Real Business Cycles with Capital Maintenance

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Abstract

We develop a stochastic general equilibrium model in which maintenance endogenously affects the capital depreciation rate. The model performs well in generating maintenance series that match closely existing survey-based measures for Canada. Maintenance is procyclical and comoves almost always with output. Investment-specific shocks are the only disturbances that induce a negative correlation between output and maintenance. This feature is crucial for the identification of such shocks in the short run. We use Bayesian estimation to obtain the time profile of equipment capital depreciation in Canadian manufacturing. The depreciation rate has been quite volatile and procyclical over the last 50 years.


Keywords: real business cycle, technology shocks, endogenous capital depreciation, maintenance.

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1 Introduction

In a study that largely stated the research agenda on RBC modeling, Prescott (1986) emphasized that “a sizeable part of the investment component of output [like a firm’s major maintenance expenditures] is hard to measure and therefore [...] not included in GNP. [...] In good times, namely when output is above trend, firms may be more likely to undertake major repairs of a not fully depreciated asset [...] and hence] these types of unmeasured investments fluctuate in percentage terms more than output. [...] A careful study is needed to determine whether the correction for currently unmeasured investment is small or large.” These remarks suggest that unmeasured capital expenditures in the form of capital maintenance, which have been left mainly unexplored in existing Dynamic Stochastic General Equilibrium (DSGE) models, can provide a promising route in understanding macroeconomic fluctuations.

McGrattan and Schmitz (1999) were the first to provide a detailed picture on the magnitude of capital maintenance and its importance for economic fluctuations using data from the Canadian survey on Capital and Repair Expenditures. Despite the apparent importance of capital maintenance for the dynamics of capital accumulation, so far very few DSGE macroeconomic models have attempted to endogenize maintenance activities. Early contributions in this literature can be found in Licandro and Puch (2000) and Collard and Kollintzas (2000). However, in both studies maintenance moves countercyclically, which contradicts the aforementioned intuition by Prescott (1986). Boucekkine and Ruiz-Tamarit (2003) adopt the standard neoclassical investment firm model allowing for a more flexible relationship between depreciation, maintenance expenditures and the rate of capital utilization to investigate how investment and maintenance decisions respond to various neutral technological shocks. They show that the response of the marginal efficiency of maintenance in the depreciation function to changes in capital utilization is important for the complementarity/substitutability between maintenance and investment. Saglam and Veliov (2008) and Boucekkine et al. (2009) study further these comovements in a firm problem and in a general equilibrium set-up, respectively. Recently, Boucekkine
et al. (2010) extend this framework to study the short-run responses of investment and maintenance and find that they move in the same direction following technology shocks, thus, suggesting that they act complementary to each other.

In this paper we introduce within an otherwise standard Real Business Cycle (RBC) model the assumption that capital outlays include maintenance expenditures that affect the decay rate of the existing capital stock.\(^1\) We employ a general specification for the depreciation function that also embeds the effect of capital utilization on depreciation, as in Burnside and Eichenbaum (1996). The model includes five types of disturbances in the analysis: a Total Factor Productivity (TFP) shock, an investment-specific shock, a labor supply shock, a preference shock and a government spending shock. In turn, we use Bayesian techniques to estimate the structural parameters of the model with aggregate data from Canadian manufacturing covering the period 1956-2005.

We find that the response of maintenance to investment-specific shocks is crucial to identify this type of shocks in the short run. After an investment-specific shock maintenance expenditures are reduced on impact while output increases, while for the rest of the disturbances considered the two variables always comove. This difference in the sign responses of maintenance and output remains robust to changes in the model specificities and is confirmed in the Canadian data. Maintenance and output move in opposite directions in responses to investment-specific shocks and in the same direction in response to TFP shocks when we use long run restrictions for the identification of these shocks, as in Fisher (2006), in a VAR that includes maintenance series. This result provides important information to decision makers and contributes to the literature about the identification of shocks, in which sign restrictions are used to identify technology, fiscal and monetary shocks (see e.g. Gali 1999; Erceg et al., 2005; Dedola and Neri, 2007; Canova and Pappa, 2007; Canova et al., 2007; 2008; Pappa, 2009).

\(^1\)Several studies have investigated the relationship between the optimal maintenance level and the maintenance-dependent depreciation rate at the firm level; see, among others, Schmalensee (1974), Nickell (1978), Schworm (1979) and Parks (1979) for early contributions in this literature. Also, some empirical studies at the sectoral level have confirmed that capital deterioration is endogenous and, in particular, associated with maintenance expenditure. See Nelson and Caputo (1997) and the references cited therein for a brief survey of the empirical findings.
Finally, our model generates estimates for maintenance that match quite well the existing survey-based measures for Canada. In turn, using the estimated series we are able to analyze the cyclical behavior of capital depreciation in Canada. The empirical literature on capital depreciation has mostly used US data to assess the magnitude of capital depreciation by estimating econometric variants that are based on the firm’s problem or the law of motion for capital accumulation (see Epstein and Denny, 1980; Hulten and Wykoff, 1981; Nadiri and Prucha, 1996a, 1996b; Jorgenson, 1996; Huang and Diewert, 2011). Instead, our model allows us to assess, for the first time, the time profile of endogenous capital depreciation based on a general equilibrium framework.\(^2\) Our results indicate that the implied depreciation rate for equipment capital in Canadian manufacturing has exhibited substantial volatility over the last 50 years with a highly procyclical pattern.

The rest of the paper is organized as follows. Section 2 describes some stylized facts about maintenance and repair expenditures. Section 3 presents the model and section 4 discusses the results from the Bayesian estimation. Section 5 presents the model’s dynamics and Section 6 presents the estimates for capital depreciation in Canada. Finally, section 7 concludes.

### 2 Capital maintenance series

Few studies have attempted to assess the size of spending on capital maintenance since standard sources, like national accounting systems, do not report explicitly most activities related to maintenance and repair, as they are performed by employees and, hence, there are no recorded market transactions. Moreover, maintenance and repair services purchased by firms in the market are typically treated as transactions involving intermediate goods. Thus, although maintenance activities are included in measured real output, their magnitude cannot be recovered by national accounts.

\(^2\)An exception that uses time-varying depreciation within a general equilibrium setup is the study by Chen et al. (2006) who calibrate the Japanese economy in order to investigate the driving forces of the saving rate. The time profile of their reported (exogenous) depreciation rates indicates that they were exceptionally high in the 50s and 60s, but declined substantially over the following decades.
McGrattan and Schmitz (1999) were the first to provide a detailed picture on the size of aggregate capital maintenance. The authors report evidence from the only source of aggregate long-run data on capital expenditures in newly purchased assets (‘new’ investment) and maintenance, namely the Canadian survey on ‘Capital and Repair Expenditures’, indicating that maintenance expenditures are too big for economists to ignore. According to this survey, total (private and public) maintenance and repair expenditures in Canada amounted on average to around 6.3% of GDP for the period 1956-93. This number was roughly equal to one third of spending on ‘new’ investments and, when compared to other so called ‘engines of growth’, was somewhat lower than education spending (6.8% of GDP), but far above the average spending on R&D (1.4% of GDP) over the same period.

In this paper we use the ‘Capital and Repair Expenditures’ survey to obtain series on maintenance and ‘new’ investment of equipment capital in the Canadian manufacturing sector, which cover a period of 50 years (1956-2005). During this period total expenditures in ‘new’ investment and maintenance amounted on average to 16.7% of manufacturing output with an average share of maintenance over total investment of 36.1%, accounting for 6% of output and 4.9% of the capital stock.3

Turning to the cyclical properties of the data, we observe that maintenance expenditures are procyclical, in accordance with McGrattan and Schmitz (1999). Figures 1(a) and 1(b) plot spending on capital maintenance and the associated maintenance to capital ratio ($MK$ ratio) versus manufacturing output.4 Both measures of maintenance are procyclical. Indeed, descriptive statistics point towards a contemporaneous correlation between maintenance and $MK$ ratio with output of 0.63 and 0.60, respectively. This correlation seems to be higher in the first part of the sample: for the period 1956-1983 the

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3A full description of the Canadian data on capital maintenance is presented in the Data Appendix. Notice that recently the U.S. Census has added in the Annual Survey of Manufacturers entries on Repair and Maintenance services of buildings and/or machinery for the years 2007, 2008, 2009. The data confirm that maintenance forms a substantial part of expenditures by firms. The amounts provided are (in current million USDs) 43.337, 39.860 and 36.048 for 2007, 2008 and 2009 respectively. ‘New’ investments and maintenance account on average for 8.7% of US manufacturing output, with maintenance amounting to 20.9% of total investment. Given the short span of these series we opted for not applying our empirical exercise to the US economy.

4All series are in logs and have been detrended with the HP filter ($\lambda = 100$ for annual data).
corresponding correlation coefficients amount to 0.85 and 0.86. The cross-correlations, presented in Figures 2(a) and 2(b), remain high for lags (-1) to (-3) of output for both maintenance and $MK$ ratio.

3 The Model

In the model economy households maximize a utility function with two arguments (goods and labor effort) over an infinite life horizon. Households rent effective capital services to firms and allocate their spending on capital between ‘new’ investment, which adds directly on the capital stock, and capital maintenance, which affects along with capital utilization the depreciation rate. In this section we present the main features of the model and its solution.

3.1 Households

The economy is inhabited by infinitely lived agents that derive utility from consumption, $C_t$, and disutility from hours worked, $h_t$, at each period $t$. The present-value utility of the household is given by:

$$E \sum_{t=0}^{\infty} \beta^t \eta^u_t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \lambda_n \eta^h_t \frac{h_t^{1+\theta_n}}{1+\theta_n} \right]$$

where $\sigma > 0$ is the risk aversion coefficient, $\theta_n > 0$ determines the supply elasticity of hours, and $\lambda_n > 0$ is a preference parameter. Parameter $\beta$ is a subjective discount factor with $0 < \beta < 1$ and $E$ is the expectation operator. $\eta^u_t$ and $\eta^h_t$ represent a preference shock and a labor supply shock, respectively; both shocks are assumed to follow an AR(1) process with i.i.d. normal error term: $\log(\eta^u_t / \eta^u) = \rho_u \log(\eta^u_{t-1} / \eta^u) + \epsilon^u_t$ and $\log(\eta^h_t / \eta^h) = \rho_h \log(\eta^h_{t-1} / \eta^h) + \epsilon^h_t$. The literature has indicated both labor supply and preference shocks as key determinants for business cycle fluctuations and for that reason we include them as possible sources of fluctuations in our analysis.

The representative household owns the capital stock and receives income from renting
the effective capital stock (capital services), \( U_tK_t \), where \( U_t \) is the utilization rate of the capital stock \( K_t \), to the firm at a rate \( r_t \) and from working at a wage rate \( w_t \). The household allocates her income stream between consumption \( C_t \), ‘new’ investment \( I_t \), and capital maintenance \( M_t \):

\[
C_t + I_t + M_t \leq w_t h_t + r_t U_t K_t
\]  

(2)

The rate at which capital depreciates depends positively on its utilization and negatively on maintenance expenditures. ‘New’ investment, \( I_t \) is related to the capital stock accumulation by:

\[
Z_t I_t = K_{t+1} - \left( 1 - \delta \left( \frac{U_t}{K_t} \right) \right) K_t + v \left( \frac{K_{t+1}}{K_t} \right) K_t
\]  

(3)

where \( \delta (\cdot) \) is the depreciation function and \( v(\cdot) \) is a function of gross investment regulating capital adjustment costs. The variable \( Z_t \) represents an investment-specific technology shock that affects the capital law of motion and can be embodied either in the investment good (like technology advances) or in the process for producing it, thus affecting the real price of investment. Greenwood et al. (1997, 2000) have shown that technology shocks involving investment-specific rather than neutral technological change can be a major source of the business cycle. Fisher (2006) shows that the combined impact of neutral and investment-specific shocks is important in explaining fluctuations of output and labor in the US with investment-specific shocks mattering more than TFP shocks. As a result, including investment-specific shocks in the analysis is crucial for studying the dynamics of maintenance. We model the investment-specific shock as an AR(1) process with i.i.d. normal error term: \( \log(Z_t/Z) = \rho_z \log(Z_{t-1}/Z) + \epsilon^z_t \).

Following the standard approach, we adopt a quadratic specification for the capital adjustment costs function:

\[
v \left( \frac{K_{t+1}}{K_t} \right) = \frac{b}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2
\]  

(4)
where \( b > 0 \) is a parameter measuring the degree of capital adjustment costs\(^5\).

As in McGrattan and Schmitz (1999), we assume that depreciation is a decreasing function of maintenance expenditure, so that as maintenance services per unit of the capital stock increase, the rate at which capital depreciates decreases. At the same time, following Greenwood et al. (1988) and Burnside and Eichenbaum (1996) we allow depreciation to be an increasing function of capital utilization. Given these assumptions, the depreciation function is parameterized as:

\[
\delta \left( U_t, \frac{M_t}{K_t} \right) = \xi \left[ \psi U_t^\phi + \left( 1 - \psi \right) e^{-\gamma \frac{M_t}{K_t}} \right]^\theta
\]  

(5)

The parameters \( \phi \) and \( \gamma \) assess the effect of utilization and maintenance on the rate of depreciation of capital, respectively. In particular, \( \phi > 0 \), so that \( \frac{\partial \delta}{\partial U} > 0 \). In the next section, we estimate the value of \( \gamma \) with Bayesian techniques. If \( \gamma > 0 \), we have that \( \frac{\partial \delta}{\partial M} < 0 \), and \( \frac{\partial^2 \delta}{\partial M^2} > 0 \). When \( \phi = 0 \), capital utilization does not affect the rate at which capital depreciates, while with \( \gamma = 0 \) maintenance expenditures are ineffective in reducing the capital depreciation rate. Moreover, when the capital stock is not utilized and maintenance expenditures are very high, there is no depreciation, i.e. \( \delta(0, \infty) = 0 \). Notice that the specification adopted in (5) nests the one in McGrattan and Schmitz (1999) for \( \psi = 0 \) and the one in Burnside and Eichenbaum (1996) for \( \psi = 1 \).\(^6\)

Given the trade-off between the production benefits and the depreciation costs of capital utilization, the agent will in general not find it optimal to fully utilize the capital stock. Under our assumptions there is also a trade-off in allocating resources between ‘new’ investment \( I_t \) and capital maintenance \( M_t \), which will be determined by their respective returns and the effects of the various shocks in the model. As described in Boucekkine and Ruiz-Tamarit (2003) and Boucekkine et al. (2010), the sign of the cross derivative \( \frac{\partial^2 \delta}{\partial M \partial U} \) is crucial to determine the degree of complementarity or substitutability between investment series and maintenance. The sign of this derivative is determined by the size

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\(^5\)Note that our results hold when adjustment costs are assumed to depend on investments.

\(^6\)When the benchmark model without endogenous maintenance is considered, the depreciation function takes the form: \( \delta(U_t) = \delta U_t^\phi \).
of parameter $\theta$: when $\theta > 1$ ($\theta < 1$) the cross derivative is negative (positive). In the steady state the value of $\theta$ depends on the estimated values of parameters $\gamma$, $\phi$ and $\alpha$. When we move to posting our priors we opt for priors that allow for values of $\theta$ that can be bigger or smaller than one and let the data decide on its magnitude.

### 3.2 Production side and market clearing

Firms use capital services and labor hours to produce a final good, $Y_t$, that can be used for consumption, investment and maintenance activities. The representative firm then chooses its factor inputs, hours worked, $h_t$, and capital services, $U_tK_t$, to produce a given level of $Y_t$ in order to minimize the production costs:

$$w_t h_t + r_t U_t K_t$$

subject to the technological constraint:

$$Y_t = (U_t K_t)^{1-\alpha} (X_t h_t)^{\alpha}$$

where the variable $X_t$ represents a neutral labor-augmenting technology (TFP) shock, with an AR(1) process with i.i.d. normal error term: $\log(X_t/X) = \rho_x \log(X_{t-1}/X) + \epsilon^x_t$.

In equilibrium the goods market clears and we have:

$$Y_t = C_t + I_t + M_t + G_t$$

where $G_t$ is a public spending shock, whose logarithm follows an AR(1) process with i.i.d. normal error term: $\log(G_t/G) = \rho_y \log(G_{t-1}/G) + \epsilon^g_t$.

### 3.3 Model solution

The representative agent chooses a sequence of $C_t$, $h_t$, $U_t$, $I_t$, and $M_t$, to maximize (1) subject to (2) and (3). The first-order conditions of the model are given by the following
Equations:

\[ \eta^h h^\theta_n = \alpha C_t^{-\sigma} Y_t \]  

(9)

\[ (1 - \alpha) \frac{Y_t}{U_t} = \xi \theta \phi \psi \left[ \psi U_t^\phi + (1 - \psi) e^{-\gamma \frac{M_t}{K_t}} \right]^{\theta-1} \frac{K_t}{Z_t} U_t^{\phi-1} \]  

(10)

\[ \xi \theta \gamma (1 - \psi) \left[ \psi U_t^\phi + (1 - \psi) e^{-\gamma \frac{M_t}{K_t}} \right]^{\theta-1} e^{-\gamma \frac{M_t}{K_t}} = Z_t \]  

(11)

\[ \beta E_t \left\{ \eta^u C_t^{-\sigma} r_{t+1} U_{t+1} - \frac{M_{t+1}}{K_{t+1}} + 1 - \delta \left( \frac{U_{t+1}}{Z_{t+1}}, \frac{M_{t+1}}{K_{t+1}} \right) \right\} = \eta^u \left[ 1 + b \left( \frac{K_{t+1}}{K_t} - 1 \right) \right] \]  

(12)

Equation (9) gives the first-order condition for hours worked and equation (10) sets the marginal return of a rise in the capital utilization rate equal to its opportunity cost measured by the increased capital depreciation rate. Equation (11) is the optimality condition with respect to maintenance and sets the marginal benefit of maintenance arising through the depreciation rate equal to its cost. Finally, equation (12) modifies the usual optimality condition that equates the marginal productivity with the user cost of capital, as a marginal increase in the capital stock implies a rise in its required maintenance cost. Firms set the marginal products of effective capital and hours worked equal to the return of capital and the wage rate, respectively.

In order to investigate the dynamics of the model, we log-linearize the equilibrium conditions around the steady state. The detailed steady-state conditions and the log-linear equations are presented in the Appendix.
4 Bayesian estimation

The log-linearized model is estimated with Bayesian techniques. We follow the method of Smets and Wouters (2007) and estimate the mode of the posterior distribution by maximizing the log posterior function, which combines the prior information on the parameters with the likelihood of the data, using the numerical method by Sims (1999). The Metropolis-Hastings algorithm is then used to get the complete posterior distribution with a sample of 250000 draws (dropping the first 20% draws) and a scale for the jumping distribution of 0.4.

The set of observable variables for Canada is comprised by five annual series over the period 1956-2005, namely manufacturing output, utilization, ‘new’ investment, consumption, and hours worked. Output, ‘new’ investment and consumption are deflated with the Industrial Selling Price index and divided by total working population. Hours are adjusted for total working population.

We fix the discount factor at 0.98, in line with a steady-state real interest rate of 2%. The steady-state depreciation rate, which corresponds to the steady-state ‘new’ investment to capital ratio is set to 0.0882 and the steady-state $MK$ ratio is set at 0.0494. Both figures equal the corresponding averages of the series from the Canadian Survey on Capital and Repair Expenditures.\footnote{The figure for the steady-state depreciation rate is in line with the estimate reported by Hwang (2002/3) on the average depreciation rate for machinery-equipment in Canadian manufacturing (8.2%).} The ratio of public spending on GDP is fixed at 17%. In turn, we use steady-state relationships to determine the values of parameters $\theta$, $\xi$ and $\psi$, and estimate the values for $\phi$ and $\gamma$ (see Appendix A.1 for details). Table 1 displays the fixed parameters together with steady-state values.

All other deep parameters and the processes governing the five structural shocks are estimated with Bayesian techniques. We set most of the priors following Smets and Wouters (2007) and describe positive parameters with normal or Gamma distributions. In particular, the intertemporal elasticity of substitution $\sigma$, the inverse of the Frisch elasticity of labor supply $\theta_n$, the parameter of the depreciation function $\gamma$, the adjustment cost parameter, $b$, and the share of labor $\alpha$ in the production function are represented by
normal distributions. Parameter $\phi$ is described by a Gamma distribution. The persistence parameters of the AR(1) processes are Beta distributed and the standard errors of the innovations are assumed to follow an Inverse-gamma distribution. Prior shapes, prior means and standard deviations are collected in Table 2.

Table 3 displays the estimation results, reporting the posterior mean and confidence bands for each estimated parameter. The data appear to be very informative. Regarding the shocks considered, all exhibit low persistence with the labor supply shock displaying the highest persistence and the preference and government spending shocks being the least persistent. Standard deviations of the shocks are estimated to be low.

Data are also very informative concerning the structural parameters of the model and it turns out that the mean of the posterior distribution is typically not far from the mean of the prior assumptions. For example, the posterior mean of the intertemporal elasticity of substitution equals 2.90, while the bands we have postulated prior to estimation include values in the (0,6) interval. Moreover, all the posterior estimates assume economically plausible values. The posterior mean for $\theta_n$ equals 1.89, which implies a Frisch elasticity of 0.53. This number is in the interval (0.01,0.85) of the values estimated in microeconomic studies for Canada. The posterior value for the labor share is somewhat higher than the initial value but remains within reasonable bounds. The degree of capital adjustment costs, measured by parameter $b$, is estimated to be equal to 8.6. There is substantial controversy surrounding the estimated magnitude of adjustment costs and, in particular, their estimates are found to be implausibly high. The estimated value for this parameter is here relatively lower, mainly because endogenous maintenance expenditures provide an additional mechanism for dampening investments.

Turning to the parameters of the depreciation function, the posterior value of parameter $\phi$ is slightly lower than the prior value, while the posterior estimate for parameter $\gamma$ is positive and above its postulated prior, confirming that as maintenance expenditures

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8Overall, the posterior distributions are normally shaped. We assess the reliability of our estimates through Monte Carlo Markov Chains univariate and multivariate diagnostics (Brooks and Gelman, 1998). This analysis proves that the results are sensible, as the moments of the parameters appear to be constant and converging.

9See Evers et al. (2008) for a summary of such estimates.
increase the depreciation rate decreases. For these values of $\gamma$, $\phi$, and $\alpha$, the implied value for parameter $\theta$ equals 2.55, whereas the implied values for $\psi$ and $\xi$ are, respectively, 0.57 and 0.21. These estimates imply that when capital is fully utilized and there are no expenditures in maintenance, the capital stock depreciates at the rate $\xi = 21\%$. When capital is not utilized and there are no maintenance expenditures, the depreciation rate equals $\xi(1 - \psi)^\theta = 2.5\%$, which forms the estimated “natural” rate of depreciation. When the capital stock is fully utilized and maintenance expenditures are very high the depreciation rate of the capital stock equals $\xi\psi^\theta = 5.2\%$.

5 Dynamics

5.1 Impulse responses

In this section we present the dynamics of our model economy. To gain some intuition on the workings of the model we compare its impulse response functions (IRFs) with those generated by a standard RBC model with variable utilization calibrated with the parameter estimates obtained in section 4. In general, responses are similar and the presence of endogenous maintenance induces a relatively higher volatility of utilization and the depreciation rate.

5.1.1 TFP shock

The first row of Figure 3 plots the estimated IRFs of the variables of interest to a one-standard-deviation shock to TFP. The productivity shock raises the marginal product of the production factors and, as a result, capital utilization, investment and capital increase in response to the shock generating a surge in output and consumption. Given the increase in utilization, the depreciation rate increases. Maintenance also increases to balance the detrimental effects of the surge in capital utilization on depreciation. Overall, the higher volatility of utilization in the maintenance model implies a higher volatility of depreciation, which is not counterbalanced by the rise in maintenance and translates also
in higher volatility of capital.

5.1.2 Investment-specific shock

The investment-specific shock affects the production of investment goods. The second row of Figure 3 presents IRFs of the two models in response to shocks in the price of investment. The fall in the price of investment does surge investment in the impact period, increasing capital and, due to complementarities in production, hours and capital utilization. In the model with maintenance, the fall in the price of investment increases also the relative price of maintenance. Agents find it optimal to decrease maintenance expenditures on impact, which further increases the depreciation rate.

5.1.3 Labor supply shock

The third row of Figure 3 plots the IRFs to a negative labor supply shock. The shock reduces hours on impact and, due to factor complementarity in the production function, also reduces capital utilization and investment. The fall in capital utilization reduces maintenance expenditures and the induced movements in utilization and maintenance reduce the depreciation rate in equilibrium.

5.1.4 Preference shock

The forth row of Figure 3 shows that a positive preference shock crowds out investment and, as a result, reduces hours, capital, and capital utilization. Consequently output also falls in equilibrium. The fall in utilization decreases capital depreciation and the need for capital maintenance and maintenance falls also in equilibrium. In comparison to the standard RBC model, we notice again that the presence of maintenance makes the responses of utilization and depreciation more volatile.
5.1.5 Government spending shock

Finally, the last row of Figure 3 presents the IRFs to a government spending shock. The increase in government spending crowds out investment, but, due to the induced negative wealth effect, it increases labor supply and capital utilization in equilibrium. The rise in capital utilization raises the depreciation of capital and maintenance expenditures increase as well.

5.2 Maintenance and investment-specific shocks

What emerges from the impulse response analysis is that the presence of maintenance is important in distinguishing an investment-specific shock from other disturbances. In particular, all shocks, apart from the government spending disturbance, have similar effects on all macroeconomic variables on impact as investment-specific shocks and imply qualitatively similar comovements of output with the production inputs. However, the response of maintenance to a shock in the price of investment is different. A careful look at Figure 3 reveals that the behavior of maintenance in the impact period following the investment-specific shock can help to identify such shocks in the short run.

Notably, this implication is general and does not depend on the exact parameterization of the model. In Figure 4 we present the confidence bands for the responses of the different variables to a shock in the price of investment. The depicted IRFs are obtained by the Bayesian estimation and represent posterior distributions of the IRFs. They are built on parameter and shock values from the corresponding estimated distributions. The plots depict the mean response (that also appears in Figure 3) together with confidence bands. Hence, the prediction of the model is robust in the sense that it is independent from the exact parameterization of the model. In addition, the fact that maintenance uniquely moves in opposite direction from output in response to investment-specific shocks applies also to more general frameworks. The negative comovement between the $MK$ ratio and output in response to investment-specific shocks remains if the model is enriched with variable labor effort or sticky prices.
Hence, we get a robust theoretical prediction that can be tested in the data. Next, using the Canadian series on maintenance we examine how this variable reacts to investment-specific shocks identified as shocks that affect the price of investment and labor productivity in the long run (see Fisher, 2006; Baleer, 2009; Justiniano et al., 2011). In Figure 5 we present the responses of output and maintenance to a permanent positive shock in the price of investment and a permanent positive shock in productivity in a VAR system that includes the log differences of the real price of investment, the log differences of labor productivity, log differences of real GDP per capita and the log of $MK$ ratio when Fisher’s (2006) assumptions are used to identify the shocks. The data support the predictions of our model: a positive shock to the price of investment, i.e. a negative investment shock, reduces output and increases maintenance expenditures, while maintenance reacts positively in response to a permanent increase in productivity.

6 The time profile of capital depreciation

Our results so far indicate that capital depreciation is a model driven variable, which depends on capital maintenance and utilization. Given the importance of capital depreciation in empirical exercises and applications, in this section we use our model to obtain the time profile of variable capital depreciation in Canadian manufacturing. Although several studies have attempted to estimate the depreciation rate (especially in US manufacturing) using various econometric approaches within single or multi-equation setups (see Epstein and Denny, 1980; Hulten and Wykoff, 1981; Nadiri and Prucha, 1996a, 1996b; Jorgenson, 1996; Huang and Diewert, 2011), there is no study that has provided estimates for depreciation series in a general equilibrium framework.

To perform the exercise, as a first step we assess the fit of our model by comparing model estimates for capital maintenance and the actual series from the Canadian Survey.

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10 The series are annual and run from 1959 to 2005. The VAR is estimated with Bayesian techniques. We have also tried alternative specifications in which we have included series for inflation and/or the interest rate in the VAR. The results presented in the figure are robust to those changes, although their significance changes due to the limited sample available. The full set of results is available upon request.
The Bayesian estimation uses the Kalman filter to obtain a state-space representation of the dynamic system and, through a recursive procedure, to derive the log-likelihood, conditional on the set of observables. The same recursive algorithm enables to sequentially update a linear projection for the system and, thus, to obtain smoothed estimates for the endogenous variables. Figure 6(a) presents the estimated trend deviations of the series for maintenance to capital ratio versus the actual trend deviations of the series from the Canadian Survey on Capital and Repair Expenditures. The model fits well the pattern for $MK$ ratio for most of the period covered with most of the peaks captured well by the estimated series, which are less volatile in general. The contemporaneous correlation between the actual and the estimated series amounts to 0.57. In line with their data counterpart, the estimated series are highly procyclical with the contemporaneous correlation of actual output and estimated maintenance equal to 0.79. Moreover, Figure 6(b) shows that the cross-correlations remain high for lags (-1) to (-3) and for lead (+1) of output, similarly to the actual series. To further assess the fit of our model we also calculate the ratio of maintenance to ‘new’ investment series for Canada, which are two key variables in our setup. The result is depicted in Figure 7. Again, our estimates give a very good approximation of the actual series. The correlation between actual and simulated series is 0.82. We therefore conclude that our generated series for maintenance yield a good approximation of the actual ones and replicate the key characteristics in the relationship of maintenance with output and ‘new’ investment.

Using our theoretical framework we provide an estimate of the time profile of the depreciation rate of equipment capital in the Canadian manufacturing sector over the period 1956-2005 (centered at 8.82%), which is depicted in Figure 8 along with actual output trend deviations. The depreciation rate of equipment capital is found to have a standard deviation of 1% and ranges between 5.8% and 10.9% over the period with a strongly procyclical profile: the correlation coefficient with output trend deviations amounts to 0.58. The correlation is higher (0.72) in the 1956-83 period of the sample, when output and $MK$ ratio exhibit a high correlation, and drops substantially (0.39) in the 1984-2005 period. This picture indicates that the long-run depreciation rate of
equipment capital in Canadian manufacturing has exhibited substantial swings following the cycle of the economy and the associated pattern of capital maintenance.

7 Conclusions

McGrattan and Schmitz (1999) have suggested that maintenance expenditures are too big to ignore in models of aggregate economic activity. In this paper we first showed that they are also potentially too important to ignore since their behavior in response to investment-specific shocks can uniquely characterize these shocks in the short run relative to other economic disturbances. This prediction is vital to both academics and policymakers for determining the nature of productivity shocks and stabilizing the economy through the proper policy after identifying the shock.

Next, we used the theoretical general equilibrium model with endogenous maintenance to derive series for maintenance for Canada, which match pretty well the actual series, in order to obtain estimates for the time profile of the depreciation rate in Canada. Our findings indicate that depreciation rate of equipment capital in Canadian manufacturing has been volatile and procyclical. Previous studies, based mainly on US data, have aimed at identifying the depreciation rate over a longer time span using single or multi-equation set-ups, and have implied a fairly constant path for the depreciation rate. Our results coming from a general equilibrium setup raise a note of caution to the existing evidence, which should also be taken into account in the estimation of capital stocks.

Nevertheless we should emphasize that our estimates are in no way intended to provide definitive estimates of depreciation. There is a great deal of room for further research, particularly in the areas of using more disaggregated sectoral data for the assessment of depreciation rates. Our findings should thus be viewed, first, as a basis of comparison with existing methodologies and results and, second, as an example of what can be achieved with a DSGE approach that accounts for capital maintenance. In this vein, the model can also be used to estimate unmeasured capital expenditures, like spending on capital maintenance, in other countries, as they form an important part of economic activity.
References


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A Appendix

A.1 Steady state and determination of $\theta$, $\phi$ and $\gamma$

Using (11) and (10) we get that in the steady state it must hold:

$$\psi \phi U^\phi = (1 - \alpha) \gamma (1 - \psi) \frac{Y}{K} e^{-\gamma \frac{M}{K}}$$

(A.1)

From the capital law of motion it also holds:

$$\frac{I}{K} = \xi \left[ \psi U^\phi + (1 - \psi) e^{-\gamma \frac{M}{K}} \right]^\theta$$

(A.2)

Equation (12) implies that:

$$\frac{1}{\beta} = \left[ r^* U + (1 - \frac{I}{K}) - \frac{M}{K} \right]$$

(A.3)

Moreover, (10) can be rewritten as:

$$\xi \theta \phi \psi \left[ \psi U^\phi + (1 - \psi) e^{-\gamma \frac{M}{K}} \right]^{\theta - 1} U^{\phi - 1} = r^*$$

(A.4)

Using the above equations, and imposing $U = 1$ in the steady state, we can derive the following conditions for the values of $\theta$, $\psi$ and $\xi$:

$$\theta = \frac{r^* \gamma + \phi}{\phi \gamma \frac{I}{K}}$$

(A.5)

$$\psi = \frac{r^* \gamma e^{-\gamma \frac{M}{K}}}{r^* \gamma e^{-\gamma \frac{M}{K}} + \phi}$$

(A.6)

$$\xi = \left( \frac{I}{K} \right)^{1-\theta} \left( \frac{r^*}{\theta \psi \phi} \right)^{\theta}$$

(A.7)
A.2 Log-linear conditions

The log-linear first order conditions are given by the following set of equations (hatted variables denote log-deviations from steady-state values):

\[
\theta_n \hat{h}_t + \hat{\eta}_t^h = -\sigma \hat{C}_t + \hat{Y}_t - \hat{h}_t \tag{A.8}
\]

\[
\left[ \gamma \frac{M}{K} + \frac{\theta - 1}{\theta} \frac{M}{I} \right] (\hat{K}_t - \hat{M}_t) + \frac{\theta - 1}{\theta} (1 - \alpha) \frac{Y}{I} \hat{U}_t - \hat{Z}_t = 0 \tag{A.9}
\]

\[
\frac{(\theta - 1)}{\theta} \frac{M}{I} (\hat{K}_t - \hat{M}_t) + \left[ \phi + \frac{\theta - 1}{\theta} (1 - \alpha) \frac{Y}{I} \right] \hat{U}_t - \hat{Z}_t - \hat{Y}_t + \hat{K}_t = 0 \tag{A.10}
\]

\[
- \sigma \hat{C}_t - \hat{Z}_t - b \hat{K}_t + \hat{\eta}_t^v = -\sigma \hat{C}_{t+1} + \beta b \hat{K}_{t+2} + \beta (1 - \alpha) \frac{Y}{K} \hat{Y}_{t+1}
\]

\[
- \beta (1 - \alpha) \frac{Y}{K} \hat{U}_{t+1} - \beta (1 - \delta) \hat{Z}_{t+1} - \left[ b(1 + \beta) + \beta (1 - \alpha) \frac{Y}{K} \right] \hat{K}_{t+1} + \hat{\eta}_{t+1}^v \tag{A.11}
\]

\[
\frac{I}{K} \hat{I}_t + \frac{I}{K} \hat{Z}_t = \hat{K}_{t+1} - \left[ (1 - \delta) - \frac{M}{K} \right] \hat{K}_t + (1 - \alpha) \frac{Y}{K} \hat{U}_t - \frac{M}{K} \hat{M}_t \tag{A.12}
\]

\[
\hat{Y}_t = (1 - \alpha) (\hat{K}_t + \hat{U}_t) + \alpha (\hat{X}_t + \hat{h}_t) \tag{A.13}
\]

\[
\hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{I}{Y} \hat{I}_t + \frac{M}{Y} \hat{M}_t + \frac{G}{Y} \hat{G}_t \tag{A.14}
\]
Equations (A.8) to (A.14) describe the path of the seven endogenous variables of the model: output $\hat{Y}_t$, utilization $\hat{U}_t$, capital $\hat{K}_t$, hours $\hat{h}_t$, consumption $\hat{C}_t$, investment $\hat{I}_t$, and maintenance $\hat{M}_t$.

\section*{B Data Appendix}

The Data Appendix describes briefly first the dataset from the Canadian survey on ‘Capital and Repair Expenditures’. Private firms, households and government organizations in Canada were asked in an annual survey over the period 1956-1993 about their capital and repair expenditures on equipment and structures. The survey (conducted after 1993 in an updated form) is a census with a cross-sectional design and a sample size of 27,000 units; the target population is all Canadian businesses and governments from all the provinces and territories in Canada and the response rate is roughly 85%. Prior to the selection of a random sample, establishments are classified into homogeneous groups (i.e. groups with the same NAICS codes, same province/territory etc).

Capital expenditures are gross expenditures on fixed assets, which are assumed to cover spending devoted to ‘new’ investment, in accordance to the broad definition given earlier. These include expenditures on (i) fixed assets (such as new buildings, engineering, machinery, and equipment) which normally have a life of more than 1 year, (ii) modifications, additions, major renovations, and additions to work in progress (iii) capital costs such as feasibility studies and general (architectural, legal, installation and engineering) fees, (iv) capitalized interest charges on loans with which capital projects are financed, (v) work by own labor force. On the other hand, repair expenditures cover spending devoted to ‘maintenance’ cost, again in accordance to the broad definition given earlier. These expenditures cover (i) maintenance and repair of nonresidential buildings, other structures, and on vehicles and other machinery, (ii) building maintenance (janitorial services, snow removal, sanding), (iii) equipment maintenance (such as oil changes and lubrication of vehicles and machinery), (iv) repair work by own and outside labor force machinery and equipment.
The following variables from the Canadian Survey on Capital and Repair Expenditures of Canada Statistics were used for capital and repair expenditures. Backward values for the manufacturing sector up to 1956 were obtained by using the growth rates for capital expenditures (the growth rates for 1992 and 1993 are common for the two surveys) and then by extrapolating the series for repair expenditures through their share in total capital and repair expenditures over 1956 to 1993.

• Capital expenditures in manufacturing, machinery and equipment: variable v754442 [D878253], 1994 to 2005, and variable v62547 [D842202], 1956 to 1993.

• Repair expenditures in manufacturing, machinery and equipment variable v754445 [D878256], 1994 to 2005, and variable v62550 [D843232], 1956 to 1993.

The rest of the Canadian variables used in the paper and their sources are as follows.

• Manufacturing capital stock in machinery and equipment: manufacturing sector end-year capital stock, variable v1071437 [D819523], 1955 to 2007 (Canada Statistics, Table 031-0002, current prices).

• Manufacturing output: Gross value added in manufacturing (source: OECD, Gross Domestic Product: B1GD), available from 1981 onwards. Backward values were extrapolated by using the industrial production index (source: International Financial Statistics, variable: 15666..CZF...) and the industrial selling price index (source: International Financial Statistics, variable: 15663...ZF...).

• Manufacturing employment: index 2000=100 (source: International Financial Statistics, variable 15667EY.ZF...).

• Hours worked: Annual average number of hours worked for all jobs; Non-durable manufacturing, index 1992=100 (source: Canada Statistics). The number of hours worked in all jobs is the annual average for all jobs times the annual average hours worked in all jobs. According to the retained definition, hours worked means the total number of hours that a person spends working, whether paid or not. In
general, this includes regular and overtime hours, breaks, travel time, training in
the workplace and time lost in brief work stoppages where workers remain at their
posts. Time lost due to strikes, lockouts, annual vacation, public holidays, sick
leave, maternity leave or leave for personal needs are not included in total hours
worked.

- **Consumption**: Annual nominal consumption in million Canadian dollars (source:
  Penn World Tables). It is obtained by multiplying consumption share of GDP per
capita (variable: cc) times GDP per capita (variable: cgdp) and total population
(variable: POP).

- **Capital utilization**: Industrial (total non-farm goods producing industries) capacity
  utilization rate (source: Canada Statistics, variables v142812, Table 028-0001, per-
cent), averaged from quarterly data available from 1962 onwards. Backward values
  were extrapolated by fitting a linear regression on total fixed non-residential capital
  stock for all industries (source: Canada Statistics, variable: D99027311000) divided
  by Canada Gross National Product (source: International Financial Statistics, vari-
  able 15699A.CZF).

- **Industrial selling price index**: Industrial selling price index (source: International
  Financial Statistics, variable: 15663...ZF...)

- **Population**: Population 15-64 (source: OECD, ALFS Summary Tables).

Maintenance, ‘new’ investment, capital stock, consumption and output were deflated
with the industrial selling price index and divided by total working population 15-64.
Hours per worker were obtained by multiplying manufacturing employment and hours
worked and dividing by total working population 15-64, as in Smets and Wouters (2007).
All variables were transformed in logs and filtered with Hodrick-Prescott filter with \( \lambda = 100 \) for annual data.
C Figures and Tables

Figure 1: Maintenance, capital and output: Canada, 1956-2005.

(a) Maintenance vs output

(b) Maintenance to capital ratio vs output
Figure 2: Cross-correlogram of maintenance, capital and output.

(a) Maintenance and output

(b) Maintenance to capital ratio and output
Figure 3: Impulse responses of variables to all shocks (in rows).

Note: solid line for endogenous maintenance model, dotted line for standard RBC model.
Figure 4: Impulse responses of variables to a positive investment-specific shock.

Note: solid lines for mean responses, dotted lines for confidence bands.
Figure 5: Impulse responses to output and MK ratio to investment-specific and TFP shocks.
Figure 6: Estimated MK ratio: Canada, 1956-2005.

(a) MK ratio (estimated vs actual)

(b) Cross-correlogram of estimated MK ratio and output
Figure 7: Actual and estimated maintenance to 'new' investment ratio: Canada, 1956-2005.
Figure 8: Output (trend deviations) and estimated depreciation rate (equipment capital): Canadian manufacturing, 1956-2005.
Table 1: Calibrated parameters and steady state values

<table>
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<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\beta$</td>
<td>discount factor</td>
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<td>$I/K$</td>
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<td>$M/K$</td>
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<tr>
<td>$\delta$</td>
<td>SS depreciation</td>
<td>$I/K$</td>
</tr>
<tr>
<td>$M/I$</td>
<td>SS maintenance to investment ratio</td>
<td>$\frac{M}{K} / \frac{I}{K}$</td>
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<td>$r^*$</td>
<td>SS net real interest rate</td>
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<tr>
<td>$Y/K$</td>
<td>SS output to capital ratio</td>
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<td>SS output to investment ratio</td>
<td>$\frac{Y}{K} / \frac{I}{K}$</td>
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<td>$G/K$</td>
<td>SS public spending to capital ratio</td>
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<td>$C/K$</td>
<td>SS consumption to capital ratio</td>
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<td>$C/I$</td>
<td>SS consumption to investment ratio</td>
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<td>$M/Y$</td>
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<td>$X$</td>
<td>SS TFP</td>
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Table 2: Prior distribution of structural parameters and shock processes

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<th>parameters</th>
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<th>prior std deviation</th>
<th>lower bound</th>
<th>upper bound</th>
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<tbody>
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<td>10</td>
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</tr>
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<td>10</td>
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<td>0.2</td>
<td>0.01</td>
<td>0.99</td>
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<tr>
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<td>0.2</td>
<td>0.01</td>
<td>0.99</td>
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<td>$\rho_h$</td>
<td>Beta</td>
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<td>0.2</td>
<td>0.01</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Beta</td>
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<td>0.2</td>
<td>0.01</td>
<td>0.99</td>
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<td>$\sigma_x$</td>
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<td>$\sigma_z$</td>
<td>Inv-gamma</td>
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<td>Inf</td>
<td>0.01</td>
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<tr>
<td>$\sigma_u$</td>
<td>Inv-gamma</td>
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<td>Inf</td>
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<td>$\sigma_h$</td>
<td>Inv-gamma</td>
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<td>Inf</td>
<td>0.01</td>
<td>3</td>
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<tr>
<td>$\sigma_g$</td>
<td>Inv-gamma</td>
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<td>Inf</td>
<td>0.01</td>
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Table 3: Posterior distribution of structural parameters and shock processes

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<tr>
<th>Model Parameters</th>
<th>Canada (Post. mean, Confidence Interval)</th>
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<tr>
<td>$\gamma$</td>
<td>23.606 (12.866, 33.897)</td>
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<tr>
<td>$\phi$</td>
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<tr>
<td>$b$</td>
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<tr>
<td>$\sigma$</td>
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<tr>
<td>$\theta_n$</td>
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<tr>
<td>$\alpha$</td>
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<tr>
<td>$\rho_z$</td>
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<tr>
<td>$\rho_z$</td>
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<tr>
<td>$\rho_h$</td>
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<tr>
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<tr>
<td>$\sigma_z$</td>
<td>0.056 (0.045, 0.067)</td>
</tr>
<tr>
<td>$\sigma_u$</td>
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</tr>
<tr>
<td>$\sigma_h$</td>
<td>0.088 (0.044, 0.133)</td>
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<td>$\sigma_g$</td>
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