Real Wage Rigidities and Disinflation Dynamics: Calvo vs. Rotemberg Pricing

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Real Wage Rigidities and Disinflation Dynamics: Calvo vs. Rotemberg Pricing

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Abstract
Calvo pricing implies output gains, while Rotemberg pricing implies output losses after a disinflation. Introducing real wage rigidities has opposite effects: it generates a long-lasting boom in output in Calvo, and a moderate output slump in Rotemberg.

Keywords: Disinflation, Sticky Prices, Real Wage Rigidity, Non-linear Simulations

JEL classification: E31, E5.

1 Introduction

The Calvo (1983) price-setting mechanism produces relative-price dispersion among firms, while the Rotemberg (1982) model is consistent with a symmetric equilibrium. Despite this economic difference to a first order approximation the two models are equivalent and, as shown by Rotemberg (1987) and Roberts (1995), imply the same reduced form New Keynesian Phillips curve. Moreover, Nisticò (2007), shows that up to a second order approximation, if the steady state is efficient, both models imply the same welfare costs of inflation. Only recently, Lombardo and Vestin (2008) show that they might entail different welfare costs at higher order of approximation. Therefore, except for welfare consideration, there is widespread agreement in the literature that the two models imply the same dynamics. Furthermore, both models deliver the well-known result of immediate adjustment of the economy to the new steady state following a disinflation, despite nominal rigidities in price-setting (see, e.g., Ball, 1994 and Mankiw, 2001). In a very insightful paper Blanchard and Gali (2007), suggest that real wage rigidities is an important feature that restores realistic output cost of disinflation in the linearized Calvo model. Ascarì and Merkl (2009), instead, show that studying the non-linear dynamics of the Calvo model, real wage rigidities actually create a boom in output, rather than a slump. A result which thus seems to be strongly at odds with the conventional view. In this paper, we show that the non-linear dynamics of the Rotemberg model restores results similar to the standard Blanchard and Gali (2007) log-linear disinflation dynamics: (i) flexible real wage imply an immediate adjustment of output to its new steady state after a permanent disinflation; (ii) real wage rigidities imply a significant output slump along the adjustment path. Results on which there seems to be consensus in the literature. It follows that the non-linear disinflation dynamics implied by the Calvo and Rotemberg pricing model is very different. In particular, the Rotemberg model seems to be more robust to non-linearities and inferring the effects of permanent shocks through log-linearized model would not lead to big mistakes, as in the Calvo model.

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2 The Model

2.1 Household

Given the separable utility function

\[ U(C_t(h), N_t(h)) = \frac{C_t^{1-\sigma}}{1-\sigma} - d_n \frac{N_t^{1+\varphi}(h)}{1+\varphi}, \]

and the budget constraint: \( P_tC_t + (1+i_t)^{-1}B_t = W_tN_t - T_t + \Pi_t + B_{t-1} \), where \( C_t \) is the standard Dixit-Stiglitz consumption basket, \( P_t \) is the CPI index, \( i_t \) is the nominal interest rate, \( B_t \) are one-period bond holdings, \( W_t \) is the nominal wage rate, \( N_t \) is the labor input, \( T_t \) are lump sum taxes, and \( \Pi_t \) is the profit income. The first order conditions with respect to \( C_t, B_t \) and \( N_t \) are:

\[ \frac{1}{C_t^\gamma} = \beta E_t \left( \frac{P_t}{P_{t+1}} \right) (1+i_t) \left( \frac{1}{C_{t+1}^\gamma} \right) \]

\[ \frac{W_t}{P_t} = \frac{U_N}{U_C} = \frac{d_n N_t^\varphi}{1/\gamma C_t^\gamma} = d_n N_t^\varphi C_t^\sigma. \]

which represent the consumption Euler equation and the labor supply. We introduce real wage rigidities in the same way as Blanchard and Gali (2007), that is

\[ \frac{W_t}{P_t} = \left( \frac{W_{t-1}}{P_{t-1}} \right)^\gamma \left( -\frac{U_N}{U_C} \right)^{1-\gamma}, \]

which means that for sufficiently high value of \( \gamma \), the model implies a sluggish adjustment of real wages.

2.2 Firms and Price Settings

Final good producers use the following technology: \( Y_t = \left[ \int_0^1 Y_{i,t}^{1-\varepsilon} \, d\varepsilon \right]^{1/\varepsilon} \). Their demand for intermediate inputs is therefore equal to \( Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t \).

The intermediate good sector is monopolistically competitive and the production function of each firm is given by: \( Y_{i,t} = N_{i,t} \).

The Calvo model

The Calvo model assumes that each period there is a fixed probability \( 1-\theta \) that a firm can re-optimize its nominal price, i.e., \( P_{i,t}^* \). The price setting problem becomes:

\[
\max_{\{P_{i,t}\}_{i=0}^{\infty}} E_t \sum_{j=0}^{\infty} \Delta_t(t+j) \theta^j \left( \frac{P_{i,t}^*}{P_{i,t+j}} - MC_{t+j} \right) Y_{i,t+j},
\]

s.t. \( Y_{i,t+j} = \left( \frac{P_{i,t}^*}{P_{i,t+j}} \right)^{-\varepsilon} Y_{i,t+j} \)

where real marginal costs are \( MC_{t+j} = \frac{W_{i+j}}{P_{i,j}} \). The equation for the optimal price is:

\[
P_{i,t}^* = \frac{\varepsilon}{\varepsilon-1} \left( \frac{E_t \sum_{j=0}^{\infty} \theta^j \Delta_t(t+j) Y_{i,t+j} MC_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \Delta_t(t+j) P_{i,t+j}^{-1} Y_{i,t+j}} \right).
\]
while the aggregate price dynamics is given by: 
\[ P_t = \left[ \theta P_{t-1}^{1-\varepsilon} + (1 - \theta) \left( P_{t-1}^* \right)^{1-\varepsilon} \right]^\frac{1}{1-\varepsilon}. \]

The Calvo model is characterized by the presence of price dispersion which results in an inefficiency loss in aggregate production. In fact

\[ N_t^d = \int_0^1 N_{t,i}^d di = \int_0^1 Y_{t,i} di = Y_t \int_0^1 \left( \frac{P_{t,i}}{P_t} \right)^{-\varepsilon} di = s_t Y_t. \]  

(6)  

where price dispersion is defined as

\[ s_t = \int_0^1 \left( \frac{P_{t,i}}{P_t} \right)^{-\varepsilon} di \]  

(7)

Schmitt-Grohé and Uribe (2007) show that price dispersion is bounded below at one, so that \( s_t \) represents the resource costs due to relative price dispersion under the Calvo mechanism. Indeed, the higher \( s_t \), the more labor is needed to produce a given level of output. To close the model, the aggregate resource constraint is simply given by: \( Y_t = C_t \).

**The Rotemberg model**

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost of adjusting nominal prices, that can be measured in terms of the final-good and given by

\[ \frac{\varphi}{2} \left( \frac{P_{t,t}}{P_{t,t-1}} - 1 \right)^2 Y_t, \]  

(8)  

where \( \varphi > 0 \) determines the degree of nominal price rigidity. The adjustment cost increases in magnitude with the size of the price change and with the overall scale of economic activity, \( Y_t \). The problem for the firm is then:

\[
\begin{align*}
\max_{\{P_{t,i}\}_{t=0}^{\infty}} & \sum_{j=0}^{\infty} E_t \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j} \left\{ \left( \frac{P_{t,t+j}}{P_{t,t}^*} - MC_{t+j}^r \right) Y_{t,t+j} - \frac{\varphi}{2} \left( \frac{P_{t,t+j}}{P_{t,t+j-1}} - 1 \right)^2 Y_{t+j} \right\}, \\
\text{s.t.} & \ Y_{t,t+j} = \left[ \frac{P_{t,t+j}}{P_{t,t}^*} \right]^{-\varepsilon} Y_{t+j}.
\end{align*}
\]

where \( \mathcal{D}_{t,t+j} \equiv \beta^j \mathcal{U}_{t+j} U_{t+j} \) is the stochastic discount factor, \( MC_{t+j}^r = \frac{W_{t+j}}{P_{t,t+j}} \) is the real marginal cost function. Firms can change their price in each period, therefore, from the first order condition, after imposing the symmetric equilibrium, we get

\[ 1 - \varphi (\pi_t - 1) \pi_t + \varphi \beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( (\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right) = (1 - MC_t^r) \varepsilon. \]  

(9)  

where \( \pi_t = \frac{P_t}{P_{t-1}} \). Since all the firms will employ the same amount of labor, the aggregate production function is simply given by \( Y_t = N_t \). The aggregate resource constraint should take the adjustment cost into account, that is: \( Y_t = C_t + \frac{\varphi}{2} (\pi_t - 1)^2 Y_t \). For what follows, it is important to note that the Rotemberg adjustment cost model creates an inefficiency wedge, \( \Psi_t \), between output and consumption:

\[ Y_t = \left[ \frac{1}{1 - \frac{\varphi}{2} (\pi_t - 1)^2} \right] C_t = \Psi_t C_t. \]  

(10)
In the Rotemberg model, the cost of nominal rigidities, i.e., the adjustment cost, creates a wedge between aggregate consumption and aggregate output, (10), because part of the output goes in the price adjustment cost. In the Calvo model, instead, the cost of nominal rigidities, i.e., price dispersion, creates a wedge between aggregate employment and aggregate output, (6), making aggregate production less efficient. Both of these wedges are non-linear functions of inflation. They are minimized at one when steady state inflation equals zero, while both wedges increase as trend inflation moves away from zero.

3 Disinflation

3.1 The Steady State and the Long-run Phillips Curve

We first look at the steady state of the two models, and in particular at the implications for the long-run Phillips Curve.

The Calvo model

Ascar (2004), Yun (2005), show that the long-run Phillips Curve is negatively sloped: positive long-run inflation reduce output, because it increases price dispersion. Higher price dispersion acts as a negative productivity shift, because \( Y = \left( \frac{\bar{\pi}}{\bar{\pi}} \right) \). Thus, the steady state real wage lowers with trend inflation, and so does consumption and leisure, so that actually steady state employment increases. As a consequence, steady state welfare decreases.

The Rotemberg model

The long-run Phillips Curve in the Rotemberg model is equal to:

\[
Y = \left[ \frac{\varepsilon - 1}{\varepsilon} + \frac{(1 - \beta)}{\varepsilon} \varphi (\bar{\pi} - 1) \bar{\pi} \right] ^{\frac{1}{\sigma}}.
\]

It is easy to show that this implies that \( \bar{\pi} \geq 1 \implies \frac{dY}{d\bar{\pi}} > 0 \), so that the minimum of output occurs at negative rate of steady state inflation, unless \( \beta = 1 \). This is a "time discounting effect": in changing the price, a firm would weight relatively more today adjustment cost of moving away from yesterday price, than the tomorrow adjustment cost of fixing a new price away from the today’s one. As in the Calvo model, the discounting effect tends to reduce average mark-up. But unlike the Calvo model, there is no price dispersion that interacts with trend inflation, and thus this is the only effect of trend inflation on the price setting decision. Indeed, the steady state mark-up is given by

\[
\text{markup} = \left[ \frac{\varepsilon - 1}{\varepsilon} + \frac{(1 - \beta)}{\varepsilon} \varphi (\bar{\pi} - 1) \bar{\pi} \right] ^{-1}
\]

which is monotonically decreasing in \( \bar{\pi} \), for positive trend inflation (\( \bar{\pi} > 1 \)). The fact that the mark-up decreases with trend inflation makes output to increase with trend inflation. However, a fraction of output is not consumed, but it is eaten up by the adjustment cost. The higher trend inflation, the more output is produced, but the less is consumption. Opposite to the Calvo model, then, output is increasing with trend inflation, but, as in the Calvo model, employment is increasing, while consumption and welfare are decreasing with trend inflation (see Figure 1).\(^1\)

\(^1\)We consider the following rather standard parameters specification (see Section 3.2): \( \sigma = 1, \beta = 0.99, \varepsilon = 10, \)
- Figure 1 about here -

As we will see, the opposite slope of the long-run Phillips Curve between the two models determines a very different short-run adjustment in the non-linear dynamics following a permanent shift in the central bank inflation target.²

3.2 Disinflation and Real Wage Rigidities

We now look at an unanticipated and permanent reduction in the inflation target of the Central Bank (CB) from 4% to zero. We plot the path for output, inflation, nominal interest rate, and real wages under different degrees of real wage rigidities.³ The CB follows a standard Taylor rule, i.e.,

\[
(1 + \iota_t) = \left(\frac{\pi_t}{\pi}\right)^{\alpha_y} \left(\frac{Y_t}{Y}\right)^{\alpha_y}.
\]

(13)

We consider the parameters specification, as in (1), which coincides with the one used by Ascani and Merkl (2009). We set \(\alpha_y = 1.5\) and \(\alpha_y = 0.125\).

The Calvo model

Figure 2 replicates Ascani and Merkl (2009) experiment, i.e. a disinflation from \(\bar{\pi} = 4\%\) to \(\bar{\pi} = 0\). Real wage rigidities have a rather surprising implication on the economy dynamics: output increases after disinflation and overshoots above its new permanent natural level. The higher the degree of real wage rigidities \(\gamma\), the more likely is the overshooting of output.⁴

- figure 2 about here -

The intuition is straightforward. As shown by Ascani and Merkl (2009), unlike in the log-linear model, a disinflation experiment increases the permanent steady state level of output. With flexible real wages a disinflation leads to a short-run overshooting of the real wage over its new higher

\[\phi = 1, \theta = 0.75. \text{ We set the cost of adjusting prices } \varphi = \left(\frac{1}{1 - \varphi}\right)^{\frac{\gamma}{\gamma - 1}}, \text{ to generate a slope of the log-linear Phillips curve equal to one we get in the Calvo model. However, none of the results qualitatively depends on the parameters values.}\]

²We thank an anonymous referee for noticing that Figure 1 implies that the steady-state consumption increases under disinflation in the Rotemberg model. Our result on consumption is due to the fact that the adjustment cost goes in the resource constraint and creates a wedge between aggregate output and consumption. Even if this results seems to be at odds with empirical evidence we maintain the assumption simply because we want to compare the two common pricing mechanisms with trend inflation and real wage rigidities as they are usually presented. Nevertheless, we also consider the case in which the adjustment costs are rebated to consumers. This assumption implies that the aggregate resource constraint becomes \(C_t = Y_t\). In this case the qualitative results of the Rotemberg model remain unchanged even if the effects of trend inflation are mitigated, both in the long-run and during the transition to the new steady state. Results are available upon request.

³Figures 2-3 are obtained using the software DYNARE. The paths of all variables display the movement from a deterministic steady state to another one. DYNARE stacks up all the equations of the model for all the periods (we set equal to 100). The resulting system is solved en bloc by using the Newton-Raphson algorithm. The non-linear model thus is solved in its full-linear form, without any approximation.

⁴In the literature it is not still clear which is the empirical plausible value of \(\gamma\). Moreover, \(\gamma\) changes by changing the country considered and the period of time. Blanchard and Gali (2007), use illustrative values for \(\gamma\) between 0.5 and 0.9. Estimates seems to be something in between 0.5 and 0.9, that’s why we calibrate with \(\gamma = 0.5\) and \(\gamma = 0.9\). In very recent paper, Knell (2010), use macroeconomic time series data (mostly from 1990 to 2007) to provide estimates of RWR for the same group of 15 European countries. The average quarterly value of \(\gamma\) that comes out is 0.7.
long-run value. With real wage rigidities, real wage adjusts sluggishly and cannot overshoot on impact. Real wage is thus lower along the adjustment, and this spurs output. Thus, the real wage overshooting is transferred to output.

The Rotemberg model
Under Rotemberg pricing the result is the other way round. Figure 3 shows that sluggish real wages cause an output slump along the adjustment path. The slump of output becomes more significant the higher the parameter of real wage rigidities, $\gamma$. Notice that only very high values of $\gamma$ tend to have sizeable effects on the output response, since for values of $\gamma = 0.5$, the quantitative effects of real rigidities are very small.

- figure 3 about here -

To give an intuition for these results, we need to look at the interplay between long-run effects and the short-run dynamic adjustment in the nonlinear models. Unlike the Calvo model, the non-linear Rotemberg model is completely forward-looking. Hence, in the Rotemberg model a disinflation implies an immediate adjustment to a permanently lower level of output, hours and real wage. Real wage rigidities again prevent the immediate adjustment of the real wage, that sluggishly decreases towards the new lower long-run level, thus depressing output. Hence, contrary to the Calvo model, the Rotemberg model exhibits a dynamics in line both with the conventional wisdom and the empirical evidence that real wage rigidities cause a significant output slump along the adjustment path (see, e.g., Blanchard and Galf, 2007).

3.2.1 The Cost of Disinflation and the Sacrifice Ratio
We now consider the cost of disinflation by measuring it in terms of the sacrifice ratio, which is generally defined as the ratio of the loss in output to the fall in trend inflation. Given that the sacrifice ratio can be affected by the degree of wage rigidities, we discuss the impact on the sacrifice ratio of wage rigidities. As a measure of the sacrifice ration we consider the following model consistent measure,

$$SR = -\frac{\sum_{t=0}^{T} \left( \frac{Y_t - \hat{Y}_{new}}{\hat{p}_{new}} \right)}{\hat{p}_{old} - \hat{p}_{new}}$$

where $Y_{new}$ represents the new steady state level of output following a disinflation experiment, i.e. at the new steady state inflation level $\hat{p}_{new}^*$. Analogously, $\hat{p}_{old}^*$ is the steady state inflation level before disinflation occurs. Thus, $SR$ measures the cumulative percentage output loss the economy has to incur for each percentage point of permanent reduction of steady state inflation. Since the economy converges to a new steady state after disinflation, we define the sacrifice ratio by calculating the output loss in terms of the deviations from the new steady state. The value of $T$ is set equal to 100, which is enough to guarantee the convergence of output to the new steady state.

Notice that, at time $t = 0$ the economy is in its initial steady state and therefore $Y_0 = Y_{old}$, where $Y_{old}$ represents the steady state level of output at $\hat{p}_{old}^*$. Thus, the sacrifice ratio $SR$ can be split in two components as follows:

$$SR = -\left( \frac{\sum_{t=1}^{T} \left( \frac{Y_t - \hat{Y}_{new}}{\hat{p}_{new}} \right)}{\hat{p}_{old}^* - \hat{p}_{new}^*} \right)_{SR_{LR}} + \left( \frac{Y_{old} - \hat{Y}_{new}}{\hat{p}_{new}^* - \hat{p}_{new}^*} \right)_{SR_{LR}}$$

6
SR$_{SR}$ (short run sacrifice ratio) measures the cumulative percentage output loss the economy incurs during the transition to the new steady state. On the other hand SR$_{LR}$ measure the long run output loss, due to the fact that after disinflation the economy converges to a new steady state. Notice that if the economy immediately adjusts to the new steady state SR$_{SR} = 0$ and SR = SR$_{LR}$. Moreover, whenever SR$_{SR}$ and SR$_{LR}$ are positive (negative), disinflation implies an output loss (gain).

Table 1 considers the sacrifice ratios SR$_{SR}$ and SR$_{LR}$ for different disinflation policies and different degrees of real wage rigidity. Contrary to what happens to SR$_{SR}$, the long-run SR$_{LR}$ is not affected by the degree of real wage rigidities. Indeed, according to our model economy, real wage rigidities do not affect the long run. Notice that, as expected in the Calvo model, disinflation implies that the long-run SR is negative, i.e. it implies an output gain, while in the Rotemberg model, disinflation implies an output loss and the long-run SR is always positive. Analogously, in the short run, the two pricing models responds in the opposite way to an increase in real wage rigidities. Indeed, the sacrifice ratio decreases as the real wage rigidities increase in the Calvo model and becomes negative (output gains) for very high value of the degree of real wage rigidity, $\gamma = 0.9$. On the other hand the higher is the degree of real wage rigidity $\gamma$, the higher is the sacrifice ratio and the cumulative short-run loss in output in the Rotemberg model. In particular notice that, for $\gamma = 0$ (flexible real wages), the Rotemberg model implies an immediate adjustment to the new steady state $\pi_{new}^*$ and therefore the short-run sacrifice ratio is equal to zero, no matter the value of $\pi_{old}$.

- Table 1 about here -

4 Conclusion

We study the effect of a permanent disinflation in a New Keynesian model with real wage rigidities under the Rotemberg and the Calvo pricing models. We show that, if the Central Bank permanently and credibly reduces the inflation target, the Calvo model implies output gain, rather than cost, of disinflation, while the Rotemberg model implies output losses. Furthermore, in the Calvo model, real wage rigidities delivers the odd result of an overshooting of output above its new higher steady state level. On the contrary, in the Rotemberg model, sluggish real wages cause a significant output slump along the adjustment path, implying a significant trade-off between stabilizing inflation and output. This last result restores a conventional result on which there seems to be consensus in the literature (see, e.g., Blanchard and Gali, 2007). Our model do not consider the effect of price indexation. We are aware that price indexation may play an important role, above all when trend inflation is high. In a companion paper, Ascani and Rossi 2010, consider the effects of price indexation on the long-run properties and on the dynamics of the two pricing models. We show that in both models, price indexation is able to dampen the effects of trend inflation and therefore to reduce the differences between the two pricing mechanism. In the very particular case of full price indexation the two models are again equivalent as in the case of zero trend inflation. Therefore, the two model imply the same dynamics under two vary particular case: i) the steady state inflation level is zero; ii) full indexation. Both the two assumptions are against the empirical evidence. First, a low and positive trend inflation seems to be much more realistic since, as the post-war economic history of industrialized countries shows. Second, the recent empirical evidence find a very low degree of price indexation. For examples, Cogley and Shordone (2008) and Benati (2008), among others, estimate a very low degree of indexation in a model with trend inflation.
5 References

Fig. 1 Steady state deviations from zero inflation s.s. in the Rotemberg model

Fig. 2. Disinflation and real wage rigidities in the Calvo model. From $\bar{\pi} = 4\%$ to $\bar{\pi} = 0$.

Fig. 3 Disinflation and real wage rigidities in the Rotemberg model. From $\bar{\pi} = 4\%$ to $\bar{\pi} = 0$. 
Table 1: Disinflation and the Sacrifice Ratio

<table>
<thead>
<tr>
<th>Disinflation</th>
<th>Calvo Model</th>
<th>Rotemberg model</th>
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<tbody>
<tr>
<td>TI</td>
<td>$\pi_{old}$</td>
<td>$\pi_{new}$</td>
</tr>
<tr>
<td>$\gamma = 0$</td>
<td>$\gamma = 0.5$</td>
<td>$\gamma = 0.9$</td>
</tr>
<tr>
<td>2%</td>
<td>0%</td>
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</tr>
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