Endogenous Growth, Monetary Shocks and Nominal Rigidities

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Endogenous Growth, Monetary Shocks and Nominal Rigidities*

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Abstract

We introduce endogenous growth in an otherwise standard NK model with staggered prices and wages. Some results follow: (i) monetary volatility negatively affects long-run growth; (ii) the relation between nominal volatility and growth depends on the persistence of the nominal shocks and on the Taylor rule considered; (iii) a Taylor rule with smoothing increases the negative effect of nominal volatility on mean growth.

Keywords: Growth, volatility, business cycle, monetary policy.

JEL codes: E32, E52, O42.

1 Introduction

Traditionally, macroeconomists considered growth and business cycles are separated research areas. However, following the seminal paper by Ramey and Ramey (1995) many theoretical and empirical contributions have studied the relationship between volatility and long-run growth (for a comprehensive treatment of the literature see Steindl and Tichy 2009). The existence of a relationship between growth and volatility has important policy implications as it suggests the possibility that policies designed to stabilize short-run fluctuations might affect the long-run performance of the economy.

In this paper we study the relationship between output growth and volatility of money shocks in a New Keynesian (NK) model characterized by endogenous growth à la Romer (1986) and

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*A detailed appendix is available from the authors upon request.
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nominal rigidities due to both staggered price and wage setting.\(^1\) The monetary rule is of the Taylor-type. To the best of our knowledge we are the first to study the relationship between volatility and growth using this setup. Indeed, very few papers analyze this relationship in the context of monetary models, thus taking into account the implications of nominal volatility (see e.g. Evans and Kenc 2003, Dotsey and Sarte 2000 and Varvarigos 2008). An even smaller subset introduce nominal rigidities (see e.g. Blackburn and Pelloni 2004, 2005 and Annicchiarico \textit{et al.} 2010), but only in the form of one-period nominal wage contracts. Third, we are the first to consider staggered prices together with staggered wages. In this way, we are able to distinguish the role the two rigidities play in affecting the relationship between nominal volatility and growth. Finally, as far as we know, our paper is the first to study the relationship between volatility and growth with a monetary authority which smooths the business cycle by adopting a Taylor-type rule. Our main results are: (i) the model implies a non-negligible negative relationship between nominal volatility and growth; this is interesting because explaining the negative correlation between the volatility and the mean rate of output found in the data has proved a challenge for the theoretical literature.\(^2\) This is true even if we adopt a logarithmic specification of utility under which precautionary saving induces, by itself, a positive effect of uncertainty on growth;\(^3\) (ii) this effect depends upon the type of the Taylor rule considered and upon the interaction between the two staggering mechanisms; (iii) by implementing a Taylor-rule with smoothing the monetary authority increases the (negative) effects of nominal volatility on long-run growth. The key point is that the literature on monetary policy cannot disregard the role that nominal rigidities and the type of monetary rule adopted have in the transmission of uncertainty on long-term growth.

The paper is organized as follows: section two describes the model setup. Section three analyzes the relationship between nominal rigidities and growth under different Taylor-type rules and concludes.

\(^1\)Vaona (2010) also uses a NK model with endogenous growth to study the relationship between inflation and growth.

\(^2\)For different but by no means not alternative explanations, see Aghion \textit{et al.} (2010), who introduce credit constraints, and Krebs (2003), who introduces idiosyncratic shocks to human capital accumulation.

\(^3\)See de Hek and Roy (2001) and Jones \textit{et al.} (2005).
2 The model

The economy is described by a standard NK model with prices and wage rigidities (see Galí 2008), extended to include endogenous capital accumulation, convex investment adjustment costs and an endogenous growth mechanism with serendipitous learning by doing à la Romer (1986). The monetary authority follows a Taylor-type rule.

2.1 Firms and Endogenous Growth

In each period, the final good $Y_t$ is produced by perfectly competitive firms, using the intermediate inputs produced by the intermediate sector, with the standard technology: $Y_t = \left[ \int_0^1 Y_j^{(\theta_p-1)/\theta_p} dj \right]^{\theta_p/(\theta_p-1)}$, with $\theta_p > 1$. There is a continuum of monopolistic competitive intermediate good-producing firms $j \in (0,1)$ each of which produces a differentiated output $Y_{j,t}$ using the following technology

$$Y_{j,t} = AK_{j,t}^{1-\alpha} (Z_t N_{j,t})^\alpha, \quad \alpha \in (0,1), \quad A > 0. \tag{1}$$

$K_{j,t}$ is physical capital (final good) and $N_{j,t}$ are labor hours from the aggregator combining household-specific labor services supplied in a monopolistic competitive market. $A$ is a constant term. $Z_t$ represents an index of knowledge, taken as given by each firm, which is freely available to all firms and which is acquired through learning-by-doing. In particular we assume $Z_t = K_t$, where $K_t = \int_0^1 K_{j,t} dj$.

Prices are modeled à la Calvo (1983). In each period there is a fixed probability $1 - \xi_p$ a firm in the intermediate sector can set its optimal price $P_{j,t}^*$, otherwise the price is unchanged. The
first-order conditions of the firm’s problem with respect to $K_{j,t}$, $N_{j,t}$, and $P_{j,t}^*$ are:

\[ R^K_t = (1 - \alpha)MC^N_{j,t} \frac{Y_{j,t}}{K_{j,t}}, \quad (2) \]
\[ W_t = \alpha MC^N_{j,t} \frac{Y_{j,t}}{N_{j,t}}, \quad (3) \]
\[ P_{j,t}^* = \frac{\theta_p}{\theta_p - 1} E^t \sum_{i=0}^{\infty} \xi^i_t Q_{t,t+i} MC^N_{j,t+i} P^0_{t+i} Y_{t+i}, \quad (4) \]

where $R^K_t$ is the nominal rental rate of capital, $MC^N_{j,t}$ denotes the nominal marginal cost, $W_t$ is the nominal wage rate and $Q_{t,t+i}$ is the stochastic discount factor used at time $t$ by shareholders to value date $t + i$ profits. The zero-profit condition in the final good sector is respected as the price of the final good (aggregate price index) is defined as

\[ P_t = \left( \int_0^1 P_{j,t}^{1-\theta_p} dy \right)^{1/(1-\theta_p)}. \]

### 2.2 Households

The typical household $h \in (0, 1)$ maximizes the following lifetime utility:

\[ E^0 \sum_{t=0}^{\infty} \beta^t \left( \log C_{h,t} - \mu_n \frac{N_{h,t}}{1+\phi} \right), \quad \phi, \mu_n > 0 \text{ and } \beta < 1, \]
\[ s.t. \quad P_t C_{h,t} + Q_{t,t+1} B_{h,t} = B_{h,t-1} + W_t(h)N_{h,t} + D_{h,t} + R^K_t K_{h,t} - P_t \left[ \Gamma_I(I_{h,t}) + I_{h,t} \right] - T_{h,t}, \quad (5) \]

where $C_{h,t}$ is consumption and $N_{h,t}$ denotes specific labor services of type $h$ at time $t$. In each period $t$ the representative household $h$ carries $K_{h,t}$ units of physical capital from the previous period. $Q_{t,t+1}$ is a vector of prices of state-contingent assets that will pay one unit of currency if a particular state of nature occurs in period $t+1$, while each corresponding element of the vector $B_{h,t}$ represents the quantity of such contingent claims purchased at time $t$. Given the current state of nature, $B_{h,t-1}$ is the market value of such claims. $T_{h,t}$ denotes lump-sum taxation. Firms are owned by consumers, and $D_{h,t}$ are dividends in nominal terms. Finally, physical capital accumulates according to: $K_{h,t+1} = (1 - \delta)K_{h,t} + I_{h,t}$. The investment decisions are subject to convex adjustment costs $\Gamma_I(I_t)$ given by: $\Gamma_I(I_t) = \frac{\varphi_I}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t$ with $\varphi_I > 0$. From the first
order conditions, dropping index $h$:

\[
R_t = \frac{1}{E_t} \left\{ \frac{C_t P_t}{C_{t+1} P_{t+1}} \right\},
\]

\[
\varphi \left( \frac{I_t}{K_t} - \delta \right) = q_t - 1,
\]

\[
C_{t-1} q_t = \beta E_t C_{t+1}^{-1} \left\{ \tilde{R}_t^k + (1 - \delta) q_{t+1} + \frac{\varphi t}{2} \left( \delta + \frac{I_{t+1}}{K_{t+1}} \right) \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \right\},
\]

where $R_t = 1/E_t Q_{t,t+1}$ is the nominal interest factor on an asset that pays one unit of currency under every state of nature in period $t+1$, $\tilde{R}_t^k = R_t^k / P_t$ and $q_t$ denotes the Tobin’s marginal q.

As in Erceg et al. (2000), households supply differentiated labor services to the intermediate good-producing sector and set nominal wages in staggered contracts à la Calvo. A representative labor aggregator combines households’ labor hours in the same proportions firms would choose,

\[
N_t = \int_0^1 \frac{N_t^{(\theta_w-1)/\theta_w}}{\theta_w} dh
\]

with $\theta_w > 1$. In each period a constant fraction $1 - \xi_w$ of households reset their wage contracts. Households maximize their utility function subject to the budget constraint and the labor demand schedule. Let $W_t^*$ denotes the value of $W_t$ set by an household that can reoptimize its wage at time $t$, we then have:

\[
\frac{W_t^*}{P_t} = \frac{\theta_w}{\theta_w - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta \xi_w)^{i+1} N_{t+i}^{1+\phi} + N_{t+i}^{1+\phi} P_{t+i} / C_{t+i} P_{t+i}}{E_t \sum_{i=0}^{\infty} (\beta \xi_w)^{i} N_{h,t+i} P_{t+i} / C_{t+i} P_{t+i}},
\]

where we use the fact that all households resetting their wage at time $t$ will choose the same wage, since they face and identical optimization problem given the existence of a complete set of securities market.
2.3 Aggregation

In equilibrium all markets clear. The aggregate resource constraint and the aggregate production function are:

\[ Y_t = C_t + I_t + G_t + \frac{\varphi I}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t, \]  
\[ Y_t = AK_t N_t^\alpha (D_{p,t}D_{w,t})^{-1}, \tag{10} \]

where \( D_{p,t} = \int_0^1 \left( \frac{P_{j,t}}{P_t} - \theta_p \right) dj \) and \( D_{w,t} = \int_0^1 \left( \frac{W_{h,t}}{W_t} - \theta_w \right) dh \) measure price and wage dispersion and \( G_t \) denotes public spending fully financed by lump-sum taxes and assumed to be equal to a constant fraction of income.

2.4 Monetary Policy

The monetary policy is described by a Taylor-type interest rate rule subject to exogenous disturbances, i.e.:

\[ R_t = \left( \frac{R_{t-1}}{R} \right)^{\tau_r} \left( \frac{\hat{y}_t}{\hat{y}} \right)^{\tau_y} \left( \frac{\hat{\pi}_t}{\pi} \right)^{\tau_{\pi}} \left( \frac{\pi^w}{\pi^w} \right)^{\tau_w} u_t, \tag{12} \]

where \( \pi_t = \frac{P_t}{P_{t-1}} \), \( \pi \) is the deterministic balanced growth path (BGP) value of \( \pi_t \), \( \hat{y} \) is the deterministic BGP value of \( \hat{y}_t = Y_t/K_t \), \( \pi^w_t \) is wage inflation and \( \pi^w \) its deterministic steady state value. \( R \) is the deterministic BGP value of \( R_t \) and \( \tau_r, \tau_{\pi}, \tau_y, \tau_w \) and are policy parameters. The term \( u_t \) is defined as \( u_t = \exp \left[ \xi_{u,t} - \frac{\sigma_u^2}{2(1-\rho_u^2)} \right] \), \( \xi_{u,t+1} = \rho_u \xi_{u,t} + \varepsilon_{u,t+1} \), and \( \varepsilon_{u} \sim N \left( 0, \sigma_u^2 \right) \). The specification of the exogenous process \( u_t \) is relatively standard and allows us to study the effects of a mean-preserving spread increase in the volatility of the monetary shock since \( E(u_t) = 1 \).

3 Price and Wage Rigidity, Nominal Volatility and Growth

To evaluate the effects of the volatility of monetary shock on output growth we solve the model by following the ‘pure’ perturbation method by Schmitt-Grohé and Uribe (2004). As a number of variables, such as output, consumption, investments and wages are not stationary along the BGP,
we first perform a change of variables, so to obtain a set of equilibrium conditions involving only stationary variables. We set the benchmark parameter in line with the existing literature. The quarterly discount factor $\beta$ equals 0.99, the inverse of the Frisch elasticity $\phi$ is set equal to 1, the capital depreciation rate $\delta$ equals 0.025 and the labor share $\alpha$ is $2/3$. The steady-state inflation is equal to zero, $\pi = 1$, $N = 0.3$ while $\theta_p = \theta_w = 5$. We calibrate the remaining parameters to have $I/Y = 0.2$ and $C/Y = 0.65$ in steady state and an annual growth rate of output of 2% along a BGP.

To disentangle the role of nominal rigidities we consider three alternative economies: i) a NK model with staggered prices and flexible wages ($\xi_p = 2/3$, $\xi_w = 0$); ii) a model with staggered wages and flexible prices ($\xi_p = 0$, $\xi_w = 2/3$); iii) a model which embeds both staggered prices and staggered wages ($\xi_p = \xi_w = 2/3$).

Our variable of interest is the mean rate of output growth, that is $Mean(y_t - y_{t-1})$ where $y_t = \log(Y_t)$. We conduct two types of sensitivity analysis: we vary the standard deviation, $\sigma_{\varepsilon_u}$, and the persistence, $\rho_{\xi_u}$, of the monetary shocks. Finally, to better understand the interaction between nominal rigidities, monetary shocks volatility and monetary policy rules, six different Taylor rules, as indicated in Table 1, will be considered.

Figures 1 and 2 show the mean annual rate of output growth for increasing from 0 to 4% monetary policy variability $\sigma_{\varepsilon_u}$ under sticky prices and flexible wages (solid lines), flexible prices and sticky wages (dotted lines) and sticky prices and wages (dashed lines). The persistence of the monetary shock is set to $\rho_{\xi_u} = 0.5$ in Figure 1 and to $\rho_{\xi_u} = 0$ in Figure 2. The monetary authority follows the rules indicated in Table 1.

Monetary policy volatility always decreases the expected long-run growth: however the magnitude of the effect is very different depending on the source of nominal rigidities and on the conduct
of the monetary authority: in particular it is negligible with staggered wages and strong with staggered prices.

In the case of an economy with staggered prices and flexible wages, Figure 1 shows that an increase in the volatility of the monetary policy shock increases the volatility of prices and, therefore, price dispersion. This will increase firms uncertainty on the future value of the aggregate price index. With monopolistic competition and price staggering firms will try to reduce the risk of uncertainty by increasing their desired markup. Therefore, firms markup increases and real wages, output and investment decrease. This will translate into a reduction in average growth by virtue of a fall in the rate of knowledge accumulation.

With staggered wages and flexible prices the stochastic growth rate of output is not particularly affected by the volatility of the monetary policy shocks. In this case firms markup is constant and therefore firms are not able to reduce the risk of uncertainty by reducing their desired markup. On the other hand, uncertainty will induce agents to desire a higher real wage rate, thus pushing the average employment down: however for realistic values of the compensated elasticity of labor supply the effect will be small.4

With staggered prices and wages a higher nominal uncertainty increases both prices and wages dispersion inducing firms and workers to increase their desired markups, so implying a lower level of economics activity, a lower accumulation of capital and slower growth. On the other hand, given that both prices and nominal wages are staggered, real wages become more sluggish and less volatile and, as a consequence, the impact on the real marginal costs is lower and so the effects on output growth. Depending on which of the two effects prevails, the increase in the volatility of the shocks may have stronger or weaker effects on growth than those observed in an economy with sticky prices only.

Remarkably, rules characterized by interest rate smoothing strongly amplify the effects of uncertainty. This is due to the fact that by smoothing the interest rate, the monetary authority is much more accommodative and therefore its stabilizing effect is lower. This last result seems to

4This result is consistent with Blackburn and Pelloni (2004), Galindev (2009), Annicchiarico et al. (2010).
be of particular interest since, as documented by many empirical papers (among others Sahuc and Smets 2008, Smets and Wouters 2003) both the FED and the ECB seem to adopt Taylor rule with very high degree of smoothing- approximately equal to 0.8, as in our benchmark calibration.

Further, notice that a lower persistence of the monetary shock implies a lower negative effect on expected growth (see Figure 2). This is explained by the fact that in a model where prices and/or wages are set in a staggered fashion, a lower serial correlation implies less persistent effects of monetary shocks reducing the effects on nominal uncertainty.

Overall, we observe the following. Monetary shocks volatility has negative effects on the stochastic growth of output. Rules targeting both price and wage inflation reduce the negative impact of volatility on growth through the moderating effects these rules have on price and wage dispersion.

References


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Figure 1: Mean Annual Rate of Output Growth and Monetary Volatility, $\rho_{\xi_u} = 0.5$

\[\sigma_{\epsilon} \downarrow, \rho_{\epsilon} = 0, \rho_{\xi_u} = 0.5, \rho_{\eta} = 0, \rho_{\pi} = 1.5, \rho_{\pi w} = 1.5, \rho_{\pi y} = 0, \rho_{\pi r} = 0, \rho_{\pi y} = 0, \rho_{\eta} = 0, \rho_{\pi r} = 0.8, \rho_{\pi y} = 0, \rho_{\eta} = 0, \rho_{\pi r} = 0.8, \rho_{\pi y} = 0, \rho_{\eta} = 0\]
Figure 2: Mean Annual Rate of Output Growth and Monetary Volatility, $\rho_{\xi_u} = 0$

- $\tau_\pi = 1.5, \tau_{\pi_u} = \tau_y = t_r = 0$
- $\tau_\pi = \tau_{\pi_u} = 1.5, \tau_y = t_r = 0$
- $\tau_\pi = \tau_{\pi_u} = 1.5, \tau_y = 0.8, t_r = 0$
- $\tau_\pi = \tau_{\pi_u} = 1.5, \tau_y = 0.125, \tau_{\pi_u} = 0$