Monopolistic Competition: CES Redux?

Paolo Bertoletti
(Università di Pavia and IEFE)

Paolo Epifani
(Università Bocconi and IGIER)

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Via San Felice, 5
I-27100 Pavia
http://epmq.unipv.eu/site/home.html

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Monopolistic Competition: CES Redux?*

Paolo Bertoletti†
Pavia University and IEFE

Paolo Epifani‡
Bocconi University and IGIER

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Abstract

We investigate competitive, selection and reallocation effects in monopolistic competition trade models. We argue that departing from CES preferences in an otherwise standard Dixit-Stiglitz setup with additive preferences seems to involve implausible assumptions about consumer behavior and inconsistent competitive effects. In the presence of trade costs, selection effects à la Melitz (2003) are instead generally robust to the assumptions about preferences. However, they are unambiguously associated to aggregate productivity gains only when preferences are CES. We also study competitive effects in alternative monopolistic competition settings featuring non-additive preferences, strategic interaction and consumers’ preference for an ideal variety. We find that none of these setups delivers a compelling pro-competitive mechanism. Overall, our results suggest that in monopolistic competition, consistent with CES preferences, larger markets select more aggressively on productivity rather than forcing firms to move down their average cost curves.

JEL Classification: F1; Keywords: Monopolistic Competition; CES Preferences; International Trade; Competitive, Selection and Reallocation Effects.

1 Introduction

Although Dixit-Stiglitz (D-S) monopolistic competition cum constant elasticity of substitution (CES) preferences is still the workhorse of trade economists, it is currently under

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†Department of Economics and Business, Università di Pavia, Via San Felice 5, 27100 Pavia (Italy), and IEFE, Bocconi University. E-mail: paolo.bertoletti@unipv.it

‡Department of Economics, IGIER and Baffi Centre, Università Bocconi, Via Röntgen 1, 20136 Milan (Italy). E-mail: paolo.epifani@unibocconi.it.
fire.¹ The traditional critique that CES preferences make monopolistic competition little interesting due to the implied invariance of markups and firm size to market size has been recently revived by Zhelobodko, Kokovin, Parenti and Thisse (2011). In their influential paper, they argue that CES preferences are a knife-edge between cases yielding opposite results, and thus propose a more general D-S monopolistic competition setup with additive preferences and a variable elasticity of substitution.² In this paper we argue instead that, although CES preferences are clearly special, their special features may be a strength rather than a weakness when compared to the available modeling alternatives. In particular, although a variable elasticity yields competitive effects in monopolistic competition, such effects seem to stem from ad hoc assumptions about consumer behavior and lead to inconsistent results.

In a celebrated paper, Krugman (1979) developed a simple model featuring additive preferences and D-S monopolistic competition to show that trade opening between identical countries leads to lower markups (the pro-competitive effect of trade) and higher welfare. Crucially, Krugman assumed that the elasticity of substitution is decreasing in individual consumption (henceforth, DES preferences). He claimed (p. 476) that: "This assumption [...] seems plausible. In any case, it seems necessary if this model is to yield reasonable results, and I make the assumption without apology." In this paper we argue instead that DES preferences are implausible, and that they yield no more reasonable competitive effects than in the opposite case in which the elasticity of substitution is increasing in individual consumption (IES preferences).

To motivate our analysis, we propose the following exercise of introspection. Consider a situation in which you are endowed with two red pencils and two blue pencils, and compare it with a situation in which you are endowed instead with ten red and ten blue pencils. The key question is: do you perceive a red and a blue pencil as more substitutable in the former or in the latter situation? If you think, as we do, that varieties become no more substitutable when consumption of each shrinks, then the DES assumption is violated.

In Section 2, we explore the implications of a variable elasticity of substitution in a framework à la Krugman (1979). IES preferences imply that a rise in market size leads to

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¹In this paper, we refer to monopolistic competition as a market structure characterized by product differentiation, a downward sloping average cost curve and free entry. By Dixit-Stiglitz assumption we refer instead to the absence of strategic interaction among firms.

²For a critique of monopolistic competition cum CES see also Neary (2003, 2004, 2009). Another recent paper by Arkolakis, Costinot and Rodriguez-Clare (2012) argues that, provided that certain conditions that crucially exploit the properties of CES preferences are satisfied, gains from trade are invariant to the microeconomic details of the underlying model (conditional on observed trade flows). It also shows, however, that with CES preferences two statistics (the expenditure share on domestic goods and the elasticity of substitution) are sufficient to compute gains from trade.
smaller firms and higher markups. DES preferences, instead, in addition to building on dubious assumptions about consumer behavior, yield contrasting competitive effects that are hard to rationalize. In particular, a rise in market size is pro-competitive when due to (frictionless) trade opening, and anti-competitive when due to productivity growth. Just the opposite is true with IES preferences. More generally, trade liberalization (specifically, a fall in the marginal cost of exporting) leads firms to charge higher markups in either the domestic market (IES preferences) or in the foreign market (DES preferences). Next, we show that the above results hold also in the presence of heterogeneous firms. More importantly, we show that, when trade costs are large enough to induce a partitioning of firms into exporters and non-exporters (arguably, the empirically relevant case), selection effects à la Melitz (2003) hold independently of the assumptions about preferences. However, they are unambiguously associated to aggregate productivity gains only when preferences are CES.

In Krugman’s (1979) model, the number of firms \( n \) does not directly affect markups. This is because: 1) consumers share the same preferences over a number of characteristics that equals the number of varieties; 2) the Dixit-Stiglitz assumption that firms do not interact strategically implies that the perceived demand elasticity \( \varepsilon \) equals the elasticity of substitution \( \sigma \), and 3) preference additivity implies that \( \sigma \) is independent of \( n \). Assuming that \( \sigma \) varies with individual consumption is therefore the only way to obtain competitive effects in this setup. One may suspect, however, that a pro-competitive effect would arise more naturally when relaxing one of the above assumptions, which prevent \( n \) from directly impacting \( \varepsilon \).

In Section 3 we explore these possibilities. We start by relaxing the assumption that preferences are additive. We argue that, even with non-additive preferences, there is no compelling reason for the elasticity of substitution to be directly increasing in \( n \). For instance, when preferences are quasi-linear and quadratic, as in Melitz and Ottaviano (2008), the elasticity of substitution is decreasing in \( n \) for given level of individual consumption. This also implies that, as in the case of DES preferences, the induced pro-competitive effect is entirely driven by the indirect negative impact of \( n \) on individual consumption, namely, by the assumption that \( \partial \sigma / \partial c < 0 \). Instead, when preferences are translog, as in Feenstra (2003), the elasticity of substitution is directly increasing in \( n \). In both cases, however, just as with DES preferences, a fall in the marginal cost of exporting leads firms to charge higher markups in the foreign markets. Interestingly, as shown by Arkolakis et
al. (2012), this kind of anti-competitive effect implies that trade liberalization may lead to smaller welfare gains than in the CES case.

Demand elasticity may positively depend on the number of firms also because a trade-induced increase in $n$ may "crowd" the variety space, thus making varieties closer substitutes. As first noted by Pettengill (1979), this effect cannot be captured by a Dixit-Stiglitz setting, as the latter implicitly assumes that the number of characteristics/varieties equals the number of firms. We therefore revert to Lancaster’s (1979) ideal variety approach to monopolistic competition, where the space of characteristics is fixed and an increase in the number of firms makes available varieties closer to one another. Contrary to Pettengill’s view, we show that even in this setting demand elasticity need not be increasing in $n$: in order for Lancaster’s model to deliver a competitive effect, additional ad hoc assumptions unrelated to the basic framework are required. Otherwise, the ideal variety approach yields no competitive effects, just like the "love for variety" approach cum CES preferences.

Finally, a pro-competitive effect may naturally arise by simply relaxing the D-S assumption that firms do not interact strategically. We therefore consider Bertrand and Cournot extensions of the basic setup with additive preferences and a variable elasticity, in which $n$ has a direct pro-competitive impact for given $\sigma$. We show that this pro-competitive effect can only be relevant when the number of firms is small, an assumption at odds with monopolistic competition. Moreover, the assumption of CES preferences seems to dominate over the alternative assumptions, both in terms of analytical simplicity and plausibility of the results.

Section 4 briefly concludes. Our paper is related to the vast theoretical literature on monopolistic competition and international trade, initiated by Dixit and Stiglitz (1977), Krugman (1979, 1980), Lancaster (1979), Helpman (1981), and whose early contributions are systematized in Helpman and Krugman (1985). It is also closely related to Zhelobodko et al. (2011), which independently studies how departing from CES preferences in a standard D-S setting leads to opposite results. Their paper does not discuss, however, the plausibility and robustness of these results, a key contribution of our work, which leads us to opposite conclusions. Finally, our work is related to the recent heterogeneous-firm extensions of the monopolistic competition trade model, and in particular to Melitz (2003), Melitz and Ottaviano (2008) and Arkolakis et al. (2012), the latter studying welfare gains from trade in the presence of a Pareto distribution of firm productivity and DES-type

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5 More recent contributions include Bertoletti (2006) and Behrens and Murata (2007, 2012), which discuss a specific functional form for DES preferences.

6 Moreover, Zhelobodko et al. (2011) does not study how trade costs affect competitive and selection effects, nor the impact of reallocations on aggregate productivity and the case of strategic interactions.
preferences featuring a choke price.$^7$

2 Dixit-Stiglitz Monopolistic Competition with Additive Preferences

We first consider the case of symmetric firms, as in Krugman (1979), then extend the analysis to a setup with heterogeneous firms, as in Melitz (2003).

2.1 Symmetric Firms

Consider an economy populated by $L$ workers, whose wage is $w = 1$. Consumers share the same additive and symmetric preferences, represented by the following utility function:

$$U = \sum_{i=1}^{N} u(c_i), \quad (1)$$

where $c_i$ is consumption of variety $i$, defined over a large number $N$ of potential varieties. Only varieties indexed by $i = 1, \ldots, n$, with $n < N$, are actually produced. The subutility function $u(\cdot)$ is strictly increasing and concave, and is at least thrice continuously differentiable. In particular, we assume that $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u(0) = 0$.$^8$

Firm $i$ produces a differentiated variety with the total cost function $TC_i = \alpha + \beta q_i$, where $q_i = c_i L$ is its output, and $\alpha$ and $\beta$ are the fixed and marginal cost, both in terms of labor. Firms are symmetric on the cost and demand side, and therefore solve the same problem. In the following, we therefore drop the variety index $i$.

Utility maximization subject to a budget constraint implies $u'(c) = \lambda(p)p$, where $p$ is the price charged by an individual firm, $\lambda > 0$ is the marginal utility of income, and $p = [p_1, \ldots, p_n]$ is the price vector. Preference additivity implies the elasticity of $\lambda$ with respect to each price $p$ to be of the same order of magnitude as $1/n$, provided that prices are not disproportionate (see, e.g., Deaton and Muellbauer, 1980, Section 5.3). Thus, under the Dixit and Stiglitz (1977) assumption that $n$ is large enough to induce each firm to treat $\lambda$ as a constant, the inverse individual demand for a variety is

$$p = \frac{u'(c)}{\lambda}, \quad (2)$$

$^7$Dhingra and Morrow (2012) and Mrazova and Neary (2011) also consider a monopolistic competition setting with heterogeneous firms and a variable demand elasticity. The former paper focuses on the welfare properties of frictionless trade integration, and the latter on the conditions ensuring supermodularity of firms’ profit functions with respect to production and market-access costs.

$^8$Note that $U = nu(c)$ at a symmetric consumption pattern, with $nu(c) > u(nc)$: thus, $U$ embeds a Chamberlinian “taste for variety”. Moreover, $U$ is well defined over the positive orthant of the relevant Euclidean space: according to standard results, this implies regular and well-behaved demand functions for strictly positive prices and income.
and the perceived price elasticity of demand for an individual firm is
\[ \varepsilon(c) = -\frac{p(c)}{p'(c)c} = -\frac{u'(c)}{u''(c)c} = \sigma(c), \] (3)
where \(\sigma(c)\) is the elasticity of substitution between any two varieties when they are consumed in the same amount \(c\) (see the discussion and derivations in the Appendix).\(^9\) The assumption \(\sigma'(c) = 0\) is the well-known CES case. Krugman (1979) assumed \(\sigma'(c) < 0\), i.e., DES preferences, whereas \(\sigma'(c) > 0\) corresponds to IES preferences.\(^10\)

The revenue of an individual firm is:
\[ R(c) = p(c)cL = \frac{u'(c)}{\lambda}cL. \] (4)
We denote marginal revenue and the derivative of marginal revenue, respectively, by
\[ r'(c) = u'(c) + u''(c)c, \] (5)
\[ r''(c) = 2u'(c) + u''(c)c. \] (6)
The first-order condition for profit maximization implies:
\[ r'(c) = \lambda\beta. \] (7)
To obtain a unique and well-behaved solution to the problem of profit maximization, we assume that the marginal revenue is everywhere positive and decreasing. That is, we assume that \(r'(c) > 0\) and \(r''(c) < 0\) for all \(c\), with \(\lim_{c \to \infty} r'(c) = 0\) and \(\lim_{c \to 0} r'(c) = \infty\).

Equations (2), (3), (5) and (7) imply that the profit maximizing price can be written as
\[ p = \frac{u'(c)}{r'(c)} \beta = \frac{\sigma(c)}{\sigma(c) - 1} \beta = m(c)\beta, \] (8)
where \(\sigma > 1\), \(m\) is the price-marginal cost markup, and
\[ m'(c) = -\frac{\sigma'(c)}{(\sigma(c) - 1)^2}. \] (9)
\(^9\)Note also that \(\varepsilon(c)\) is independent of \(\lambda\) and \(L\) for given \(c\), and that it equals the reciprocal of the elasticity of marginal utility with respect to individual consumption.
\(^10\)A possible rationalization for IES preferences is that, by their very nature, differentiated varieties of some product can be used to perform either generic or more specific tasks. For instance, a blue pencil can be used either to write down a laundry list (for which a red pencil would be equally appropriate) or, jointly with a red pencil, to mark different types of comments on an exam paper. Hence, a fall in the symmetric endowment of varieties, by reducing the opportunity to use varieties to perform specific tasks, may also reduce their substitutability.
Evidently, markups are increasing in individual consumption $c$ with DES preferences, decreasing in $c$ with IES preferences, and constant with CES preferences. Differentiating (3) and using (5) and (6) yields the following expression for $\sigma'$:

$$\sigma'(c) = \frac{u'(c)r''(c) - u''(c)r'(c)}{[u''(c)c]^2},$$

which implies that

$$\sigma'(c) \gtrless 0 \iff \frac{r'(c)u''(c)}{r''(c)u'(c)} = \frac{\eta(c)}{\sigma(c)} \gtrless 1,$$

where $\eta(c) = -\frac{r'(c)}{r''(c)c} > 0$ is the reciprocal of the elasticity of marginal revenue. Note that $\eta = \sigma$ when preferences are CES. With non-CES preferences, instead, $\eta \neq \sigma$ and both quantities are variable.

Free entry implies zero equilibrium profits:

$$\pi = \pi_v - \alpha = (p - \beta)cL - \alpha = 0,$$

where $\pi$ and $\pi_v$ denote total and variable profits. Using (5) and (8), the free-entry condition can be written as

$$\pi_v = [m(c) - 1] \beta Lc = \left[ \frac{u'(c)}{r'(c)} - 1 \right] \beta Lc = \left[ -\frac{u''(c)}{r'(c)c} \right] \beta Lc = \alpha.$$

Differentiating $\pi_v$ with respect to $c$ yields:

$$\frac{\partial \pi_v}{\partial c} = \beta L \left[ \frac{u''(c)r'(c) - u'(c)r''(c)}{r'(c)c} - \frac{u''(c)c}{r'(c)c} \right] = \beta L \frac{m(c)}{\eta(c)} > 0.$$

Variable profit is therefore monotonically increasing in individual consumption, which ensures that the equilibrium is unique. Finally, full employment implies that labor demand, $n(\alpha + \beta cL)$, equals labor supply, $L$ (equivalently, equilibrium in the goods market implies that individual expenditure, $npc$, equals income, $w = 1$). Thus, using (12) and (8) yields:

$$n = \frac{L}{\alpha + \beta Lc} = \frac{L}{\alpha \sigma(c)} = \frac{1}{m(c)c\beta}.$$

Note that (12) implicitly defines the level of individual consumption consistent with profit maximization and free entry as a function $c(L, \beta, \alpha)$ of market size $L$, marginal cost $\beta$, and the fixed cost $\alpha$. The equilibrium number of firms, $n(L, \beta, \alpha)$, is instead determined recursively by (14).

We can now show how individual consumption depends on the model’s parameters.
Differentiating (12) with respect to $L$, $\beta$ and $\alpha$ yields:

$$\frac{\partial \pi_v}{\partial c} \frac{\partial c}{\partial L} + \frac{\partial \pi_v}{\partial L} = \frac{\partial \pi_v}{\partial c} \frac{\partial c}{\partial \beta} + \frac{\partial \pi_v}{\partial \beta} = 0 \text{ and } \frac{\partial \pi_v}{\partial c} \frac{\partial c}{\partial \alpha} = 1.$$ 

Noting that $\frac{\partial \pi_v}{\partial L} L = \frac{\partial \pi_v}{\partial \beta} \beta = \pi_v = \alpha$, we obtain the following

**Lemma 1** Individual consumption is decreasing in market size $L$ and marginal cost $\beta$, and increasing in the fixed cost $\alpha$, with:

$$\frac{\partial \ln c}{\partial \ln L} = -\frac{\partial \ln c}{\partial \ln \beta} = -\frac{\eta(c)}{\sigma(c)} \leq -1 \Leftrightarrow \sigma'(c) \geq 0.$$  (15)

Lemma 1 and $\text{sign}(m') = -\text{sign}(\sigma')$ immediately imply

**Proposition 1** With IES (DES) preferences markups are increasing (decreasing) in $(\beta L/\alpha)$, and firm size, $q(c) = cL$, is decreasing (increasing) in market size $L$.

2.1.1 Costly Trade

We now assume that exporting in an identical foreign market involves an iceberg trade cost $\tau > 1$. We denote by the subscript $x$ variables related to the export market, and by no subscript those related to the domestic market. Total profits now equal $\pi_v(c) + \pi_v(c_x) - \alpha$, where $\pi_v(c)$ is still given by (12), and:

$$\pi_v(c_x) = [m(c_x) - 1] \tau \beta c_x L.$$  (16)

By (7), profit maximization implies $r'(c_x) = \lambda \tau \beta$ and $r'(c) = \lambda \beta$. Thus:

$$r'(c_x) = \tau r'(c).$$  (17)

Eq. (17) implicitly defines $c_x = c_x(c, \tau)$, with

$$\frac{\partial \ln c_x}{\partial \ln c} = \frac{\eta(c_x)}{\eta(c)} > 0, \quad \frac{\partial \ln c_x}{\partial \ln \tau} = -\eta(c_x) < 0.$$  (18)

The zero-profit condition, which implicitly defines $c = c(L, \beta, \alpha, \tau)$, can now be written as

$$\{[m(c) - 1] c + [m(c_x) - 1] \tau c_x \} \beta L - \alpha = 0.$$  (19)

Applying the implicit function theorem to (19) and using (13), (17) and (18) yields:

$$\frac{\partial \ln c}{\partial \ln \tau} = \frac{\eta(c) c_x \tau}{cm(c) + c_x \tau m(c_x)} > 0.$$  (20)
Evidently, a fall of trade costs \( \tau \) reduces individual domestic demand for domestic firms. It follows that, with DES (IES) preferences, trade liberalization leads domestic firms to charge lower (higher) markups in the domestic market.

Consider now foreign sales. Differentiating (17) and using (18) and (20) yields:

\[
\frac{d \ln c_x}{d \ln \tau} = \eta(c_x) \left[ \frac{c_x \tau}{cm(c) + c_x \tau m(c_x)} - 1 \right] < 0.
\]

Thus, a fall of trade costs \( \tau \) increases individual demand for foreign varieties. It follows that, with DES (IES) preferences, trade liberalization leads foreign firms to charge higher (lower) markups.

2.1.2 Discussion

When preferences are IES, a rise in market size leads to higher markups and smaller firms. These results are the opposite of the pro-competitive and de-fragmentation effects delivered by Krugman’s model and by virtually all monopolistic competition trade models departing from CES preferences. Moreover, it can be shown that gains from trade and productivity growth are not ensured in this case.\(^{11}\)

When preferences are DES, however, although a rise in market size is pro-competitive when due to frictionless trade opening (which in this model is isomorphic to a rise in \( L \)), it is anti-competitive when due to productivity growth (as captured by a fall in the marginal cost \( \beta \)). Moreover, both cases yield a rise in the firm size \( q = cL \) and a fall in the market share \( (1/n) \), which therefore turn out to be phenomena unrelated the "competitive effects" identified by the model. The opposite results hold with IES preferences. These contrasting results seem hard to rationalize. In addition, trade liberalization leads firms to charge higher markups in either the domestic market (IES preferences) or in the foreign market (DES preferences). As far as we know, there is no evidence in support of any of these specific implications, which seem at odds with the conventional wisdom about the pro-competitive effects of trade liberalization.

Finally, as for the role played by fixed production costs, note that in monopolistic competition trade models endogenizing technology it is standard to assume that a lower

\(^{11}\)This is because a rise in \( L/\alpha \) has a positive welfare effect due to the induced rise in \( n \) (the standard love for variety effect), and a negative welfare effect due to the rise in markups. Conversely, a rise in \( \beta \) has a negative welfare effect due to the fall in the real wages (also due to the rise in markups), and a positive welfare effect due to the induced rise in \( n \). It can be shown that a sufficient condition for a rise in market size to be welfare increasing is that \( \phi' > 0 \), where \( \phi(c) = d \ln u/d \ln c \). Instead, \( \phi' < 0 \) is a sufficient condition for a rise in marginal cost to be welfare decreasing. See also Dhingra and Morrow (2012) for an analysis of the welfare effects of an increase in market size in the presence of a variable demand elasticity and heterogeneous firms.
marginal cost requires a higher fixed cost $\alpha$, e.g., in terms of R&D expenditures (see, among others, Yeaple, 2005, Bustos, 2011, Costantini and Melitz, 2007). This type of technical change would imply even stronger anti-competitive effects with DES preferences.\footnote{Interestingly, Mrazova and Neary (2011) have shown that, when preferences are DES (sub-convex in their terminology), it is also possible that more productive firms end up serving the foreign market through exports rather than FDI, a result at odds with the conventional wisdom and the existing empirical evidence.}

2.2 Heterogeneous Firms

In this section, we show that the above competitive effects hold also in the presence of heterogeneous firms. More importantly we show that, in the presence of trade costs, Melitz-type selection effects are robust to non-CES preferences. However, only when preferences are CES they are unambiguously associated to aggregate productivity gains. Following Melitz (2003) we assume that, upon paying a fixed entry cost $\alpha\epsilon$, firms draw their marginal cost $\beta \in [\beta, \infty)$ from a continuous cumulative distribution $G(\beta)$ with density $g(\beta)$ and $\beta > 0$. We also assume a continuum of varieties, we index firms by their marginal cost $\beta$ and denote individual demand for a $\beta$-firm by $c(\beta)$. Otherwise, the setup is the same as in the previous Section.

Firm productivity, size and markups Denote by $p(\beta) = m(c(\beta))\beta$ and $\pi_v(\beta) = [p(\beta) - \beta] c(\beta) L$ the price and variable profit of a $\beta$-firm. The first-order and second-order conditions for profit maximization, $r'(c) = \lambda \beta$ and $r''(c) < 0$, imply that:

\[
\begin{align*}
\frac{d \ln c(\beta)}{d \ln \beta} &= \frac{\lambda \beta}{r''(c(\beta)) c(\beta)} = -\eta(c(\beta)) < 0, \quad (21) \\
\frac{d \ln \pi_v(\beta)}{d \ln \beta} &= -c(\beta) L \frac{\beta}{\pi_v(\beta)} = 1 - \sigma(c(\beta)) < 0, \quad (22) \\
\frac{d \ln p(\beta)}{d \ln \beta} &= \frac{\eta(c(\beta))}{\sigma(c(\beta))} \geq 1 \iff \sigma'(c(\beta)) \geq 0. \quad (23)
\end{align*}
\]

Evidently, high-productivity (low-$\beta$) firms are larger (21) and more profitable (22), as in Melitz (2003). Unlike the Melitz model, however, where preferences are CES and markups are constant, with IES (DES) preferences larger firms charge lower (higher) markups (23). Note also that $\eta$ and $\sigma$ govern, respectively, relative firm size and profitability.

Zero cutoff profit condition Denote by $\beta^*$ the marginal cost cutoff, namely, the value of $\beta$ satisfying the zero cutoff profit condition $\pi_v(\beta^*) = 0$:

\[
\pi_v(\beta^*) = [m(c^*) - 1] \beta^* c^* L = \alpha. \quad (24)
\]
Eq. (24) implicitly defines the individual demand for the cutoff firm, \( c^* = c^*(\beta^*, L, \alpha) \). Differentiating yields:

\[
\frac{\partial \ln c}{\partial \ln L} = \frac{\partial \ln c}{\partial \ln \beta^*} = -\frac{\partial \ln c}{\partial \ln \alpha} = -\frac{\eta(c^*)}{\sigma(c^*)} < 0.
\] (25)

Eqs. (25) and (11) also imply that the size of the cutoff firm, \( q^* = c^*L \), is decreasing or increasing in \((L\beta^*/\alpha)\) depending on whether preferences are IES or DES.

**Individual demand for a \( \beta \)-firm** Profit maximization by the cutoff firm implies \( r'(c^*) = \lambda \beta^* \). Solving for \( \lambda \) and substituting into \( r'(c) = \lambda \beta \) yields:

\[
r'(c) = r'(c^*) \frac{\beta}{\beta^*}.
\] (26)

Eq. (26) is key to the characterization of the equilibrium, as it implicitly defines individual demand for a \( \beta \)-firm, \( c(\beta) = c(\beta; \beta^*, c^*) \). Using \( c^* = c^*(\beta^*, L, \alpha) \), we can now show how \( c(\beta) = c(\beta; \beta^*, c^*(\beta^*, L, \alpha)) = c(\beta; \beta^*, L, \alpha) \) varies with \( \beta^* \), \( L \) and \( \alpha \). Differentiating (26) with respect to \( \beta^* \) yields:

\[
r''(c) \frac{\partial c}{\partial \beta^*} = \frac{\beta}{\beta^*} \left[ r''(c^*) \frac{\partial c^*}{\partial \beta^*} - r'(c^*) \right].
\]

Rearranging terms and using (25) yields:

\[
\frac{\partial \ln c}{\partial \ln \beta^*} = \frac{\eta(c)}{m(c^*)} > 0.
\]

Hence, as in Melitz (2003), individual demand is increasing in the cutoff marginal cost \( \beta^* \). Similarly, differentiating (26) with respect to \( L \) and using (25) yields:

\[
\frac{\partial \ln c}{\partial \ln L} = -\frac{\partial \ln c}{\partial \ln \alpha} = -\frac{\eta(c)}{\sigma(c^*)} < 0.
\]

Thus, for given \( \beta^* \), individual demand is decreasing in market size \( L \) and increasing in the fixed cost \( \alpha \). Note also that, with IES preferences, \( \sigma(c^*) < \sigma(c) \) for \( c^* < c \); hence, by (11):

\[
\frac{\partial \ln c}{\partial \ln L} < -\frac{\eta(c)}{\sigma(c)} < -1.
\]

Thus firm size, \( q = cL \), is decreasing in \( L \) for given \( \beta^* \). In contrast, with DES preferences \( \sigma(c^*) > \sigma(c) \) for \( c^* < c \), and hence firm size is increasing in \( L \).

The following lemma summarizes
Lemma 2 Individual demand $c(\beta; \beta^*, L, \alpha)$ is increasing in $\beta^*$ and $\alpha$ and decreasing in $L$, with

$$\frac{\partial \ln c}{\partial \ln \beta^*} = \frac{\eta(c)}{m(c^*)} > 0, \quad \frac{\partial \ln c}{\partial \ln L} = -\frac{\partial \ln c}{\partial \ln \alpha} = -\frac{\eta(c)}{\sigma(c^*)} \leq -1 \Leftrightarrow \sigma'(c) \geq 0.$$ (27)

Free-entry condition Free entry implies that expected profits,

$$\pi^E = \int_\beta^{\beta^*} \pi(\beta) dG(\beta),$$

equal the sunk entry cost $\alpha_e$. Integrating $\pi^E$ by parts using (22), and noting that $\pi(\beta^*) = G(\beta_e) = 0$, yields:

$$\pi^E = \int_\beta^{\beta^*} c(\beta) L G(\beta) d\beta = \alpha_e.$$ (28)

Differentiating $\pi^E$ with respect to $\beta^*$ yields:

$$\frac{\partial \pi^E}{\partial \beta^*} = c(\beta^*) L G(\beta^*) + \int_\beta^{\beta^*} \frac{\partial c(\beta)}{\partial \beta^*} L G(\beta) d\beta > 0,$$ (29)

where the inequality follows from Lemma 2. Hence, as in Melitz (2003), expected profits are increasing in $\beta^*$ and the free-entry condition (28) uniquely pins down $\beta^*$, thereby defining the equilibrium value of $c(\beta) = c(\beta; L, \alpha, \alpha_e)$.13

2.2.1 Competitive and Selection Effects of a Rise in Market Size

Selection effects The impact of a rise in market size $L$ on the marginal cost cutoff $\beta^*$ can be computed by applying the implicit function theorem to (28):

$$\frac{d\beta^*}{dL} = -\frac{\partial \pi^E / \partial L}{\partial \pi^E / \partial \beta^*},$$ (30)

13Note that the measure of active firms $n$ is determined by the budget constraint (or, equivalently, by the full-employment condition), requiring average expenditure to equal $1/n$, and thus:

$$n = \left[ \int_\beta^{\beta^*} p(\beta) c(\beta) G(\beta) \right]^{-1}.$$
where $\partial \pi^E / \partial \beta^* > 0$ by (29) and, using Lemma 2,

$$\frac{\partial \pi^E}{\partial L} = \int_\beta^{\beta^*} \left[ \frac{\partial c(\beta)}{\partial L} L + c(\beta) \right] G(\beta) d\beta = \int_\beta^{\beta^*} \left( 1 - \frac{\eta(c(\beta))}{\sigma(c^*)} \right) c(\beta) G(\beta) d\beta \leq 0 \Leftrightarrow \sigma' \geq 0. \tag{31}$$

Thus, with DES preferences, $d\beta^*/dL < 0$: a rise in market size leads to a standard selection effect. Conversely, with IES preferences a rise in market size leads to a rise in $\beta^*$ and a consequent anti-selection effect, whereby less productive firms can survive in a larger market.\(^{14}\) Therefore, as also noted by Zhelobodko et al. (2011) and Dhingra and Morrow (2012), selection effects seem to crucially depend on the assumptions about preferences. Moreover, as implicitly argued in this recent literature, aggregate productivity gains or losses seem to uniquely depend on whether $\beta^*$ rises or falls. Below we show, however, that both conclusions are fragile.

**Competitive effects** The impact of a rise in market size on markups can be obtained by simply studying how a rise in $L$ affects individual consumption. Using (29) and (31) in (30) and rearranging yields:

$$c(\beta^*) LG(\beta^*) \frac{d\beta^*}{dL} + \int_\beta^{\beta^*} c(\beta) G(\beta) d\beta + L \int_\beta^{\beta^*} \frac{dc}{dL} G(\beta) d\beta = 0, \tag{32}$$

where $dc/dL = \partial c/\partial L + (\partial c/\partial \beta) (d\beta^*/dL)$ represents the total impact of $L$ on individual consumption. Note that the first two terms in (32) are positive, thereby implying that the last term is negative. Moreover, using Lemma 2:

$$\frac{d \ln c}{d \ln L} = -\frac{\eta(c(\beta))}{\sigma(c^*)} \left[ 1 - (\sigma(c^*) - 1) \frac{d \ln \beta^*}{d \ln L} \right],$$

where the sign of the term in square brackets is independent of $\beta$. It follows that the sign of $dc/dL$ is the same for all firms, and must therefore be negative according to (32). Thus, as in the baseline model with symmetric firms, a rise in market size leads to an anti-competitive effect with IES preferences and to a pro-competitive effect with DES preferences.\(^{15}\)

\(^{14}\)Proceeding as above, it is also possible to show that, independent of consumer preferences, a rise in the fixed production cost $\alpha$ yields a selection effect, whereas a rise in the sunk entry cost $\alpha_e$ yields an anti-selection effect.

\(^{15}\)Similarly, it is possible to show that a rise in $\alpha$ or in $\alpha_e$ are pro-competitive with IES preferences and anti-competitive with DES preferences.
2.2.2 Reallocation Effects and Aggregate Productivity

We now show that, with non-CES preferences, selection effects are no longer associated to unambiguous changes in aggregate productivity. The reason is that a rise in market size not only leads to a change in the marginal cost cuttoff, it also affects relative firm output. To see this note, from (7), that a rise in $L$ affects the output of active firms through an increase in $\lambda$.\footnote{As can be seen from (4), a rise in $\lambda$ is required to offset the demand increase induced by a rise in $L$.} Differentiating (7) yields:

$$\frac{d\ln c(\beta)}{d\ln \lambda} = -\eta(c(\beta)).$$

Thus, the reallocations induced by a rise in $L$ are in favor of more productive firms only if $\eta' < 0$, as in this case the induced rise in $\lambda$ reduces individual demand for low-$\beta$ firms relatively less. Instead, if $\eta' > 0$, reallocations are in favor of less productive firms.\footnote{Similarly, it can be shown that $\frac{d\ln \pi_v(\beta)}{d\ln \lambda} = -\sigma(c(\beta))$, which implies that a rise in market size affects relative output and profitability in the same way only in the case of CES preferences, in which $\eta = \sigma$.}

Note also that:

$$\eta'(c) = -\frac{r''(c)c - [r''(c)c + r''(c)]r'(c)}{(r''(c)c)^2} \Rightarrow \eta'(c) \geq 0 \iff -\frac{r''(c)c}{r''(c)} \geq \frac{1 + \eta(c)}{\eta(c)},$$

where $r''' = u'''c + 3u''$, thus suggesting that the behavior of $\eta$ depends on properties of preferences which are hard to pin down and do not appear to be directly related to the sign of $\sigma'$. In particular, although $\sigma' > 0$ if and only if $\eta > \sigma$, no general relation can be inferred between the sign of $\sigma'$ and $\eta'$. As a consequence, selection and reallocation effects may point in opposite directions, leading to an ambiguous overall impact on aggregate productivity. For instance, the selection effect implied by DES preferences ($\sigma' < 0$) may not be associated to aggregate productivity gains if $\eta' > 0$.

More formally, define the average marginal cost as

$$\tilde{\beta} = \int_\beta^{\beta^*} \beta \, d\Gamma(\beta),$$

where

$$\Gamma(\beta) = \int_\beta^{\beta^*} c(z) \frac{dG(z)}{dG(s)} \int_\beta^{\beta^*} c(s) \, dG(s)$$

is the distribution of $\beta$ weighted by the corresponding production levels.\footnote{Note that $\tilde{\beta}$ satisfies the property of invariance of total cost, i.e.,

$$n \left[ \int_{\beta}^{\beta^*} \beta c(\beta)L \, \frac{dG(\beta)}{G(\beta)} + \alpha \right] = n \left[ \int_{\beta}^{\beta^*} \beta c(\beta) L \, \frac{dG(\beta)G(\beta^*)}{G(\beta^*)} + \alpha \right].$$} Aggregate
productivity $1/\tilde{\beta}$ is then determined by $\Gamma(\beta)$, which has support $[\beta, \beta^*]$ and depends on $G(\beta)$ and $c(\beta)$. Let $\gamma = \Gamma'$ be the density function associated to $\Gamma$, denote by $h = d\log \Gamma/d\beta = \gamma/\Gamma$ the corresponding so-called reverse hazard rate, and recall that, if two distributions have the same reverse hazard rate, they are identical. We have:

$$h(\beta) = \frac{c(\beta)g(\beta)}{\int_\beta^\beta c(z)dG(z)} = \frac{g(\beta)}{\int_\beta^\beta \frac{c(z)}{c(\beta)}dG(z)},$$

which implies that the reverse hazard rate corresponding to a $\beta$-firm depends on its output relative to that of all other firms with a lower marginal cost. Thus, by governing relative firm size, $\eta$ affects how market size impacts on aggregate productivity.

For instance, if $\eta', \sigma' < 0$, a simple "reverse hazard rate dominance" argument (see, e.g., Shaked and Shanthikumar, 1994) shows that a rise in market size, which is associated to a rise in the marginal utility of income from, say, $\lambda$ to $\hat{\lambda}$, leads to a rise in aggregate productivity. To see this, denote by $\Gamma_\lambda$ and $\Gamma_{\hat{\lambda}}$ the corresponding distributions, with supports $[\beta, \beta^*_\lambda]$ and $[\beta, \beta^*_{\hat{\lambda}}]$, and $\beta^*_{\hat{\lambda}} < \beta^*_\lambda$ due to the selection effect implied by DES preferences ($\sigma' < 0$). For all firms active in both market situations, $h_{\hat{\lambda}}(\beta) \leq h_\lambda(\beta)$ due to $\eta' < 0$, thereby implying

$$\int_\beta^\beta \frac{c_\lambda(z)}{c_\lambda(\beta)}dG(z) \leq \int_\beta^\beta \frac{c_{\hat{\lambda}}(z)}{c_{\hat{\lambda}}(\beta)}dG(z).$$

Moreover, since $h_{\hat{\lambda}}(\beta) = 0 \leq h_\lambda(\beta)$ for all $\beta \in \left(\beta^*_\lambda, \beta^*_{\hat{\lambda}}\right]$, it follows that $h_{\hat{\lambda}}(\beta) \leq h_\lambda(\beta)$ for all $\beta \in \left[\beta, \beta^*_{\hat{\lambda}}\right]$ and thus $\tilde{\beta}_{\hat{\lambda}} < \tilde{\beta}_\lambda$. By the same reasoning, if $\eta', \sigma' > 0$, a rise in market size leads to an unambiguous reduction in aggregate productivity. On the contrary, if the sign of $\eta'$ and $\sigma'$ do not agree, the selection effect (governed by $\sigma$) and the reallocation effect (governed by $\eta$) point in opposite directions, and the impact of a rise in market size on aggregate productivity becomes ambiguous, as it will in general depend on the properties of the distribution $G$.

We summarize our results in the following

**Proposition 2** Selection effects due to a rise in market size are unambiguously associated to aggregate productivity changes only if $\eta'$ and $\sigma'$ agree in sign.
2.2.3 Costly Trade

Our previous results suggest competitive and selection effects to crucially depend on the sign of $\sigma'$. We now show how the results change in the presence of trade costs. We assume that selling in an identical foreign market involves a fixed cost of exporting $\alpha_x > 0$ and a variable iceberg trade cost $\tau > 1$. To save space we focus, as in Melitz (2003), on the more interesting case in which trade costs are large enough to induce a partitioning of firms into exporters and non-exporters. This requires that $\beta^* > \beta^*_x$, where $\beta^*_x$ is the marginal cost cutoff for exporting firms.

Using the same notation as in Section 2.1.1, a $\beta$-firm’s profits in the domestic and foreign markets are given by:

$$\pi(\beta) = \pi_v(\beta) - \alpha, \quad \pi_x(\beta) = \pi_v(\tau \beta) - \alpha_x,$$

where $\pi_v(\beta) = \left[ m(c(\beta)) - 1 \right] \beta c(\beta) L$, and $L$ is the size of each market. Thus, the marginal cost cutoffs for exporters and domestic firms, $\beta^*_x$ and $\beta^*$, are implicitly given by:

$$\pi_x(\beta^*_x) = \left[ m(c^*_x) - 1 \right] \beta^*_x c^*_x L - \alpha_x = 0,$$

$$\pi(\beta^*) = \left[ m(c^*) - 1 \right] \beta^* c^* L - \alpha = 0,$$

where $c^*_x = c(\tau \beta^*_x)$, $c^* = c(\beta^*)$, and $\pi_v^{-1}(\alpha_x) < \tau \pi_v^{-1}(\alpha)$ in order for the condition $\beta^* > \beta^*_x$ to be satisfied. Finally, the free-entry condition is now given by:

$$\pi^E = \int^{\beta^*}_\beta \pi(\beta) dG(\beta) + \int^{\beta^*_x}_\beta \pi_x(\beta) dG(\beta) = \alpha_e.$$

Selection effects Consider first a fall in the variable trade cost $\tau$. Applying the implicit function theorem to (41) yields:

$$\frac{\partial \beta^*}{\partial \tau} = - \frac{\partial \pi^E}{\partial \pi^E} \frac{\partial \pi^E}{\partial \beta^*}.$$

19Note that a domestic firm producing only for the foreign market would incur an overall fixed cost equal to $\alpha + \alpha_x$, which implies that this case cannot arise in equilibrium. For analytical convenience, we can therefore apportion the fixed cost $\alpha$ to domestic profits, in this following the heterogeneous-firms literature.

20The measure of active firms is given by:

$$n = \left[ \int^{\beta^*}_\beta p(\beta) c(\beta) \frac{dG(\beta)}{G(\beta)} + \int^{\beta^*_x}_\beta p(\tau \beta) c(\tau \beta) \frac{dG(\beta)}{G(\beta^*_x)} \right]^{-1}.$$
Using the envelope theorem, recalling that \( \partial \pi_e / \partial c = \beta L [m(c) / \eta] > 0 \) by (13), and that \( \partial \ln c / \partial \ln \beta^* = \eta(c) / m(c^*) > 0 \) by Lemma 2, yields:

\[
\frac{\partial \pi^E}{\partial \tau} = \int_\beta^{\beta^*} \frac{\partial \pi_e(\tau \beta)}{\partial \tau} g(\beta) d\beta = -\int_\beta^{\beta^*} c(\tau \beta) \beta L dG(\beta) < 0,
\]

\[
\frac{\partial \pi^E}{\partial \beta^*} = \int_\beta^{\beta^*} \frac{\partial \pi_e(\beta)}{\partial \beta} \frac{\partial c(\beta)}{\partial \beta^*} dG(\beta) + \int_\beta^{\beta^*} \frac{\partial \pi_e(\tau \beta)}{\partial c} \frac{\partial c(\tau \beta)}{\partial \beta^*} dG(\beta) = 0.
\]

Thus we can write:

\[
\frac{\partial \ln \beta^*}{\partial \ln \tau} = \frac{\int_\beta^{\beta^*} \frac{m(c^*)}{m(c(\tau \beta))} \beta L c(\beta) \tau \beta dG(\beta)}{\int_\beta^{\beta^*} p(\beta) c(\beta) dG(\beta) + \int_\beta^{\beta^*} p(\tau \beta) c(\tau \beta) dG(\beta)} > 0. \tag{42}
\]

Evidently, a fall of \( \tau \) leads to a fall of \( \beta^* \), a standard selection effect.

Consider now a fall in the fixed cost of exporting \( \alpha x \). Proceeding as above yields:

\[
\frac{\partial \beta^*}{\partial \alpha x} = -\frac{\partial \pi^E / \partial \alpha x}{\partial \pi^E / \partial \beta^*},
\]

where \( \partial \pi^E / \partial \beta^* > 0 \) and

\[
\frac{\partial \pi^E}{\partial \alpha x} = \int_\beta^{\beta^*} \frac{\partial \pi_e(\beta)}{\partial \alpha x} dG(\beta) = -G(\beta^*_x) < 0.
\]

Hence \( \partial \beta^* / \partial \alpha x > 0 \), implying that a fall in \( \alpha x \) leads to a fall in \( \beta^* \). We record our main results in the following

**Proposition 3** When fixed (\( \alpha x \)) and variable (\( \tau \)) trade costs induce a partitioning of firms into exporters and non-exporters, a fall in either \( \alpha x \) or \( \tau \) leads to a selection effect (a fall in \( \beta^* \)) independently of the sign of \( \sigma' \).

Thus, Melitz-type selection effects are robust to the sign of \( \sigma'(c) \). We view these results as a reassuring. As shown in the previous section, however, they are unambiguously associated to aggregate productivity gains only when preferences are CES.

**Competitive effects** To study how a fall of trade costs affects domestic and foreign markups, we just have to study how individual consumption of domestic and foreign varieties, \( c(\beta) \) and \( c(\tau \beta) \), varies with trade costs. Consider first a fall in \( \alpha x \). Note,
from (39)-(40), that $\alpha_x$ affects individual consumption only through $\beta^*$, and that $\beta^*$ is increasing in $\alpha_x$ by Proposition 3. Moreover, individual consumption is increasing in $\beta^*$ by Lemma 2. It follows that a fall in $\alpha_x$ leads to a fall in both $c(\beta)$ and $c(\tau \beta)$, thereby reducing (increasing) markups with DES (IES) preferences.

Consider now a fall in $\tau$. By Proposition 3, it leads to a fall in $\beta^*$ and therefore to a fall in individual consumption of domestic varieties $c(\beta)$. Thus, it leads to a fall (rise) of domestic markups with DES (IES) preferences. As for individual consumption of foreign varieties, $c(\tau \beta)$, a fall of $\tau$ has both a positive direct effect an indirect negative effect through $\beta^*$. The overall effect is given by:

$$\frac{d \ln c(\tau \beta)}{d \ln \tau} = \frac{\partial \ln c(\tau \beta)}{\partial \ln \tau} + \frac{\partial c(\tau \beta)}{\partial \beta^*} \frac{\partial \beta^*}{\partial \tau} \frac{\tau}{c(\tau \beta)}.$$  

Recalling that $\partial \ln c(\tau \beta)/\partial \ln \tau = -\eta(c(\tau \beta)) < 0$ by (21), and $\partial \ln c(\tau \beta)/\partial \ln \beta^* = \eta(c(\tau \beta))/m(c^*) > 0$ by Lemma 2, we can write:

$$\frac{d \ln c(\tau \beta)}{d \ln \tau} = \eta(c(\tau \beta)) \left[ \frac{1}{m(c^*)} \frac{\partial \ln \beta^*}{\partial \ln \tau} - 1 \right] < 0,$$

where the inequality follows directly from (42). Hence, as in the symmetric-firm case, a fall of variable trade costs, by increasing $c(\tau \beta)$, leads to higher (lower) foreign markups with DES (IES) preferences.

We record our main results in the following

**Proposition 4** With DES preferences, a fall of either $\alpha_x$ or $\tau$ leads to lower domestic markups; a fall of $\tau$ ($\alpha_x$) leads instead to higher (lower) foreign markups. The opposite results hold with IES preferences.

To conclude, as in the baseline model with symmetric firms, trade liberalization leads to contrasting results, which seems to cast doubt on the competitive effects identified in this Section. In the next Section, we therefore investigate the plausibility of competitive effects in alternative monopolistic competition settings.

### 3 Alternative Environments

D-S monopolistic competition with additive preferences implies that markups do not directly depend on the number of firms $n$. This is because: 1) consumers share the same preferences over a number of characteristics that equals the number of varieties (see 1); 2) the Dixit-Stiglitz assumption implies $\varepsilon = \sigma$ (see 3), and 3) additivity implies that $\sigma$ is independent of $n$ (see the Appendix). It follows that in this setup a pro-competitive
effect can arise only through variation in the level of individual consumption $c$, and only if $\sigma' \neq 0$. For this reason, a D-S setup cum additive preferences may be unsuited to study competitive effects.

In this Section, we therefore consider monopolistic competition environments in which an increase in the number of firms directly affects demand elasticity, thereby yielding a competitive effect.\footnote{See Pettengill (1979) and Dixit and Stiglitz (1979).} Building on examples drawn from the received trade literature, we first discuss the implications of relaxing the assumption of preference additivity, and then investigate Lancaster (1979)'s ideal variety approach, in which consumers are heterogeneous. Finally, we study strategic interaction à la Bertrand and Cournot in a setting with additive preferences. Perhaps surprisingly, we find that none of the above setups seems to yield a compelling pro-competitive mechanism in monopolistic competition.

3.1 Non-Additive Preferences

When preferences are additive, as in (1), the elasticity of substitution $\sigma_{ij}$ between varieties $i$ and $j$ is independent of the number $n$ of varieties available for consumption (see the Appendix for a proof). The reason is that in this case the marginal rate of substitution between any two varieties is unaffected by consumption of other varieties. Matters are different, however, if preferences are non-additive. In particular, at a symmetric consumption pattern ($c_i = c_i$, $i = 1, \ldots, n$), consumption of other varieties may affect the elasticity of substitution between varieties $i$ and $j$ through a direct impact of $n$ on $\sigma_{ij}$ (see the Appendix). Hence, non-additive preferences imply a sort of externality of $n$ on the elasticity of substitution. As shown in the following examples, however, the sign and interpretation of this externality are not obvious.\footnote{In this respect, it is suggestive that in their discussion of "diversity as a public good", Dixit and Stiglitz (1975: section 4.4) consider a non-additive case in which $n$ enters the utility function without however affecting the marginal rate of substitution between any two varieties.}

3.1.1 Quasi-Linear Quadratic Preferences

Consider first quasi-linear quadratic preferences of the type $U(c_0, u(c_{-0})) = c_0 + u(c_{-0})$, where $c_0$ is consumption of a numeraire good, $c_{-0} = [c_1, c_2, \ldots, c_n]$ is consumption of $n$ varieties of some product and

$$u(c_{-0}) = a \sum_{j=1}^{n} c_j - \frac{b}{2} \sum_{j=1}^{n} c_j^2 - \frac{t}{2} \left( \sum_{j=1}^{n} c_j \right)^2,$$  \hspace{1cm} (43)
with \(a, b, t > 0\). The above non-additive preferences have been recently used in an influential paper by Melitz and Ottaviano (2008). Their monopolistic competition framework with heterogeneous firms is widely perceived as an appealing alternative to the original Melitz (2003) model, as it delivers a trade-induced pro-competitive effect.

Maximization of (43) with respect to \(c_i\) subject to a budget constraint yields:

\[
p_i = \frac{\partial u}{\partial c_i} = a - bc_i - t \sum_{j=1}^{n} c_j, \quad i = 1, \ldots, n. \tag{44}
\]

Summing (44) across varieties and rearranging yields the direct demand function for variety \(i\):

\[
c_i(p) = \frac{a}{b + nt} - \frac{p_i}{b} + \frac{nt}{b + nt} \frac{1}{b} p, \tag{45}
\]

where \(\bar{p} = \frac{1}{n} \sum_{j=1}^{n} p_j\) is the average price of a variety. Note that firm \(i\)'s perceived demand is linear in \(p_i\) under the D-S assumption that it takes \(n\) and \(\bar{p}\) as given. Accordingly, the demand elasticity equals \((p_i/c_i)/b\) and is therefore decreasing in \(c_i\) for any given \(p_i\).

Consider now the elasticity of substitution. As detailed in the Appendix, \(\sigma_{ij} = \varepsilon_{ji} - \varepsilon_{ii}\), where \(\varepsilon_{ij} = \tilde{c}_{ij}/(p_j/\tilde{c}_i)\) and \(\varepsilon_{ii} = \tilde{c}_{ii}/(p_i/\tilde{c}_i)\) are the compensated demand-price elasticities, \(\tilde{c}_{ii} = \partial \tilde{c}_i/\partial p_i\), \(\tilde{c}_{ij} = \partial \tilde{c}_i/\partial p_j\) are the corresponding derivatives, and \(\tilde{c}_i\) is the compensated demand for variety \(i\). Recall that with quasi-linear preferences there are no income effects on the demand for non-numeraire goods, and thus compensated and uncompensated demand functions coincide, i.e., \(\tilde{c}_i = c_i\). Accordingly:

\[c_{ii} = -\frac{1}{b} + \frac{t}{b + nt b} \frac{1}{b} = c_{ij} - \frac{1}{b}, \quad i, j = 1, 2, \ldots, n \text{ and } i \neq j,
\]

and, for \(p_i = p_j\), which implies a symmetric consumption of varieties \(i\) and \(j\) \((c_i = c_j)\), the elasticity of substitution is given by:

\[
\sigma_{ij} = \varepsilon_{ji} - \varepsilon_{ii} = \frac{p_i}{c_i} (c_{ji} - c_{ii}) = \frac{1}{b} c_i \left( a - bc_i - t \sum_{h=1}^{n} c_h \right). \tag{46}
\]

Evidently, also in this setting the perceived demand elasticity equals the elasticity of substitution.

Next, to see the role played by \(n\), consider first a quasi-symmetric equilibrium in which consumption of all third varieties is equal (i.e., \(c_i = c_j = c, h = 1, \ldots, n, i \neq h \neq j\)). In this case, we obtain:

\[
\sigma_{ij} = \frac{1}{b} \left\{ \frac{a}{c_i} - b - t \left[ 2 + (n - 2) \frac{c}{c_j} \right] \right\}, \tag{47}
\]
which implies that the elasticity of substitution is decreasing in the number of firms \( n \) (as well as in \( c_i \) and \( c \)). Finally, in a fully symmetric equilibrium (i.e., for \( c_i = c, i = 1, \ldots, n \)), the expression for the elasticity of substitution between any two varieties boils down to

\[
\sigma = \frac{1}{b} \left[ \frac{a}{c} - b - tn \right].
\]

The above results show that the number of firms has a negative direct impact on the elasticity of substitution. This implies that the pro-competitive effect of an increase in market size delivered by this type of preferences in monopolistic competition is entirely driven by firms’ perceived linearity of demand, hence by the fact that, just as in Krugman (1979), the elasticity of substitution is decreasing in the level of individual consumption. Moreover, consistent with quasi-linear quadratic preferences being of the DES type, they imply an inverse relationship between markups and marginal costs in a D-S monopolistic competition equilibrium. As a consequence, a fall in the marginal cost of exporting induces higher markups on foreign sales.

3.1.2 Translog Preferences

Another prominent example of non-additive preferences is represented by the translog expenditure function, a flexible specification providing a second-order approximation to any functional form (see, e.g., Varian, 1992: ch. 12). Feenstra (2003) has shown that, when preferences are symmetric and homothetic, the log of the expenditure function \( E(p, U) \) can be written as

\[
\ln E = \ln U + a_0(n) + \sum_{i=1}^{n} b_i \ln p_i + \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} t_{ij} \ln p_j \ln p_i,
\]

where

\[
b_i = 1/n, \ t_{ii} = -t(n-1)/n, \ t_{ij} = t/n, \ t > 0, \ i, j = 1, \ldots, n, \ i \neq j,
\]

and \( a_0 \) can be shown to reflect the welfare gains from a rise in the number of available varieties. Differentiation yields:

\[
\theta_i = \frac{\partial \ln E}{\partial \ln p_i} = b_i + \sum_{j=1}^{n} t_{ij} \ln p_j,
\]

where \( \theta_i = p_i \tilde{C}_i / E \) is the expenditure share on variety \( i \). It can be shown that:

\[
\varepsilon_i = 1 - \frac{\partial \ln \theta_i}{\partial \ln p_i} = 1 - \frac{t_{ii}}{\theta_i},
\]

\[
\sigma_{ij} = 1 + \frac{\partial \ln \theta_j}{\partial \ln p_i} \frac{\partial \ln \theta_i}{\partial \ln p_i} = 1 + \frac{t_{ji}}{\theta_j} - \frac{t_{ii}}{\theta_i}.
\]

(49)
Note that, unlike the case of quasi-linear quadratic preferences, at a symmetric equilibrium (in which $\theta_i = \theta_j = 1/n$) the elasticity of substitution ($\sigma = 1 + nt$) is directly increasing in $n$, which leads to a strong pro-competitive effect of market size expansion. However, demand elasticity is decreasing in the market share $\theta_i$, which implies that, as in the case of DES-type preferences, a fall in the marginal cost of exporting leads firms to charge higher markups in the foreign markets. As shown by Arkolakis et al. (2012), this anti-competitive effect implies that trade liberalization may lead to smaller welfare gains than in the CES case.

The following proposition summarizes.

**Proposition 5** When preferences are quasi-linear and quadratic, as in Melitz and Ottaviano (2008), the elasticity of substitution is decreasing in the number of firms for given level of symmetric individual consumption (i.e., $\sigma = \sigma(n, c)$, with $\partial \sigma / \partial n < 0$). Thus, the pro-competitive effect of a rise in market size is entirely driven by the fact that the elasticity of substitution is decreasing in individual consumption (i.e., $\partial \sigma / \partial c < 0$). When preferences are translog, as in Feenstra (2003), the elasticity of substitution is instead directly increasing in $n$ at a symmetric equilibrium, i.e., $\sigma = \sigma(n)$, with $\partial \sigma / \partial n > 0$. In both cases, a fall in the marginal cost of exporting leads to higher markups in the foreign market.

### 3.2 Ideal Variety Approach to Monopolistic Competition

In a Dixit-Stiglitz setting, the introduction of new varieties does not crowd the variety space, as the number of characteristics/varieties is the same as the number of firms. One may argue, however, that a pro-competitive effect may naturally arise in a framework in which an increase in the number of available varieties reduces their distance in a fixed characteristics space, thereby increasing their substitutability.\(^{25}\)

In this Section we show that, surprisingly, this needs not be the case. To make the

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\(^{24}\)Yet, this cannot be considered a general property of non-additive preferences. Another example of flexible functional form for non-additive preferences is the so-called Leontief-Diewert expenditure function (Varian, 1992, p. 209), which yields an elasticity of substitution that is decreasing in $n$, just as in the case of quasi-linear quadratic preferences. Specifically, in the symmetric and homothetic case the Leontief-Diewert expenditure function can be written as $E(p, U) = U \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \sqrt{p_i p_j}$, where $b_{ij} = b$, $i, j = 1, \ldots, n$, $i \neq j$. If $p_i = p$ (and then $c_i = c$) for any $i = 1, \ldots, n$, the elasticity of substitution equals $\sigma(n) = \frac{nb^{2}[(n-1)b+b]}{2(n-1)b+2b}$. The free parameter $b$ can be normalized to ensure that $\sigma > 1$, in which case $\sigma' < 0$. In any case, the impact of $n$ on the elasticity of substitution is negligible when the number of firms is large enough.

\(^{25}\)See, e.g., Pettengill (1979) on this point. He claims (p. 960) that "one’s normal idea of monopolistic competition is that as the number of products in the industry increases, they become closer and closer substitutes". He therefore argues that Lancaster’s (1975) ideal variety approach to monopolistic competition is more realistic in this respect.
point, we consider Lancaster’s (1975, 1979) "ideal variety" approach to monopolistic competition. In this setting, consumer preferences are heterogeneous and the aggregate demand for each variety arises from diversity of tastes. In particular, each consumer has a most preferred ("ideal") variety. As described in Helpman and Krugman (1985, pp. 120-21), on which we build in this section, ideal variety means that "when the individual is offered a well-defined quantity of the good but is free to choose any potentially possible variety, he will choose the ideal variety independently of the quantity offered and independently of the consumption level of other goods. Moreover, when comparing a given quantity of two different varieties, the individual prefers the variety that is closest to his ideal product".

These assumptions are formalized by assuming that each variety is represented by a point $\omega$ on the unit length circumference $\Omega$ of a circle, and that preferences for the ideal product are uniformly distributed over $\Omega$ across consumers. $L$ is the size (and density) of the population. The utility function of a consumer with ideal variety $\tilde{\omega}$ is assumed to be:

$$U = \sum_{\omega \in \Omega} \frac{c(\omega)}{h(\delta(\omega, \tilde{\omega}))},$$

(50)

where $\delta(\omega, \tilde{\omega})$ is the shortest arc distance between $\omega$ and $\tilde{\omega}$, and $h(\delta)$ is the so-called Lancaster’s compensation function, assumed to be positive, non decreasing and generally normalized so that $h(0) = 1$ (see Lancaster, 1975). Moreover, it is generally assumed (see, Helpman and Krugman, 1985) that $h(\delta)$ is strictly increasing and convex, and that $h'(0) = 0$.

We now show that the above assumptions are insufficient to deliver a pro-competitive impact of entry in this setting.\footnote{See also Helpman (1981) on this point.} To this purpose, note first that individual preferences as in (50) are of the "perfect substitute" type, with a marginal rate of substitution between any two varieties $\omega$ and $\tilde{\omega}$ equal to:

$$MRS(\omega, \tilde{\omega}) = \frac{h(\delta(\omega, \tilde{\omega}))}{h(\delta(\tilde{\omega}, \tilde{\omega}))}.$$  

(51)

The above assumptions on $h(\cdot)$ imply that $MRS(\tilde{\omega}, \omega) = h(\delta(\omega, \tilde{\omega}))$ is an increasing convex function of the arc distance between $\tilde{\omega}$ and $\omega$. Utility maximization implies:

$$c(\omega) = \begin{cases} 
\frac{1}{p(\omega)} & \omega = \omega' \\
0 & \omega \neq \omega' 
\end{cases},$$

where $\omega' = \arg\min_{\omega \in \Omega} p(\omega)h(\delta(\omega, \tilde{\omega}))$. In the Appendix, following Helpman and Krug-
man (1985) we derive total demand, $q(\omega)$, for a firm selling variety $\omega$ at price $p(\omega)$, with contiguous competitors $\omega_l$ and $\omega_r$ charging prices $p(\omega_l)$ and $p(\omega_r)$. In a symmetric equilibrium in which $p(\omega) = p(\omega_l) = p(\omega_r)$ and $\delta(\omega_l, \omega) = \delta(\omega_r, \omega) = 1/n$, the price elasticity of total demand is given by

$$\varepsilon(q(\omega)) = -\frac{\partial \ln q(\omega)}{\partial \ln p(\omega)} = 1 + \frac{1}{2\epsilon_h(1/2n)},$$

where

$$\epsilon_h = \frac{h'(1/2n)}{2nh(1/2n)}$$

is the elasticity of the compensation function. Accordingly, a rise in market size $L$, by increasing the number of firms $n$, also increases $\varepsilon$ if $\epsilon'_h > 0$, thus yielding a pro-competitive effect. Instead, if $\epsilon'_h < 0$, a rise in market size decreases $\varepsilon$ and is therefore anti-competitive. Finally, if $\epsilon'_h = 0$ (i.e., if $h(\cdot)$ is isoelastic), $\varepsilon$ is independent of $n$, just as in the "love for variety" approach when preferences are CES. Thus, a pro-competitive effect is unwarranted even in a framework in which an increase in the number of firms crowds the variety space.

There are two main reasons for this result. First, given that (50) assumes that varieties are of the "perfect substitute" type, and thus their elasticity of substitution is constant, a crowding of the variety space cannot affect it. In this respect, the model fails to capture a potentially genuine pro-competitive effect of the crowding of variety space.

Second, the ideal variety approach does not impose sufficient restrictions on $h(\cdot)$ to pin down the properties of $\epsilon_h$, and therefore the relationship between $\varepsilon$ and $n$. Specifically, is the assumption $\epsilon'_h > 0$ plausible? Note that $MRS(\tilde{\omega}, \omega) = h(\delta(\omega, \tilde{\omega}))$ implies that, in order for Lancaster’s model to deliver a pro-competitive effect, consumer preferences must feature an ever increasing distance elasticity of the marginal rate of substitution between $\tilde{\omega}$ and $\omega$. It is hard to provide a rationale for this assumption, which seems no more plausible than the opposite assumption of a decreasing distance elasticity, which would however deliver an anti-competitive effect.

We summarize the main results in the following

**Proposition 6** When preferences are heterogeneous across consumers and are of the ideal variety type, as in Lancaster (1979), a rise in market size does not affect markups when Lancaster’s compensation function is isoelastic. A rise in market size is instead pro-

---

27 Recall that, in a symmetric monopolistic competition equilibrium, the relationship between $L$ and $n$ is given by $n = L/\alpha\epsilon$. 28 Only in the limit case in which $n$ goes to infinity, and due to the (rather ad hoc) assumptions $h(0) = 1$ and $h'(0) = 0$, the aggregate demand elasticity is increasing in $n$ (and goes to infinite). This requires a situation, unfeasible under a positive fixed cost, in which the circumference of the circle is full and the aggregate demand for each firm is infinitesimal.
competitive (anti-competitive) when Lancaster’s compensation function features an increasing (decreasing) distance elasticity.

3.3 Strategic Interaction

The Dixit-Stiglitz assumption that each firm treats $\lambda$ (the marginal utility of income) as a constant removes a direct channel whereby an increase in the number of firms may raise the perceived demand elasticity $\varepsilon$. Instead, in a model with strategic interactions, firms properly treat $\lambda$ as a function of the price vector (i.e., $\lambda = \lambda(p)$) and demand elasticity no longer coincides with the elasticity of substitution.

Consider first our baseline setting with Bertrand competition, in which firms set prices. In a symmetric Bertrand-Nash equilibrium, firms perceive the actual demand elasticity (see (70) in the Appendix):

$$\varepsilon^b(c, n) = \sigma(c) - \frac{\sigma(c) - 1}{n}. \quad (52)$$

Note that $\frac{\partial \varepsilon^b}{\partial n} > 0$, which captures the direct pro-competitive channel induced by strategic interaction. Thus, the pricing equation is:

$$p = m^b(c, n)\beta = \frac{(n - 1)\sigma(c) + 1}{(n - 1)\sigma(c)} m(c)\beta, \quad (53)$$

where $\frac{\partial m^b}{\partial n} < 0$, and $\frac{\partial m^b}{\partial c} \geq 0 \iff m' \geq 0 \iff \sigma' \leq 0$.

Consider now the case of Cournot competition, in which firms set production levels. Multiplying both sides of the first-order conditions for utility maximization ($u'(c_i) = \lambda p_i$) by $c_i$, adding up and using the budget constraint ($\sum p_j c_j = w = 1$) yields:

$$\lambda = \sum_j u'(c_j)c_j. \quad (54)$$

Using (54) in the first-order conditions yields the following inverse demand system:

$$p_i(c) = \frac{u'(c_i)}{\sum_j u'(c_j)c_j}, \quad i = 1, \ldots n. \quad (55)$$

Let $R_i(c) = p_i(c)c_iL$ be firm $i$’s revenue. Marginal revenue is therefore given by:

$$\frac{\partial R_i(c)}{\partial c_i} = \frac{r'(c_i)\left(\sum_{j \neq i} u'(c_j)c_j\right)}{\left(\sum_j u'(c_j)c_j\right)^2}, \quad (56)$$

and is decreasing in $c_i$ under our assumptions that $r'' < 0$ and $r' = u''c + u' > 0$. 25
In a Cournot-Nash equilibrium, each firm chooses its quantity to satisfy the first-order condition \((\partial R_i/\partial c_i)/L = \beta\) under a correct conjecture about the quantities produced by its competitors. Then, (56) implies that, in any symmetric Cournot-Nash equilibrium,\[ c = \frac{(n - 1) r'(c)}{n^2 u'(c) / \beta} = \frac{n - 1}{n^2 m(c) / \beta}. \] (57)

By (55) and (57), the pricing equation is thus:\[ p = m^c(c, n) \beta = \frac{n}{n - 1} m(c) \beta. \] (58)

where \( \partial m^c / \partial n < 0 \), and \( \partial m^c / \partial c \geq 0 \) \( \Leftrightarrow \) \( m' \geq 0 \) \( \Leftrightarrow \) \( \sigma' \leq 0 \). Accordingly, Cournotian firms perceive a demand elasticity equal to\[ \varepsilon^c(c, n) = \frac{\sigma(c) n}{\sigma(c) + n - 1}. \] (59)

We can use the pricing equations, (53) or (58), the corresponding free-entry condition \((p = \beta + \alpha/cL)\) and goods market equilibrium condition \((p = 1/nc)\) to characterize the symmetric Bertrand and Cournot equilibria. Equilibrium conditions are summarized by the two-equation system:

\[ m^s(c^s, n^s) c^s \beta = \frac{1}{n^s} = \frac{\alpha}{L} + c^s \beta, \] (60)

where \( s = b, c \). Differentiating (60) yields:

\[
\begin{bmatrix}
\frac{\partial m^s}{\partial c} c^s + m^s \\
\beta
\end{bmatrix}
\begin{bmatrix}
\frac{\partial c^s}{\partial (\alpha/L)} \\
\frac{\partial m^s}{\partial (\alpha/L)}
\end{bmatrix}
= \begin{bmatrix}
0 & -m^s c^s \\
-1 & -c^s
\end{bmatrix}.
\] (61)

Thus, applying Cramer’s rule,

\[
\begin{bmatrix}
\frac{\partial m^s}{\partial (\alpha/L)} & \frac{\partial c^s}{\partial (\alpha/L)} \\
\frac{\partial n^s}{\partial (\alpha/L)} & \frac{\partial m^s}{\partial (\alpha/L)}
\end{bmatrix}
= \frac{1}{D^s} \begin{bmatrix}
\frac{\partial m^s}{\partial c} c^s + m^s + \frac{1}{(n^s)^2} & -m^s c^s + c^s \left( \frac{\partial m^s}{\partial c} c^s + \frac{1}{(n^s)^2} \right) \\
-\left( \frac{\partial m^s}{\partial c} c^s + m^s \right) \beta & -\beta \frac{\partial m^s}{\partial c} (c^s)^2
\end{bmatrix},
\] (62)

where \( D^s \) is the determinant of the first matrix on the LHS of (61). As shown in the Appendix,\footnote{Note that \( \varepsilon^c(c, n) < \varepsilon^s(c, n) \), i.e., Cournotian firms perceive a lower demand elasticity relative to Bertrand firms.}
\[
\frac{\partial c^s}{\partial (\alpha/L)} \geq 0, \quad \frac{\partial c^s}{\partial \beta} < 0, \quad \frac{\partial n^s}{\partial (\alpha/L)} < 0 \quad \text{and} \quad \text{sign}\left\{\frac{\partial n^s}{\partial \beta}\right\} = \text{sign}\{\sigma'\}, \quad (63)
\]

where \(\partial c^s/\partial (\alpha/L) = 0\) only for \(s = c\) and \(n = 2\).\(^{30}\) Thus, the comparative statics results delivered by the baseline model hold also in the presence of Bertrand and Cournot competition. This allows us to study the competitive effects of market size expansion (a fall in \(\alpha/L\)) and productivity growth (a fall in the marginal cost \(\beta\)). Note first that

\[
\frac{\partial m^s}{\partial (\alpha/L)} = \frac{\partial m^s}{\partial c} \frac{\partial c^s}{\partial (\alpha/L)} + \frac{\partial m^s}{\partial n} \frac{\partial n^s}{\partial (\alpha/L)}, \quad (64)
\]

where \(\partial m^s/\partial n < 0\) and (63) imply that the second term on the RHS is always positive. It follows that \(\partial m^s/\partial (\alpha/L) > 0\) for \(\sigma' \leq 0\), as in this case \(\partial m^s/\partial c \geq 0\). Thus, a rise in market size is pro-competitive when preferences are DES or CES. Instead, as shown in the Appendix, \(\partial m^s/\partial (\alpha/L) < 0\) for \(\sigma' > 0\), unless \(n\) is small. Thus, as in the baseline model, an increase in market size is generally anti-competitive with IES preferences.

Finally, note that

\[
\frac{\partial m^s}{\partial \beta} = \frac{\partial m^s}{\partial c} \frac{\partial c^s}{\partial \beta} + \frac{\partial m^s}{\partial n} \frac{\partial n^s}{\partial \beta}, \quad (65)
\]

implying that the change in individual consumption and in the number of firms induced by a change in the marginal cost \(\beta\) affect markups in opposite directions. In the Appendix we show, however, that in all cases \(\text{sign}\left\{\frac{\partial m^s}{\partial \beta}\right\} = \text{sign}\{\sigma'\}\), and hence that productivity growth is anti-competitive (pro-competitive) with DES (IES) preferences.

We summarize our main results in the following

**Proposition 7** a) When firms interact strategically, a rise in market size is pro-competitive with DES or CES preferences; with IES preferences, instead, a pro-competitive effect may arise only when the number of firms is small. b) A fall in the marginal cost is anti-competitive with DES preferences, neutral with CES preferences, and pro-competitive with IES preferences.

**3.3.1 Discussion**

Strategic interaction with additive preferences seems to yield the same competitive effects illustrated in Section 2.1 for IES/DES preferences. In particular, market size expansion is still generally anti-competitive when preferences are IES. Moreover, the competitive effects of a fall in the marginal cost are still generally the opposite of those of a rise in

\(^{30}\) Although we treat \(n\) as a real number (thereby ignoring the integer problem), consistent with the assumption of strategic interaction we focus on values of \(n\) equal or greater than 2.
market size, which implies, e.g., that with DES preferences a fall in the marginal cost of exporting still leads to higher markups on foreign sales.

When preferences are CES, instead, strategic interaction naturally leads to a pro-competitive effect of market size expansion, whereas marginal costs have no impact on markups. Moreover, CES preferences lead to simple closed-form solutions, both with Bertrand competition (Yang and Heijdra, 1993; Eckel, 2008) and Cournot competition:

\[
\begin{align*}
    n^b &= \frac{L}{\sigma \alpha} + \frac{(\sigma - 1)}{\sigma}, \quad p^b = \frac{\sigma \beta L}{(L - \alpha)(\sigma - 1)}, \quad c^b = \frac{\alpha (\sigma - 1) (L - \alpha)}{\beta L (L + \alpha (\sigma - 1))}, \\
    n^c &= \frac{L \left(1 + \sqrt{1 + 4 \alpha \sigma (\sigma - 1)/L}\right)}{2 \alpha \sigma}, \quad p^c = \frac{\beta L}{L - n^c \alpha}, \quad c^c = \frac{1}{\beta} \left(\frac{1}{n^c} - \frac{\alpha}{L}\right).
\end{align*}
\]

The above results suggest that there seems to be no reason to move beyond the CES when allowing for strategic interaction.

Finally note that, by (52) and (59), independent of the assumptions about preferences, the competitive effects arising from strategic interactions can be relevant only when the number of firms is small, or else the impact on markups of a rise in \(n\) becomes negligible. However, as forcefully argued by Dixit and Stiglitz (1993), assuming that \(n\) is small is problematic in a monopolistic competition setting. In particular, if the number of firms is small enough to induce them to interact strategically, it is unclear why they do not also engage in collusion and entry deterrence, thereby preventing the free entry of firms. By the same token, it is actually unclear why we should expect significant competitive effects in a market situation in which the number of firms is large enough to make sense of the free-entry condition postulated in monopolistic competition models.

4 Conclusion

We have studied competitive, selection and aggregate productivity effects in monopolistic competition. We have argued that allowing for a variable elasticity of substitution in an otherwise standard Dixit-Stiglitz setup with additive preferences, an approach advocated by a recent literature, involves implausible assumptions about consumer behavior and inconsistent competitive effects. We have also shown that, in the presence of trade costs, Melitz-type selection effects are robust, but only when preferences are CES they are unambiguously associated to aggregate productivity gains. Finally, we have argued that allowing for non-additive preferences, for the "crowding" of the variety space and for strategic interactions does not obviously lead to more robust and plausible competitive effects in monopolistic competition.

Given that markups are hardly observable, and thus their empirical estimation is to
be closely guided by theory, it is crucial that the latter builds on plausible assumptions and mechanisms. Our current ignorance of how to plausibly model competitive effects in monopolistic competition suggests that departing from CES preferences, which overlook these effects, may turn out to be a move in the wrong direction. We view this warning as the most important message of our paper.

5 Appendix

5.1 Demand Elasticity and Elasticity of Substitution

In this Section we show how demand elasticity and the elasticity of substitution are related. Consider a general setting in which consumers share the same preferences, represented by the utility function

$$U(c_0,u(c_{-0})),$$

where $c_0$ is consumption of a numeraire good, $c_{-0} = [c_1,c_2,\ldots,c_n]$ is consumption of $n$ varieties of some good, and $u(\cdot)$ is symmetric and concave in its arguments. Denote the associated system of compensated (Hicksian) demand by

$$\tilde{c} = [\tilde{c}_0,\tilde{c}_{-0}] = \tilde{c}(p,U),$$

where $p = [p_0,p_{-0}]$ is the price vector. Moreover, denote by

$$\tilde{c}_{ij} = \partial \tilde{c}_i / \partial p_j \; (i,j = 0,1,\ldots,n)$$

the compensated demand-price derivatives, and by $\tilde{\varepsilon}_{ij} = \partial \ln \tilde{c}_i / \partial \ln p_j$ the corresponding elasticities. The elasticity of substitution between goods $i$ and $j$ ($i \neq j$) measures the substitutability between any two goods at a given consumption vector $c = [c_0,c_{-0}]$, and is given by:

$$31 \sigma_{ij} = \tilde{\varepsilon}_{ji} - \tilde{\varepsilon}_{ii}. \quad (66)$$

Specifically, $\sigma_{ij}$ measures how the marginal rate of substitution between $i$ and $j$, $MRS_{ij} = (\partial U / \partial c_i) / (\partial U / \partial c_j)$, varies with the consumption ratio $c_i / c_j$. That the elasticity of substitution is a key ingredient of demand elasticity can be seen by manipulating the Slutsky equation, which decomposes the price effect on the Marshallian (uncompensated) demand $c_i(p,Y)$ into a substitution and an income effect:

$$c_{ij} = \tilde{c}_{ij} - c_i Y c_j, \quad (67)$$

where $Y$ is income, $c_{ij} = \partial c_i / \partial p_j$ and $c_i Y = \partial c_i / \partial Y$. In elasticity terms, (67) implies:

$$\varepsilon_{ii} = \tilde{\varepsilon}_{ii} - \varepsilon_{iY} \theta_i, \; \varepsilon_{ji} = \tilde{\varepsilon}_{ji} - \varepsilon_{jY} \theta_i, \quad (68)$$

where $\theta_i = p_i c_i / Y$, $\varepsilon_{iY} = \partial \ln c_i / \partial \ln Y$, $\varepsilon_{ii} = \partial \ln c_i / \partial \ln p_i$ and $\varepsilon_{ij} = \partial \ln c_i / \partial \ln p_j$ are, respectively, good $i$’s expenditure share, income elasticity, own-price and cross-price demand.

---

31 See Blackorby and Russell (1989) for a discussion of the elasticity of substitution.
elasticity with respect to good $j$.\footnote{For expositional purposes, in this Appendix it proves convenient to define the demand elasticity as $\varepsilon_{ii} = \partial \ln c_i / \partial \ln p_i$, rather than $\varepsilon_i = -\partial \ln c_i / \partial \ln p_i$, as in the main text.} Using (66) and (67) yields:

$$
\varepsilon_{ii} = -\sigma_{ij} + \varepsilon_{ji} + (\varepsilon_{jY} - \varepsilon_{iY}) \theta_i, \quad i, j = 1, \ldots, n; \ i \neq j. \tag{69}
$$

Thus, good $i$'s demand elasticity can be written in terms of the elasticity of substitution and cross-price elasticity with respect to another good $j$, and of the difference between the corresponding income effects, $(\varepsilon_{jY} - \varepsilon_{iY}) \theta_i$.\footnote{This latter term disappears if $U(\cdot)$ is homothetic (as $\varepsilon_{jY} = \varepsilon_{iY} = 1$) or quasi-linear with respect to the numeraire (as $\varepsilon_{iY} = \varepsilon_{jY} = 0$), or if $p_i = p_j$ (as $\varepsilon_{iY} = \varepsilon_{jY}$ by the symmetry of preferences over $\mathbf{c}_{-0}$). More generally, unless either $c_i$ or $c_j$ are disproportionate, $(\varepsilon_{jY} - \varepsilon_{iY}) \theta_i$ is an order of magnitude smaller than $1/n$.}

The expression in (69) drastically simplifies when there is no numeraire (as in Krugman, 1979 and in Section 2) and $p_i = p$ for all $i$, which implies $c_i = c$ for all $i$. In this case, $\bar{\varepsilon}_{ii} = -(n - 1) \tilde{\varepsilon}_{ij}$, $\bar{\varepsilon}_{ij} = \sigma_{ij}/n$, $\varepsilon_{ji} = (\sigma_{ij} - 1)/n$ and:\footnote{The same expression holds in the presence of a numeraire, provided that preferences are Cobb-Douglas with respect to the numeraire, i.e., when $U(c_0, u(\mathbf{c}_{-0})) = a_0^\alpha u(\mathbf{c}_{-0})^{1-\alpha}$, where $0 < \alpha < 1$ and $u(\cdot)$ is homogeneous. The reason is that in this case the income and substitution effects partially cancel out (i.e., $\sigma_{oi} = \varepsilon_{iY} = 1$, see below).}

$$
\varepsilon_{ii} = -\frac{n - 1}{n} \sigma_{ij} - \frac{1}{n}. \tag{70}
$$

Another simple case arises when preferences are quasi-linear, i.e., $U(c_0, u(\mathbf{c}_{-0})) = c_0 + u(\mathbf{c}_{-0})$ (as in Section 3.1). In this case, there are no income effects on the demand for non-numeraire goods, hence $\varepsilon_{ji} = \bar{\varepsilon}_{ji} = -(\sigma_{ij} + \tilde{\varepsilon}_{i0})/n$ and:

$$
\varepsilon_{ii} = -\frac{n - 1}{n} \sigma_{ij} - \frac{\tilde{\varepsilon}_{i0}}{n}. \tag{71}
$$

More generally, it is possible to show that, for a symmetric consumption pattern over $\mathbf{c}_{-0}$, (70) and (71) are special cases of the general expression:

$$
\varepsilon_{ii} = -\frac{n - 1}{n} \sigma_{ij} - \frac{1}{n} \left[ 1 + \frac{c_0}{Y} (\sigma_{0i} - \varepsilon_{0Y}) \right].
$$

Thus, for a wide class of utility functions, demand elasticity is approximately equal to the elasticity of substitution for $n$ large.

A general result is that, if $u(\cdot)$ is additive (i.e., if $u(\mathbf{c}_{-0}) = \sum_{i=1}^n u(c_i)$), then, for $c_i = c_j$ ($i, j = 1, \ldots, n, i \neq j$),

$$
\sigma_{ij} = -\frac{u'(c_i)}{u''(c_i)c_i}, \tag{72}
$$

implying that the demand elasticity perceived under the Dixit-Stiglitz assumption is al-
ways equal to the elasticity of substitution at a symmetric consumption. To see this, note that differentiating the first-order conditions

\[ p_i = \mu U_u(c_0, \sum_i u(c_i))u'(c_i), \]

where \( \mu \) is the relevant Lagrangian multiplier and \( U_u = \partial U/\partial u \), yields:

\[
\begin{align*}
1 &= \mu_i U_u u'(c_i) + \mu U_u u''(c_i)\tilde{c}_{ii} + \mu u'(c_i) \left[ U_u0\tilde{c}_{0i} + U_{uu}\sum_{h=1}^{n} u'(c_h)\tilde{c}_{hi} \right], \\
0 &= \mu_j U_u u'(c_i) + \mu U_u u''(c_i)\tilde{c}_{ij} + \mu u'(c_i) \left[ U_u0\tilde{c}_{0j} + U_{uu}\sum_{h=1}^{n} u'(c_h)\tilde{c}_{hj} \right],
\end{align*}
\]

where \( U_u0 = \partial^2 U/\partial u \partial c_0 \), \( U_{uu} = \partial^2 U/\partial u^2 \) and \( \mu_i = \partial \mu/\partial p_i \). Subtracting the second expression from the first, exploiting the symmetry of price effects implied by the compensated demand functions (i.e., \( \tilde{c}_{ij} = \tilde{c}_{ji} \)) and manipulating yields:

\[
1 = p_i \left[ \frac{\mu_i - \mu_j}{\mu} + \frac{U_u0}{U_u} (\tilde{c}_{0i} - \tilde{c}_{0j}) + \frac{U_{uu}}{U_u} \sum_{h=1}^{n} u'(c_h)(\tilde{c}_{hi} - \tilde{c}_{hj}) \right] + \frac{u''(c_i)c_i}{u'(c_i)} (\tilde{c}_{ii} - \frac{c_i}{c_i} \tilde{c}_{ji}). \tag{73}
\]

Note that, for \( p_i = p_j \), then, due to symmetry of preferences over \( \omega \), \( c_i = c_j \), \( \mu_i = \mu_j \), \( \tilde{c}_{hi} = \tilde{c}_{hj} \) \( (h = 0, ..., n, i \neq h \neq j) \) and \( \tilde{c}_{ii} = \tilde{c}_{jj} \). Thus (73) boils down to (72).

### 5.2 Demand in Lancaster’s (1979) Model

Following Helpman and Krugman (1985), we now derive total demand, \( q(\omega) \), for a firm selling variety \( \omega \) at the price \( p(\omega) \), with contiguous competitors \( \omega_l \) and \( \omega_r \) charging prices \( p(\omega_l) \) and \( p(\omega_r) \). The cliente of firm \( \omega \) is a compact set ranging from \( \omega \) to \( \overline{\omega} \), where \( \omega \) and \( \overline{\omega} \) are the locations of consumers just indifferent between \( \omega \) and \( \omega_l \), and between \( \omega \) and \( \omega_r \). The values of \( \omega \) and \( \overline{\omega} \) are therefore implicitly defined by:

\[
\begin{align*}
p(\omega_l)h(\delta(\omega_l, \omega)) &= p(\omega)h(\delta(\omega, \omega)), \\
p(\omega_r)h(\delta(\omega_r, \overline{\omega})) &= p(\omega)h(\delta(\omega, \overline{\omega})).
\end{align*}
\]

Denote by \( d^* = \delta(\omega_l, \omega_r) \) the distance between firm \( \omega \)'s competitors, by \( d = \delta(\omega_l, \omega) \) the distance between \( \omega_l \) and \( \omega \), and by \( d = \delta(\omega, \omega) \) and \( \overline{d} = \delta(\overline{\omega}, \omega) \) firm \( \omega \)'s distance from its marginal consumers. It follows that \( \delta(\omega_l, \omega) = d - d^* \) and \( \delta(\omega_r, \overline{\omega}) = d^* - \overline{d} \).

31
Substituting into (74) yields:

\[
p(\omega_l)h(d - d^*) = p(\omega)h(d), \tag{75}
\]
\[
p(\omega_r)h(d^* - d - d) = p(\omega)h(d).
\]

A firm’s market width is obtained by inverting the above implicit conditions:

\[
d = \delta(p(\omega), p(\omega_l), p(\omega_r), d^*, d), \tag{76}
\]
\[
d = \delta(p(\omega), p(\omega_l), p(\omega_r), d^*, d).
\]

Total demand for firm \( \omega \) is therefore given by:

\[
q(\omega) = \left[ \delta(\cdot) + \delta(\cdot) \right] c(\omega) L = \left[ \delta(\cdot) + \delta(\cdot) \right] \frac{L}{p(\omega)}. \tag{77}
\]

Implicit differentiation of the two-equation system in (75) yields:

\[
\frac{\partial \delta(\cdot)}{\partial p(\omega)} = -\frac{h(d)}{p(\omega_l)h'(d - d^*) + p(\omega)h'(d)}, \tag{78}
\]
\[
\frac{\partial \delta(\cdot)}{\partial p(\omega)} = -\frac{h(d)}{p(\omega_r)h'(d^* - d - d) + p(\omega)h'(d)}.
\]

Note that, at a symmetric equilibrium, \( p(\omega_l) = p(\omega_r) = p(\omega) \), \( d = \delta = d - d^* = d^* - d = d/2 \), and \( d = 1/n \). Substituting into (77) and (78) yields:

\[
\frac{\partial \delta(\cdot)}{\partial p(\omega)} = \frac{\partial \delta(\cdot)}{\partial p(\omega)} = \frac{h(\frac{d}{n})}{2p(\omega)h'(\frac{d}{2n})},
\]
\[
\frac{\partial q(\omega)}{\partial p(\omega)} = \left[ \frac{\partial \delta(\cdot)}{\partial p(\omega)} + \frac{\partial \delta(\cdot)}{\partial p(\omega)} \right] L \frac{p(\omega)}{p(\omega)} - \left[ \delta(\cdot) + \delta(\cdot) \right] \frac{L}{p(\omega)^2}
\]
\[
= -\frac{h(\frac{d}{2n})}{p(\omega)h'(\frac{d}{2n})} \frac{L}{p(\omega)} - \left[ \delta(\cdot) + \delta(\cdot) \right] \frac{L}{p(\omega)^2},
\]

thus leading to the expression for the demand elasticity \( \varepsilon(q(\omega)) \) reported in the main text.

5.3 Comparitive Statics of the Bertrand and Cournot Equilibria

We now prove the results in Proposition 7.
Bertrand competition. Differentiating (53) yields:
\[
\frac{\partial m^b}{\partial c} = \frac{n}{n-1}m',
\]
\[
\frac{\partial m^b}{\partial n} = -\frac{1}{(n-1)^2(\sigma - 1)} = -\frac{m}{(n-1)^2\sigma} = -(n-1)(n-1)\sigma + 1.
\] (79)

Next, using (79) in (62) yields:
\[
\begin{bmatrix}
\frac{\partial c^b}{\partial (\alpha/L)} & \frac{\partial c^b}{\partial \beta} \\
\frac{\partial n^b}{\partial (\alpha/L)} & \frac{\partial n^b}{\partial \beta}
\end{bmatrix} = \frac{1}{D^b} \begin{bmatrix}
\frac{c}{(n^b)^2} \left( 1 - m^b - \frac{n^b}{(n^b-1)(n^b-1)\sigma + 1} \right) > 0 \\
-\frac{\beta}{n^b-1} \left[ \frac{m}{n^b} \left( m'c^b + m \right) - 1 \right] < 0
\end{bmatrix},
\]
\[
\text{where } D^b = \frac{\beta}{n^b(n^b-1)} \left[ m'c^b + m - 1 + \frac{m}{\sigma(n^b-1)m^b} \right] > 0,
\]
and the latter inequality follows from (13), implying that $m'c + m - 1 > 0$. Similarly, the inequalities in (80) are straightforward implications of $m'c + m - 1 > 0$ and of the definition of $m^b$. This proves that the inequalities in (63) hold for $s = b$.

Next, using (80) in (64) yields:
\[
\frac{\partial m^b}{\partial (\alpha/L)} = \frac{n^b m'}{n^b - 1} \frac{\partial c^b}{\partial \alpha} - \frac{m^b}{(n^b-1)(n^b-1)\sigma + 1} \frac{\partial n^c}{\partial \alpha} = \frac{1}{D^b} \left\{ \frac{m'}{n^b-1} \left[ \frac{1}{n^b} - \frac{m}{\sigma(n^b-1)^2m^b} \right] + \frac{m^b \beta \left[ n^b (m'c^b + m) - 1 \right]}{(n^b-1)^2((n^b-1)\sigma + 1)} \right\} = m + m'c^bg(n^b) - \frac{1}{n^b} = \frac{c^b (n^b-1)^2 [\sigma(n^b-1)+1]}{D^b},
\] (81)

where $g(n) = \frac{(n-1)(n-1)\sigma + 1}{n} > 0$ is a monotonically increasing function, with $g(1 + \frac{1}{\sqrt{\sigma}}) = 1$. Note that $\partial m^b/\partial (\alpha/L) > 0$ if and only if $m + m'c^bg(n^b) > 1/n^b$, a condition that is always satisfied for $m' \geq 0$ (i.e., with DES or CES preferences). Instead, for $m' < 0$ (namely, with IES preferences), $\partial m^b/\partial (\alpha/L) < 0$ unless $n^c$ is small, i.e., sufficiently close to $1 + \frac{1}{\sqrt{\sigma}}$. 

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Finally, using (80) in (65) yields:

\[
\frac{\partial m^b}{\partial \beta} = m'^b \left( \frac{n^b}{n^b - 1} \frac{\partial c^b}{\partial \beta} - \frac{m^b}{(n - 1)(n - 1)\sigma + 1} \frac{\partial n^b}{\partial \beta} \right) - \frac{1}{(n^b - 1) D^b} \left\{ m'^b \left( 1 - m^b - \frac{n^b}{(n^b - 1)\sigma + 1} \right) + \frac{m^b m'^b (c^b)^2}{(n^b - 1)(n - 1)\sigma + 1} \right\}
\]

which implies that \( \text{sign} \left\{ \frac{\partial m^b}{\partial \beta} \right\} = -\text{sign} \{ m' \} = \text{sign} \{ \sigma' \}.

Cournot competition. Differentiating (58) yields:

\[
\frac{\partial m^c}{\partial c} = \frac{n}{n - 1} m', \quad \frac{\partial m^c}{\partial n} = -\frac{m}{(n - 1)^2} = -\frac{m^c}{n(n - 1)}.
\]

Next, using (83) in (62) yields:

\[
\begin{bmatrix}
\frac{\partial c}{\partial (\alpha/L)} \\
\frac{\partial \sigma}{\partial (\alpha/L)} \\
\frac{\partial n}{\partial (\alpha/L)} \\
\frac{\partial \sigma}{\partial (\alpha/L)}
\end{bmatrix}
= \frac{1}{D^c} \begin{bmatrix}
\frac{m'^c}{2} \\
\frac{(n^c)^2 (n^c - 1)}{n^c (n^c - 1) - 1 + \frac{1}{n^c - 1}} \\
\frac{\sigma^c}{n^c (n^c - 1) - 1 + \frac{1}{n^c - 1}} \\
\frac{\sigma^c (n^c - 1)}{n^c (n^c - 1) - 1 + \frac{1}{n^c - 1}}
\end{bmatrix},
\]

where

\[
D^c = \frac{\beta}{(n^c)^2} \left[ \frac{n^c (m'^c + m)}{n^c - 1} - 1 + \frac{1}{n^c - 1} \right] > 0.
\]

Note that \( \frac{\partial \sigma^c}{\partial (\alpha/L)} = 0 \) only for \( n^c = 2 \), and that the inequalities in (84) and (85) are straightforward when recalling that \( m + m' c - 1 > 0 \). Thus, the inequalities in (63) hold also for \( s = c \).

Next, using (84) in (64) yields:

\[
\frac{\partial m^c}{\partial (\alpha/L)} = \frac{n^c m'}{n^c - 1} \frac{\partial c}{\partial (\alpha/L)} - \frac{m}{(n^c - 1)^2} \frac{\partial n}{\partial (\alpha/L)} = m + m' c (n^c - 1) \frac{\partial (\alpha/L)}{n^c (n^c - 1)^2 D^c c}.
\]

Note that \( \frac{\partial m^c}{\partial (\alpha/L)} > 0 \) if and only if \( m + m' c (n^c - 1) > 0 \), a condition that is always satisfied for \( m' \geq 0 \) (i.e., with DES or CES preferences). Instead, for \( m' < 0 \) (IES preferences), \( \frac{\partial m^c}{\partial (\alpha/L)} < 0 \) unless \( n^c \) is small, i.e., sufficiently close to 2.

Finally, using (84) in (65) yields:

\[
\frac{\partial m^c}{\partial \beta} = \frac{n^c m'}{n^c - 1} \frac{\partial c}{\partial \beta} - \frac{m^c}{n^c (n^c - 1) \frac{\partial n}{\partial \beta}} = \frac{c^c m' (1 - m - \frac{1}{n^c})}{(n^c - 1)^2 D^c c},
\]
implying that \( \text{sign} \left\{ \begin{array}{c} \frac{\partial m}{\partial \beta} \\ \frac{\partial m}{\partial c} \end{array} \right\} = -\text{sign} \left\{ m' \right\} = \text{sign} \left\{ \sigma' \right\}. \)

**References**


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