thus obtain:

$$L = \sum_{i=t}^{\infty} (1+\theta)^{i-1} \left\{ a \left[ \varphi(t_i, b_{i-1}) \right]^2 + [f(t_i) - Y^*_i]^2 \right\} =$$

$$= a \left[ \varphi(t_t, b_{t-1}) \right]^2 + [f(t_t) - Y^*_t]^2 + \sum_{i=t+1}^{\infty} (1+\theta)^{i-1} \left\{ a \left[ \varphi(t_i, b_{i-1}) \right]^2 + [f(t_i) - Y^*_i]^2 \right\}$$  \hspace{1cm} (25)

The Government chooses in the initial period $t$ the sequence of all current and future tax rates (in periods from $t$ to $\infty$) by minimizing eq. (25). The first-order conditions for these optimal tax rates yield a set of infinite conditions described by the following equation:

$$a \frac{\partial \varphi_j}{\partial t_j} \varphi(t_j, b_{j-1}) + \sum_{i=j+1}^{\infty} (1+\theta)^{i-1} a \frac{\partial \varphi_i}{\partial b_{i-1}} \frac{\partial b_{i-1}}{\partial t_j} \varphi(t_i, b_{i-1}) = -f'(t_j)(f(t_j) - Y^*_j) \quad \text{for} \quad j = t, ..., \infty$$  \hspace{1cm} (26)

Let us now examine the optimal choice of the tax rate in the static framework (eq. (18)) and in the multi-period framework (eq. (25)). Comparing the loss functions in the two cases we see that the function to be minimized in the static case is equal to first two terms of the equation to be minimized in the dynamic context. Furthermore, in the third term of the loss function (25),
the effect of taxes is given by its influence on the sequence of \( b_{i-1} \) only.\(^{34}\) This effect is clearly negative, since a lower level of current taxation implies a larger debt growth. So, in a multi-period framework, a lower rate of taxation will generate an extra-loss with respect to the static model, due to its negative influence on future debt growth.

This observation leads to a clear indication for the comparison between the optimal level of taxation in the static \( (t^{oc}) \) and in the multi-period framework (where the “dynamic” \( t^{oc} \) may be labelled \( t^{doc} \)). Since lower taxation generates an additional loss in the multi-period framework, we have that the optimal level of taxation is higher in the dynamic case (i.e., \( t^{oc} < t^{doc} \)).

A second conclusion concerning the optimal dynamic fiscal rule for commitment can be obtained by examining the sequence of \( t^{doc}_j \) for \( j = 1, \ldots, \infty \), determined by eq. (26). In particular, each term of the summation in the left-hand side of eq. (26) includes the function \( \varphi(t_j, b_{i-1}) \) which is increasing in \( b_{i-1} \). Since the summation has infinite terms, a value of \( b_{i-1} \) increasing over time would imply that the summation explodes. This would violate condition (26), since its right-hand side is finite. On the other hand, a level of \( t_j \) ensuring \( \Delta b = 0 \) in every period would imply, by eq. (17), that the left-hand side of eq. (26) would be null; this would again violate condition (26), since its right-hand side is strictly positive.

These conclusions exclude two possible paths for \( t^{doc}_j \): the path implying endless debt growth and the path where public debt is immediately stabilized. The same findings also suggest a possible solution path, entailing some (possibly several) periods of commitment to a predetermined positive debt growth followed by a commitment to debt stabilization.\(^{35}\)

Finally, the consideration of a multi-period framework also raises the issue of the possible existence of reputational equilibria. In a static framework, in fact, the equilibrium level of taxation \( t^{oc} \) is not sustainable without commitment because of time inconsistency. As in the

\(^{34}\) Notice that the level of \( t_i \) will affect the whole sequence of \( b_{i-1} \) since, by changing the current level of public debt, the Government will influence the levels of debt in all future periods.

\(^{35}\) Graphically, in Figures 1 and 2, since, by construction, points OC and CS coincide only when the EL locus is vertical, it follows that public debt will be stabilized at the level given by \( b^* = 1/\gamma' \) (or \( 1/\gamma \) in the linear case).
traditional Barro-Gordon framework, however, a different conclusion is possible if the game is repeated. In this case, the Government could announce a sequence of tax rates $\tau_{ij}^{\text{doc}}$ and choose not to deviate from it in order to acquire reputation and avoid the losses related to agents’ punishment strategy if the discretionary outcome is chosen.

In order to study this reputational problem, we consider a standard framework where we assume that the Government is trusted until it deviates from its announcements, and is no longer trusted forever once it deviates. Under this assumption and in the case of an infinite time horizon, the Government chooses not to deviate from $\tau_{ij}^{\text{doc}}$ if and only if

$$
\sum_{i=j+1}^{\infty} (1+\theta)^{i-j} \left( L_{ij}^{\text{doc}} - L_{ij}^{d} \right) < (1+\theta)^{j} \left( L_{j}^{f} - L_{j}^{\text{doc}} \right)
$$

for $j = 1, \ldots, \infty$ (27)

where $L_{ij}^{\text{doc}}$ is the value of the loss suffered at time $j$ in the case where the Government chooses the sequence of tax rates $\tau_{ij}^{\text{doc}}$, $L_{ij}^{f}$ is the value of the loss suffered at time $j$ in case of fooling, and $L_{ij}^{d}$ is the value of the loss suffered at time $j$ in case of discretion. If condition (27) is satisfied, the optimal fiscal sequence $\tau_{ij}^{\text{doc}}$ is a sustainable equilibrium without commitment (i.e. a reputational equilibrium).

It is in general not possible to establish when condition (27) is satisfied. One simple result is however evident. Since the right-hand side of inequality (27) is positive and since the left-hand side tends to $\infty$ for $\theta \to 0$ and to 0 for $\theta \to \infty$, and is a decreasing function of $\theta$, we can state the following:

**Proposition 7.** Ceteris paribus, $\exists \hat{\theta}:$ if $0 < \hat{\theta}$ then the reputational equilibrium exists and if $\theta > \hat{\theta}$ then the reputational equilibrium does not exist.

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36 For a standard textbook analysis of the Barro-Gordon dynamic inconsistency problem in a multi-period framework see Blanchard and Fischer (1989), Ch. 11.

37 A standard backward induction argument ensures that the equilibrium cannot be sustained without commitment if the time horizon is finite.

38 It should be stressed that the variables $L_{ij}^{\text{doc}}$, $L_{ij}^{f}$ and $L_{ij}^{d}$ in eq. (27) correspond to the values of the loss suffered at time $j$ only, and are different from the total loss suffered in all periods of time (from $j$ to $\infty$) described by eq. (25).
6. Conclusions

This paper studied the relative advantages and costs of rules versus discretion in the conduct of fiscal policy within a framework in which agents expectations about Government choices affect the risk premium paid on public bonds. The results are relevant from many perspectives.

In the first place, our model shows that under discretion the Government will choose a lower tax rate than the rate announced because of its desire to increase output beyond the current equilibrium level. Since this time-inconsistent behaviour is correctly anticipated by rational agents, it will generate a “deficit bias” due to the increase in the risk premium paid on public bonds, as a consequence of the debt growth entailed by the increased deficit. This “deficit bias” is similar to the “inflation bias” generated by monetary policy in the traditional time inconsistency literature and provides a new argument in the analysis of the reasons behind the emergence of public deficits. In fact, as this paper shows, another potential source of a deficit bias is the ability of financial markets to incorporate the effects of time inconsistent Government behaviour into the risk premium paid on public bonds.

Secondly, the existence of time inconsistency in fiscal policy suggests that the introduction of a commitment would enable Government to reach a better equilibrium. According to the literature on fiscal sustainability, translated into the standard prescriptions of international institutions, the most common suggestion is a commitment to stabilize public debt. Our analysis, however, shows that this kind of commitment is not always superior to discretion, because in our model there is a trade-off between output growth and debt restraint. In particular, since the value of the deficit depends upon the level of the initial public debt, the relative desirability of a commitment to debt stabilization versus discretion depends upon that level as

39 As already noted in the introduction, within the political economy approach to the analysis of fiscal deficits, Alesina and Tabellini (1990) show that a “deficit bias” can arise when there are differences in political parties preferences on the composition of public expenditure, such that an incumbent Government will not fully internalize the cost of bequeathing debt to the new entrant. Persson and Svensson (1989) show that a conservative Government will run a larger deficit if it knows that it will be replaced by a liberal one.
well. Consideration of an optimal fiscal rule leads however to different conclusions, since in this case the superiority of commitment can be established. But this optimal rule, which entails a positive (rather than null) predetermined debt growth, if not supported by a formal commitment, is still subject to time inconsistency. Furthermore this rule entails a very complex formulation, implying some practical problems of implementation.

The introduction of uncertainty into the model only partially alters these conclusions. First, the threshold level of public debt that makes commitment preferable to stabilization is higher in the presence than in the in the absence of random shocks. Second, as in the Barro-Gordon framework for monetary policy, uncertainty implies that discretion can generate a smaller expected loss than the optimal fiscal rule, since it makes it possible for the Government to react to shocks, partially counteracting their effects.

The consideration of a dynamic framework yields some further indications about the features of optimal Government choices. In a multi-period horizon, the optimal fiscal rule is characterized by a tax rate that is larger than in the static model and implies debt stabilization after some periods of time. Debt stabilization is thus suboptimal in the short-run but desirable in the long-run. Finally, in a multi-period framework, if the intertemporal discount rate is not too high, the possibility of repeating the strategic game between the Government and rational agents may generate reputational equilibria, which implies that the optimal fiscal rule could also be implemented without commitment.

7. References


Appendix A. Proof of Proposition 1.

A simple inspection of eq. (15) shows that the difference $L(t^d) - L(t^{cs})$ is positive for $b_{t-1} \to \infty$ and is an increasing function of $b_{t-1}$.

The further conclusion that the difference $L(t^d) - L(t^{cs})$ is negative for $b_{t-1} = 0$ can be derived from the following observations. First, for $b_{t-1} = 0$ we have no effect of the risk premium on debt growth (the term $\gamma (\Delta b_t) b_{t-1}$ is zero); this implies that commitment does not reduce the interest payments with respect to discretion. Given this result, and since $t^d$ is, by assumption, the optimal choice of the Government given the loss function, we have that $L(t^d) \leq L(t^{cs})$.

Finally, under the hypothesis that $Y^* > L(t^{sl})$ and since $t^{cs} = t^{sl}$, $t^{cs}$ cannot be the optimal level of $t$. This excludes the possibility that $L(t^d) = L(t^{cs})$.

The three results we have just shown, according to which the difference $L(t^d) - L(t^{cs})$ is negative for $b_{t-1} = 0$, positive for $b_{t-1} \to \infty$ and increasing in $b_{t-1}$, prove Proposition 1.

Appendix B. Proof of Proposition 5.

We first prove that ceteris paribus, for any given value of $b_{t-1}$, $E[L(t^u)] - E[L(t^{cs})]$ is lower than $L(t^d) - L(t^{cs})$. Given the expected loss function (17) we have:

$$E[L(t^u)] = E \left[ a \left[ h(t^u) + \gamma (\Delta b_t^c) b_{t-1} \right]^2 + \left[ f(t^u) - Y^* + \tilde{\varepsilon} \right]^2 \right]$$

where $t^u$ satisfies condition (18) in Section Four. The expected loss under commitment is instead:

$$E[L(t^{cs})] = E \left[ f(t^{cs}) - Y^* + \tilde{\varepsilon} \right]^2 = L(t^{cs}) + \sigma^2$$

where the last equality holds since $E(\varepsilon) = 0$. 

32
In order to determine $E\left[ L(t^u) \right] - E\left[ L(t^{cs}) \right]$ we first compute the expected loss obtained when the Government chooses $t^d$ (determined by condition (12)) under uncertainty. Since $E(\varepsilon) = 0$ we get:

$$E\left[ L(t^d) \right] = E\left[ a \left[ h(t^d) + \gamma (\Delta b^d) b_{t-1} \right]^2 + \left[ f(t^d) - Y^* + \varepsilon \right]^2 \right] = L(t^d) + \sigma^2$$  \hspace{1cm} (29)

Eqs. (28) and (29) imply that $E\left[ L(t^d) \right] - E\left[ L(t^{cs}) \right] = L(t^d) - L(t^{cs})$. However, since $t^u$ is different from $t^d$ (cf. eq. (23)) and is the optimal choice under uncertainty we have that $E\left[ L(t^d) \right] > E\left[ L(t^u) \right]$ implying that $L(t^d) - L(t^{cs}) > E\left[ L(t^u) \right] - E\left[ L(t^{cs}) \right]$ for any given value of $b_{t-1}$.

This conclusion has relevant implications for the comparison between the value of $\hat{b}$ under certainty ($\hat{b}^{cs}$) or uncertainty ($\hat{b}^u$). Indeed, for $b_{t-1} = \hat{b}^{cs}$, we have $L(t^d) = L(t^{cs})$ and $E\left[ L(t^{cs}) \right] > E\left[ L(t^u) \right]$. Since $E\left[ L(t^u) \right]$ is increasing in $b_{t-1}$ this implies that $\hat{b}^u > \hat{b}^{cs}$.

**Appendix C. Proof of Proposition 6**

Substituting condition (23) (with $\varphi(\bar{\varepsilon})$ instead of $\varphi(\varepsilon)$, since the expected loss is computed ex-ante) in eq. (27) we get:

$$E\left[ L(t^u) \right] = E\left[ a \left[ h(t^d + \varphi(\bar{\varepsilon})) + \gamma (\Delta b^d) b_{t-1} \right]^2 + \left[ f(t^d + \varphi(\bar{\varepsilon})) - Y^* + \bar{\varepsilon} \right]^2 \right] =$$

$$= a E\left[ h(t^d + \varphi(\bar{\varepsilon}))^2 \right] + a \left[ \gamma (\Delta b^d) b_{t-1} \right]^2 + 2a \gamma (\Delta b^d) b_{t-1} E\left[ h(t^d + \varphi(\bar{\varepsilon})) \right] + E\left[ f(t^d + \varphi(\bar{\varepsilon}))^2 \right] + Y^* + \sigma^2 + 2Y^* E\left[ f(t^d + \varphi(\bar{\varepsilon})) \right] + 2E\left[ f(t^d + \varphi(\bar{\varepsilon})) \bar{\varepsilon} \right]$$  \hspace{1cm} (30)

Furthermore it is easy to see that the expected loss associated with the optimal fiscal rule is equal to:

$$E\left[ L(t^{oc}) \right] = L(t^{oc}) + \sigma^2$$  \hspace{1cm} (31)
From the form of functions \( f(.) \) and \( \varphi(.) \) we have that the term \( 2E[f(t^d + \varphi(\epsilon))\epsilon] \) in (30) is certainly negative. Comparing \( E[L(t^u)] \) and \( E[L(t^{\omega})] \) it is easy to see that this is sufficient to ensure that the sign of \( E[L(t^{\omega})] - E[L(t^u)] \) is ambiguous.
Figure 1. Possible outcomes when commitment to stabilisation is preferable
Figure 2. Possible outcomes when discretion is preferable