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Endogenous Entry, Banking, and Business Cycle

Carla La Croce
(Università di Pavia)

Lorenza Rossi
(Università di Pavia)

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Via San Felice, 5
I-27100 Pavia
http://epmq.unipv.eu/site/home.html

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Carla La Croce and Lorenza Rossi

University of Pavia

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Abstract

We consider a DSGE model with monopolistic competitive banking sector together with endogenous firms entry decisions. We find that economies characterized by endogenous firms dynamics imply: i) higher volatilities of both real and financial variables than those implied by a DSGE model with monopolistic banking sector and a fixed number of firms; ii) slower recovery in the aftermath of a financial crisis. Finally, we show that a macroprudential policy rule targeting capital requirements according to Basel III is more stabilizing in an economies characterized by endogenous firms creation.

1Correspondence: Lorenza Rossi, Department of Economics and Management, via San Felice al Monastero, 5. 27100 - Pavia. Phone: +39 0382 986483. Email: lorenza.rossi@eco.unipv.it. The authors thank Alice Albonico, Guido Ascari, Paolo Bertolletti, Alessandra Bonfiglioli, Andrea Caggese, Davide Debortoli, Giovanni Di Bartolomono, Stefano Fasani, Patrizio Tirelli and Lauri Vilmi, for their helpful comments and suggestions. We also are grateful to all the participants of: the Workshop on "Macro Banking and Finance" held at the University of Milano "Bicocca" in September 2013, the Workshop on "Macroeconomics, Financial Frictions and Asset Prices", held at the University of Pavia, October 2013, the Riccardo Faini - CEIS seminar, University of Tor Vergata, the CRENOS seminar, University of Sassari, the Universitat Autonoma de Barcelona (UAB seminar) and CREI-UPF seminar (Barcelona). The authors are particularly grateful to Lilia Cavallari, for her helpful discussion of the paper presented at the workshop on "Current Macroeconomic Challenges", held at the University "La Sapienza", March 2014.
1 Introduction

An important link between the financial market and the real economy is created by firms that finance their activity by borrowing from banks. Studying this link helps to understand one of the most important transmission channel of the financial market to the real economy. Further, as shown in the recent financial crisis, the interaction between the banking sector and the good market sector, may affect not only the intensive margin of the good market but also its extensive margin, that is firm entry and exit decisions. Following these insights, this paper investigates the relationship between firms dynamics and banking, in a DSGE model characterized by flexible prices, monopolistic competitive banking and sticky interest rates together with endogenous firms’ entry decisions, modelled as in Bilbiie Ghironi and Melitz (2012 - BGM henceforth). Using this framework, we seek to understand: i) the transmission channel of both real and financial shocks to the real economic activity, disentangling the role played by endogenous firms dynamics; ii) the role played by financial market regulation in affecting business cycle dynamics, through macroprudential policy rules based on Basel II versus Basel III capital requirements. We contribute to the literature by finding the following main results. First, in response to both real and financial shocks, economies characterized by endogenous firms creation imply a strong amplification of the business cycle and higher volatilities of both real and financial variables than those implied by a DSGE model with monopolistic banking and a fixed number of firms. Second, the extensive margin of the good market implies a slower recovery in the aftermath of a financial crisis. Third, we show that a macroprudential policy rule implemented through Basel III seems to have a stronger stabilizing effect than in a model with a constant number of firms, in response to a financial crisis simulated through a shock to bank capital. Finally, in line with the evidence, the model features a countercyclical interest rate spread between the interests rate on loans and deposits. To assess the robustness of our results we consider two different way of measuring firms sunk cost of entry. We show that the main results remain unchanged.

Our paper is motivated by two main empirical facts. First, the big role played by the banking sector in the recent financial crisis both the US and in Europe. Adrian, Colla and Shin (2012) for example have shown that depletion of bank capital from sub-prime losses has forced banks to reduce lending and to raise to costs of credit. Similarly, Neri (2012) shows the EU GDP contraction started in 2008 was almost entirely due to shocks to the banking sector. Second, the strong contraction of the GDP has been accompanied by a strong credit crunch and a reduction in firms entry as well as an increase in exit, which also contributed to deteriorate the quality of the banks balance sheets.

So far, theoretical DSGE models used for business cycle analysis do not investigate the interaction between firms dynamics and banking. Recently, BGM (2012) consider a model with endogenous firms entry and show that the sluggish response of the number of producers (due to the sunk entry costs) generates

\footnote{See for example, Aliaga-Diaz and Olivero (2007), and Airaudo and Olivero (2014), among others.}
a new and potentially important endogenous propagation mechanism for real business cycle models. Etro and Colciago (2010) characterize endogenous good market structure under Bertrand and Cournot competition in a DSGE model and show that their model improves the ability of a flexible price model in matching impulse response functions and second moments for US data. Colciago and Rossi (2012) extend this model accounting for search and matching frictions in the labor market. Bergin and Corsetti (2008) and Cavallari (2013) analyze the role of entry in an open economy framework. Nevertheless, all these models embed a perfect financial market. At the same time DSGE models embedding financial market frictions as for example Bernanke, Gertler and Gilchrist (1999), do not consider the direct central bank intermediation as an instrument of monetary policy. An exception are Kiyotaki and Moore (2009), Curdia and Woodford (2009) and more recently, Gertler and Karadi (2011), Gerali et al. (2010) and De Walque (2011), among others. All these models however, consider a constant number of firms and do not investigate the role played by the interaction between firms dynamics and banking. Thus, to the best of our knowledge we are the first to introduce a structured banking sector in a DSGE model characterized by endogenous firms entry decision. Overall, we show that theoretical models cannot disregard the role played by endogenous monopolistic market structure since they would underestimate the effects of both real and financial shocks.

The remainder of the paper is organized as follows. Section 2 presents the model economy. Section 3 contains the main results. Section 4 presents some robustness and Section 5 concludes. Technical details are left in the supplemental Appendix available online.

2 The model

2.1 Firms

The supply side of the economy is composed by an intermediate good-producing sector and a retail sector that aggregates the intermediate goods. The latter operate under perfect competition, while the former under monopolistic competition.

2.1.1 Firms: the intermediate sector

We assume a continuum of firms producing a differentiated intermediate goods \( i \in N \), so that \( N \) represents both the mass of available goods and the number of firms. \( P_{i,t} \) being the nominal price of good \( i \). The intermediate good is sold under fully flexible prices to the retail sector. The production function of firm \( i \) is

\[
y_{i,t} = A_t l_{i,t} \tag{1}
\]

where \( l_{i,t} \) is the amount of labor hours employed by firm \( i \), and \( A_t \) is the
aggregate productivity, such that

$$\log \left( \frac{A_t}{A} \right) = \rho \log \left( \frac{A_{t-1}}{A} \right) + \varepsilon_{a,t}$$

where $\varepsilon_{a,t}$ is a standard white noise with zero mean and a standard deviation $\sigma_a$.

Real profits of the intermediate goods firm, are given by:

$$j_{i,t} = P_{i,t} y_{i,t} - f^E + b_{i,t} - w_t l_{i,t} - \left( 1 + r^b_{t-1} \right) \frac{1}{\pi_t} b_{i,t-1}$$

where $y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t$ is the demand for the intermediate good $i$, with $P_t$ being the CPI index, and $b_{i,t}$ is the real amount of borrowing of firm $i$ from the banking sector to pay the fixed cost $f^E$, that is $b_{i,t} = f^E$ for each $t$. This is paid back to the bank at the net interest rate $r^b_{t-1}$. The variable $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate, while $w_t l_{i,t}$ is total wage bill paid for the labor input $l_{i,t}$. The intermediate goods firm chooses the amount of labor and the optimal price in order to maximize its expected profits. First order conditions yield the optimal demand for labor and the optimal price,

$$w_t = m c_{i,t} A_t$$

$$\frac{P_{i,t}}{P_t} = \frac{\theta}{\theta - 1} m c_{i,t}$$

with $m c_{i,t}$ being firm $i$ real marginal costs. Notice that, although the assumption that $b_{i,t} = f^E$ could be seen as a strong simplification, it will allow us to capture the role played by the endogenous market structure, since as we will see, in aggregate terms the dynamics of loans will coincide with that of firms. Section 4 in the paper, shows that our results remain unaltered, when departing from the assumption of constant borrowing.

### 2.1.2 Endogenous Entry

As in BGM (2012), prior to entry firms are identical and face a fixed sunk entry cost $f^E$. At the beginning of each period $N_t^E$ new firms enter in the economy. As in Ghironi and Melitz (2005) entry occurs until the average firm value is equalized with the fixed entry cost, $f^E$, leading to the following firm entry condition

$$v_t = f^E$$

Entrants at time $t-1$ will only start producing at time $t$, so that a one-period time-to-build lag is introduced in the model. After production has occurred, as in BGM (2012) a constant fraction $\eta$ of firms exit from the market. Thus, the law of motion of number of firms in the economy at period $t$ is:

$$N_t = (1 - \eta) N_{t-1} + N_{t-1}^E$$
where $\eta$ is the exogenous probability of exiting the market.\footnote{We consider also the case in which new entrants can be separated before start producing. We find the main results unchanged. Results are available upon request.}

### 2.1.3 Firms: Retail Sector

The retail sector aggregates the intermediate goods of each intermediate firm at no cost according to the CES technology

\[
Y_t = \left[ \int_{i\in N} \left( y_{i,t}^{I} \right)^{\frac{\theta-1}{\theta}} \, di \right]^\frac{\theta}{\theta-1}
\]

at the price level

\[
P_t = \left[ \int_{i\in N} \left( P_{i,t}^{I} \right)^{(1-\theta)} \, di \right]^\frac{1}{1-\theta}
\]

As in BGM (2012), the price level of the retail firm can be rewritten as:

\[
P_t = N_t^{\frac{1}{1-\theta}} P_t^{I}
\]

where the average price in the retail sector can be expressed also in terms of average producer intermediate price. The aggregate output is:

\[
Y_t = N_t^{\frac{\theta}{\theta-1}} y_t^{I} = N_t^{\frac{1}{1-\theta}} N_t A_t L_t = \rho_t A_t L_t
\]

where we define $\rho_t = N_t^{\frac{1}{1-\theta}}$.

### 2.2 Households

Households maximize their expected utility which depends on consumption and labor hours as follows

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t - \frac{L_t^{1+\phi}}{1+\phi} \right]
\]

where $\beta \in (0,1)$ is the discount factor and the variable $L_t$ represents hours worked, while $C_t$ is the consumption index for a set of goods bundled by the retail sector as follows:

\[
C_t = \left[ \int_{i\in N} C_{i,t} \left( y_{i,t}^{I} \right)^{\frac{\theta-1}{\theta}} \, dj \right]^\frac{\theta}{\theta-1}
\]

The parameter and $\theta > 1$ is the elasticity of substitution between the goods produced in each sector. Households consume and work. They also decide how much to invest in new firms and in the shares of incumbent firms and how much to lend to the banking sector. The households budget constraint is
According to BGM (2012), we denote with \( \gamma_t \) the share in a mutual fund of firms held by the representative household. During period \( t \), the representative household buys \( \gamma_{t+1} \) shares in a mutual fund of \( N_{H,t} \) firms, where \( N_{H,t} = N_t + N_t^E \) represents firms already operating at time \( t \) and the new entrants. The mutual fund pays \( N_t j^i_t \) profits in each period, which is equal to the total profit of all firms that produce in that period. The main difference between new and old firms is that establishing a new firm requires an entry cost while the shares of an old firm are traded on the stock market. Households’ resources are composed by wage earnings \( (w_t L_t) \), the total amount of fixed costs paid by firms is \( F^F \), net interest income on previous deposits \( (r^d_{t-1} D_{t-1} / \pi_t) \), the value of the shares of firms they own \( (v_t N_t \gamma_t) \) and firms’ dividends from firms survived from the previous period \( (N_t j^i_{t-1}) \) in the same sector. The flow of expenses includes consumption \( (C_t) \), deposits to be made this period \( (D_t - D_{t-1} / \pi_t) \) and financial investments in firms already operating in the market and in new firms \( (N_{H,t} v_t \gamma_{t+1}) \).

Combining households FOCs, we get

\[
C_t = \frac{w_t}{L_t^\sigma} \quad (13)
\]

\[
E_t \beta \left( \frac{\varepsilon^P_{t+1}}{\varepsilon^P_t} \left( \frac{C_{t+1}}{C_t} \right)^{-1} \right) = \frac{\pi_{t+1}}{(1 + r^d_t)} \quad (14)
\]

\[
v_t = E_t \beta \frac{\varepsilon^P_{t+1}}{\varepsilon^P_t} \frac{C_t}{C_{t+1}} \left[ v_{t+1} + j^i_{t+1} \right] \quad (15)
\]

which respectively are the households’ labor supply, the Euler equation for consumption and the Euler equation for share holding.

### 2.3 Banking Sector

#### 2.3.1 Loans and Deposits Demand

The structure of the banking sector follows Gerali et al. (2010). We assume that deposits from households and loans to entrepreneurs are a composite CES basket of slightly differentiated products, each supplied by a single bank with elasticities of substitution equal to \( \varepsilon^b_t \) and \( \varepsilon^d_t \) respectively. More in detail we assume that the retail branch of banks are monopolistic competitive, so that they enjoy market power in setting interest rates on deposits and loans.\(^4\) As in

\(^4\)The assumption of imperfect competition in the banking sector is not unrealistic. There seems to be a consensus in the literature on the existence of market power in banking. See for example Freixas and Rochet (1997).
the standard Dixit–Stiglitz (1977) framework, loans and deposits demands are respectively:

\[ b_{j,t} = \left( \frac{r_{j,t}^b}{r_t^b} \right)^{-\varepsilon_t^b} b_t \quad \text{and} \quad d_{j,t} = \left( \frac{r_{j,t}^d}{r_t^d} \right)^{-\varepsilon_t^d} d_t \] (16)

where \( b_{j,t} \) is the aggregate demand for loans at bank \( j \), that is \( b_{i,t} = \sum_{i \in N} b_{i,t} d_i \), and \( b_t \) is the overall volume of loans to firms. Then \( d_{j,t} \) is the households aggregate demand for deposits to bank \( j \), while \( d_t \) is the households overall demand for deposits.

Further, following Gerali et al. (2010) and in line with Smets and Wouters (2003), we assume that the elasticities of substitution in the banking industry follow an AR(1) stochastic process.

\[ \varepsilon_t^d = (1 - \rho_d) \varepsilon_{t-1}^d + \varepsilon_t^d \] \[ \varepsilon_t^b = (1 - \rho_b) \varepsilon_{t-1}^b + \varepsilon_t^b \] (17) (18)

where \( \varepsilon_t^d \) and \( \varepsilon_t^b \) are normally distributed white noises with zero mean and variance \( \sigma_d^2 \) and \( \sigma_b^2 \) respectively.

The assumption of exogenous shocks is motivated by our interest in analyzing how and to what extent these shocks hitting the banking sector affect the real economy.5

2.3.2 Wholesale and Retail

The financial agents are banks, which are divided in three branches: the wholesale branch and the retail branches for loans and deposits. At the wholesale level they operate in perfect competition, while as mentioned above, at the retail level they operate in a regime of monopolistic competition.

The amount of loans issued by each bank can be financed through the amount of deposits collected from households, and through bank capital, which is accumulated out of retained earnings.

Banks play a key role in determining the conditions of credit supply. Assuming monopolistic competition between banks, we allow retail banks to have a certain degree of market power in setting or adjusting interest rates on deposits and loans in response to shocks.

Wholesale banks have to obey a balance sheet constraint,

\[ B_t = D_t + K_t^b \] (19)

We assume that the wholesale branch issues loans \( (B_t) \) to the loans branch of the retail banks by using both deposits collected by deposit branch of the retail

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5As claimed by Gerali et al (2010) the innovations to the elasticities of substitution in the banking sector may be interpreted as changes to the banking interest rate spreads arising independently of monetary policy, as for example the exogenous increase in the loan spread occurred during the recent financial crisis.
banks from households ($D_t$) and bank capital ($K^b_t$). All variables are expressed in real terms.

$$\pi_t K^b_t = (1 - \delta^b) K^b_{t-1} + j^b_{t-1} - \eta B_{t-1}$$  \hspace{1cm} (20)

where $\delta^b$ represents resources used in managing bank capital and $j^b_{t-1}$ are overall profits made by the retail branches of the bank and $\eta B_t$ represents the amount of loans not repaid by firms that exit the market, that will decrease the bank networth.

**Wholesale Branch**  The wholesale branch operates in a competitive way and combines bank capital and deposits to issue wholesale loans. Through the balance sheet constraint it manages the capital position of the bank. Moreover, there is an exogenous constraint, which is an optimal value for capital-to-asset ratio ($\nu^b$), and identifies decisions about how much own resources to hold, or an easy way to study the consequences of capital regulation. Since it is costly to deviate from this optimal value, the bank pays a quadratic cost whenever the capital-to-asset ratio moves away from the target value. This gives the banks a key role in determining the credit supply and put them in a crucial position between real and financial sector. Indeed, when economic conditions become tighter, bank profits and then bank capital decrease, weakening banks’ financial position. To keep capital-to-asset ratio at the optimal level, banks have to reduce loans, thus worsening the initial negative shock.

The problem for the wholesale branch is to choose the amount of loans and deposits to maximize the discounted sum of real cash flows, subject to the balance sheet constraint:

$$\max_{\{D_t, B_t\}} \sum_{t=0}^{\infty} \beta^t \left[ (1 + R^w_t) B_t - \pi_{t+1} B_{t+1} + \pi_{t+1} D_{t+1} - (1 + R^d_t) D_t + \Delta K^b_{t+1} - \frac{\kappa K^b_t}{B_t} (K^b_t - \nu^b) \right]^2 K^b_t$$  \hspace{1cm} (21)

s.t.  \hspace{1cm} $B_t = D_t + K^b_t$

where $\Delta K^b_{t+1} = (\pi_{t+1} K^b_{t+1} - K^b_t)$, $B_t$ and $D_t$ are the total amount of wholesale loans and wholesale deposits, $R^w_t$ and $R^d_t$ are the net wholesale loans rate and the net wholesale deposit rate, exogenously given. $\frac{K^b_t}{B_t}$ is the capital-to-asset ratio and $\nu^b$ represents its optimal value. Bank capital is not a choice variable for the bank since bank profits are totally re-invested.

The objective function reduces to

$$\max_{\{D_t, B_t\}} \left[ R^w_t B_t - R^d_t D_t - \frac{\kappa K^b_t}{2} \left( \frac{K^b_t}{B_t} - \nu^b \right)^2 \right] K^b_t$$

The first order condition of the wholesale bank gives us the spread between rates on wholesale loans and on wholesale deposits to the degree of leverage ($\frac{B_t}{K^b_t}$).

$$R^w_t = R^d_t - \kappa K^b \left( \frac{K^b_t}{B_t} - \nu^b \right)^2 \left( \frac{K^b_t}{B_t} \right)^2$$  \hspace{1cm} (22)
Moreover, we assume that wholesale branch has access to a lending facility at the central bank, so it can have access to funds when they are needed, at the policy rate $r_t$. Then the deposit rate is pinned down in the interbank market and it is equal to the policy rate ($R^d_t = r_t$).

$$S^{WB} = R^b_t - r_t = \kappa_{K^b} \left( \psi^b - \frac{K^b_t}{B_t} \right) \left( \frac{K^b_t}{B_t} \right)^2 \quad (23)$$

This equation defines the role of bank capital in determining loan supply conditions: the spread between rates on wholesale loans and wholesale deposits is inversely related to the capital-to-assets ratio. In particular, when banks are scantily capitalized ($K^b_t$ decreases) and leverage ($\frac{B_t}{K^b_t}$) increases, the difference between $R^b_t$ and $r_t$ increases and margins become wider. In this case, as leverage increases, the spread between the wholesale loan rate and the policy rate increases, banks increase loan supply because of the greater interest rate on wholesale loans, and thus increase their profits. But on the other hand, as leverage increases further, the deviation from the optimal value of capital-to-asset ratio ($\psi^b$) becomes more costly, reducing bank profits. In this case, the result given by the first order condition suggests that banks have to choose a level of loans (and thus of leverage, given a level of $K^b_t$) that keeps the marginal cost of reducing the capital-to-asset ratio equal to the spread between rates on wholesale loans and deposits.

### 2.3.3 Retail Branches (Loans and Deposits)

Retail banks compete under monopolistic competition with other banks. As in Gerali et al (2010), we use a standard Dixit-Stiglitz aggregator for loans and deposits. This implies that all banks essentially serve all firms, providing slightly differentiated loan contracts. Similarly, banks offer differentiated deposits to the household. Both loans and deposits of banks are indexed to a continuum interval ($j = 0, 1$). Imperfect substitutability between the contracts of different banks will additionally lead to explicit monopolistic mark-ups and mark-downs on these rates.

The loan branch can borrow from the wholesale unit at a rate $R^b_t$, it differentiates the loans at no cost and resells them to the firms applying a markup. Each retail bank faces a quadratic adjustment cost for changing the loan rates. This cost introduces sticky bank rates in the model.

We assume that banks do not observe the borrower’s financial situation, they only observe if the borrower repays the loan. So that banks profits maximization problem is:
max \( E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ b_{j,t} (1 - \eta) - R_{t} B_{j,t} - \frac{\kappa_b}{2} \left( \frac{r_{j,t} - r_{j,t-1}}{r_{j,t}^b} \left( \frac{r_{j,t}^b}{r_{j,t}^d} - 1 \right) \right)^2 r_{t} B_{t} \right] \) 

s.t. 
\[ b_{j,t} = \left( \frac{r_{j,t}^b}{r_{t}^b} \right)^{-\varepsilon_t^b} b_t \text{ and } b_{j,t} = B_{j,t} \] 

(25)

where \( b_{j,t} = \left( \frac{r_{j,t}^b}{r_{t}^b} \right)^{-\varepsilon_t^b} b_t \) is the demand for loans of bank \( j \). From the FOC, after imposing symmetry across banks, i.e. \( r_{j,t}^b = r_{t}^b \), and thus \( b_{j,t} = b_t \) and \( B_{j,t} = B_t = N_{t+1} f^F \), we get the equation for the optimal interest rate:

\[
\varepsilon_t^b \frac{R_t^b}{r_t^b} = \kappa_b \left( \frac{r_t^b}{r_t^{b-1}} - 1 \right) \frac{r_t^d}{r_t^b} - E_t \Lambda_{t+1} \kappa_b \frac{B_{t+1}}{B_t} \left( \frac{r_{t+1}^b}{r_t^b} - 1 \right) \left( \frac{r_{t+1}^b}{r_t^b} \right)^2 (1 - \eta) (1 - \varepsilon_t^b) \]

(26)

notice that under flexible rates it becomes,

\[
r_t^b = \frac{\varepsilon_t^b}{(\varepsilon_t^b - 1) (1 - \eta)} R_t^b \]

(27)

the interest rate on loans is a mark-up over marginal costs, represented by \( R_t^b \). Notice that the newness is that, with respect to Gerali et al (2010), firms dynamics affects the value of the mark-up. As the probability of exit \( \eta \) increases, retail banks set higher interest rate. The intuition is straightforward. Indeed, an higher probability of exit increases the probability of a firm of not repaying the loan, bank that issued that loan faces lower profits and is forced to increase the interest rates.

The retail deposit branch collects deposits from households and gives them to the wholesale unit. The wholesale unit pays them at rate \( r_t \), which is the same rate at which wholesale unit have access to the funds of the Central Bank. The problem for the deposit branch is

\[
\max E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ r_t^d D_{j,t} - r_t^d d_{j,t} - \frac{\kappa_d}{2} \left( \frac{r_{j,t}^d}{r_{j,t-1}^d} - 1 \right) \right]^2 \] 

s.t. 
\[ d_{j,t} = \left( \frac{r_{j,t}^d}{r_t^d} \right)^{-\varepsilon_t^d} d_t \text{ and } D_{j,t} = d_{j,t} \] 

(29)

where \( d_{j,t} = \left( \frac{r_{j,t}^d}{r_t^d} \right)^{-\varepsilon_t^d} d_t \) is the demand for deposits of bank \( j \). From the FOC, after imposing symmetry across banks, i.e. \( r_{j,t}^d = r_t^d \), and thus \( d_{j,t} = d_t \) and
$D_{j,t} = D_t$, we get the optimal interest rate for deposits,
\[
ed_t^d r_t = E_t A_{t,t+1} \kappa_d \left( \frac{r_{t+1}^d}{r_t^d} - 1 \right) \left( \frac{r_{t+1}^d}{r_t^d} \right)^2 d_{t+1} \kappa_d \left( \frac{r_t^d}{r_{t-1}^d} - 1 \right) \frac{r_t^d}{r_{t-1}^d} + (\varepsilon_t^d - 1)
\]
(30)
under flexible rates the equation becomes,
\[
r_t^d = \frac{\varepsilon_t^d}{\varepsilon_t^d - 1} r_t
\]
the interest rate on deposits is mark-down over the policy rate $r_t$.

2.3.4 Bank Profits
Bank profits are also affected by the probability of exit, since they are the sum of the profits of the wholesale and the retail sector. Bank profits now become:
\[
j_t^b = r_t^b B_t (1 - \eta) - r_t^d D_t - Adj_t^B
\]
(32)
where
\[
Adj_t^B = \frac{\kappa K_t^b}{2} \left( \frac{K_t^b}{B_t} - \phi_b \right)^2 K_t^b + \frac{\kappa_b}{2} \left( \frac{r_t^b}{r_{t-1}^b} - 1 \right)^2 r_t^b B_t + \frac{\kappa_d}{2} \left( \frac{r_t^d}{r_{t-1}^d} - 1 \right)^2 r_t^d D_t
\]
(33)
indicates adjustment costs for changing interest rates on loans and deposits and changes in capital-to-asset ratio.

2.4 Monetary Policy
To close the model we need to specify an equation for the Central Bank behavior, i.e. we need to introduce an equation for the nominal interest rate prevailing in the interbank market. In this respect we assume that the monetary authority simply follows a standard Taylor rule given by
\[
\ln \left( \frac{1 + r_t}{1 + \pi_t} \right) = \phi_R \ln \left( \frac{1 + r_{t-1}}{1 + \pi_t} \right) + (1 - \phi_R) \left[ \phi_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \phi_y \ln \left( \frac{Y_t}{Y} \right) \right] + \ln \varepsilon_t^\pi
\]
(34)
where $\varepsilon_t^\pi$ is an AR(1) interest rate shock with zero mean and a standard deviation equal to $\sigma_r$.

2.5 Market Clearings
For the aggregation procedure we follow Melitz (2003). The market clearing condition implies
\[
Y_t = C_t + \delta^b \frac{K_{t-1}^b}{\pi_t} + \left( \frac{1 + r_{t-1}^b}{\pi_t} \right) B_{t-1} \eta + \frac{Adj_{t-1}^B}{\pi_t}
\]
where $\left( \frac{1 + r_{t-1}^b}{\pi_t} \right) B_{t-1} \eta$ represents the loss of banks from defaulted firms.

6See the Technical Appendix for all the derivations.
3 Business Cycle Analysis

In what follows we study the impulse response functions to a productivity shock, to a shock to bank capital and to a shock to bank markup. In order to disentangle the role played by the endogenous firms dynamics, we compare the performance of two models: i) a standard DSGE model with a fixed number of firms, that we label as Constant Firms model; ii) a model with firms endogenous entry, labelled as EEM model.

3.1 Calibration

Calibration is set on a quarterly basis. The elasticity of substitution among intermediate goods, $\theta$, is set equal to 4, a value which is in line to that of BGM (2012). Analogously, as in BGM (2012), we set the inverse of Frisch $\phi = 1/4$, the entry cost $f^E = 1$ and we set the size of the exogenous exit shock $\eta$ to be 0.025, to match the U.S. empirical level of 10% of firms destruction per year. We set fixed costs $f^F$ such that in all the economies considered they correspond to 5% of total output produced. The steady state of productivity $A = 1$.

We calibrate the banking parameter $\varepsilon^b = 3.12$ as in Gerali et al. (2010) so as to replicate their markup, we calibrate the discount factor $\beta$ to 0.9878 to ensure that the target value of capital-to-asset ratio $\nu^b$ is 0.09 and we calibrate the elasticity of deposits $\varepsilon^d$ to have a depreciation rate of the bank capital $\delta^b$ equal to 0.025. For the Taylor rule parameters, we set $\phi_R = 0.8$, $\phi_\pi = 2.25$ and $\phi_y = 0.125$, which are in the range of the parameters usually estimated for both US and EU.\(^7\)

3.2 Impulse Response Functions

Figure 1-3 show the impulse response functions (IRFs) to a positive technology shock, a negative shock to bank capital, and to a shock to bank markup. In all figures, the dashed-dotted line represents the Constant Firms model\(^8\) whereas the solid line represents the EEM model.

3.2.1 Technology shock

As shown in Figure 1, the economy characterized by endogenous firms dynamics (i.e. EEM model henceforth) shows higher volatilities of output, bank capital, interest rates on loans and policy rate than those implied by a standard model with a constant number of firms. A positive technology shock creates expectations of higher future profits which lead to the entry of new firms. Given that entry is subject to a one period time-to-build lag the total number of firms, $N_t$, does not change on impact, but builds up gradually.

The entry margin leads to a much stronger and more persistent increase in output and to a high and persistent increase in the demand for loans. Since

\(^7\)The main results are not affected by change in the parameters.

\(^8\)We normalized the number of firms to 1.
the banking sector is imperfectly competitive, interest rates on loans are related to the policy rate. The decline in the policy rate leads to a decline in the interest rate on loans, leading in turn to a wider access to credit for firms, and thus implying an increase in the number of firms asking for loans. Given our assumption on fixed loans borrowed by firms, the growth rate of loans coincides with that of firms. Lower interest rates on loans has two effects: lower loan rates, ceteris paribus, imply higher firms profits and thus higher entry, which gives an additional boost to output.

The reduction in bank profits, followed by a fall in bank capital is mainly due to endogenous countercyclical response of the interest rate margin, i.e. the spread between the interest rate on loans and deposits. After the initial increase of loans, due to the more favorable credit access, lower interest rates generate lower bank profits and lower bank capital and an higher bank leverage ratio. After some periods, higher leverage costs force banks to reduce loans, reduce credit access, so that the number of firms asking for loans decreases and, consequently, all variables turn back to their steady state values.

![Figure 1: IRFs to 1% positive TFP shock. Constant Firms Model (dashed-dotted line). Exogenous exit model (solid line).](image1)

### 3.2.2 Bank Capital Shock

In Figure 2 we present the IRFs to a negative shock to bank capital. As before, the economy characterized by endogenous firms dynamics shows higher volatilities of output, inflation, interest rates and policy rate than those implied by a standard model with a constant number of firms.

Notice that, since bank capital contraction decreases banks’ profits, banks are forced to increase interest rates on loans and, as a result, firms marginal costs increase and profits decrease. Given the expectations of lower profits new entrants decrease, so that the total amount of firms and loans decrease. The persistent increase in the interest rate on loans drags real activity down. The higher financing costs push inflation up, so that the policy rate increases.
3.2.3 Bank Markup Shock

We now show the IRFs to a negative shock to bank markup, obtained through a positive shock to the interest rate elasticity of loans. Figure 3 compares the IRFs of the model with endogenous entry with those of the model with a constant number of firms. The increase in the substitutability between loans leads to an increase in the competition between banks, which implies lower banks markups and thus profits, and a decrease in bank capital. Incumbent firms face more convenient credit conditions which leads to an increase in firms new entrants. Inflation jumps on impact so that the monetary authority increases the policy rate in the first periods. The lower interest rate on loans leads to a decrease in the interest rate spread and to an increase in the demand for loans. Since the interest rate on loans decreases for several periods firms costs keep on decreasing so that after the initial increase inflation undershoots its long run level, and so does the policy rate. The reduction in the policy rate, increases the households opportunity costs of savings in new firms and consequently firms entry undershoots its long run value and so does output.\footnote{Notice that this undershooting effect will almost disappear in presence of sticky prices.}

Figure 2: IRFs to 1% negative bank capital shock. Constant Firms Model (dashed-dotted line). Exogenous exit model (solid line).
3.3 Macroprudential Policies

After the US 2007 financial crisis there has been a change in the perception of how financial regulation should behave. It appeared that the traditional formal requirements for banks (asset quality and adequate capital) were no longer sufficient to ensure the stability of financial institutions. Moreover, while in the pre-crisis period a common belief was that monetary policy had to pursue price stability, disregarding financial questions, now researchers in both academics and institutions seem to agree that monetary policy should contribute in controlling system stability, through an interaction between monetary policy and financial market regulation.

In this section we introduce financial market regulation and we study the effect of alternative macroprudential policies in response to the three shocks analyzed so far.

In particular, as in Angelini et al (2010), we introduce Basel capital requirements, considering a time varying capital-to-asset ratio $b_t$. In particular, we will consider both a procyclical and countercyclical capital requirements rule, which is specified as follows:

$$v_t^b = (1 - \phi_v) v_t^b + \phi_v v_{t-1}^b + (1 - \phi_v) \lambda_b \log \left( \frac{Y_t}{Y} \right)$$

where with $\lambda_b > 0$, capital-to-asset ratio are countercyclical, as for Basel III, while with $\lambda_b < 0$, capital-to-asset ratio are procyclical, as for Basel II. Finally with $\lambda_b = \phi_v = 0$, the equation reproduces the Basel I regime, in which the capital-to-asset-ratio is fixed to 9% (as a matter of fact, Basel I shows a small degree of procyclicality, but in this paper we will refer to Basel I for indicating a macroprudential policy requiring fixed capital-to-asset ratios).

In what follows we will show the responses of the model with endogenous firms entry (EEM) and the model with a constant number of firms (Constant Firms Model) (dashed-dotted line). Exogenous exit model (solid line).
Firms) to a bank capital shock under both Basel II (procyclical capital requirements) and Basel III (countercyclical capital requirements).

### 3.3.1 Bank Capital Shock

We now compare the responses to the bank capital shock of the two models considered so far, under the two alternative macroprudential policy rules: i) Basel II, that is procyclical capital requirements; and ii) Basel III, that is a policy rule implementing countercyclical capital requirements. Similarly to Angelini et al. (2010) we set $\lambda_b = 20$ in the case of Basel III and $\lambda_b = -20$ for Basel II and $\phi_v = 0.9$ in both cases. Under this parameterization the capital-to-asset ratio change by almost 0.25 percentage point in the EEM model, passing from 9% to 8.75% in the medium term, while for the model with constant number of firms the change in the capital-to-asset ratio is almost 0.10 percentage points, changing from 9% to 8.80%. The Taylor rule parameters are set as in the baseline calibration.

Figure 4 shows the difference between Basel II and Basel III in response to a 1% bank capital shock in the model with a constant number of firms. The solid line represents the IRFs under Basel II while the dotted line represents the IRFs under Basel III of the constant firms model. Notice that, there is no substantial difference in the model dynamics under the two different macroprudential policies. This is quite surprising since we expected that Basel III should have stronger stabilizing effects than Basel II. However we are not the first to find similar results in a model with a constant number of firms and an imperfect banking sector.\(^\text{10}\)

Figure 4. IRFs to 1% Bank Capital shock in Basel 2 (solid line) and 3 (dashed line) for COnstant Firms model.

Figure 5 shows the same IRFs for the model with EEM. Notice that, in this case, when the economy is hit by a bank capital shock, even a small change in

\(^{10}\)See Gale et al (2011) for a brief survey of the recent literature.
the capital-to-asset-ratio, $v^b$ implies a significant difference between Basel II and Basel III. Overall, Basel II shows a greater volatility and Basel III is much more stabilizing. Thus, the endogenous firms creation mechanism seems to strongly interact with the macroprudential rules.

![Figure 5. IRFs to 1% Bank Capital shock in Basel 2 (solid line) and 3 (dashed line) for Endogenous Entry model.](image)

4 Robustness: Considering Different Entry Costs

In this section we examine the effect of having different entry costs. We consider two alternatives: i) a non-constant entry cost defined in terms of labor units, modelled as in BGM (2012); ii) a non-constant entry cost defined in terms of consumption units. We show impulse response functions to three shocks considered so far.

To introduce an entry cost defined in labor units we define total labor as $L_t = L^C_t + L^E_t$, where $L^C_t$ is the amount of labor used to produce consumption goods, whereas $L^E_t$ is the amount of labor used to create new firms in the intermediate sector. The sunk entry cost is now defined as

$$v_t = \frac{w_t}{A_t} f^E$$

the rest of the model remains unchanged.

The non-constant entry cost defined in terms of consumption units is instead modelled as follows:

$$v_t = \frac{f^E}{A_t}$$

In this case, the entry costs decreases in face of an increase of average productivity. Thus, there is no share of the labour supply occupied to build new intermediate firms, so that $L^E_t = 0$, which implies that all labour is used to produce consumption goods ($L_t = L^C_t$).
4.1 Impulse Response Functions

In what follows we study the IRFs to a total factor productivity shock, to a bank capital shock and to a shock to bank markup. We compare the performance of the three models: i) the baseline EEM with a constant cost of entry; ii) the EEM with an entry cost measured in labor units; and iii) the EEM with an entry cost defined in terms consumption units.

In all figures, the solid line represents the model with constant cost of entry (labelled as CC), the dashed-dotted line represents the model with entry cost in consumption units, (labelled as CU), whereas the dotted line represents the model with entry cost in labor units (labelled as LU).

Figure 6 shows the IRFs to a positive technology shock. As shown in the Figure the exogenous exit model with entry cost in terms of consumption units shows a higher volatility with respect to the other two models. This is not surprising since an increase in productivity directly decreases the cost of entry. Despite this, the three models show very similar dynamics, but for the LU model, which is characterized by a decrease of hours used in the good-producing sector (labelled as $L_c$ in the figure). Labor used to produce new firms strongly increases so that total hours worked increase also in the LU economy.

Figure 7 presents the IRFs to a negative shock to bank capital. As before the LU model show a countercyclical response of hours used in the good producing sector and, consequently, an increase on impact of output instead of a decrease. However, after the first period also the LU economy enters into a downturn. Notice that since productivity is constant the IRFs of the CC and the CU model clearly overlap.
Figure 7: IRFs to 1% negative bank capital shock. The model with Constant Entry Cost (CC) (solid line), entry cost in Consumption Units (CU) (dashed-dotted line), Entry Cost in Labor Units (LU) (dotted line).

Figure 8 presents the IRFs to a negative shock to bank markup. In this case CC model and the CU model show identical pattern. As before, the LU model show a decrease in output on impact and a countercyclical response of hours in the good-producing sector and also of output which decreases on impact instead of increasing. However, from the first period on also the LU economy enters into a boom.

Figure 8: IRFs to 1% negative bank markup shock. The model with Constant Entry Cost (CC) (solid line), entry cost in Consumption Units (CU) (dashed-dotted line), Entry Cost in Labor Units (LU) (dotted line).

A second robustness check, not reported in the paper, has been done by introducing sticky prices à la Rotemberg (1982), in the intermediate good sector. We find that the main results and the message of the paper remain unchanged. Finally, we consider also the case of flexible interest rates on loans and deposits. Also in this case the main results and message of the paper are unchanged.  

11 Results on both the robustness checks are available upon request.
5 Conclusion

We consider a DSGE model with flexible prices, monopolistic competitive banking sector and sticky interest rates together with endogenous firms’ dynamics. We show that in response to both real and financial shocks, economies characterized by endogenous firms dynamics imply higher volatilities of both real and financial variables than those implied by a DSGE model with monopolistic banking sector and a fixed number of firms. Also, the extensive margin of the good market implies a slower recovery in the aftermath of a financial crisis. Further we show that a policy rule implementing Basel III capital requirements seems to have an important role in stabilizing the economy in periods of financial distress, when the economy feature endogenous firms dynamics. Our result are robust to the introduction of alternative entry costs, as well as to the introduction of sticky prices in the retail sector. Overall, we show that theoretical models cannot disregard the role played by endogenous firms dynamics since they would underestimate the effects of both real and financial shocks.

Our model can be extended along several dimensions. First, we can introduce borrowing against a collateral in order to evaluate the role of the collateral constraints as well as the role of alternative rules on the loan-to-value ratio. Considering non-conventional monetary policy is also part of our research agenda. Finally, investigating the role of firms endogenous exit decisions is also a possible extension and is our work in progress.

6 References


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