This version: January, 2007

MONOPOLISTIC PRICE FLEXIBILITY AND SOCIAL WELFARE: THE LINEAR CASE

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Abstract. We explore the case for monopolistic price flexibility in a linear setting. With different marginal costs, by changing the production mix price differentiation can improve welfare and also aggregate consumer surplus even if total output does not increase (as in the linear case). This depends on the variability of cost and demand parameters, and on their correlation. We also show that the welfare criterion (developed for the case of common marginal costs) based on total output can be replaced by the computation of well-defined price indexes. These results possibly pave the way for a more optimistic assessment of monopolistic pricing.

Keywords: price flexibility, uniform pricing, welfare bounds, third-degree price discrimination, price and quantity indexes.

JEL *Classification*: D11, D42, L51.

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Financial support from MIUR is gratefully acknowledged. I am thankful for useful comments to participants at seminars in Milan and Naples, and in particular to P. Battigalli. The usual disclaimer applies.

I. Introduction

Consider a setting in which a monopolistic firm sells in several markets. In particular, we have in mind the case in which the products sold in the different markets are alike, so that the units of output are commensurate and in principle the rule of a uniform price could be (and in practice sometimes is) imposed by an antitrust authority: see e.g. Cabral (2000: paragraph 10.5).[1] It is well known that imposing uniform pricing on a multi-market monopolist instead of allowing complete price flexibility has ambiguous welfare implications. However, Schmalensee (1981), Varian (1985) and Schwartz (1990) proved that, if marginal costs are common, a necessary condition for the monopolistic so-called (in such a case) "third-degree price discrimination" to raise aggregate welfare is that *total* output increases under discriminatory pricing. Moreover, in the benchmark setting with linear demand and cost functions, total output does not change if the markets served by the monopolist are the same both under uniform and differentiated pricing:[2] see Schmalensee (1981) and Tirole (1988: p. 139) for the case with a common (marginal) cost, and Appendix A. Since the linear setting provides a first-order approximation to the general case and is usually adopted because it allows an explicit computation of the results, the literature on the welfare effect of monopolistic price flexibility tends to be rather pessimistic:[3] see e.g. Schmalensee (1981: p. 246) and Varian (1989: pp. 622-623).

In this paper we reconsider the case for monopolistic pricing by departing from the textbook set-up, which assumes that there is a common cost of serving different markets. Accordingly, we are not really concerned with a case of (third-degree) price discrimination as narrowly defined, since this would require that the *same* commodity be sold at different prices to different consumers: see e.g. Phlips (1981: p. 5). Nevertheless, we think our setting interesting for several reasons. First, the standard set up with equal marginal costs is a textbook idealization with little empirical content. For example, what guarantees that the marginal cost of serving a physically identical product to geographically distant markets is the same? Thus the case of (possibly small) marginal cost differences seems empirically the most relevant. Second, indeed many standard definitions of price discrimination encompasse the case of different marginal costs: see Tirole (1988: pp. 133-4) and Clerides (2004). For example, perhaps the most appealing among them (attributed to George Stigler, 1987) says that a firm price discriminates when the ratio in prices is different from the ratio in marginal costs for two "similar" (possibly identical) goods offered by it: see e.g. Varian (1989: p.

[1] This case is also the subject of the so-called "average revenue regulation": see e.g. Armstrong and Sappington (2005: paragraph 3.1.2).

[2] For the sake of simplicity, we make this assumption throughout this paper. Notice that in fact price differentiation may lead to a Pareto improvement if it causes some market to open: see e.g. Schmalensee (1981), Varian (1985) and Tirole (1988: pp. 137-9).

598) and Stole (2003: pp. 1-2). Here the issue is twofold: on the one hand, (marginal) cost differences are operationally difficult to verify. On the other hand, in contrast to regulatory bodies, antitrust authorities usually lack the power to regulate the prices according to the costs. Thus, it seems to us interesting to test the use of “uniform pricing” as a policy prescription against the possibility of *unaccounted* marginal cost differences. Moreover, it is unclear whether in our second-best setting a no-price-discrimination prescription *à la* Stigler, even when feasible, would turn out to be optimal:[4] see below.

Third, while we should perhaps refer to them as examples of price discrimination (once the latter is properly defined: see e.g. the comments in Tirole, 1988: pp. 133-4), there are sectors with acknowledged differentiated marginal costs, such as the mail or telephone ones, in which uniform rates are imposed by public regulation (avoiding, for example, tariff discrimination between rural and urban areas), sometimes on a political basis invoking “horizontal equity” (this is the case, for instance, of the Italian residential electricity sector). In these cases, uniform pricing entails cross-subsidies whose welfare consequences have not yet been entirely understood. Fourth, the use of a uniform price may serve as a (potentially anticompetitive) strategic practice not to reveal the underlying cost structure, while price flexibility might on the contrary be used by an incumbent firm to deter entry (see e.g. Armstrong and Sappington, 2005: paragraph 3.1.3). While these aspects are not considered here, the related social costs and benefits should in principle be weighed against the other welfare consequences of avoiding price differentiation, which are the subject of this paper. Finally, even if the monopolistic results we are concerned with do not necessary apply to other imperfectly competitive markets, it is sometimes suggested that they hold more generally under the assumption of equal marginal costs (see e.g. Galera and Zaratiegui, 2006, p. 606). Since the assumption of identical costs would seem definitely too strong to be realistic in most cases with different firms, we also hope that our framework can serve as a useful starting point for future investigations of non-monopolistic markets.[5]

Not surprisingly, it turns out that in our setting the condition of an increase in output is no longer necessary to achieve an improvement in welfare through price differentiation. In particular, we are able to discuss several cases in which monopolistic price flexibility increases total welfare and aggregate consumer surplus even if, as in the linear case, total output does not increase. This happens if demands are close enough, as is to some extent intuitive since this reduces the scope for

[3] This comment only applies to the case of an otherwise unregulated monopoly: see e.g. Armstrong and Vickers (1991).
[4] Indeed, our results provide some theoretical foundations to an alternative price-discrimination definition based on price-cost margins: see Clerides (2004) for the empirical relevance of the distinction among different formulations.
[5] After the first version of this paper was written in 2003, we had the opportunity of reading the papers by Aguirre (2004) and Galera and Zaratiegui (2006), which show by the way of examples that even with linear demands price

the surplus-extraction purpose of price "discrimination", and also in some cases in which there is no systematic relation across markets between the demand and the cost structures (in general, a seemingly reasonable assumption). The intuition is straightforward: the economic rationale for imposing uniform pricing in the setting with a common marginal cost rests on the result that a *given* quantity of the *same* good should be distributed according to a common price (a result which is implied by the optimality of marginal cost pricing). But with different marginal costs no general principle can be invoked to support uniform pricing (goods are economically different and *first-best* optimality would rather call for the weaker condition of having the prices ratios equal to the marginal cost ratios),[6] which has no special properties. In other words, as a policy prescription, uniform pricing is not robust even to possibly *very* small marginal cost differences.

Indeed, the socially efficient production of a given total amount of output (a somewhat unusual *second-best* problem if goods are not identical) would require that the *differences* between prices and the relative marginal costs be equal across markets. This price structure, which will never be achieved by uniform pricing if marginal costs are different, to some extent is more similar[7] to the one that would be chosen by an unregulated monopolist (a special case of Ramsey pricing, as is well known). In fact, in order to maximize his profit, a monopolist could use the alleged price flexibility to increase the price of the more costly goods, thereby decreasing *average* total cost and also increasing welfare. Indeed, he actually does that if the demand elasticities are not "adversely" correlated with the unit costs (this should be the case if demands are "close enough" with respect to the cost differences). Some part of the cost reduction so achieved could then be "passed" trough prices to the consumers but, of course, the monopolist would also try to use his pricing authority to extract more surplus from them. Notice that this might give rise to a second-best conflict between social welfare and consumer surplus concerns.

In general, welfare effects unfortunately maintain their ambiguity (as should be expected): however, we show that while the welfare criterion based on total output (developed for the case with a common marginal cost) fails in general, it can be replaced by the computation of well-defined price indexes. Moreover, our results for the linear case show that indeed price flexibility reduces overall costs in some interesting cases (if there is no systematic covariance between cost and demand parameters), and provide a few sufficient conditions for such a cost reductions to

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discrimination can be welfare improving if there is more than one firm. We believe that our results, which apply to the monopolistic case, nicely complement (and somehow generalize) the point made by those papers.

[6] Notice that this is the condition underlying the broad definition (quoted above) of price discrimination due to Stigler. Also notice that the unregulated monopolist would use this first-best price *structure* if demand elasticities were equal across markets.

[7] Phlips (1981: pp. 3-4) commented negatively on this similarity, but by referring to the case of a unique, common marginal cost.

become welfare (or even consumer surplus) improvements. Overall, they also suggest that price differentiation by an unregulated monopolist should perhaps be considered more optimistically.

The paper is organized as follows: in section II the model by Schmalensee (1981) is generalized to the case of different (constant) marginal costs, and the results by Varian (1985) are applied to it. Then, by discussing the special (but dual to the standard) case of equal demand elasticities (at the uniform price), we show that monopolistic pricing can be welfare and (aggregate) consumer surplus improving even if it does not increase total output. In this section we also introduce a form of monopolistic "piecemeal" price differentiation. Section III explores the linear case, and deals with the use of price indexes to check conditions for welfare improvements and the role of cost and demand variability. Technical results are summarized in Proposition 1: section IV discusses them and concludes. More technical derivations are confined to the Appendixes.

II. The model

We refer to the model in Schmalensee (1981), which can be seen as a special case of Varian (1985). In particular, a monopolist is selling in N distinguishable markets. Let $q_i(p_i)$ be the demand function in market i (i = 1, …, N), where p_i is the price charged by the monopolist and q_i the quantity he sells. It is assumed that the relevant cost function is given by $C(\boldsymbol{q}) = \boldsymbol{c'q} = \Sigma_i c_i q_i$, where $\boldsymbol{q}' = [q_1,q_2,\ldots,q_N]$ is the vector[8] of the monopolist's output quantities, c_i indicates the constant unit cost of output q_i and $\boldsymbol{c}' = [c_1,c_2,\ldots,c_N]$. We generalize Schmalensee's (1981) setting by assuming that c_i is possibly different from c_j, $i \neq j$, i,j = 1, …, N. The net profit generated in market i can then be written $\pi_i(p_i) = (p_i - c_i)q_i(p_i)$: following most of the literature on price discrimination,[9] it is assumed that the $\pi_i(p_i)$ are smooth, strictly concave functions. The total profit function is thus given by $\Pi(\boldsymbol{p}) = (\boldsymbol{p} - \boldsymbol{c})'\boldsymbol{q}(\boldsymbol{p}) = \Sigma_i \pi_i(p_i)$, where $\boldsymbol{p}' = [p_1,p_2,\ldots,p_N]$ is the vector of prices that the monopolist charges. It is also assumed that consumers have quasi-linear preferences. Thus, since there are no income and distributional effects, we can think in terms of a representative consumer with indirect utility function $V(\boldsymbol{p}) = v(\boldsymbol{p}) + y_0$, where y_0 is the total endowment of the *numeraire* (the Marshallian composite commodity), who consumes all the goods produced by the monopolist. By quasi-linearity, $v(\cdot)$ is convex and the demand system faced by the monopolist is given by $\boldsymbol{q}(\boldsymbol{p}) = -\boldsymbol{D}_{\boldsymbol{p}}v(\boldsymbol{p})$, where $\boldsymbol{D}_{\boldsymbol{x}}f(\boldsymbol{x})$ is the gradient of $f(\cdot)$. Aggregate (social) welfare (the Marshallian indicator) can then be written as $W(\boldsymbol{p}) = \Pi(\boldsymbol{p}) + v(\boldsymbol{p})$.

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[8] Bold characters are used to indicate vectors; conventionally, $\boldsymbol{x}$ is a column vector and $\boldsymbol{x}'$ indicates its transpose (a row vector).

[9] Without concavity of the profit (of the demand) functions, price discrimination might even move all prices in the same direction: see Nahata et alii (1990).

Clearly, the unregulated monopolist would adopt a price vector $\boldsymbol{p}^*$ uniquely characterized by the FOCs $\boldsymbol{D_p}\Pi(\boldsymbol{p}^*) = \boldsymbol{0}$, where $\boldsymbol{0}$ is the relevant null vector; i.e., $i = 1, \ldots, N$:

$$\pi_i'(p_i^*) = (p_i^* - c_i)q_i'(p_i^*) + q_i(p_i^*) = 0\,, \tag{1}$$

which is just the familiar condition of marginal revenue and marginal cost equality in each market. On the contrary, if the monopolist is forced to use a uniform price he would charge all buyers p^*, which is uniquely defined by:

$$\sum_{i=1}^{N} \pi_i'(p^*) = \sum_{i=1}^{N} [(p^* - c_i)q_i'(p^*) + q_i(p^*)] = 0\,. \tag{2}$$

It is standard to adopt Joan Robinson's terminology and to call a market i "strong" if $p_i^* > p^*$: conventionally, we write that in such a case $i \in S$. Accordingly, a market i is "weak" if $p_i^* < p^*$ (and thus $i \in W$) and "intermediate" if $p_i^* = p^*$ (and thus $i \in I$). Schmalensee (1981: pp. 243-4) noted that the removal of the uniform pricing constraint could be usefully characterized as follows. Consider the problem of maximizing $\Pi(\boldsymbol{p})$ under the linear constraint:

$$\sum_{i=1}^{N} \pi_i'(p^*)(p_i - p^*) \le t\,, \tag{3}$$

where t is some nonnegative real number. The solution $\boldsymbol{p}(t)$ to such a problem is uniquely defined by the FOCs $\boldsymbol{D_p}\Pi(\boldsymbol{p}(t)) = \lambda(t)\boldsymbol{D_p}\Pi(p^*\boldsymbol{\iota})$, where $\boldsymbol{\iota}$ is the relevant unit vector and $\lambda(t)$ is the nonnegative relevant Lagrangean multiplier (it measures the marginal increase in profit due to a marginal increase in t: i.e., $\lambda(t) = d\Pi(\boldsymbol{p}(t))/dt = \Sigma_i \pi_i'(p_i(t))p_i'(t)$).

The price system $\boldsymbol{p}(t)$ has a number of properties:[10] in particular, $\boldsymbol{p}(0) = p^*\boldsymbol{\iota}$ and $\lambda(0) = 1$ (this value depends on the normalization of the price differences implicit in (3)), and there exits a finite $\underline{t} \ge 0$ such that $\boldsymbol{p}(t) = \boldsymbol{p}^*$ and $\lambda(t) = 0$ for all $t \ge \underline{t}$. Moreover, $\lambda'(t) < 0$ and $sign\{p_i'(t)\} = sign\{\pi_i'(p_i(t))\} = sign\{\pi_i'(p^*)\}$ for all $t < \underline{t}$, $i = 1, \ldots, N$. Thus, $i \in S$ ($i \in W$) only if $\pi_i'(p^*) > 0$ ($\pi_i'(p^*) < 0$) and, unless all markets are intermediate (in which case the imposition of uniform pricing would have no effect at all), the removal of the uniform pricing constraint monotonically increases some prices and decreases others. Analogously, let us introduce the "virtual" unit cost of market i at t, $c_i(t)$, uniquely characterized by the condition:

$$(p_i(t) - c_i(t))q_i'(p_i(t)) + q_i(p_i(t)) \equiv 0\,. \tag{4}$$

[10] Interestingly, one might think of $(\boldsymbol{p}(t),\lambda(t))$ as a dynamic system for which $-\Pi(\boldsymbol{p}(t))$ results in a Liapunov function and $(\boldsymbol{p}^*,0)$ is a globally stable equilibrium.

Notice that $c_i(t)$ is obviously equal to market i's marginal revenue at $q_i(p_i(t))$: i.e., $\pi_i'(p_i(t)) = (c_i(t) - c_i)q_i'(p_i(t))$. Clearly, $i \in S$ ($i \in W$) if and only if $c_i > c_i(t)$ ($c_i < c_i(t)$), for $t < \underline{t}$ ($c_i(\underline{t}) = c_i$), $i = 1, ..., N$. This says that price rises (decreases) are "naturally" associated to high (low) marginal costs and low (high) marginal revenue at t. Finally, notice that $\boldsymbol{\iota}'\boldsymbol{D_p}\boldsymbol{\Pi}(\boldsymbol{p}(t)) = \Sigma_i\pi_i'(p_i(t)) = 0$ for all t (a sort of "average" equality of marginal revenues and marginal costs applies to all t), and thus, by differentiation:

$$\sum_{i=1}^{N}\pi_i''(p_i(t))p_i'(t) = \sum_{i=1}^{N}[2q_i'(p_i(t)) + (p_i(t) - c_i)q_i''(p_i(t))]p_i'(t) = 0 \qquad (5)$$

(Schmalensee, 1981: p. 244, provides a graphical analysis for the case with $N = 2$ and $c_1 = c_2$; see below our Figure 2 for a linear case).

Very generally, Varian (1985) established the following welfare bounds for any change in (linear) prices from $\boldsymbol{p}_0$ to $\boldsymbol{p}_1$ (the result follows from the convexity of $v(\cdot)$):

$$\boldsymbol{p}_0'\boldsymbol{\Delta q} - \Delta C \geq \Delta W \geq \boldsymbol{p}_1'\boldsymbol{\Delta q} - \Delta C, \qquad (6)$$

where $\boldsymbol{\Delta q} = \boldsymbol{q}(\boldsymbol{p}_1) - \boldsymbol{q}(\boldsymbol{p}_0)$, $\Delta C = C(\boldsymbol{q}(\boldsymbol{p}_1)) - C(\boldsymbol{q}(\boldsymbol{p}_0))$ and $\Delta W = W(\boldsymbol{p}_1) - W(\boldsymbol{p}_0)$.[11] Basically, (6) says that the price systems $\boldsymbol{p}_0$ and $\boldsymbol{p}_1$ provide useful information to assess the welfare effect of the change $\boldsymbol{\Delta q}$. In our setting, i.e. with $\boldsymbol{p}_0 = p^*\boldsymbol{\iota}$ and $\boldsymbol{p}_1 = \boldsymbol{p}^*$, (6) specializes to the linear (with respect to $\boldsymbol{\Delta q}$) expression:

$$(p^*\boldsymbol{\iota} - \boldsymbol{c})'\boldsymbol{\Delta q} \geq \Delta W \geq (\boldsymbol{p}^* - \boldsymbol{c})'\boldsymbol{\Delta q}, \qquad (7)$$

which immediately says, as Varian (1985: p. 872) noted, that *if* marginal cost is the same across markets (i.e., $\boldsymbol{c} = c\boldsymbol{\iota}$) an increase in total output ($\boldsymbol{\iota}'\boldsymbol{\Delta q} = \Delta Q > 0$) is a necessary condition for welfare to increase following a change from uniform pricing (notice that this does not depend on p^* and $\boldsymbol{p}^*$ being chosen by a maximizing monopolist). This result, already established by Schmalensee (1981) a few years earlier, confirmed a conjecture that dates back to the seminal works of Arthur C. Pigou (1920: chapter 16) and Joan Robinson (1933: chapter 13).[12]

In fact, it is easy to see that, if total output does not change, the change in welfare can be written $\Delta W = \Delta v + \Delta_P p - \Delta C$, where $\Delta_P p = \boldsymbol{q}(\boldsymbol{p}^*)'(\boldsymbol{p}^* - p^*\boldsymbol{\iota}) = \boldsymbol{q}(\boldsymbol{p}^*)'\boldsymbol{\Delta p}$ is the relevant Paasche price variation for the representative consumer, and $-\Delta v = v(p^*\boldsymbol{\iota}) - v(\boldsymbol{p}^*)$ is the relevant Hicksian equivalent variation (see Bertoletti, 2002). And it is a well-known result of consumer theory (see

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[11] Notice that $\boldsymbol{p}_0'\boldsymbol{\Delta q}$ and $\boldsymbol{p}_1'\boldsymbol{\Delta q}$ are respectively the Laspeyres and Paasche quantity variations.

e.g. Deaton and Muellbauer, 1980: chapter 7) that the former variation is never larger than the latter (unless in the very special case of zero substitution effects, in which case they are equal). Thus, to avoid a welfare decrease following price differentiation (which causes a decrease in (gross) consumer surplus if total output does not change), we need a (strict) decrease in total cost. Notice that this result is due to the underlying preferences (and to the use of linear prices), and does not depend on the cost structure.

Following Schmalensee (1981: p. 244), by using (5) it is also easy to see that the change in total output, $Q(\boldsymbol{p}(t)) = \Sigma_i q_i(p_i(t))$, according to t is given by:

$$\begin{aligned} \frac{dQ(\boldsymbol{p}(t))}{dt} &= \sum_{i=1}^{N} q_i'(p_i(t))p_i'(t) = -\frac{1}{2}\sum_{i=1}^{N}(p_i(t)-c_i)q_i''(p_i(t))p_i'(t) \\ &= \frac{1}{2}\sum_{i=1}^{N}\left[\frac{q_i(p_i(t))}{q_i'(p_i(t))} - (c_i(t)-c_i)\right]q_i''(p_i(t))p_i'(t). \end{aligned} \tag{8}$$

This shows that total output does not change (i.e., $\boldsymbol{\iota}'\boldsymbol{\Delta q} = 0$) if the demand system is linear ($q_i'' = 0$, $i = 1, ..., N$), as Schmalensee (1981) noted for the case with a common unit cost. This result, that again goes back to Pigou and Robinson, also generalizes to the case of interdependent demands, and even to the case of *common* variable marginal costs: see Appendix A. Notice that the second term in the last expression of (8) can be signed according to the concavity of demand: i.e., it is positive if and only if demand of market i is strictly convex ($q_i'' > 0$).

The previous results concerning total output are at the basis of the skeptical view of monopolistic pricing that we mentioned in section I. That view is rather widespread in spite of being in striking contrast with the related literature on price regulation, which tends to suggest that to allow a monopolist some price flexibility is welfare enhancing: see e.g. Armstrong and Vickers (1991) and Vickers (1997). In fact, we believe that a negative assessment of monopolistic price differentiation is generally unwarranted, simply because the welfare criterion based on total output is not robust to the introduction of even small differences in marginal costs. To illustrate this weakness, consider the special case, somehow "dual" to the standard setting of equal marginal costs, which arises if demands have the same elasticity at the uniform price p^*. Intuition suggests that this is a favorable situation for allowing price flexibility to the monopolist, since he should be willing to make the prices to reflect the cost differences. Moreover, one can show that, if demands are concave, $p^*\boldsymbol{\iota}$ *minimizes* $v(\boldsymbol{p})$ over the set $\{\boldsymbol{p} \mid Q(\boldsymbol{p}) \geq Q(p^*\boldsymbol{\iota})\}$. Thus, any differentiated price vector actually chosen by a profit-maximizing monopolist *without decreasing total output* (as it

[12] Varian (1985: p. 875) also showed that the result generalizes to the case of (common) increasing marginal costs, and Schwartz (1990) proved that it applies to the case of (common) decreasing marginal costs as well.

happens in the linear case) would actually *increase* consumer surplus, and accordingly social welfare by a trivial "revealed preference" result.

In the 2-goods case the situation is depicted in Figure 1, where $\Delta Q = 0$ indicates the locus of prices which corresponds to the same total output $Q(p^* \boldsymbol{\iota})$, and $V = v(p^* \boldsymbol{\iota})$ is the relevant consumer surplus indifference curve. The vector $\boldsymbol{q}(p^* \boldsymbol{\iota})$ is orthogonal to the plane $\Delta Q = 0$ due to the assumption of equal demand elasticities. This property of the uniform pricing might come as a surprise, but it is just due to the familiar substitution effect. In particular, it is well-known (again see e.g. Deaton and Muellbauer, 1980: chapter 7) that the Laspeyres price variation $\Delta_L p = \boldsymbol{q}(p^* \boldsymbol{\iota})'(\boldsymbol{p}^* - p^* \boldsymbol{\iota}) = \boldsymbol{q}(p^* \boldsymbol{\iota})'\boldsymbol{\Delta p}$ is never larger than the Hicksian equivalent variation: in Figure 1 the price locus $\Delta_L p = 0$ just describes the hyperplane tangent to $V = v(p^* \boldsymbol{\iota})$ at $p^* \boldsymbol{\iota}$. Thus, a nonpositive Laspeyres price variation is a sufficient condition for a consumer surplus (and then a welfare) increase.

The situation is less clear-cut if demands are convex: however, for another example suppose that demands are isoelastic, i.e., $q_i(p_i) = k_i p_i^{-\varepsilon}$, with $k_i > 0$ and $\varepsilon_i > 1$. It is easy to see that, under the assumption of equal demand elasticities ($\varepsilon_i = \varepsilon$, $i = 1, ..., N$), $\Delta_L p = 0$: see Appendix B. Accordingly, in such a case monopolistic price differentiation increases total output (by demand convexity, $\Delta Q = 0$ must lie above $\Delta_L p = 0$), aggregate consumer surplus and welfare. Notice that, in such a case, a fortiori elasticities are the same even at prices $\boldsymbol{p}^*$, which indeed should not be considered discriminatory according to the Stigler's quoted definition.

We conclude this section by formally showing that, when elasticities are the same at $p^* \boldsymbol{\iota}$, if the monopolistic departure from uniform pricing is "small" (a so-called "piecemeal policy") <u>and</u> output does not decrease, a welfare improvement is generally (whatever the concavity of demand) achieved. To see it, consider the change in welfare associated with $t < \underline{t}$:

$$\begin{aligned}\frac{dW(\boldsymbol{p}(t))}{dt} &= \sum_{i=1}^{N}[(c_i(t) - c_i)q_i'(p_i(t))p_i'(t) - q_i(p_i(t))p_i'(t)] \\ &= \sum_{i=1}^{N}(p_i(t) - c_i)q_i'(p_i(t))p_i'(t) \qquad (9) \\ &= \sum_{i=1}^{N} p_i(t)\frac{dq_i(p_i(t))}{dt} - \frac{dC(\boldsymbol{q}(\boldsymbol{p}(t)))}{dt}.\end{aligned}$$

The first term in the square bracket in (9) reflects the change in profit in market i and is always positive, while the second is the correspondent change in consumer surplus, which is positive only if $i \in W$. Since for large enough t it will be $p_i(t) > c_i$ for all i,[13] the second expression in (9) makes it

[13] Note that ($p_i(t)$ - c_i) may even be negative in some markets for some t, but in such a case the correspondent element of the welfare change would certainly be positive.

clear that in general it is not possible to sign the overall welfare impact of the increase in t between 0 and $\underline{t}$. But note that welfare is increasing at t if, geometrically, the projection of $d\boldsymbol{q}(\boldsymbol{p}(t))/dt$ on $(\boldsymbol{p}(t) - \boldsymbol{c})$ points in the same direction as the latter vector (i.e., if the inner product $(\boldsymbol{p}(t) - \boldsymbol{c})'d\boldsymbol{q}(\boldsymbol{p}(t))/dt$ is positive). Since the monopolist will choose the price change $d\boldsymbol{p}(t)/dt$ which achieves the maximum (available) value for $(\boldsymbol{c}(t) - \boldsymbol{c})'d\boldsymbol{q}(\boldsymbol{p}(t))/dt$, this may adversely affect welfare only if $(\boldsymbol{p}(t) - \boldsymbol{c}(t))'d\boldsymbol{q}(\boldsymbol{p}(t))/dt = -\Sigma_i q_i(p_i(t))p_i'(t) < 0$. That is, only if price rises are associated with the largest demands (so that consumer surplus decreases).

The third expression in (9) is just a continuous version of (7). It implies:

$$\frac{dW(\boldsymbol{p}(0))}{dt} = p^* \frac{dQ(\boldsymbol{p}(0))}{dt} - \sum_{i=1}^{N} c_i \frac{dq_i(p_i(0))}{dt}, \tag{10}$$

which means that the introduction of (small) price differentiation causes a welfare improvement by reducing the *average* total cost if it does not also reduce total output, except in the adverse case in which quantity rises are on average associated with large marginal costs.[14] Notice that $c_i(0) = p^*/m_i(p^*)$, where $m_i(p_i) = \varepsilon_i(p_i)/(\varepsilon_i(p_i) - 1)$ is the monopolistic mark up in market i. Thus, if demands have the same elasticity, $d\boldsymbol{q}(\boldsymbol{p}(0))/dt$ and $(\boldsymbol{p}(0) - \boldsymbol{c}(0)) = (p^* - c(0))\boldsymbol{\iota}$ (where $c_i(0) = c(0)$, $i = 1, ..., N$) are at worst orthogonal if $dQ(\boldsymbol{p}(0))/dt \geq 0$. In particular, in such a case, by using:

$$\frac{dC(\boldsymbol{q}(\boldsymbol{p}(t)))}{dt} = \sum_{i=1}^{N} c_i(t) \frac{dq_i(p_i(t))}{dt} - \lambda(t), \tag{11}$$

and $\lambda(0) = 1$, we get:

$$\frac{dW(\boldsymbol{p}(0))}{dt} = \frac{p^*}{\varepsilon(p^*)} \frac{dQ(\boldsymbol{p}(0))}{dt} + 1. \tag{12}$$

Thus, if demands have the same elasticity, a non-decreasing output is a *sufficient* condition for a profit-maximizing "piecemeal policy" of price differentiation to raise welfare. Notice that the first term on the right-hand side of (12) is just $dv(\boldsymbol{p}(t))/dt$, which says that (aggregate) consumer surplus increases if and only if total output increases.[15] Of course, if marginal costs are all equal as well, as in the standard set up, the unregulated monopolist adopts $\boldsymbol{p}^* = p^*\boldsymbol{\iota}$, and the removal of the uniform pricing constraint has neither a positive nor negative effect.

These examples suggest that a profit-maximizing monopolist should be willing to use price flexibility to decreases his *average* unit cost, even when leaving total output unchanged (as happens in the benchmark linear case). And this could then be welfare improving, as shown by (7) in the

[14] Notice that $dW(\boldsymbol{p}(0))/dt = 0$ if the demand system is linear and marginal cost is the same across markets.
[15] One can also show that a piecemeal policy always increases welfare under *equal demands* (if demand is strictly convex total output increases, raising consumer surplus as well).

case of discrete price variations. However, if elasticities at the uniform price are different, he might increase prices in markets which are more costly to serve (if demand elasticities are positively correlated to marginal cost), and the directions of cost and output variations become difficult to establish even for a small price differentiation. Moreover, consumer surplus might decrease more rapidly than profit increases, thus reducing total welfare. This tends to happen if prices rise in markets where quantities are larger, which may well be the case since in those markets the demand elasticities tend to be smaller. However, intuition suggests that the real issue at stake is the correlation across markets among the demand and the cost differences: why should we think of any specific correlation in the general case? We investigate the linear case in next section.

III. The linear case

Since we have shown in section II that the output criterion is generally invalid, we start the analysis of the linear case by discussing some alternative welfare tests that could be usefully applied to it. Following once again Varian (1985: pp. 873-4), we can notice that, by (1), the right-hand side of (7) can be written: $\Sigma_i c_i \Delta q_i/(\varepsilon_i(p_i^*) - 1)$, where $\Delta q_i = q_i(p_i^*) - q_i(p^*)$ (note that, if $c_i/(\varepsilon_i(p_i^*) - 1) = (p_i^* - c_i) = \rho$, $i = 1, ..., N$, then a weak increase in total output would be sufficient for monopolistic price differentiation to produce a welfare improvement). It can be proven that: *i*) for *concave* demand functions, a *sufficient* condition for the right-hand side of (7) being non negative is that the Paasche price variation $\Delta_P p$ is non positive;[16] *ii*) if the demand system is *linear*, $\Sigma_i c_i \Delta q_i/(\varepsilon_i(p_i^*) - 1) \geq 0$ and $\Delta_P p \leq 0$ are *equivalent*, sufficient conditions for a welfare improvement under monopolistic pricing, but the latter is informatively less demanding, since it does not involve costs and elasticities. We also know from the discussion in section II that, under linear demands, a *necessary* condition for avoiding a welfare decrease due to price differentiation is that total cost does decrease. In fact, in the case of a linear demand system the welfare bounds in (7) become: $-\Delta C \geq \Delta W \geq -\Delta_P p$.[17]

Since $\Delta_L p \leq 0$ is always (for any price variation) a *sufficient* condition for a consumer surplus (and then, if profit does not decreases, a welfare) improvement, it thus turns out that under linear demands we can simply replace the invalid output criterion by checking the sign of the price variations $\Delta_L p$ and $\Delta_P p$. Negative value for those variations are indeed sufficient conditions respectively for even a consumer surplus or just a welfare improvement. Note that these conditions

[16] Notice that this does not guarantee that consumer surplus does not decrease: again see e.g. Deaton and Muellbauer (1980: chapter 7).
[17] Notice that the latter disequality *assumes* monopolistic behavior (there seems to be a misprint in Varian, 1985: p. 875 concerning the expression of the welfare change when the demand system is linear).

are valid also for the case of strictly concave demands, and that their verification does not require cost knowledge.[18]

Of course, in principle one can also directly compute the welfare variations: it is easily obtained that $p^* = (Cov\{c_i,b_i\} + \underline{a} + \underline{bc})/(2\underline{b})$ and $p_i^* = (a_i + b_ic_i)/(2b_i)$, where $q_i(p_i) = a_i - b_ip_i$ ($a_i,b_i > 0$, $a_i/b_i > c_i$, $i = 1, ..., N$), $Cov\{c_i,b_i\} = (\Sigma_i c_ib_i)/N - \underline{cb}$ is the covariance between c_i and b_i across markets, and $\underline{a} = (\Sigma_i a_i)/N$, $\underline{b} = (\Sigma_i b_i)/N$ and $\underline{c} = (\Sigma_i c_i)/N$ are respectively the average value of a_i, b_i and c_i. Notice that the previous expressions imply that the average value of $-b_i\Delta p_i = \Delta q_i$ is null, as we already knew. Also note that $(p_i^* - c_i) = a_i/(2b_i) - c_i/2$. Unfortunately, in general it is difficult to draw unambiguous conclusions: we present the relevant statistics in Appendix C. However, as a general result, if the demand parameters are uncorrelated with the marginal costs (perhaps the interesting case), price differentiation implies $\Delta C < 0$ (unless there is no cost variability at all): see (C.1) and the analysis below.

The simplest (unambiguous) case arises if a_i/b_i is the same across markets (notice that this limits demand variability only at a single point, perhaps very far from the quantity-price couples observed in the market).[19] In that case $(p_i^* - p_j^*) = (c_i - c_j)/2$ and thus the only reason for monopolistic price differentiation is to reflect the marginal cost differences (in the standard setting with a marginal cost common across markets, one gets $p^* = p_i^*$ and allowing price differentiation has no effect at all).[20] In our setting, one should then expect that relaxing the constraint of uniform pricing would increase welfare. Indeed, allowing the monopolist price flexibility increases both welfare and consumers surplus. To see why, consider that in the linear case:

$$\Delta W = \frac{1}{2}[\Delta\Pi + \sum_{i=1}^{N}(\frac{a_i}{b_i} - c_i)\Delta q_i], \tag{13}$$

where $\boldsymbol{\Delta\Pi = \Pi(p^*)} - \Pi(p^*\boldsymbol{\iota})$, and then:

$$\Delta v = \frac{\sum_{i=1}^{N}(\frac{a_i}{b_i} - c_i)\Delta q_i - \Delta\Pi}{2}. \tag{14}$$

[18] It is worth noting that the actual computation of these price variations is common practice in the industries regulated by price caps: see e.g. Armstrong and Sappington (2005: section 3).

[19] The “vertical” intercept parameter of a linear demand function is sometimes identified with the “reservation price” (willingness to pay) of the “richest” consumer: see for example Tirole (1988: pp. 143-4). But in our setting the assumption of quasi-linear preferences and no income effect does preclude such an interpretation.

[20] Notice that this is nevertheless a case of price discrimination according to the Stigler’s broad definition we quoted in section I.

Thus, the condition $\Sigma_i(a_i/b_i - c_i)\Delta q_i > 0$ is sufficient for a welfare improvement and necessary for a consumer surplus improvement. Moreover, that condition is equivalent to $\Delta C < 0$ if $a_i/b_i = \kappa$; $i = 1, ..., N$: i.e., if all demands have the same vertical intercept, a decrease in total cost is a necessary *and* sufficient condition for a welfare improvement. In such a case (see Appendix C for details):

$$\Delta W = \frac{3N}{8}[\sum_{i=1}^{N}\frac{(c_i b_i)^2}{N b_i} - \frac{1}{\underline{b}}\left(\frac{\sum_{i=1}^{N} c_i b_i}{N}\right)^2] = 3\Delta v\,, \tag{15}$$

which, since the function $g(x,y) = x^2/y$ is convex on $\Re^2_+$, is necessarily non negative by the Jensen's inequality (notice that the right-hand side of (15) becomes $(3bN/8)Var\{c_i\}$ if the demand functions are just equal across markets).

To fully grasp the intuition behind this result, consider the case of a piecemeal price differentiation. We already know that total output does not change along the path from 0 to $\underline{t}$ (see (8)), which implies that $\Sigma_i b_i p_i'(t) = 0$. Moreover, (4) implies that $c_i(t) = 2p_i(t) - a_i/b_i$: accordingly, marginal revenues (demand elasticities) are equal at $t = 0$, i.e., $c_i(0) = c(0)$, $i = 1, ..., N$, *if* all demands have the same vertical intercept. We can immediately deduce from our analysis in section II that in such a case welfare increases and consumer surplus does not change. Note that, more generally, we get from (10) and (11):

$$\frac{dW(\boldsymbol{p}(0))}{dt} = -\sum_{i=1}^{N} c_i \frac{dq_i(p_i(0))}{dt} = \sum_{i=1}^{N}(a_i/b_i)\frac{dq_i(p_i(0))}{dt} + 1, \tag{16}$$

which says that a necessary and sufficient condition for a "piecemeal" welfare improvement is a negative correlation between c_i and $dq_i(p_i(0))/dt$, while a positive correlation between a_i/b_i (and thus $c_i(0)$) and $dq_i(p_i(0))/dt$ would be necessary and sufficient for a consumer surplus improvement. Note that while the former condition appears quite natural, the latter seems to require some additional correlation between the demand parameters. What happens if marginal cost is the same across markets is that the increase in profit is exactly matched by the decrease in consumer surplus, so that there is no welfare change. If, on the contrary, a_i/b_i is the same across markets, then there is no consumer surplus change and the increase in welfare is just due to cost reduction.

Figure 2 illustrates the situation with *equal marginal costs* for the 2-markets case. Without loss of generality, the price in the strong market is indicated on the horizontal axis. Points *u*, *d*, *f* indicate respectively the uniform pricing, unconstrained monopolistic pricing and first-best prices. We show three iso-profit loci (surrounding *d*) indicated by *Π*, and three iso-welfare loci (surrounding f) indicated by W. The portion ud of the plane $\Delta Q = 0$ is the locus of the $\boldsymbol{p}(t)$ price vectors.[21] The line fd is instead the locus of the Ramsey price vectors (tangency points between the iso-profit and iso-welfare curves). Notice that at u the relevant iso-welfare curve is tangent to the $\Delta Q = 0$ plane (since the slope of the former is given by $[-\ b_s(p_s - c_s)]/[b_w(p_w - c_w)]$): accordingly, u dominates any other point of that locus with respect to social welfare (since $a_s/b_s > a_w/b_w$, it is the case that at u the plane $\Delta Q = 0$ is steeper than the relevant representative consumer indifference curve (not shown for the sake of simplicity), whose slope is given by $-\ q_s(p_s)/q_w(p_w)$). Figure 3 illustrates instead the "dual" situation with *equal demand elasticities* at p^* ($a_s/b_s = a_w/b_w$). We show three iso-welfare loci and, for simplicity, a single iso-profit locus. Notice that the $\Delta Q = 0$ plane is steeper than the relevant welfare locus at u, while they are tangent at σ (thus $p_s - p_w = c_s - c_w$ at that point).[22] Since $p_s^* - p_w^* = (c_s - c_w)/2$, it must be that point d lies between u and σ. Accordingly, it necessarily welfare-dominates u.

Note that a *very* special case arises if $a_i/b_i - c_i = 2\rho$, $i = 1, ..., N$. In such a case (13) and (14) imply that $\Delta W = \Delta\pi/2 = -\ \Delta v > 0$, and of course $\Delta C < 0$, unless there is no market variability at all and $\boldsymbol{p}^* = p^*\boldsymbol{\iota}$ (notice that (16) implies $dC(\boldsymbol{q}(\boldsymbol{p}(0))/dt = -\ ½ = dv(\boldsymbol{p}(0)/dt = -\ dW(\boldsymbol{p}(0)/dt)$. The reason is simple: in this case the Ramsey price vector $\boldsymbol{p}^*$ satisfies the second-best conditions $(p_i^* - c_i) = \rho$, $i = 1, ..., N$, we mentioned in section I, and thus $\boldsymbol{p}^*$ *maximizes* $W(\boldsymbol{p})$ over the set $\{\boldsymbol{p} \mid Q(\boldsymbol{p}) = Q(p^*\boldsymbol{\iota})\}$. But, at the same time, $\boldsymbol{p}^*$ *minimizes* $v(\boldsymbol{p})$ over the previous set, since it equalizes q_i/q_i' across markets. The situation is illustrated in Figure 4 for the two-markets case (notice that points d and σ coincide). These results neatly illustrate the potential conflict between welfare versus consumer surplus concerns in our second-best setting. However, they require a good deal of cross demand and cost parameter correlation, which in general is hardly plausible.

Two other cases can be analyzed in more depth. Suppose first that $a_i = a$, $i = 1, ..., N$: i.e., all demands have the same "horizontal" intercept. In such a case we get:

$$\Delta C = \frac{aN}{2\underline{b}} Cov\{c_i, b_i\} + \frac{N}{2}[\frac{1}{\underline{b}}\left(\frac{\sum_{i=1}^{N} c_i b_i}{N}\right)^2 - \sum_{i=1}^{N}\frac{(c_i b_i)^2}{N b_i}]. \qquad (17)$$

Thus, a non positive covariance of demand slope and marginal cost would be sufficient to get a cost decrease by price differentiation (if marginal costs differ). Notice that a negative covariance means

[21] Notice that ud is the locus of tangencies of the relevant iso-profit curves with 45° lines of the form $p_s - p_w = T$ (this follows from (3)).
[22] Also notice that in Figure 2 point σ would coincide with u.

that at p^* (at $t = 0$) higher marginal costs tend to be associated with smaller marginal revenues, so that average total cost should indeed be expected to decrease under price differentiation.

The problem is that, under such a negative covariance, price rises would also be associated with larger demands, so harming the representative consumer (as is suggested by (14) and (16)). Actually, this is the way monopolistic price discrimination operates if marginal costs are equal across markets. In fact, even assuming no cross correlation between cost and demand parameters (why should there be any in the general case?), the sign of the welfare change stays ambiguous (see (C6) in Appendix C). Clearly, in the two-markets case, a *necessary* condition for price differentiation to cause a welfare increase is that $c_s > c_w$ (market i is strong if and only if $a_i/b_i - a_j/b_j > c_j - c_i$, $i \neq j$, $i,j = 1,2$). It is easy to see that a *sufficient* condition for a welfare improvement is then $c_s - c_w > a_s/b_s - a_w/b_w$; i.e., $(c_s - c_w)/a > 1/b_s - 1/b_w$, if the horizontal intercept is common across markets (things are in such a case very much as in Figure 3). Indeed, $a_s/b_s < a_w/b_w$ (i.e., $b_w < b_s$) is a *sufficient* condition for monopolistic price differentiation to deliver even an aggregate consumer surplus increase (in fact, in such a case the $\Delta Q = 0$ locus is steeper than the relevant representative consumer indifference curve at u). However, Figure 5 shows an adverse case in which $c_s < c_w$ and (any degree of) monopolistic price differentiation decreases social welfare with respect to the case of uniform pricing (notice that, in such a case, necessarily $a_s/b_s > a_w/b_w$ and thus price differentiation decreases aggregate consumer surplus).[23]

Second, suppose now that $b_i = b$, $i = 1, ..., N$: in such a case:

$$\Delta C = -\frac{N}{2}[Cov\{a_i, c_i\} + bVar\{c_i\}], \qquad (18)$$

where $Cov\{a_i,c_i\} = (\Sigma_i a_i c_i)/N - \underline{a}\,\underline{b}$ is the covariance between a_i and c_i across markets. Notice that a nonnegative covariance of demand intercept and marginal cost is a sufficient condition to get a cost decrease (if all demands have the same slope and costs differ). Indeed, in this case a positive covariance means that at p^* higher marginal costs tend to be associated to smaller marginal revenue, so that, once again, average total cost should be expected to decrease. But again a welfare improvement cannot be guaranteed, because even if a_i and c_i were uncorrelated, prices would rise in the markets with the largest intercept values, and this would have a negative first-order effect on consumer surplus.

[23] Notice that Figures 3 and 5 can be thought of as the results of "perturbations" of the market fundamentals (the demand and cost parameters) with respect to the case of demand (elasticity) and marginal cost equal across markets. In the latter case the iso-profit loci would be centered on the uniform price, points u, d and σ would coincide (at that point the relevant indifference curve of the representative consumer would be tangent to the plane $\Delta Q = 0$, as in Figure 1) and of course the uniform pricing rule would have no hold.

In fact, a simple computation shows that (again see Appendix C for details), with equal demand slope:

$$\Delta W = \frac{N}{8b}[4b^2 Var\{c_i\} - Var\{a_i - bc_i\}], \tag{19}$$

and

$$\Delta v = \frac{N}{8b}[Var\{a_i - bc_i\} - 4Var\{a_i\}]. \tag{20}$$

(19) and (20) confirm that a welfare improvement cannot be guaranteed, since it depends on the *relative* variability of a_i and c_i and on their correlation. In particular, notice in (19) that if the quantity demanded at the competitive equilibrium level is the same across markets, then ΔW achieves its maximum (positive) value (given marginal cost variability). Of course this can only happen (with identical slope) if marginal costs are "perfectly" correlated with the intercept parameters (indeed, it requires $a_i = b(c_i + 2\rho)$, $i = 1, ..., N$: see above), and then Δv gets its minimum (negative) value, as shown by (20). However, assuming $Cov\{a_i,c_i\} = 0$, it can also be shown that $Var\{c_i\} > Var\{a_i\}/(3b^2)$ becomes a necessary and sufficient condition for a welfare improvement due to price differentiation, while $Var\{c_i\} > 3Var\{a_i\}/b^2$ becomes a necessary and sufficient condition for a consumer surplus increase (see (C7) in Appendix C). It is easy to see that, in the two-markets case, if the slope is common across markets, $c_s - c_w > (1/b)(a_s - a_w)$ is a *sufficient* condition for price differentiation to cause a welfare increase (it must also be the case that $c_s - c_w > (1/b)(a_w - a_s)$: then things are once again very much as in Figure 3; in addition, if $a_w > a_s$ monopolistic price differentiation also raises aggregate consumer surplus).

The technical results of this section are summarized in Proposition 1.

PROPOSITION 1. *In the linear case, monopolistic price differentiation:*

1. *decreases total cost if cost and demand parameter are uncorrelated; it also raises welfare* (or *even consumer surplus) if demands have the same slope* and *marginal cost variability is large enough with respect to demand variability;*
2. *increases both consumer surplus and welfare if demands have the same vertical intercept.*

IV. Discussion and conclusions

One lesson of our investigation is that the traditional output criterion for denying price flexibility to a monopolist is invalid, unless one can be sure that marginal costs are equal. On the contrary, one can robustly refer to the *Paasche* price index to check whether sufficient conditions for a welfare improvement to follow monopolistic price differentiation are satisfied (this actually requires demand concavity), or to the *Laspeyres* price index to see if even an aggregate consumer surplus can be guaranteed (this more severe condition is indeed totally general). Notice that the computation of these price indexes does not seem informatively more demanding that the application of the output criterion. On the contrary, detailed information on the relative variability and correlation among demand and cost conditions appear necessary to finely measure the welfare impact of allowing price flexibility.

Notwithstanding these ambiguous welfare results, we have also shown that to allow price flexibility tends to produce socially valuable (average) cost reductions. This applies to the benchmark case of differentiated linear costs and demands, under null demand and cost parameter *co*variance (a possibly realistic assumption). In particular, we have proven that cost reductions become full-fledged welfare (or even consumer surplus) improvements if demands have (a) the same vertical intercept, or (b) an equal demand slope *if* in addition cost variability is large enough with respect to demand variability. It may even happen (admittedly in a hardly realistic case) that the Ramsey pricing freely chosen by the profit-maximizing monopolist is a second-best welfare "optimum". We believe that these results are of some interest: in applications, they generally suggest which dimensions (parameter variability) should be investigated and are easily used in the case of a small number of markets (as in the two-market prototype of our figures). Moreover, they should also be useful in theoretical models, which often refer to linear setting and restrict parameter variability (Valletti, 2006 and Galera and Zaratiegui, 2006 are two up-to-date examples).

In a sense, the message of this paper comes as no surprise. Phlips (1981: p. 1) observed twenty-five years ago that, due to the presence of some market power: "generally, discriminatory prices will be required for an optimal allocation of resources in real life situation". It turns out that we cannot guarantee a welfare improvement by imposing a uniform pricing constraint on a monopolist, unless we are sure that the commodity sold is *exactly* the same. That is, the assumption of a *single* commodity is crucial to the textbook analysis. "Similar" commodities are not the same commodity if they have different marginal costs, and several authors noticed this difficulty: see e.g. Phlips (1981: pp. 5-9) and Varian (1989: p. 599). At the same time, by now many years of "new" regulation theory have taught that cost differences are not easily disclosed, not to say incorporated in a regulation practice. Indeed, the recent literature, from which this paper draws inspiration, stresses the (at least potential) social usefulness of allowing some price flexibility to a regulated monopolist (see e.g. Vickers, 1997).

However, it is also worth emphasizing that our results do not sustain a policy of complete price de-regulation. On the one hand, we have discussed no strategic (anti-competitive and/or anti-regulative) monopolistic tactics: thus our conclusions do not necessarily conflict with the rules of the Robinson-Patman Act. On the other hand, we have not considered the use of uniform pricing in "competitive" environments, a topic only very recently investigated by the economic literature (see e.g. Stole, 2003, Aguirre, 2004 and Galera and Zaratiegui, 2006). In the latter case, in particular, it is for example well known that the European Authorities have reasons to endorse, to some extent, a policy of uniform pricing that relate to the making of the European common market and are not based on welfare considerations (even when they do not explicitly run counter to them: see e.g. Cabral, 2000: paragraph 10.5). And a similar comment would apply to the political constraints (mentioned in section I) based on "horizontal equity" concerns (even though these may be weakened by the result that price differentiation can also increase aggregate consumer surplus).

Finally, we also stress that throughout the paper it has been assumed that the monopolist possesses all the demand and cost information needed to maximize profit, very much as in the "Ramsey- Boiteux" tradition, and that indeed he maximizes it (also see Schwartz, 1990: p. 1262). If this were not the case, it seems to us that it would be even more difficult to sustain the use of a uniform pricing constrain: why should the policymaker know more than the monopolist? In particular, if his differentiated pricing structure just reflects the cost knowledge of the monopolist, who is unable to get reliable demand (elasticity) information (as in the case of the so-called "Allais doctrine"), it is hard to imagine any economically sound reasons to deny him price flexibility.

Appendix A

We wish to show that even if the demands are not independent (i.e., $\partial q_i/\partial p_j$ is possibly non zero for some $i \neq j$, $i,j = 1, ..., N$), relaxing the constraint of a uniform pricing does not change monopolistic total output if the demand system is linear and marginal costs are either constant or common. We will assume that the correspondent FOC conditions do characterize the solution of the relevant profit maximization programs. Equations (1) generalize to:

$$\frac{\partial \Pi(\boldsymbol{p}^*)}{\partial p_i} = q_i(\boldsymbol{p}^*) + \sum_{j=1}^{N} (p_j^* - \frac{\partial C(\boldsymbol{q}(\boldsymbol{p}^*))}{\partial q_j}) \frac{\partial q_j(\boldsymbol{p}^*)}{\partial p_i} = 0 \,, \tag{A1}$$

and by adding up we get

$$\sum_{i=1}^{N}\frac{\partial \Pi(\boldsymbol{p}^*)}{\partial p_i}=\sum_{i=1}^{N}q_i(\boldsymbol{p}^*)+\sum_{i=1}^{N}\sum_{j=1}^{N}(p_j^*-\frac{\partial C(\boldsymbol{q}(\boldsymbol{p}^*))}{\partial q_j})\frac{\partial q_j(\boldsymbol{p}^*)}{\partial p_i}=0\,; \qquad \text{(A2)}$$

i.e., by using linearity of the demand system

$$2Q(\boldsymbol{p}^*)-Q(\boldsymbol{0})-\sum_{i=1}^{N}\sum_{j=1}^{N}q_{ji}\frac{\partial C(\boldsymbol{q}(\boldsymbol{p}^*))}{\partial q_j}=0\,, \qquad \text{(A3)}$$

where $q_{ji} = \partial q_j/\partial p_i$, i,j =1, ..., N. Similarly, (2) generalizes to:

$$\frac{d\Pi(p^*\boldsymbol{\iota})}{dp^*}=\sum_{i=1}^{N}[(p^*-\frac{\partial C(\boldsymbol{q}(p^*\boldsymbol{\iota}))}{\partial q_i})\sum_{j=1}^{N}\frac{\partial q_i(p^*\boldsymbol{\iota})}{\partial p_j}+q_i(p^*\boldsymbol{\iota})]=0\,, \qquad \text{(A4)}$$

i.e.,

$$2Q(p^*\boldsymbol{\iota})-Q(\boldsymbol{0})-\sum_{i=1}^{N}\sum_{j=1}^{N}q_{ij}\frac{\partial C(\boldsymbol{q}(p^*\boldsymbol{\iota}))}{\partial q_j}=0 \qquad \text{(A5)}$$

by linearity. Finally, notice that, since $q_{ji} = q_{ij}$,[24] the left-hand side of (A3) and (A5) are identical functions with respect to Q if either marginal costs $\partial C(\boldsymbol{q})/\partial q_j$ are constant or they are equal and only depend on total output Q. QED.

Appendix B

Assuming isoelastic demands, equations (1) become:

$$\pi_i'(p_i^*)=-\varepsilon_i k_i(p_i^*-c_i)(p_i^*)^{-\varepsilon_i-1}+k_i(p_i^*)^{-\varepsilon_i}=0\,, \qquad \text{(B1)}$$

which imply $p_i^* = (\varepsilon_i c_i)/(\varepsilon_i - 1)$. Similarly, equation (2) becomes:

$$\sum_{i=1}^{N}\pi_i'(p^*)=\sum_{i=1}^{N}(p^*)^{-\varepsilon_i}[-\varepsilon_i k_i(1-\frac{c_i}{p^*})+k_i]=0\,, \qquad \text{(B2)}$$

which implies, assuming $\varepsilon_i = \varepsilon$ (i =1, ..., N), $p^* = (\varepsilon Cov\{c_i,k_i\}/\underline{k} + \varepsilon\underline{c})/(\varepsilon - 1)$, where $Cov\{c_i,k_i\} = (\Sigma_i c_i k_i)/N - \underline{ck}$ is the covariance between c_i and k_i across markets, and $\underline{k} = (\Sigma_i k_i)/N$ is the average value of k_i. Since $\Delta p_i = \varepsilon[c_i - \underline{c} - Cov\{c_i,k_i\}/\underline{k}]/(\varepsilon - 1)$, it follows that:

[24] The symmetry property $q_{ji} = q_{ij}$ follows from the quasi linearity of consumers' preferences: also see Layson (1998).

$$\Delta_L p = \sum_{i=1}^{N} q_i(p^*)\Delta p_i$$

$$= \left[\frac{\varepsilon}{\varepsilon - 1}\right]^{1-\varepsilon} \left[\frac{Cov\{c_i, k_i\}}{\underline{k}} + \underline{c}\right]^{-\varepsilon} \left[\sum_{i=1}^{N}(k_i c_i) - N\underline{kc} - NCov\{c_i, k_i\}\right] = 0. \qquad \text{(B3)}$$

Appendix C

In the case of a linear demand system, direct computation shows that the following equalities hold:

$$\Delta C = \frac{N}{2\underline{b}}[Cov\{b_i, c_i\}(\underline{a} + 2\underline{bc} + Cov\{b_i, c_i\}) - \underline{b}Cov\{a_i, c_i\} + \underline{b}(\underline{bc}^2 - \sum_{i=1}^{N}\frac{b_i c_i^2}{N})], \qquad \text{(C1)}$$

$$\Delta_L p = \frac{N}{2\underline{b}}[\underline{b}Cov\{a_i, c_i\} - \underline{a}Cov\{b_i, c_i\} + \underline{b}(\sum_{i=1}^{N}\frac{a_i^2}{Nb_i} - \frac{\underline{a}^2}{\underline{b}})], \qquad \text{(C2)}$$

$$\Delta_P p = \frac{N}{4\underline{b}}[(Cov\{b_i, c_i\})^2 + 2\underline{bc}Cov\{b_i, c_i\} + \underline{b}(\sum_{i=1}^{N}\frac{a_i^2}{Nb_i} - \frac{\underline{a}^2}{\underline{b}}) + \underline{b}(\underline{bc}^2 - \sum_{i=1}^{N}\frac{b_i c_i^2}{N})]. \qquad \text{(C3)}$$

Notice that the last term in the square bracket in (C2) (the third term in the square bracket in (C3))[25] is nonnegative by Jensen's inequality. Also notice that, if the demand parameters are not correlated with the marginal costs (perhaps the interesting case), we get $\Delta C \leq 0$ and $\Delta_L p \geq 0$, while $\Delta_P p$ cannot be signed (the last term in the square bracket in (C1) and (C3) is then certainly non positive). And remember that, by a general property, $-\Delta_P p \geq \Delta v \geq -\Delta_L p$. A little bit more can be said if demand parameters, in addition to not being correlated to marginal costs, are also uncorrelated between them. In particular, if (without demand functions and marginal costs being all equal) $a_i = a$ (i = 1, ..., N), we get $\Delta C = (N/2)[\underline{bc}^2 - (\Sigma_i b_i c_i^2/N)] < 0$, $\Delta_L p = (a^2/2)[(\Sigma_i 1/b_i) - N/\underline{b}] > 0$ and $\Delta_P p = (N/4)\{a^2[\Sigma_i(Nb_i)^{-1} - 1/\underline{b}] + \underline{bc}^2 - (\Sigma_i b_i c_i^2/N)\}$; while, if $b_i = b$ (i = 1, ..., N), we get $\Delta C = -(bN/2)Var\{c_i\} < 0$, $\Delta_L p = (N/(2b))Var\{a_i\} > 0$ and $\Delta_P p = (N/(4b))[Var\{a_i\} - b^2 Var\{c_i\}]$, which neatly illustrate the role of the relative variance of demand and cost parameters. Note that, in the latter case, $b^2 Var\{c_i\} > Var\{a_i\}$ is a sufficient condition for price differentiation to deliver a welfare improvement.

Moreover, if (without assuming away any cross demand and cost parameter correlation) a_i/b_i is the same across markets, then we get $\Delta_P p = \Delta C/2 = (N/(4\underline{b}))[(\Sigma_i b_i c_i/N)^2 - \underline{b}(\Sigma_i b_i c_i^2/N)] \leq 0$ and $\Delta_L p = 0$, which implies that consumer surplus (and thus welfare) increases if the monopolist actually

[25] This term is clearly related to the variability of a_i/b_i.

differentiates prices (i.e., if marginal costs are different). In the very special case in which the demand functions are equal (i.e., $a_i = a$ and $b_i = b$, $i = 1, ..., N$) it turns out that $\Delta C = -(bN/2)Var\{c_i\} = 2\Delta_P p \le 0$: notice that this means that half of the cost decrease is "passed" to the representative consumer in the form of a revenue reduction. Also notice that, if $c_i = c$, $i = 1, ..., N$ (the standard set up in the price discrimination literature), $\Delta C = 0$ (and thus welfare decreases if the monopolist does differentiate prices), and $\Delta_P p = \Delta_L p/2 = 1/4[(\Sigma_i a_i^2/b_i) - N(\underline{a}^2/\underline{b})] \ge 0$.

In addition, one can compute that:

$$
\begin{aligned}
\Delta W &= \sum_{i=1}^{N} \frac{[(p_i^* - c_i) + (p^* - c_i)]\Delta q_i}{2} \\
&= \frac{1}{2}\sum_{i=1}^{N}[(p^* - c_i)(q_i(c) - q_i(p^*)) - (p_i^* - c_i)(q_i(c) - q_i(p_i^*))] \\
&= \frac{-1}{2}\sum_{i=1}^{N} b_i \Delta p_i [p_i^* + p^* - 2c_i] = \frac{-1}{2}\sum_{i=1}^{N} b_i \Delta p_i [p_i^* - 2c_i] = \frac{1}{4}\sum_{i=1}^{N} b_i \Delta p_i [3c_i - \frac{a_i}{b_i}] \qquad \text{(C4)} \\
&= \frac{N}{8\underline{b}}[\underline{b}(\frac{\underline{a}^2}{\underline{b}} - \sum_{i=1}^{N}\frac{a_i^2}{Nb_i}) - 2\underline{a}Cov\{b_i, c_i\} + 2\underline{b}Cov\{a_i, c_i\} + 3\underline{b}(\sum_{i=1}^{N}\frac{(c_i b_i)^2}{Nb_i} - \frac{1}{\underline{b}}\left(\frac{\sum_{i=1}^{N} c_i b_i}{N}\right)^2)],
\end{aligned}
$$

and, finally:

$$
\begin{aligned}
\Delta v &= \frac{1}{2}\sum_{i=1}^{N}[(\frac{a_i}{b_i} - p_i^*)q_i(p_i^*) - (\frac{a_i}{b_i} - p^*)q_i(p^*)] \\
&= \frac{1}{2}\sum_{i=1}^{N} b_i \Delta p_i [p_i^* + p^* - \frac{2a_i}{b_i}] = \frac{1}{2}\sum_{i=1}^{N} b_i \Delta p_i [p_i^* - \frac{2a_i}{b_i}] = \frac{1}{4}\sum_{i=1}^{N} b_i \Delta p_i [c_i - \frac{3a_i}{b_i}] \qquad \text{(C5)} \\
&= \frac{N}{8}[3(\frac{\underline{a}^2}{\underline{b}} - \sum_{i=1}^{N}\frac{a_i^2}{Nb_i}) + \frac{2\underline{a}}{\underline{b}}Cov\{b_i, c_i\} - 2Cov\{a_i, c_i\} + (\sum_{i=1}^{N}\frac{(c_i b_i)^2}{Nb_i} - \frac{1}{\underline{b}}\left(\frac{\sum_{i=1}^{N} c_i b_i}{N}\right)^2)].
\end{aligned}
$$

Notice that the first terms in the square brackets of the last expressions in (C4) and (C5) are nonpositive, while the last terms are nonnegative. Also note that, as anticipated above, if a_i/b_i is the same across markets, allowing price flexibility to the monopolist delivers a consumer surplus and welfare improvement: $\Delta W = (3N/8)[\Sigma_i(c_i b_i)^2/Nb_i) - (1/\underline{b})(\Sigma_i(c_i b_i)/N)^2] = 3\Delta v = 3/2\ \Delta\Pi \ge 0$ ($\Delta W = (3bN/8)Var\{c_i\}$ if demand functions are just equal across markets). If, on the contrary, marginal costs are equal across markets, monopolistic price discrimination causes a welfare decrease, as is well known: $\Delta W = (N/8)[(\underline{a}^2/\underline{b}) - \Sigma_i(a_i^2/Nb_i)] = \Delta v/3 = -\Delta\Pi/2 \le 0$.

More generally, however, even with a null cross demand and cost parameter correlation, (C4) and (C5) have ambiguous signs. In particular, in such a case they reduce to:

$$\varDelta W = \frac{N}{8\underline{b}}[a^2(1-\sum_{i=1}^{N}\frac{\underline{b}}{Nb_i}) + 3\underline{b}(\sum_{i=1}^{N}\frac{{c_i}^2 b_i}{N} - \underline{c}^2\underline{b})],$$
$$\varDelta v = \frac{N}{8\underline{b}}[3a^2(1-\sum_{i=1}^{N}\frac{\underline{b}}{Nb_i}) + \underline{b}(\sum_{i=1}^{N}\frac{{c_i}^2 b_i}{N} - \underline{c}^2\underline{b})], \qquad \text{(C6)}$$

and

$$\varDelta W = \frac{N}{8b}[-Var\{a_i\} + 3b^2 Var\{c_i\}],$$
$$\varDelta v = \frac{N}{8b}[-3Var\{a_i\} + b^2 Var\{c_i\}], \qquad \text{(C7)}$$

respectively if $a_i = a$ or $b_i = b$ (i = 1, ..., N). Note that, in the latter case, (C7) says that $Var\{c_i\} > Var\{a_i\}/(3b^2)$ is a necessary and sufficient condition for a welfare improvement due to price differentiation, while $Var\{c_i\} > 3Var\{a_i\}/b^2$ is a necessary and sufficient condition for a consumer surplus increase.

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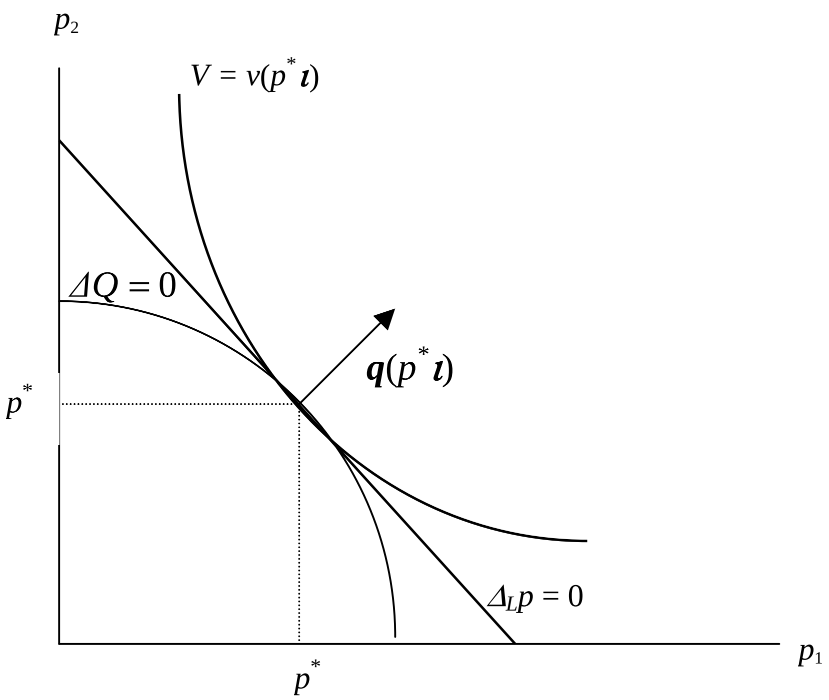

Figure 1: Uniform pricing with *concave* demands, equal demand *elasticites*

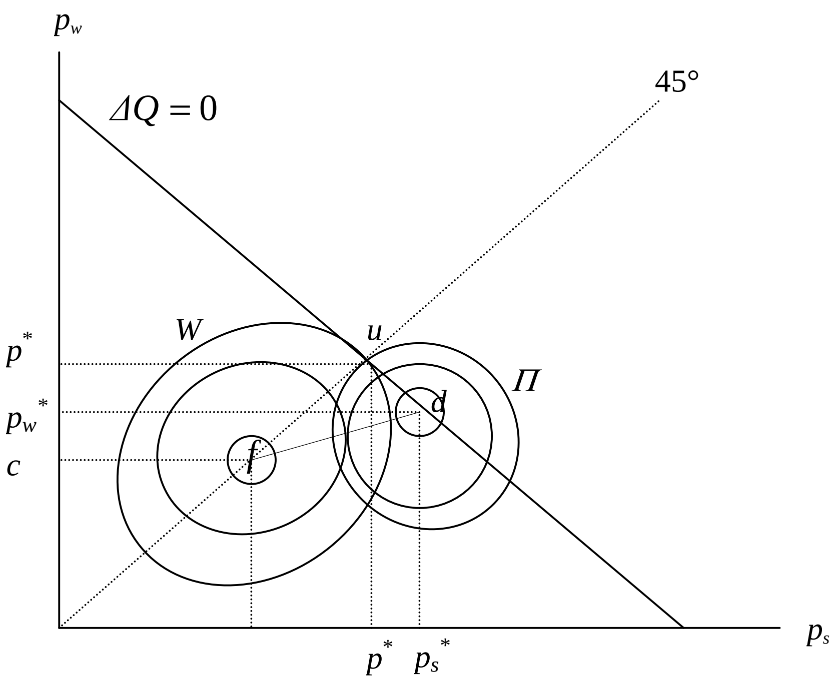

Figure 2: Monopolistic Price differentiation with equal marginal costs

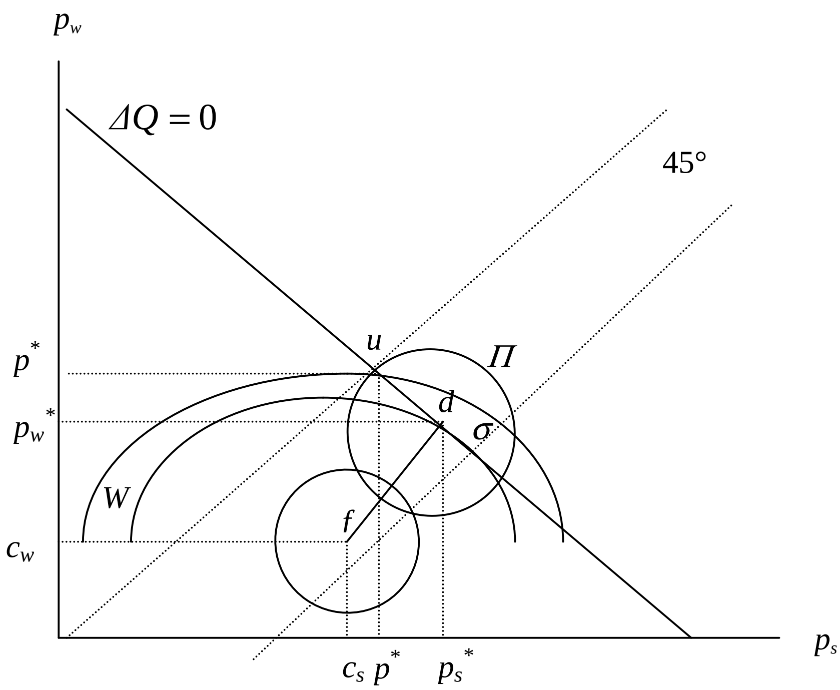

Figure 3: Monopolistic price differentiation with equal demand elasticities at p^*

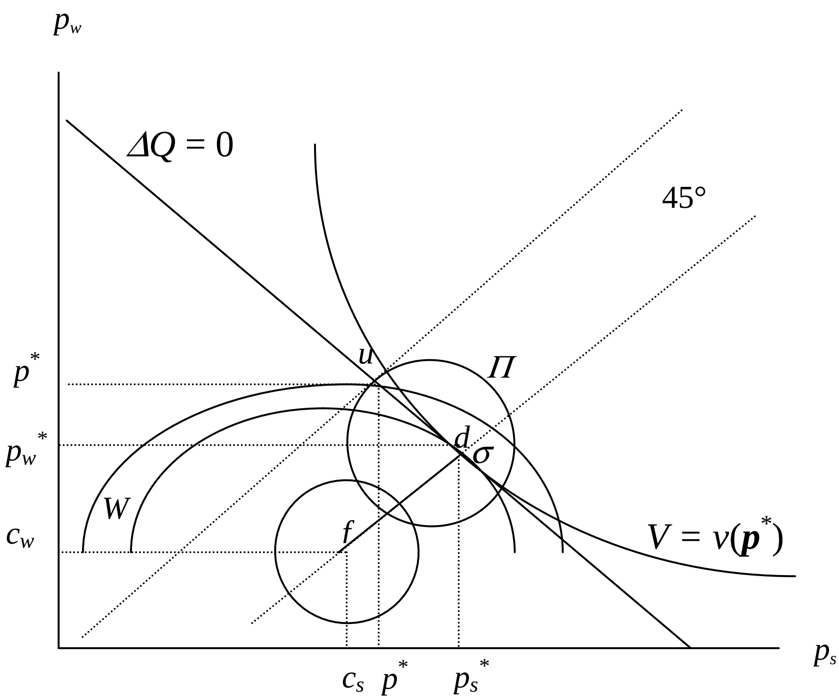

Figure 4: Second-best efficient monopolistic price differentiation

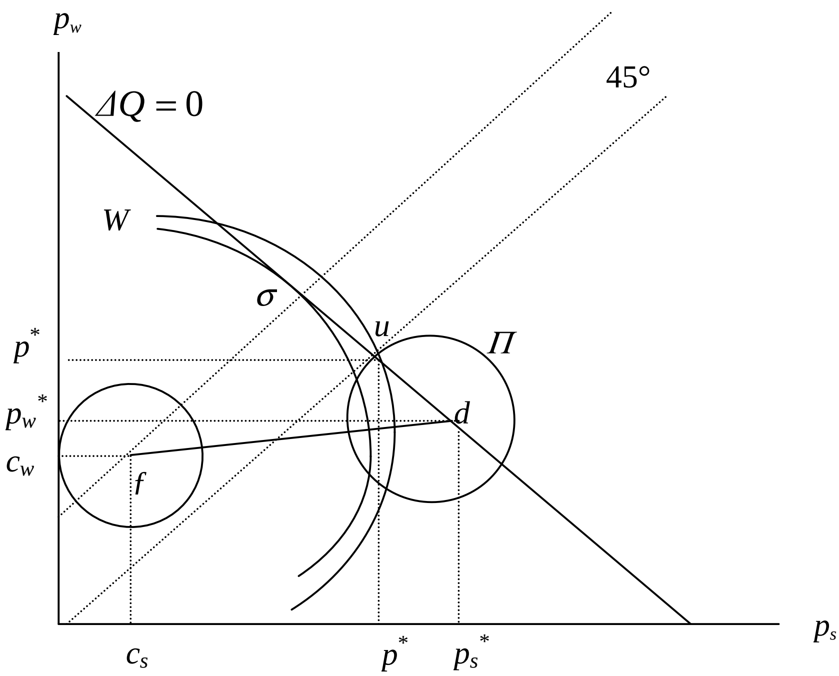

Figure 5: Socially inefficient monopolistic price differentiation