Uniform Pricing and Social Welfare

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Abstract. We re-examine the case for uniform pricing in a monopolistic third-degree price-discrimination setting by introducing differentiated costs. Intuitively, the monopolist would like to use differentiate pricing to decrease the average total cost, unless marginal costs are “perversely” correlated with demand elasticities. Indeed, monopolistic price differentiation can improve welfare and also aggregate consumer surplus even if, as in the benchmark linear case, total output does not increase. Accordingly, the welfare criterion based on total-output fails and should be replaced by the computation of well-defined price indexes. These results pave the way for a more optimistic assessment of monopolistic pricing.

Keywords: uniform pricing, third-degree price discrimination, welfare bounds, price and quantity indexes.

JEL Classification: D11, D42, L51.
I. Introduction

It is well known that imposing uniform pricing instead of allowing third-degree price discrimination to a monopolist has ambiguous welfare implications. However, Schmalensee (1981), Varian (1985) and Schwartz (1990) proved that, if (marginal) costs are common, a necessary condition for monopolistic third-degree price discrimination to raise aggregate welfare is that total output increases under discriminatory pricing. Moreover, in the benchmark case of linear demand and cost functions, total output does not change if the markets served by the monopolist are the same both under uniform and differentiated pricing: see Schmalensee (1981) and Tirole (1988: p. 139). Since the linear setting is usually adopted because it allows an explicit computation of the results and provides a first-order approximation to the general case, the literature on the welfare effect of allowing monopolistic price differentiation tends to be rather pessimistic: see e.g. Schmalensee (1981: p. 246) and Varian (1989: pp. 622-623).

In this paper we reconsider the case for uniform pricing by departing from the standard set-up, which assumes that there is a common (marginal) cost of serving different markets. Accordingly, we are not really concerned with (third-degree) price discrimination narrowly defined, since this would require that the same commodity be sold at different prices to different consumers: see e.g. Philips (1981: p. 5). Nevertheless, we think our setting interesting for several reasons. First, the standard set up with equal marginal costs is a textbook idealization with little empirical content. For example, what ensures that the marginal cost of serving geographically distant markets is the same? Second, the most appealing definition of price discrimination (attributed to George Stigler, 1987), which says that a firm price discriminates when the ratio in prices is different from the ratio in marginal costs for two “similar” (possibly identical) goods offered by the firm (see e.g. Varian, 1989: p. 598 and Stole, 2001: p. 2), encompasses the case of different marginal costs. But since they are difficult to verify, it seems clear that, as a policy prescription, the use of uniform pricing should be tested against the possibility of unaccounted marginal cost differences. Moreover, in our second-best setting even the no-price-discrimination condition à la Stigler does not need to be optimal: see below.

Third, while we should perhaps refer to them as examples of price discrimination (once the latter is properly defined), there are sectors with acknowledged differentiated marginal costs, such as the mail or telephone ones, in which uniform rates are imposed by public regulation (avoiding for example tariff discrimination between rural and urban areas), sometimes on a political basis

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1 For the sake of simplicity, we make this assumption throughout this paper. Notice that in fact price differentiation may lead to a Pareto improvement if it causes some market to open: see e.g. Schmalensee (1981), Varian (1985) and Tirole (1988: pp. 137-9).

2 This comment only applies to the case of an otherwise unregulated monopoly: see e.g. Armstrong and Vickers (1991).
invoking “horizontal equity” (this is the case, for instance, of the Italian residential electricity sector). In these cases, uniform pricing entails cross-subsidies whose welfare consequences have not been yet entirely understood. Fourth, in an imperfectly competitive market the use of a uniform price may serve as a (potentially anticompetitive) strategic practice not to reveal the underlying cost structure. While this aspect is not considered here, the related additional social costs should in principle be weighted against the other welfare consequences of avoiding price differentiation, which are the subject of this paper.

Not surprisingly, it turns out that in our enlarged setting the condition of an increase in output is no longer necessary for a welfare improvement. In particular, we are able to find many cases in which monopolistic price differentiation increases total welfare and aggregate consumer surplus even if, as in the benchmark linear case, total output does not increase. This happens if demands are close enough, as it is to some extent intuitive, and also in some cases in which there is no systematic relation between the demand and the cost structures (in general, a reasonable assumption). The intuition is straightforward: the economic rationale for imposing uniform pricing in the standard setting rests on the result that a given quantity of the same good should be distributed according to a common price (a result which is implied by the optimality of marginal cost pricing). But with different marginal costs no general principle can be invoked to support uniform pricing (goods are economically different and first-best optimality would rather call for the weaker condition of having the prices ratios equal to the marginal cost ratios),\(^3\) which has no special properties.

Indeed, the socially efficient production of a given total amount of output (an unusual second-best problem if goods are not identical) would ask for the difference between prices and the relative marginal costs being equal across markets. This price structure, which will never be achieved by uniform pricing if marginal costs are different (nor by imposing a no-price-discrimination condition \(à la\) Stigler), to some extent is more similar\(^4\) to the one that would be chosen by an unregulated monopolist (a special case of Ramsey pricing, as it is well known). In fact, intuition also suggests that, with respect to uniform pricing, a monopolist would like to use the alleged price authority to increase the price of the more costly goods, so decreasing average total cost and increasing welfare. And that he would actually do that if the demand elasticities were not “perversely” correlated with the unit costs (this should be the case if demands are close enough). Some part of the so-achieved cost reduction is sometimes “passed” trough prices to the consumers but, of course, the monopolist will also try to use his pricing flexibility to extract more surplus from them. Accordingly, this might

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\(^3\) Notice that this is the condition underlying the Stigler’s price discrimination broad definition quoted above. Also notice that the unregulated monopolist would use this first-best price structure if demand elasticities were equal across markets.

\(^4\) Philips (1981: pp. 3-4) commented negatively on this similarity, but by referring to the narrowly defined case of a unique, common marginal cost.
give rise to a conflict between social welfare and consumers surplus concerns. In general, welfare effects maintain unfortunately their ambiguity (as it should be expected): however, from our results price differentiation by an unregulated monopolist should perhaps be considered more optimistically.

II. The model

We refer to the model in Schmalensee (1981), which can be seen as a special case of Varian (1985). In particular, a monopolist is selling in $N$ distinguishable markets. Let $q_i(p_i)$ be the demand function in market $i$ ($i = 1, N$), where $p_i$ is the price charged by the monopolist and $q_i$ the quantity he sells. It is assumed that the relevant cost function is given by $C(q) = c'q = \sum c_i q_i$, where $q' = [q_1, q_2, \ldots, q_N]$ is the vector of the monopolist’s output quantities, $c_i$ indicates the constant unit cost of output $q_i$ and $c' = [c_1, c_2, \ldots, c_N]$. We generalize the Schmalensee’s (1981) setting by assuming that $c_i$ is possibly different from $c_j$, $i \neq j$, $i, j = 1, N$. The net profit generated in market $i$ can then be written $\pi_i(p_i) = (p_i - c_i)q_i(p_i)$: following most of the literature, it is assumed that the $\pi_i(p_i)$ are smooth, strictly concave functions. The total profit function is thus given by $\Pi(p) = (p - c)'q(p) = \sum_i \pi_i(p_i)$, where $p' = [p_1, p_2, \ldots, p_N]$ is the vector of prices that the monopolist charges. It is also assumed that consumers have quasi-linear preferences. Thus, since there are no income and distributional effects, we can think in terms of a representative consumer with indirect utility function $V(p) = v(p) + y_0$, where $y_0$ is the total endowment of the numeraire (the Marshallian composite commodity), who consumes all the goods produced by the monopolist. By quasi-linearity, $v(\cdot)$ is convex and the demand system faced by the monopolist is given by $q(p) = -D_v(p)$, where $D_vf(x)$ is the gradient of $f(\cdot)$. Aggregate (social) welfare (the Marshallian indicator) can then be written as $W(p) = \Pi(p) + v(p)$.

Clearly, the unregulated monopolist would adopt a price vector $p^*$ uniquely characterized by the FOCs $D_p\Pi(p^*) = 0$, where $0$ is the relevant null vector; i.e., $i = 1, N$:

$$\pi_i'(p^*_i) = (p^*_i - c_i)q_i'(p^*_i) + q_i(p^*_i) = 0, \quad (1)$$

which is just the familiar condition of marginal revenue and marginal cost equality in each market. On the contrary, if the monopolist is forced to use a uniform price he would charge all buyers $p^*$, which is uniquely defined by:

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5 Bold characters are used to indicate vectors; conventionally, $x$ is a column vector and $x'$ indicates its transpose (a row vector).
\[
\sum_{i=1}^{N} \pi'_i(p^*) = \sum_{i=1}^{N} [(p^* - c_i)q'_i(p^*) + q_i(p^*)] = 0. \quad (2)
\]

It is standard to adopt Joan Robinson’s terminology and to call “strong” a market \(i\) if \(p_i^* > p^*\); we will write that in such a case \(i \in S\). Accordingly, a market \(i\) is “weak” if \(p_i^* < p^*\) (and thus \(i \in W\)) and “intermediate” if \(p_i^* = p^*\) (and thus \(i \in I\)). Schmalensee (1981: pp. 243-4) noted that the remove of the uniform pricing constraint can be usefully characterized as follows. Consider the problem of maximizing \(\Pi p\) under the linear constraint:

\[
\sum_{i=1}^{N} \pi'_i(p^*)(p_i - p^*) \leq t, \quad (3)
\]

where \(t\) is some nonnegative real number. The solution \(p(t)\) to such a problem is uniquely defined by the FOCs \(D_p\Pi p(t) = \lambda(t)D_p\Pi p^* t\), where \(t\) is the relevant unit vector and \(\lambda(t)\) is the nonnegative relevant Lagrangean multiplier (it measures the marginal increase in profit due to a marginal increase in \(t\)): i.e., \(\lambda(t) = d\Pi p(t)/dt = \Sigma_i \pi'_i(p_i(t))p_i^*(t))\).

The price system \(p(t)\) has a number of properties: in particular, \(p(0) = p^*\) and \(\lambda(0) = 1\) (this value depends on the normalization of the price differences implicit in (3)), and it exits a finite \(\tau \geq 0\) such that \(p(t) = p^*\) and \(\lambda(t) = 0\) for all \(t \geq \tau\). Moreover, \(\lambda'(t) < 0\) and \(\text{sign}\{p_i'(t)\} = \text{sign}\{\pi'_i(p_i(t))\} = \text{sign}\{\pi'_i(p^*)\}\) for all \(t < \tau\), \(i = 1, N\). Thus, a market is strong (weak) only if \(\pi'_i(p^*) > 0\) (\(\pi'_i(p^*) < 0\) and, unless all markets are intermediate (in such a case the imposition of uniform pricing would cause no effect at all), the remove of the uniform pricing constraint monotonically increases some prices and decreases others. Analogously, let us introduce the “virtual” unit cost of market \(i\) at \(t\), \(c_i(t)\), uniquely characterized by the condition:

\[
(p_i(t) - c_i(t))q'_i(p_i(t)) + q_i(p_i(t)) = 0. \quad (4)
\]

Notice that \(c_i(t)\) is obviously equal to market \(i\)’s marginal revenue at \(q(p_i(t))\): i.e., \(\pi'_i(p_i(t)) = (c_i(t) - c_i)q'_i(p_i(t))\). Clearly, \(i \in S\) (\(i \in W\)) if and only if \(c_i > c_i(t)\ (c_i < c_i(t))\), for \(t < \tau\ (c_i(t) = c_i)\), \(i = 1, N\). This says that price rises (decreases) are “naturally” associated to high (low) marginal costs and low (high) marginal revenue at \(t\). Finally, notice that \(tD_p\Pi p(t)) = \Sigma_i \pi'_i(p_i(t)) = 0\) for all \(t\) (a sort of “average” equality of marginal revenues and marginal costs applies to all \(t\)), and thus, by differentiation:

\(^4\text{Without concavity of the profit (of the demand) functions, price discrimination might even move all prices in the same direction: see Nahata et alii (1990).}

\(^5\text{Interestingly, one might think of (}\p(t),\lambda(t))\text{ as a dynamic system for which -}\Pi p(t)\text{ results a Liapunov function and (}\p^*,0)\text{ a globally stable equilibrium.}
\[
\sum_{i=1}^{N} \pi_i''(p_i(t))p_i'(t) = \sum_{i=1}^{N} [2q_i'(p_i(t)) + (p_i(t) - c_i)q_i''(p_i(t))]p_i'(t) = 0
\]  

(Schmalensee (1981: p. 244) provides a graphical analysis for the case with \( N = 2 \) and \( c_1 = c_2 \).)

Very generally, Varian (1985) established the following welfare bounds for any change in (linear) prices from \( p_0 \) to \( p_1 \) (the result follows from the convexity of \( v(\cdot) \)):

\[
p_0' \Delta q - \Delta C \geq \Delta W \geq p_1' \Delta q - \Delta C,
\]  

where \( \Delta q = q(p_1) - q(p_0) \), \( \Delta C = C(q(p_1)) - C(q(p_0)) \) and \( \Delta W = W(p_1) - W(p_0) \).\(^8\) Basically, (6) says that the price systems \( p_0 \) and \( p_1 \) provides useful information to assess the welfare effect of the change \( \Delta q \). In our setting, i.e., with \( p_0 = p^* t \) and \( p_1 = p^* \), (6) specializes to the linear (with respect to \( \Delta q \)) expression:

\[
(p^* t - c)' \Delta q \geq \Delta W \geq (p^* - c)' \Delta q,
\]  

which immediately says, as Varian (1985: p. 872) noted, that if marginal cost is the same across markets (i.e., \( c = c t \)) an increase in total output (\( t' \Delta q > 0 \)) is a necessary condition for welfare to increase following a change from uniform pricing (notice that this does not depend on \( p^* \) and \( p^* \) being chosen by a maximizing monopolist). This result, already established by Schmalensee (1981) a few years earlier, confirmed a conjecture that dates back to the seminal works of Arthur C. Pigou (1920: chapter 16) and Joan Robinson (1933: chapter 13).\(^9\)

In fact, it is easy to see that, if total output does not change, the change in welfare can be written \( \Delta W = \Delta v + \Delta_v p - \Delta C \), where \( \Delta_v p = q(p^*)'(p^* - p^* t) = q(p^*)' \Delta p \) is the Paasche price variation relevant for the representative consumer, and \( - \Delta v = v(p^* t) - v(p^*) \) is the relevant Hicksian equivalent variation. And it is a well-known result of consumer theory (see e.g. Deaton and Muellbauer, 1980: chapter 7) that the former variation is never larger than the latter (unless in the very special case of zero substitution effects, in which case they are equal). Thus, to avoid a welfare decrease following price differentiation (which causes a decrease in (gross) consumer surplus if total output does not change), we need a (strict) decrease in total cost. Notice that this result is due to the underlying preferences (and to the use of linear prices), and does not depend on the cost structure.

\(^8\) Notice that \( p_0' \Delta q \) and \( p_1' \Delta q \) are respectively the Laspeyres and Paasche quantity variations.

\(^9\) Varian (1985: p. 875) also showed that the result generalizes to the case of (common) increasing marginal costs, and Schwartz (1990) proved that it applies to the case of (common) decreasing marginal costs as well.
Following Schmalensee (1981: p. 244), by using (5) it is also easy to see that the change in total output, \( Q(p(t)) = \sum q_i(p_i(t)) \), according to \( t \) is given by:

\[
\frac{dQ(p(t))}{dt} = \sum_{i=1}^{\infty} q_i'(p_i(t)) p_i'(t) = -\frac{1}{2} \sum_{i=1}^{\infty} (p_i(t) - c_i) q_i''(p_i(t)) p_i'(t)
\]

\[
= \frac{1}{2} \sum_{i=1}^{\infty} \left[ \frac{q_i(p_i(t))}{q_i'(p_i(t))} \cdot (c_i(t) - c_i) \right] q_i''(p_i(t)) p_i'(t).
\]

This shows that total output does not change (i.e., \( i \Delta q = 0 \)) if the demand system is linear \( (q_i'' = 0, i = 1,N) \), as Schmalensee (1981) noted for the case with a common unit cost. This result, that again goes back to Pigou and Robinson, generalizes to the case of interdependent demands, and even to the case of common variable marginal costs: see Appendix A. Notice that the second term in the last expression of (8) can be signed according to the concavity of demand: i.e., it is positive if and only if demand of market \( i \) is strictly convex \( (q_i'' > 0) \).

### III. Results: equal elasticities

The two results concerning total output illustrated in the previous section are at the basis of the skeptical view of monopolistic differentiated pricing that we mentioned in section I. That view is rather widespread in spite of being in striking contrast with the related literature on price regulation, which tends to suggest that to allow some price flexibility to a monopolist is welfare enhancing: see e.g. Armstrong and Vickers (1991) and Vickers (1997). In fact, we believe that a negative assessment of price differentiation is generally unwarranted. Consider the following motivating example. Suppose that demands have the same elasticity at the uniform price \( p^* \), i.e., consider a case somehow “dual” with respect to the standard setting. Admittedly, as it turns out, this is the most favorable situation for allowing price differentiation, but consider that it should be difficult to defend the use of uniform pricing if demand were very different. Moreover, intuition (and results below) suggests that the real point is the correlation among the demand and the cost differences: why should we think of any specific correlation in the general case? Accordingly, we start with the no-correlation simplest case. Now suppose demands are linear. Then it must be the case that imposing uniform pricing does not change total output. In the 2-goods case the situation is depicted in Figure 1, where \( \Delta Q = 0 \) indicates the locus of prices which corresponds to the same total output \( Q(p^*t) \), and \( V = v(p^*t) \) is the relevant consumer surplus indifference curve. The vector \( q(p^*t) \) is orthogonal to the plane \( \Delta Q = 0 \) due to the assumption of equal demand elasticities. In fact, the no total-output change condition can be written: \(-\sum (q_i(p^*)e_i(p^*)/p^*) dp_i = 0\), where \( e(p_i) = -\)
\(q_i'(p_i) p_i / q_i(p_i)\) is the demand elasticity in market \(i\). Notice that any differentiated price vector actually chosen by a monopolist would increase consumer surplus, and thus social welfare by a trivial “revealed preference” result.

The previous argument depends on another well-known result of consumer theory (again see e.g. Deaton and Muellbauer, 1980: chapter 7): i.e., that the Laspeyres price variation \(\Delta L_D = q(p^* t) - q(p^* t)\Delta p\) is never larger than the Hicksian equivalent variation (the price locus \(\Delta L_D = 0\) just describes the hyperplane tangent to \(V = v(p^* t)\) at \(p^* t\)). Thus, a nonpositive Laspeyres price variation is a sufficient condition for a consumer surplus (and then welfare) increase. In our example of equal elasticities and linear demands, the Laspeyres price variation from uniform pricing is given by \(\sum_i q_i'(p^*) (p_i^* - p_i^*) = -p^* \epsilon(p^*) \sum_i q_i'(p^*) \Delta p_i\) (where \(\epsilon(p^*)\) is the common value of the demand elasticity), which is nonpositive if:

\[
\sum_{i=1}^N q_i'(p^*) \Delta p_i = \sum_{i=1}^N \Delta q_i = \Delta Q \geq 0,
\]

where \(\Delta Q = Q(p^*) - Q(p^* t)\).

Now suppose that demands are (strictly) concave: in that case, depicted by Figure 2, the constant total output locus will have a concave graph. In this situation (concave demands with equal elasticities at \(p^* t\)) any price differentiation by the monopolist that does not decrease total output would clearly realize a sufficient condition for a welfare improvement. Formally, in (9): \(q_i'(p^*) \Delta q_i \geq \Delta q_i\). Indeed, note that, if demands are concave, \(p^* t\) minimizes \(v(p)\) over the set \(\{p \mid Q(p) \geq Q(p^* t)\}\). This property of the uniform pricing (which does not depend on the cost structure) might come as a surprise, but it is just due to the familiar substitution effect. Suppose that, starting from uniform pricing, prices are differentiate in such a way that to an increase \(\Delta q_i > 0\) in a market \(i\) it corresponds a decrease \(\Delta q_i = -\Delta q_i < 0\) in a market \(j\). The first-order impact on the representative consumer’s utility due to this changes is given by \(p(\Delta q_i / \epsilon(p) + \Delta q_j / \epsilon(p)) = p \Delta q_i (1/\epsilon(p) - 1/\epsilon(p))\), where \(p\) is the uniform price. The previous expression is of course null if demand elasticities are equal at the uniform price, but the overall impact will be positive because the consumers will increase the demand of good \(j\) and decrease the demand of good \(i\), trading-off them against the other goods (in our quasi-linear preferences setting, substituting them with and for the numeraire). That is, in the case of both markets the first-order term overstates the true impact on consumer surplus (this is just a verbal restatement of the convexity of \(v(\cdot)\)). Notice that the first-order impact of a price change on the consumer surplus depends on the demand elasticity and the related quantity change.
Finally, suppose that demands are convex. In the case of isoelastic demands, i.e., if \( q_i(p_i) = k_i p_i^{-\epsilon_i} \), with \( k_i > 0 \) and \( \epsilon_i > 1 \), it is easy to see that, under the assumption of equal demand elasticities \( \epsilon_i = \epsilon \) for \( i = 1, N \), \( \Delta p = 0 \): see Appendix B. Accordingly, also in such a case monopolistic price differentiation increases total output, welfare and aggregate consumer surplus. More generally, even in the case of (possibly strictly) convex demand functions, a welfare improvement is achieved if the price change from the uniform price structure is “small” and output does not decrease. To see it formally, consider the change in welfare associated to \( t < t^* \):

\[
\frac{dW(p(t))}{dt} = \sum_{i=1}^{N} [(c_i(t) - c_i)q_i'(p_i(t))p_i'(t) - q_i(p_i(t))p_i'(t)] \\
= \sum_{i=1}^{N} (p_i(t) - c_i)q_i'(p_i(t))p_i'(t) \\
= \sum_{i=1}^{N} p_i(t) \frac{dq_i(p_i(t))}{dt} - \frac{dC(q(p(t)))}{dt}.
\]

The first term in the square bracket in (10) reflects the change in profit in market \( i \) and is always positive, while the second is the correspondent change in consumer surplus, which is positive only if \( i \in W \). Since for \( t \) large enough it will be \( p_i(t) > c_i \) for all \( i \), the second expression in (10) makes it clear that in general it is not possible to sign the overall welfare impact of the increase in \( t \) between 0 and \( t^* \). But note that welfare is increasing at \( t \) if, geometrically, the projection of \( dq(p(t))/dt \) on \( (p(t) - c) \) points in the same direction as the latter vector (i.e., if the inner product \( (p(t) - c) \cdot dq(p(t))/dt \) is positive). Since, the monopolist will choose the price change \( dp(t)/dt \) which achieves the maximum (available) value for \( (c(t) - c) \cdot dq(p(t))/dt \), this may point towards a wrong direction only if \( (p(t) - c(t)) \cdot dq(p(t))/dt = \sum q_i(p_i(t))p_i'(t) < 0 \). That is, if price rises are associated to the largest demands (so that consumer surplus decreases).

The third expression in (10) is just a continuous version of (7). It implies:

\[
\frac{dW(p(0))}{dt} = \sum_{i=1}^{N} p_i^* \frac{dQ(p(0))}{dt} - \sum_{i=1}^{N} c_i \frac{dq_i(p(0))}{dt},
\]

which means that a piecemeal policy allowing the introduction of (small) price differentiation would cause a welfare improvement by reducing the average total cost (unless in the “perverse” case in which quantity rises are on average associated to larger marginal costs), if it does not also reduce total output. Notice that \( c_i(0) = p_i^* m_i(p_i^*) \), where \( m_i(p_i) = \epsilon_i(p_i)/(\epsilon_i(p_i) - 1) \) is the monopolistic

Note that \( (p_i(t) - c_i) \) may even be negative in some market for some \( t \), but in such a case the correspondent element of the welfare change would certainly be positive.

Notice that \( dW(p(0))/dt = 0 \) if the demand system is linear and marginal cost is the same across markets.
mark up in market $i$. Thus, if demands have the same elasticity, in spite of them possibly being convex, $dq(p(0))/dt$ and $(p(0) - c(0)) = (p^* - c(0))t$ (where $c_i(0) = c(0), i = 1, N$) are at worst orthogonal if $dQ(p(0))/dt \geq 0$. In particular, in such a case, by using:

$$\frac{dC(q(p(t)))}{dt} = \sum_{i=1}^{N} c_i(t) \frac{dq_i(p(t))}{dt} - \lambda(t),$$

and $\lambda(0) = 1$, we get:

$$\frac{dW(p(0))}{dt} = \frac{p^*}{\varepsilon(p^*)} \frac{dQ(p(0))}{dt} + 1.$$  

Notice that the first term on the right-hand sides of (13) is just $dv(p(t))/dt$, which says that, for a small price differentiation, (aggregate) consumer surplus increases if and only if total output increases (if demands have the same elasticity).

In fact, it is not difficult to see that, under the assumption of equal demands, total output increase only if demand is strictly convex. In fact, by evaluating (8) at $t = 0$, it follows that, in the case of equal demands:

$$\frac{dQ(p(0))}{dt} = \frac{1}{2} \frac{q''(p^*)}{q'(p^*)^2} q(p^*) \frac{dQ(p(0))}{dt} - \lambda(0)$$

$$= -\frac{q''(p^*)q'(p^*)}{2q'(p^*)^2} - q(p^*)q''(p^*).$$

(14) says that, if demands are equal across markets, when allowing a small amount of price differentiation total output decreases if demand is strictly concave ($q'' < 0$) and increases if demand is strictly convex. In turn, (14) implies that:

$$\frac{dW(p(0))}{dt} = \frac{2q'(p^*)^2}{2q'(p^*)^2 - q(p^*)q''(p^*)},$$

which says that, if demands are equal across markets, a piecemeal policy introducing small price differentiation always increases welfare in our setting (see footnote 12). Of course, if marginal costs are all equal as well, as in the standard set up, the unregulated monopolist adopts $p^* = p^* t$, and the remove of the uniform pricing constraint has no positive effect (nor it harms too).

We summarize our results in the following Proposition.
Proposition 1.

I. If under uniform pricing the demand elasticities are equal across markets, monopolistic price differentiation does increase both welfare and aggregate consumer surplus:

1) if demands are linear;
2) if demands are isoelastic;
3) if demands are concave and total output does not decrease.

II. If only small differentiations are allowed (a so-called piecemeal policy):

4) welfare always increases under equal demands (if demand is strictly convex total output increases, raising consumer surplus as well);
5) welfare and (weakly) consumer surplus increase if total output does not decrease under equal demand elasticities;

More generally, intuition suggests that a monopolist freed by the uniform pricing constraint should be willing to use this freedom to decreases the average unit cost, even when leaving unchanged total output (as it happens in the benchmark linear case). And this would then be welfare improving, as suggested by the results above, and more generally by (7) in the case of discrete price variations. But if elasticities at the uniform price are different, it might happen that prices increase in markets which are more costly to serve (if demand elasticities are positively correlated to marginal cost), and the directions of cost and output variations become difficult to establish even for a small price differentiation. Moreover, consumer surplus might decrease more rapidly that profit increases, so reducing total welfare. This tends to happen if prices rise in markets where quantities are larger, which may well be the case since in those markets the demand elasticities tend to be smaller. Will indeed some of the previous results generalize to the case of unequal demand functions?

IV. Results: The linear case

Following Varian (1985: pp. 873-4), we should notice that, by (1), the right-hand side of (7) can be written: \( \sum c_i \Delta q_i (\varepsilon(p_i^*) - 1) \), where \( \Delta q_i = q_i(p_i^*) - q_i(p^*) \) (note that, if \( c_i / (\varepsilon(p_i^*) - 1) = (p_i^* - c_i) = \rho, i = 1, N \), then a weak increase in total output is sufficient for monopolistic price differentiation to produce a welfare improvement). Moreover, for concave demand functions, a sufficient condition for the right-hand side of (7) being non negative is that the Paasche price variation \( \Delta p \) is non positive.\(^{13} \) However, while they have different information requirements (the latter being

\(^{12}\) This is true under demand convexity if \( 2\varepsilon(p^*) < \eta(p^*) \), where \( \eta(p^*) = -p^* q''(p^*) / q'(p^*) \), but such a condition must be satisfied if the profit function has to be concave. Notice that the previous condition is always satisfied if demands are isoelastic.

\(^{13}\) Notice that this does not guarantee that consumer surplus does not decrease: again see e.g. Deaton and Muellbauer (1980: chapter 7).
considerably less demanding, since it does not involve unit costs), $\Sigma c_i \Delta q_i/(e_i(p^*_i) - 1) \geq 0$ and $\Delta p \leq 0$ are equivalent, sufficient conditions for a welfare improvement if the demand system is linear. In fact, in this case the welfare bounds in (7) become: $-\Delta C \geq \Delta W \geq -\Delta p$.\(^{14}\) We also know from the discussion in section II that a necessary condition for avoiding a welfare decrease due to price differentiation is that total cost does decrease, and that $\Delta \pi \leq 0$ is a sufficient condition for a consumer surplus (and then a welfare) improvement. Thus, it would be possible to check two (increasingly stronger) sufficient conditions for a consumer surplus or just for a welfare improvement simply by calculating the two relevant price variations. Note that these conditions are valid also for the case of strictly concave demands, and that their verification does not require cost knowledge.\(^{15}\)

Of course, in principle one can check by direct computation the welfare variations, but in the general case it seems difficult to derive unambiguous conclusions (we present some relevant statistics in Appendix C). Notice that it is easily computed that $p^* = (\text{Cov}\{c_i,b_i\} + a + bc)/2b$ and $p_i^* = (a_i + b_i c_i)/(2b_i)$, where $q_i(p_i) = a_i - b_i p_i (a_i, b_i > 0, a_i/b_i > c_i, i = 1,N)$, $\text{Cov}\{c_i,b_i\} = (\Sigma c_i b_i)/N - cb$ is the covariance between $c_i$ and $b_i$ across markets, and $a = (\Sigma a_i)/N, b = (\Sigma b_i)/N$ and $c = (\Sigma c_i)/N$ are respectively the average value of $a_i, b_i$ and $c_i$. Also notice that the previous expressions imply that the average value of $-b_i \Delta p_i = \Delta q_i$ is null, as we already knew. Finally, note that $(p_i^* - c_i) = a_i/(2b_i) - c_i/2$.

The simplest (unambiguous) case arises if $a_i/b_i$ is the same across markets (notice that this restricts demand variability only at a single point, perhaps very far from the quantity-price couples observed in the market).\(^{16}\) In that case $(p_i^* - p_i^*) = (c_i - c_j)/2$ and thus the only reason for monopolistic price differentiation is to reflect the marginal cost differences (in the standard setting with a marginal cost common across markets, one gets $p^* = p_i^*$ and allowing price differentiation has no effect at all).\(^{17}\) In our setting, one should then expect that relaxing the constraint of uniform pricing would increase welfare. Indeed, allowing price flexibility to the monopolist once again increases both welfare and consumers surplus. To see why, consider that in the linear case:

$$\Delta W = \frac{1}{2} [\Delta \Pi + \sum_{i=1}^{N} (a_i/b_i - c_i) \Delta q_i], \quad (16)$$

\(^{14}\) There seems to be a misprint in Varian (1985: p. 875) concerning the expression of the welfare change when the demand system is linear.

\(^{15}\) It might be worthy to note that the actual computation of these price variations is common practice in the industries regulated by price caps: see e.g. Armstrong and Vickers (1991).

\(^{16}\) The vertical intercept parameter of a linear demand function is sometimes identified with the “reservation price” (willingness to pay) of the “richest” consumer: see for example Tirole (1988: pp. 143-4). But in our setting the assumption of quasi-linear preferences and no income effect does preclude such an interpretation.
where \( \Delta \Pi = \Pi(p^*) - \Pi'(p^*)t \), and then:

\[
\Delta v = \frac{\sum_{i=1}^{N} (\frac{a_i}{b_i} - c_i) \Delta q_i - \Delta \Pi}{2}.
\]

(17)

Thus, the condition \( \Sigma_i (a_i/b_i - c_i) \Delta q_i > 0 \) is sufficient for a welfare improvement and necessary for a consumer surplus improvement. Moreover, that condition is equivalent to \( \Delta C < 0 \) if \( a_i/b_i = \kappa, i = 1,N \): i.e., if all demands have the same vertical intercept, a decrease in total cost is a necessary and sufficient condition for a welfare improvement. In such a case (see Appendix C for details):

\[
\Delta W = \frac{3N}{8} \left[ \frac{\sum_{i=1}^{N} (c_i b_i)^2}{N b_i} - \frac{1}{b} \left( \frac{\sum_{i=1}^{N} c_i b_i}{N} \right)^2 \right] = 3 \Delta v,
\]

which, since the function \( g(x,y) = x^2/y \) is convex on \( \mathbb{R}_{+}^2 \), is necessarily non negative by the Jensen’s inequality (notice that the right-hand side of (18) becomes \((3bN/8)Var\{c_i\}\) if the demand functions are just equal across markets).

To get a complete grasp of the previous result, consider the case of a piecemeal policy with linear, possibly different, demands. We already know that total output does not change along the path from 0 to \( t \) (see (8)), which implies that \( \Sigma_{b_i} p_i'(t) = 0 \). Moreover, (4) implies that \( c_i(t) = 2p_i(t) - a_i/b_i \): accordingly, marginal revenues (demand elasticities) are equal at \( t = 0 \), i.e., \( c_i(0) = c(0), i = 1,N \). We can immediately deduce from our analysis in section II that welfare increases and consumer surplus does not change following the introduction of small price differentiation. Note that, more generally, we get from (11) and (12):

\[
\frac{dW(p(0))}{dt} = \sum_{i=1}^{N} c_i \frac{dq_i(p_i(0))}{dt} + \sum_{i=1}^{N} (a_i/b_i) \frac{dq_i(p_i(0))}{dt} + 1,
\]

(19)

which says that a necessary and sufficient condition for a welfare improvement is a negative correlation between \( c_i \) and \( dq_i(p_i(0))/dt \), while a positive correlation between \( a_i/b_i \) (and thus \( c_i(0) \)) and \( dq_i(p_i(0))/dt \) would be necessary and sufficient for a consumer surplus improvement. Note that while the former condition looks quite natural, the latter seems to require some form of correlation between the demand parameters. Moreover, what goes wrong if marginal cost is the same across markets is that the increase in profit is exactly matched by the decrease in consumer surplus, so that

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17 Notice that this is nevertheless a case of price discrimination according to the Stigler’s broad definition we quoted in
there is no welfare change (in the case of a piecemeal policy). If, on the contrary, \( a_i/b_i \) is the same across markets, then there is no consumer surplus change and the increase in welfare is just due to cost reduction.

Note that a very special case arises if \( a_i/b_i - c_i = 2\rho, i = 1,N \). In such a case (16) and (17) imply that \( \Delta W = \Delta \pi/2 = - \Delta v > 0 \), and of course \( \Delta C < 0 \), unless there is no market variability at all and \( p^* = \bar{p} \) (notice that (19) implies \( dC(q(p(0))/dt = - \frac{1}{2} = dv(p(0))/dt = - dW(p(0))/dt \)). The reason is simple: in this case the Ramsey price vector \( p^* \) satisfies the second-best conditions \( (p_i^* - c_i) = \rho, i = 1,N \), we mentioned in section I, and thus \( p^* \) maximizes \( W(p) \) over the set \( \{ p | Q(p) = Q(p^*) \} \). But, at the same time, \( p^* \) minimizes \( v(p) \) over the previous set, since it equalizes \( q_i/q_i^* \) across markets. These results neatly illustrate the potential conflict between welfare versus consumer surplus concerns in our second-best setting. However, they require a good deal of cross demand and cost parameter correlation, which in general seems little plausible.

Two other cases can be analyzed a bit more. Suppose that \( a_i = a, i = 1,N \): i.e., all demands have the same horizontal intercept. In such a case it is easily computed that:

\[
\Delta C = \frac{aN}{2b} \text{Cov}\{c,b\} + \frac{N}{2b} \left\{ \sum_{i=1}^{N} \frac{c_i b_i}{N} \right\}^2 - \sum_{i=1}^{N} \left( \frac{c_i b_i}{N} \right)^2, \tag{20}
\]

Thus, a non positive covariance of demand slope and marginal cost would be sufficient to get a cost decrease by price differentiation (if all demands have the same horizontal intercept but marginal costs differ). Notice that a negative covariance means that at \( p^* \) (at \( t = 0 \)) higher marginal costs tend to be associated to smaller marginal revenues, so that indeed average total cost should be expected to decrease. The problem is that, under a negative covariance, price rises would also be associated to larger demands, so harming the representative consumer (as it is suggested by (17) and (19)). Actually, this is the way monopolistic price discrimination operates if marginal costs are equal across markets. In fact, even assuming no cross correlation between cost and demand parameters (why there should be any in the general case?), the sign of the welfare (and also of the consumer surplus) change keeps ambiguous (see (C6) in Appendix C).

Now suppose that \( b_i = b, i = 1,N \): in such a case:

\[
\Delta C = -\frac{N}{2} [\text{Cov}\{a,c\} + b \text{Var}\{c\}], \tag{21}
\]
where \( \text{Cov}\{a_i,c_i\} = (\Sigma a_i c_i) / N – a b \) is the covariance between \( a_i \) and \( c_i \) across markets. Notice that a non negative covariance of demand intercept and marginal cost is a sufficient condition to get a cost decrease (if all demands have the same slope and costs differ). Indeed, in this case a positive covariance means that at \( p^* \) higher marginal costs tend to be associated to smaller marginal revenue, so that, once again, average total cost should be expected to decrease. But again a welfare improvement cannot be guaranteed, because even if \( a_i \) and \( c_i \) were uncorrelated, prices would rise in the markets with the largest intercept values, and this has a negative first-order effect on consumer surplus.

In fact, it is easily computed (see Appendix C for details) that, with equal demand slope:

\[
\Delta W = \frac{N}{8b} [4b^2 \text{Var}\{c_i\} - \text{Var}\{a_i - bc_i\}] .
\]  

(22)

and

\[
\Delta v = \frac{N}{8b} [\text{Var}\{a_i - bc_i\} - 4\text{Var}\{a_i\}] .
\]  

(23)

(22) and (23) confirm that a welfare improvement cannot be guaranteed, since it depends on the relative variability of \( a_i \) and \( c_i \) and on their correlation. In particular, notice in (22) that if the quantity demanded at the competitive equilibrium level is the same across markets, then \( \Delta W \) achieves its maximum value. Of course this can only happen (with identical slope) if marginal costs are “perfectly” correlated with the intercept parameters (indeed, it requires \( a_i = b(c_i + 2\rho), i = 1,N\); see above), and then \( \Delta v \) gets its minimum value, as shown by (23). However, assuming \( \text{Cov}\{a_i,c_i\} = 0 \), it can also be shown that \( \text{Var}\{c_i\} > \text{Var}\{a_i\} / (3b^2) \) becomes a necessary and sufficient condition for a welfare improvement due to price differentiation, while \( \text{Var}\{c_i\} > 3\text{Var}\{a_i\} / b^2 \) becomes a necessary and sufficient condition for a consumer surplus increase (see (C7) in Appendix C).

The main results of this section are summarized in Proposition 2.

**Proposition 2.**

*If demands are linear, monopolistic price differentiation:*

1) has ambiguous welfare effect in the general case: however, one can use Laspeyres and Paasche price indexes to check sufficient conditions respectively for an aggregate consumer surplus or just a welfare improvement;
2) increases both consumer surplus and welfare if demands have the same vertical intercept;
3) without cross cost and demand parameter correlation, it decreases total cost if demands have the same horizontal intercept or the same slope, and in the latter case also raises
welfare (or even consumer surplus) if marginal cost variability is large enough with respect to demand variability.

V. Discussion and conclusions

In this paper we have re-considered the case for imposing uniform pricing to a monopolist in a setting with (possibly) differentiated marginal costs. As it should be expected, we find that such an imposition has ambiguous welfare results. However, in the case of equal demand elasticities at the uniform price (a case somehow dual to the one considered by the literature), it is generally socially better to allow some (perhaps mild) form of price differentiation (also an aggregate consumer surplus increase can be guaranteed if total output does not decrease). The main reason is that the price flexibility granted to the monopolist tends to produce socially valuable average cost reductions. Moreover, from the social point of view there is nothing of special in a uniform price (we show that it can be the “worst” pricing option), unless it is coupled with a common marginal cost. Similar results apply to the benchmark case of differentiated linear demands, if they have the same vertical intercept or, under null cross demand and cost parameter correlation (a reasonable assumption in general) and equal demand slope, if the variability of marginal cost is large enough with respect to demand variability. We conclude that the welfare criterion based on total output to allow price differentiation is not robust. On the contrary, sufficient conditions for achieving a consumer surplus improvement, or just a welfare increase (assuming demand concavity), can be checked by using the relevant Paasche and Laspeyres price indexes.

We see the previous results as no surprise. Philips (1981: p. 1) observed twenty years ago that, due to presence of some monopoly power: “generally, discriminatory prices will be required for an optimal allocation of resources in real life situation”. In fact, it is well known that in a second-best environment the imposition of an additional constraint may improve efficiency, but this is not always the case. It turns out that, even if output does not decrease, we cannot guarantee a welfare improvement by imposing to a monopolist a uniform pricing constraint, unless we are sure that the commodity is exactly the same. And apart from that case, the use of uniform pricing is just the addition of a further constraint to a second-best setting. The crucial element is then the assumption of a single commodity. Several authors noticed this difficulty (see e.g. Philips, 1981: pp. 5-9 and Varian, 1989: p. 599), and it should be clear that “similar” commodities are just not the same commodity if they have different marginal costs. Moreover, by now many years of “new” regulation theory have taught us that we cannot assume that cost differences are easily disclosed, not to say incorporated in a regulation practice. Indeed, one message of the recent price regulation
literature stresses the (at least potential) social usefulness of allowing some price flexibility to a (regulated) monopolist (see e.g. Vickers, 1997), and it has inspired this paper.

This is not to mean that we disregard the contributions of the by now classic works by Schmalensee (1981), Varian (1985) and Schwartz (1990), or, needless to say, of the original intuitions by Pigou (1920) and Robinson (1933). In particular, we have used the model by Schmalensee (1981), and ideas and results of all the authors quoted above, to argue that, as a policy prescription, uniform pricing is not robust, even when corroborated by the requirement that total output should not increase by allowing price differentiation. Nowadays, it should be easy to maintain the idea that possible social gains in the form of cost reductions should not be dismissed. And that the computation of well-defined Laspeyres or Paasche (under demand concavity) price indexes should replace the measurement of total output changes to serve consumer surplus or social welfare concerns (indeed, we owe these results to standard consumer theory, as first shown by Varian, 1985).

It is also worth emphasizing that our results would not sustain a policy of complete liberalization of monopolistic price behavior. We have just argued that imposing price uniformity to an otherwise unregulated monopolist is generally not a promising strategy. Moreover, the monopolistic strategic (anti-competitive and anti-regulative) tactics are probably too rich to be described by a simple model, and we did not even attempted here (so our conclusions do not necessarily conflict with the rules of the Robinson-Patman Act). Nor we have considered the use of uniform pricing in “competitive” environments, a topic only very recently investigated by the economic literature (see e.g. Stole, 2001). In the latter case, in particular, it is for example well known that the European Authorities have reasons to somewhat endorse a policy of uniform pricing that relate to the making of the European common market and are not based on welfare considerations (even when they do not explicitly disagree with: see e.g. Cabral, 2000: paragraph 10.5). And a similar comment would apply to the political constraints (mentioned in section I) based on “horizontal equity” concerns (even if the latter may be weakened by the result that price differentiation can increase aggregate consumer surplus).

Finally, we stress that we have assumed throughout the paper that the monopolist possesses all the demand and cost information needed to maximize profit, very much as in the “Ramsey-Boiteux” tradition (also see Schwartz, 1990: p. 1262). Was this not the case, it seems to us that it would be even more difficult to sustain the use of a uniform pricing constraint: why should the policymaker be assumed to know more than the monopolist? In particular, if the differentiated pricing structure just reflects the cost knowledge of the monopolist, who is unable to get reliable
demand (elasticity) information (as in the case of the so-called “Allais doctrine”), it is hard to imagine some economically sound reasons to defend the use of a uniform price.

Appendix A

We wish to show that even if the demands are not independent (i.e., \( \frac{\partial q_i}{\partial p_j} \) is possibly non zero for some \( i \neq j, i,j = 1,N \)), relaxing the constraint of a uniform pricing does not change total output if the demand system is linear and marginal costs are either constant or common. We will assume that the correspondent FOC conditions do characterize the solution of the relevant profit maximization programs. Equations (1) become:

\[
\frac{\partial \Pi(p^*)}{\partial p_i} = q_i(p^*) + \sum_{j=1}^{N} \left( p_j^* - \frac{\partial C(q(p^*))}{\partial q_j} \right) \frac{\partial q_j(p^*)}{\partial p_i} = 0, \quad (A1)
\]

and by adding up

\[
\sum_{i=1}^{N} \frac{\partial \Pi(p^*)}{\partial p_i} = \sum_{i=1}^{N} q_i(p^*) + \sum_{i=1}^{N} \sum_{j=1}^{N} \left( p_j^* - \frac{\partial C(q(p^*))}{\partial q_j} \right) \frac{\partial q_j(p^*)}{\partial p_i} = 0, \quad (A2)
\]

i.e., by using linearity of the demand system

\[
2Q(p^*) - Q(0) - \sum_{i=1}^{N} \sum_{j=1}^{N} q_{ij} \frac{\partial C(q(p^*))}{\partial q_j} = 0, \quad (A3)
\]

where \( q_{ij} = \frac{\partial q_i}{\partial p_j}, i,j = 1,N \). Similarly, (2) becomes:

\[
\frac{d \Pi(p^*)}{dp^*} = \sum_{i=1}^{N} \left( p^* - \frac{\partial C(q(p^*)}{\partial q_i} \right) \sum_{j=1}^{N} \frac{\partial q_j(p^*)}{\partial p_j} + q_i(p^*) = 0, \quad (A4)
\]

i.e.,

\[
2Q(p^*) - Q(0) - \sum_{i=1}^{N} \sum_{j=1}^{N} q_{ij} \frac{\partial C(q(p^*)}{\partial q_j} = 0 \quad (A5)
\]

by linearity. Finally, notice that, since \( q_{ji} = q_{ij} \), the left-hand side of (A3) and (A5) are identical function with respect to \( Q \) if either marginal costs \( \frac{\partial C(q)}{\partial q} \) are constant or they are equal and only depend on total output \( Q \). QED.

\(^{18}\) The symmetry property \( q_{ji} = q_{ij} \) follows from quasi linearity of consumers’ preferences: also see Layson (1998).
Appendix B

Assuming isoelastic demands, equations (1) become:

\[ \pi_i^*(p_i^*) = -\varepsilon_i k_i (p_i^* - c_i)(p_i^*)^{\varepsilon_i - 1} + k_i (p_i^*)^{-\varepsilon_i} = 0, \]  

(B1)

which imply \( p_i^* = (\varepsilon_i c_i)/(\varepsilon_i - 1) \). Similarly, equation (2) becomes:

\[ \sum_{i=1}^{N} \pi_i^*(p_i^*) = \sum_{i=1}^{N} (p_i^*)^{-\varepsilon_i} [-\varepsilon_i k_i (1 - c_i/p_i^*) + k_i] = 0, \]  

(B2)

which implies, assuming \( \varepsilon_i = \varepsilon \) (\( i = 1, N \)), \( p_i^* = (\varepsilon Cov\{c_i, k_i\}/k + \varepsilon c_k)/(\varepsilon - 1) \), where \( Cov\{c_i, k_i\} = (\sum c_i k_i)/N - c_k \) is the covariance between \( c_i \) and \( k_i \) across markets, and \( k = (\sum k_i)/N \) is the average value of \( k_i \). Since \( \Delta p_i = \varepsilon c_i - Cov\{c_i, k_i\}/k)/(\varepsilon - 1) \), it follows that:

\[ \Delta_L p = \sum_{i=1}^{N} q_i(p_i^*) \Delta p_i = \left[ \frac{\varepsilon}{\varepsilon - 1} \right] \left[ Cov\{c_i, k_i\} k \right] + \left[ \varepsilon \right] \left[ \sum_{i=1}^{N} (k_i c_i) - N k c - NCov\{c_i, k_i\} \right] = 0. \]  

(B3)

Appendix C

In the case of a linear demand system, direct computation shows that the following equalities hold:

\[ \Delta C = \frac{N}{2b} [Cov\{b_i, c_i\} (a + 2bc + Cov\{b_i, c_i\}) - b Cov\{a, c_i\} + b (bc - \sum_{i=1}^{N} b c_i^2)/N]. \]  

(C1)

\[ \Delta_L p = \frac{N}{2b} [b Cov\{a, c_i\} - a Cov\{b_i, c_i\} + b (bc - \sum_{i=1}^{N} a_i^2)/N]. \]  

(C2)

\[ \Delta_P p = \frac{N}{4b} [(Cov\{b_i, c_i\})^2 + 2bc Cov\{b_i, c_i\} + b (b c^2 - \sum_{i=1}^{N} a_i^2)/N]. \]  

(C3)

Notice that the last term in the square bracket in (C2) (the third term in the square bracket in (C3)) is non negative by Jensen’s inequality. Also notice that, if the demand parameters are not correlated with the marginal costs (perhaps the interesting case), we get \( \Delta C \leq 0 \) and \( \Delta_L p \geq 0 \), while \( \Delta_P p \) cannot
be signed (the last term in the square bracket in (C1) and (C3) is then certainly non positive). And remember that, by a general property, \(-\Delta p \geq \Delta v \geq -\Delta \bar{p}\). A little bit more can be said if demand parameters, in addition of not being correlated to marginal costs, are also uncorrelated between them. In particular, if (without demand functions and marginal costs being all equal) \(a_i = a\) (\(i = 1, N\), we get \(\Delta C = (N/2)[bc^2 - (\Sigma b_i c_i^2/N)] < 0, \Delta \bar{p} = (a^2/2)[(\Sigma a_i /b_i) - N/b] > 0\) and \(\Delta \bar{v} = (N/4)[a^2[\Sigma (Nb_i)^{-1} - 1/b] + bc^2 - (\Sigma b_i c_i^2/N)];\) while, if \(b_i = b\) (\(i = 1, N\), we get \(\Delta C = -(bN/2)\text{Var}\{c_i\} < 0, \Delta \bar{p} = (N/2)b\text{Var}\{a_i\} > 0\) and \(\Delta \bar{v} = (N/4)b^2\text{Var}\{c_i\} > 0\). This neatly illustrates the role of the relative variance of demand and cost parameters. Note that, in the latter case, \(b^2\text{Var}\{c_i\} > \text{Var}\{a_i\}\) is a sufficient condition for price differentiation delivering a welfare improvement.

Moreover, if (without assuming away any cross demand and cost parameter correlation) \(a_i/b_i\) is the same across markets, then we get \(\Delta \bar{p} = \Delta \bar{v}/2 = (N/4)[(\Sigma a_i^2/b_i) - N(a^2/b)] \geq 0\) and \(\Delta v = 0\), which implies that consumer surplus (and thus welfare) increases if the monopolist actually differentiates prices (i.e., if marginal costs are different). In the special case in which the demand functions are equal (i.e., \(a_i = a\) and \(b_i = b\), \(i = 1, N\); see section III) it turns out that \(\Delta C = -(bN/2)\text{Var}\{c_i\} = 2\Delta \bar{v} \leq 0\): notice that this means that half of the cost decrease is “passed” to the representative consumer in the form of a revenue reduction. Also notice that, if \(c_i = c\), \(i = 1, N\) (the standard set up in the literature), \(\Delta C = 0\) (and thus welfare decreases if the monopolist does not differentiate prices), and \(\Delta \bar{p} = \Delta \bar{v}/2 = 1/[4(\Sigma a_i^2/b_i) - N(a^2/b)] \geq 0\).

In addition, one can compute that:

\[
\Delta W = \sum_{i=1}^{N} \frac{[(p_i^* - c_i) + (p^* - c_i)]\Delta q_i}{2} = \frac{1}{2} \sum_{i=1}^{N} [(p^* - c_i)(q_i(c) - q_i(p^*)) - (p_i^* - c_i)(q_i(c) - q_i(p^*))]
\]

\[
= -\frac{1}{2} \sum_{i=1}^{N} b_i \Delta p_i (p_i^* + p^* - 2c_i) = -\frac{1}{2} \sum_{i=1}^{N} b_i \Delta p_i \sum_{i=1}^{N} (p_i^* - 2c_i) = \frac{1}{4} \sum_{i=1}^{N} b_i \Delta p_i [3c_i - a_i]
\]

\[= \frac{N}{8b} \left[ b \frac{a_i^2}{b} - \sum_{i=1}^{N} \frac{a_i^2}{Nb_i} \right] + 2b \text{Cov} \{b, c_i\} + 2b \text{Cov} \{a, c_i\} + 3b \left( \sum_{i=1}^{N} \frac{(c_i b_i)^2}{Nb_i} \right) - \frac{1}{b} \left( \sum_{i=1}^{N} \frac{c_i b_i}{N} \right)^2.
\]

and, finally:

\[19\text{ This term is clearly related to the variability of } a/b_i.\]
\[ \Delta v = \frac{1}{2} \sum_{i=1}^{N} \left[ \frac{a_i}{b_i} - p_i^* q_i(p_i^*) - \frac{a_i}{b_i} - p^* q(p^*) \right] \]

\[ = \frac{1}{2} \sum_{i=1}^{N} b_i \Delta p_i [p_i^* + p^* - 2a_i/b_i] = \frac{1}{2} \sum_{i=1}^{N} b_i \Delta p_i [p_i^* - 2a_i/b_i] = \frac{1}{4} \sum_{i=1}^{N} b_i \Delta p_i [c_i - 3a_i/b_i] \]

\[ = \frac{N}{8} \left[ 3 \left( \frac{a^2}{b} - \sum_{i=1}^{N} \frac{a_i^2}{Nb_i} \right) + \frac{2a}{b} \text{Cov} \{b_i, c_i\} - 2 \text{Cov} \{a_i, c_i\} + \left( \sum_{i=1}^{N} \frac{c_i b_i}{Nb_i} - \frac{1}{b} \left( \frac{1}{N} \sum_{i=1}^{N} c_i b_i \right) \right)^2 \right]. \] (C5)

Notice that the first terms in the square brackets of the last expressions in (C4) and (C5) are nonpositive, while the last terms are non-negative. Also note that, as anticipated above, if \(a_i/b_i\) is the same across markets, allowing monopolistic price differentiation delivers a consumer surplus and welfare improvement: \(\Delta W = (3N/8)\text{Var} \{c_i\} - (1/N)\text{Var} \{a_i\}/b_i \) if demand functions are just equal across markets: see section III). If, on the contrary, marginal costs are equal across markets, monopolistic price differentiation (discrimination) causes a welfare decrease, as it is well known: \(\Delta W = (N/8)(\Sigma_i (a_i^2/b_i) - 1/N)\text{Var} \{c_i\} \text{Var} \{a_i\}/b_i \)

More generally, however, even with a null cross demand and cost parameter correlation, (C4) and (C5) have ambiguous signs. In particular, in such a case they reduce to:

\[ \Delta W = \frac{N}{8b} \left[ a_i^2 \left( 1 - \sum_{i=1}^{N} \frac{b_i}{Nb_i} \right) + 3b \left( \sum_{i=1}^{N} \frac{c_i^2 b_i}{N} - c^2 b \right) \right], \] (C6)

\[ \Delta v = \frac{N}{8b} \left[ 3a_i^2 \left( 1 - \sum_{i=1}^{N} \frac{b_i}{Nb_i} \right) + b \left( \sum_{i=1}^{N} \frac{c_i^2 b_i}{N} - c^2 b \right) \right], \]

and

\[ \Delta W = \frac{N}{8b} \left[ -\text{Var} \{a_i\} + 3b^2 \text{Var} \{c_i\} \right], \] (C7)

\[ \Delta v = \frac{N}{8b} \left[ -3\text{Var} \{a_i\} + b^2 \text{Var} \{c_i\} \right], \]

respectively if \(a_i = a\) or \(b_i = b\) \((i = 1, N)\). Note that, in the latter case, (C7) says that \(\text{Var} \{c_i\} > \text{Var} \{a_i\}/(3b^2)\) is a necessary and sufficient condition for a welfare improvement due to price differentiation, while \(\text{Var} \{c_i\} > 3\text{Var} \{a_i\}/b^2\) is a necessary and sufficient condition for a consumer surplus increase.
References


Figure 1: Uniform pricing with linear demands, equal elasticites

Figure 2: Uniform pricing with concave demands, equal elasticites
Dipartimento di economia politica e metodi quantitativi
Università degli studi di Pavia

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