Firm Entry, Endogenous Markups and the Dynamics of the Labor Share

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Abstract

Recent U.S. evidence suggests that the response of the labor share to a productivity shock is characterized by countercyclicality and overshooting. These findings cannot be easily reconciled with existing business cycle models. We extend the standard model of search and matching in the labor market by considering strategic interactions among an endogenous number of producers. This leads to countercyclical price markups. While Nash bargaining is sufficient to capture the labor share countercyclicality, we show that countercyclical markups are key to address the overshooting.

\textit{JEL classification: E24, E32, L11.}

Keywords: Endogenous Market Structures, Oligopolistic Competition, Firms’ Entry, Search and Matching Frictions, Labor Share Overshooting.

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1 Introduction

In this paper we develop a theory of the joint dynamics of the labor share and technology shocks. We show that a search and matching model with entry of firms and oligopolistic competition in the goods market reproduces the countercyclicality and the overshooting of the labor share in response to a productivity innovation, identified by Rios-Rull and Santaeulàlia-Llopis (2010) for the U.S. economy.

![Image](image1.png)

Figure 1. The dynamics of the Labor Share. Source: Rios - Rull and Santaeulàlia-Llopis (JME 2010)

Figure 1, shows the labor’s share response to orthogonalized productivity innovations for the U.S. in the period 1954:1–2004:4. Notice that after falling by more than one percent on impact, the labor share shows an hump-shaped response, overshooting its long-run level after five quarters, and peaking at the fifth year at a level larger in absolute terms than the initial drop. Seven years after the peak the labor share is still half-way toward its steady state value.

As noticed by Rios-Rull and Santaeulàlia-Llopis (2010) standard business cycle models cannot explain both the countercyclicality and the overshooting of the labor share. In particular in the RBC model the wage and labor productivity move identically, so that the labor share displays no deviations from its steady state value in response to a productivity shock. Models with search and matching in the labor market brake the close connection between the wage and labor productivity resorting to Nash bargaining. This explains the countercyclicality, but does not help replicating the labor share overshooting. As a result the

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2Rios-Rull and Santaeulàlia-Llopis (2010) show that the labor share is also weakly correlated with output.
latter remains a puzzle. We argue that a search and matching model augmented with endogenous entry and oligopolistic competition in the goods market matches both the countercyclicality and the overshooting of the labor share. The key mechanism to replicate the overshooting result is the countercyclicality of price markups originating from oligopolistic competition.

To the best of our knowledge we are the first to present a model addressing the overshooting of the labor share resorting to countercyclical markups. Choi and Rios-Rull (2010) obtain overshooting considering a model with putty-clay technology, decentralized non-competitive wage setting (bilateral Nash bargaining) and an aggregate technological shock that has a stronger effect for newer hires. In this case, the overshooting is due to the particular specification of the process for technology. The process for technology that we adopt is, instead, fully standard. Shao and Silos (2011) derive their result using a rather different framework. They also consider a model with costly entry of firms and a frictional labor market. However, their model is characterized by monopolistic competition and thus by constant price markups. In their framework overshooting is due to the countercyclical value of vacancies. Unfortunately, it is difficult to find an empirical measure of the asset value of a vacancy. On the contrary, our transmission mechanism is well supported by the empirical evidence. Bils (1987), Rotemberg and Woodford (2000) and Gali et al. (2007) forcefully document price markup countercyclicality. Importantly, to save space we make our point using Cournot competition, but the same results hold under Bertrand competition.

2 The Model

2.1 Labor and Goods Markets

There are two main building blocks in the model: oligopolistic competition with endogenous entry in the goods market and search and matching frictions in the labor market. In this paragraph we outline their main features.

As in Colciago and Etro (2010) the economy features a continuum of sectors, or industries, on the unit interval. Sectors are indexed with \( j \in (0, 1) \). Each sector \( j \) is characterized by different firms \( i = 1, 2, ..., N_j \) producing the same good in different varieties. At the beginning of each

\(^3\)Choi and Rios-Rull (2009), consider alternative search and matching models with Nash bargaining and show that none of these models can replicate the labor share overshooting.

\(^4\)Results on Bertrand competition are available upon request. See also Colciago and Rossi (2011).
period $N_j^{e_t}$ new firms enter into sector $j$, while at the end of the period a fraction $\delta \in (0, 1)$ of market participants exits from the market for exogenous reasons.

The labor market is characterized by search and matching frictions, as in Andolfatto (1996) and Mertz (1995). A fraction $u_t$ of the unit mass population is unemployed at time $t$ and searches for a job. Firms producing at time $t$ need to post vacancies in order to hire new workers. Unemployed workers and vacancies combine according to a CRS matching function and deliver $m_t$ new hires, or matches, in each period. The matching function reads as $m_t = \gamma_m (v_t^{tot})^{1-\gamma} u_t^{\gamma}$, where $\gamma_m$ reflects the efficiency of the matching process, $v_t^{tot}$ is the total number of vacancies created at time $t$ and $u_t$ is the unemployment rate. The probability that a firm fills a vacancy is given by $q_t = \frac{m_t}{v_t^{tot}}$, while the probability to find a job for an unemployed worker reads as $z_t = \frac{m_t}{u_t}$. Firms and individuals take both probabilities as given. Matches become productive in the same period in which they are formed. Each firm separates exogenously from a fraction $1 - \phi$ of existing workers each period, where $\phi$ is the probability that a worker stays with a firm until the next period.

As a result a worker may separate from a job for two reasons: either because the firm where the job is located exits from the market or because the match is destroyed. Since these sources of separation are independent, the evolution of aggregate employment, $L_t$, is given by $L_t = (1 - \delta) q L_{t-1} + m_t$. Thus, the number of unemployed workers searching for a job at time $t$ is $u_t = 1 - L_{t-1}$.

### 2.2 Households and Firms

Using the family construct of Mertz (1995) we can refer to a representative household consisting of a continuum of individuals of mass one. Members of the household insure each other against the risk of being unemployed. The representative family has lifetime utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln C_{jt} dj - \chi L_t \frac{h_{t}^{1+1/\varphi}}{1+1/\varphi} \right\} \chi, \varphi \geq 0$$

where $\beta \in (0, 1)$ is the discount factor and the variable $h_t$ represents individual hours worked. Note that $C_{jt}$ is a consumption index for a set of goods produced in sectors $j \in [0, 1]$, defined as

$$C_{jt} = \left[ \sum_{i=1}^{N_{jt}} C_{jt}(i) \frac{\varepsilon^{-1}}{\varepsilon} \right]^{\frac{1}{\varepsilon-1}}$$

where $C_{jt}(i)$ is the production of firm $i$ of this sector, and $\varepsilon > 1$ is the elasticity of substitution between the goods belonging to the same sec-
tor. The distinction between different sectors and different goods within a sector allows to realistically separate limited substitutability at the aggregated level, and high substitutability at the disaggregated level.\footnote{Contrary to many macroeconomic models with imperfect competition, our focus will be on the market structure of disaggregated sectors: intrasectoral substitutability (between goods produced by firms of a same sector) is high, while intersectoral substitutability is low.} The family receives real labor income \( w_t h_t L_t \) and profits from the ownership of firms. Further, we assume that unemployed individuals receive an unemployment benefit \( b \) in real terms, leading to an overall benefit for the household equal to \( b(1 - L_t) \). This is financed through lump sum taxation by the government. Notice that the household recognizes that employment is determined by the flows of its members into and out of employment according to

\[
L_t = (1 - \delta) q L_{t-1} + z_t u_t
\]  

Households choose how much to save in riskless bonds and in the creation of new firms through the stock market according to standard Euler and asset pricing equations.\footnote{For a detailed description and derivation of all the equations of the model see Colciago and Rossi (2011).}

Each firm \( i \) in sector \( j \) produces a good with a linear production function. We abstract from capital accumulation issues and assume that labor is the only input. Output of firm \( i \) in sector \( j \) is then:

\[
y_{jt}(i) = A_t n_{jt}(i) h_{jt}(i)
\]  

where \( A_t \) is the, common to all sectors, total factor productivity at time \( t \), \( n_{jt}(i) \) is firm \( i \)'s time \( t \) workforce and \( h_{jt}(i) \) represent hours per employee. Since each sector can be characterized in the same way, in what follows we will drop the index \( j \) and refer to the representative sector.

### 2.3 Endogenous Market Structures

Following BGM (2007) we assume that new entrants at time \( t \) will only start producing at time \( t + 1 \). Given the exogenous exit probability \( \delta \), the average number of firms per sector, \( N_t \), follows the equation of motion:

\[
N_{t+1} = (1 - \delta)(N_t + N_t^e)
\]  

where \( N_t^e \) is the average number of new entrants at time \( t \). Households share equally their expenditure among sectors due to the unit intersectoral elasticity of substitution. We define the common level of nominal
expenditure as $EXP_t$. In this case, the direct demand function for good $i$ reads as:

$$y_t(i) = Y_t \left( \frac{p_t(i)}{P_t} \right)^{-\varepsilon} = \frac{p_t(i)^{-\varepsilon} EXP_t}{P_t^{1-\varepsilon}} \quad i = 1, 2, ..., N_t \quad (6)$$

where $P_t$ is the price index $P_t = \left[ \sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)} \right]^{-\frac{1}{\varepsilon-1}}$, such that $EXP_t = \sum_{j=1}^{N_t} p_t(j)C_t(j) = C_tP_t$.\(^7\) Period $t$ real profits of an incumbent producer are defined as

$$\pi_t(i) = \rho_t(i) y_t(i) - w_t(i) n_t(i) h_t(i) - \kappa v_t(i) \quad (7)$$

where $\rho_t(i) = \frac{p_t(i)}{P_t}$ is the real price of firm $i$’s output, $v_t(i)$ represents the number of vacancies posted at time $t$ and $\kappa$ is the output cost of keeping a vacancy open.

Incumbent firms which do not exit from the market have a time $t$ individual workforce given by

$$n_t(i) = n_{t-1}(i) + v_t(i) q_t \quad (8)$$

We consider competition in quantities à la Cournot, which has been largely neglected in general equilibrium macroeconomic models with imperfect competition. In this case firms maximize the expected discounted value of its profits choosing their production $y_t(i)$ beside $n_t(i)$ and $v_t(i)$, taking as given the production of the other firms. As shown in Colciago and Rossi (2011) the symmetric Cournot equilibrium generates the individual output

$$y_t = \frac{\varepsilon - 1}{\varepsilon} \frac{N_t - 1}{N_t^2} \frac{EXP_t}{MC_t} \quad (9)$$

where $MC_t$ is the nominal marginal cost.\(^8\) The associated equilibrium mark up is

$$\mu(\varepsilon, N_t) = \frac{\varepsilon N_t}{(\varepsilon - 1)(N_t - 1)} \quad (10)$$

which remains positive for any degree of substitutability and is decreasing and convex in the number of firms with elasticity $\varepsilon_N = 1/(N - 1)$.

\(^7\)The demand of the individual good and the price index are the solution to the, usual, consumption expenditure minimization problem.

\(^8\)See Colciago and Rossi (2011) for the Bertrand equilibrium.
Notice that the individual level of output is common between incumbent firms and firms which produce for the first time. In the symmetric equilibrium, all producing firms have the same size no matter their period of entry. It follows that the optimal hiring policy of time $t - 1$ new entrants, which have no initial workforce, consists in posting at time $t$ as many vacancies as required to reach the size of incumbent producers. Given vacancy posting is costly they will suffer lower profits, pay lower dividends and grow faster with respect to incumbent producers. This is consistent with the U.S. empirical evidence in Haltiwanger et al. (2010) and Cooley and Quadrini (2001), which suggests that start-ups creates on average more new jobs than an incumbent firm and distribute lower dividends.

2.4 Entry and Job Creation

We assume that entry requires a fixed cost $\psi$, which is measured in units of output. In each period the level of entry is determined endogenously to equate the value of a new entrant, $V^e_t$, to the entry cost$^{10}$

$$V^e_t = \psi$$

(11)

Profits maximization implies the following Job Creation Condition (JCC)

$$\frac{\kappa}{q_t} = \left(\frac{p_t}{m_t} - \frac{w_t}{A_t}\right)A_t h_t + \sigma E_t N_{t,t+1} \frac{\kappa}{q_{t+1}}$$

(12)

The JCC equates the real marginal cost of hiring a worker, the left hand side, with the marginal benefit, the right hand side. Importantly, the marginal benefit depends positively on the ratio $\frac{p_t}{m_t}$, which is a positive function of the number of firms in the market, $N_t$. As the number of competitors increases, agents consume more goods and enjoy higher welfare for any given level of nominal expenditure. For this reason the welfare based price level must decrease and the relative price of variety $i$ increases. This increases the profitability of the marginal worker. At the same time, stronger competition leads to a lower markup, stimulating demand by consumers and thereby increasing output. As shown by Colclough and Rossi (2011), a positive technology shock leads to entry of new firms and thus to an increase in $\frac{p_t}{m_t}$. In equilibrium, since hiring depends on the current and expected future values of the marginal product of labor, this boosts hiring and employment.$^{11}$

$^9$For a proof see Colclough and Rossi (2011).

$^{10}$The value of a new entrant is determined by the discounted value of its future profits. The discount factor takes into account the exit probability $\delta$.

$^{11}$Notice that the model features love for variety. This means that a rise in the range
2.5 Bargaining over Wages and Hours

As in the bulk of the literature we assume Nash wage bargaining, which results in the following wage equation

\[ w_t = (1 - \eta) \frac{b}{h_t} + \eta \frac{\rho_t}{\mu_t} A_t + (1 - \eta) \chi c_t \frac{h_t^{1/\varphi}}{(1 + 1/\varphi)} + \frac{\eta \kappa}{(1 - \delta) h_t} E_t A_{t,t+1} \theta_{t+1} \]

(13)

where \( \frac{\mu_t}{q_t} = \theta_t \), \( A_{t,t+1} = (1 - \delta) \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \) and, importantly, \( \frac{\mu_t}{q_t} = m c_t \). The direct effect of entry on the real wage is captured through the term \( \eta \frac{\mu_t}{q_t} A_t \). Notice that \( \frac{\mu_t}{q_t} A_t \) represents the marginal revenue product (MRP) of labor, while \( \eta \) represents the share of the MRP which goes to workers.

As described above, entry leads to an increase in the MRP of labor. Thus, \textit{ceteris paribus}, stronger competition shifts the wage curve up. This result is similar to that in Blanchard and Giavazzi (2003), who find a positive effect of competition on the real wage.

Hours are set to maximize the joint surplus of the match. The FOC with respect of \( h_t \) is

\[ \chi c_t h_t^{1/\varphi} = \frac{\rho_t}{\mu_t} A_t. \]

(14)

Hours worked are such that the marginal rate of substitution between hours and consumption equals the MRP of labor. Stronger competition leads to an increase in hours bargained between the workers and firms for the same reasons for which competition positively affects the wage schedule.

2.6 Aggregation and Market Clearing

Considering that the individual workforce, \( n_t \), is identical across producers leads to

\[ L_t = n_t N_t \]

(15)

To obtain aggregate output notice that \( P_t Y_t = \sum_{i=1}^{N_t} p_t y_t = N_t p_t y_t \), further given \( \rho_t = \frac{\mu_t}{q_t} \) and the individual production function it follows that

\[ Y_t = \rho_t N_t y_t = \rho_t A_t L_t h_t \]

(16)

Technology is assumed to follow a first order autoregressive process given by \( \ln (A_t/A) = \rho_A \ln (A_{t-1}/A) + \varepsilon_A \), where \( \rho_A \in (0, 1) \) and \( \varepsilon_A \) is a white noise disturbance, with zero expected value and standard deviation \( \sigma_A \).
Aggregating the budget constraints of households we obtain the aggregate resource constraint of the economy
\[ C_t + \psi N_t^e = W_t h_t L_t + \Pi_t \] (17)
which states that the sum of consumption and investment in new entrants must equal the sum between labor income and aggregate profits, \( \Pi_t \), distributed to households at time \( t \). Aggregate profits are defined as
\[ \Pi_t = (1 - \delta) N_{t-1} \pi_t + [N_t - (1 - \delta) N_{t-1}] \pi_t^{NP} \] (18)
where \( (1 - \delta) N_{t-1} \) is the number of mature incumbent producers, and \( N_t - (1 - \delta) N_{t-1} \) is the number of time-\( t \) first period incumbent firms. Goods’ market clearing requires
\[ Y_t = C_t + N_t^E \psi + \kappa v_t^{tot} \] (19)
Finally, the dynamics of aggregate employment reads as
\[ L_t = (1 - \delta) g L_{t-1} + q_t v_t^{tot} \] (20)
which shows that workers employed to a firm which exits the market join the mass of unemployed.

3 Labor Share Dynamics

In what follows we study the impulse response functions of the labor share and its components to a one percent increase in technology. To disentangle the role of entry from that of the endogenous price markup we compare the performance of the Cournot model with two alternatives: (i) a standard search model à la Shimer (2005), augmented with monopolistic competition in the goods market; (ii) a search model with entry and monopolistic competition. Importantly, under monopolistic competition firms do not interact strategically but set a constant markup over marginal costs equal to \( \mu = \frac{\epsilon}{\epsilon - 1} \).

Independently of the model considered, the labor share is defined as
\[ l_s_t = \frac{w_t h_t}{Y_t} = \frac{w_t H_t}{Y_t} \], where \( H_t \) are total hours worked and \( \frac{H_t}{Y_t} \) is the productivity of labor.\(^\text{12}\) In log-deviations
\[ \hat{l}_s_t = \hat{w}_t - \left( \hat{y}_t - \hat{H}_t \right) = \hat{w}_t - \hat{A}_t, \] (21)
\(^\text{12}\)As argued in BGM (2007) when comparing model properties to empirical evidence it is important to net out the effect of changes in the range of available products. In particular, CPI data are closer to \( p_t \) than \( P_t \). For this reason we focus on real variables deflated a data consistent of price index. So for any variables \( X_t \) measured in units of consumption the data consistent counterpart is \( P_t X_t / p_t = X_t / \rho_t \).
equation (21) simply states that the log-deviation of the labor share is the difference between the log-deviation of the real wage and that of labor productivity. Notice that in a standard RBC model, the real wage is equal to \( \dot{y}_t - \dot{H}_t \), that is, real wages and labor productivity move identically. Hence, there is no labor share dynamics. In order to obtain a non constant labor share the allocative role of the real wage has to be broken.\(^{13}\) In the search and matching framework this is obtained through Nash bargaining.

We present our result through numerical simulations. We calibrate the model on a quarterly basis as follows. The discount factor, \( \beta \), is set to 0.99. As in BGM (2007) the rate of business destruction, \( \delta \), equals 0.025 and the entry cost is \( \psi = 1 \). With no loss of generality, the value of \( \chi \) is such that steady state labor supply equals one. The Frisch elasticity of labor supply is \( \varphi = 1 \). The intrasectoral elasticity of substitution is \( \varepsilon = 6 \), as estimated by Christiano, Eichenbaum and Evans (2005). As standard in the literature we set the steady state marginal productivity of labor, \( A \), to 1. As in BGM (2007) and King and Rebelo (2000) we set \( \rho_a = 0.979 \). We set the separation rate \( \varrho \) equal to 0.1, as suggested by estimates provided by Hall (1995) and Davis et al. (1996). The elasticity of matches to unemployment, \( \gamma \), is set equal to the worker bargaining power \( \eta \) and is equal to \( \frac{1}{2} \), as in the bulk of the literature. The efficiency parameter in matching, \( \gamma_m \), and the steady state job market tightness are calibrated to target an average job finding rate, \( z \), equal to 0.7 and a vacancy filling rate, \( q \), equal to 0.9. We draw the latter value from Andolfatto (1996) and Dee Haan et al. (2000), while the former from Blanchard and Gali (2010).\(^{14}\) Finally, we calibrate the overall replacement ratio to have unemployment benefit in real terms, \( b \), such that the replacement ratio \( \frac{w}{w_t} \) equals 0.60. This value is consistent with the monetary replacement rate reported in the OECD Economic Outlook of 1996 for the US. Given these parameters we can recover the cost of posting a vacancy \( \kappa \) by equating the steady state version of the JCC and the steady state wage setting equation.

Figure 2 shows the dynamics of the labor share together with its components, i.e. output, real wage and total hours worked, in response

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\(^{13}\) For example, Gomme and Greenwood (1995) break the relationship between the real wage and productivity augmenting the RBC model with long term labor contracts, which insure workers against income fluctuations. In this case workers are paid more than the marginal product of labor in bad times, and vice versa, leading to a countercyclical labor share. Nevertheless, the authors do not tackle the issue of the overshooting of the labor share.

\(^{14}\) A job finding rate equal to 0.7 corresponds, approximately, to a monthly rate of 0.3, consistent with US evidence.
to a productivity shock under the three economies considered.

![Graphs showing IRFs to a 1% increase in technology.](image)

**Figure 2.** IRFs to a 1% increase in technology.

In the standard search model and the model with entry but constant markups, bargaining over wages implies that only a fraction \( \eta \) of the increase in productivity goes to workers. As a result \( \hat{\omega}_t - \hat{A}_t < 0 \) and the labor share is countercyclical on impact. However, the labor share goes back to its long run level without overshooting.

In the Cournot model, the labor share is countercyclical for the same reasons mentioned above, but the labor share overshoots its long run level after about five quarters, it peaks at about the fifth year at a level much larger than its long-run value and seven years after the shock has hit the economy is still halfway toward its average. This pattern resembles very closely that in the data (see Figure 1).

The overshooting in the Cournot model is related to the dynamics of the price markup. The intuition is the following. Recall that entry is subject to a one period time-to-build lag, which implies that the number of producing firms, \( N_t \), does not change on impact. This translates into an initially muted response of the markup in response to a technology shock. In particular, the price markup finds its negative peak after few periods and then gradually reverts to its long run value.\(^{15}\) This in turns implies, through the wage equation, a positive, and delayed effect on real wages, which remains persistently above labor productivity. Since \( \hat{\ell}_t = \hat{w}_t - \hat{A}_t \), this explains the overshooting of the labor share. As a result the dynamic response of the markup to the technology shock is key for the overshooting.

\(^{15}\)This correlation pattern is consistent with the analysis of prices and costs in Rotemberg and Woodford (2000) and with the VAR evidence for the U.S. in Colciago and Etro (2010).
4 Conclusion

Recent U.S. evidence suggests that the response of labor share to a productivity shock is characterized by countercyclicality and overshooting. This findings cannot be easily reconciled with existing business cycle models.

Our model characterized by endogenous entry, oligopolistic competition in the good market and search and matching frictions in the labor market addresses this evidence. The countercyclicality of price markup is at the basis of our result.

5 References


